Activity-Based Pricing in a Monopoly

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Abstract

This paper studies the interaction between cost accounting systems and pricing decisions in a setting where a monopolist sells a base product and related support services to customers whose preference for support services is known only to them. The paper considers two pricing mechanisms—Activity-Based Pricing (ABP) and traditional pricing, and two cost-accounting systems—Activity-Based Costing (ABC) and traditional costing. Under traditional pricing, only the base product is priced while support services are provided free because detailed cost-driver volume information on the consumption of support services by each customer is unavailable. Under ABP, customers pay based on the quantities consumed of both the base product and the support services because detailed cost-driver information is available for each customer. Likewise, under traditional costing, the firm knows only the distribution of the cost-driver rates for the base product and support services while under ABC the firm knows the actual cost-driver rates for the base product and support services. The paper compares the equilibrium quantities of the base product and support services sold, the information rent paid to the customers, and the expected profits of the monopolist under all four combinations of cost-driver volume and cost-driver rate information.

The paper shows that ABP helps reduce control problems, such as moral hazard and adverse selection problems, for the supplier and increases its ability to engage in price discrimination. We show that firms would adopt ABP when their customer base is very diverse and the variable costs of providing customer support is high. Firms adopt ABC when their priors on cost-driver rates under traditional costing are very diffuse. The paper also shows that cost-driver rate information and cost-driver volume information are complements.

While prior literature has viewed ABC and Activity-Based Management (ABM) as facilitating better decision-making, this paper shows that ABC and ABP (a form of ABM) are useful tools for addressing control problems in supply chains.

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1 Introduction

Foster, Gupta, and Sjoblom (1996) predicted that Customer Profitability Measurement (CPM) would be an important future direction of management accounting. There are several reasons for this shift in focus from product profitability to customer profitability. Firms are becoming more customer-centric rather than product-centric. When selling a portfolio of products to a customer they use some of their products as loss leaders. They sell a mix of customized and standardized products to the same customer. Hence, it becomes important to understand not just product profitability, but also customer profitability.

All customer profitability measurement systems involve allocating the cost of resources consumed in serving customers (mostly Selling, General, and Administrative costs of providing support services) to cost pools. These costs are then allocated from cost pools to customers using cost-drivers. For example, Owens and Minor, a distributor of medical supplies, uses the number of Electronic Data Interchange (EDI) orders and the number of non-EDI orders as cost drivers to allocate order processing costs to customers.\(^1\) Thus, CPM can be viewed as an application of Activity-Based Costing (ABC) to allocate costs to customers rather than to products. These allocated costs are then matched with revenues from each customer to determine individual customer profitability.

In this paper, we will focus on using customer profitability information for pricing decisions - a practice labeled Activity-Based Pricing (ABP) in the practitioner literature.\(^2\) ABP involves harnessing the cost-driver volume and rate information, used to compute customer profitability, to price the products and services offered to those customers. Owens and Minor, for example, charges separately for the number of line-items in an order, the number of EDI orders, the number of non-EDI orders, the number of deliveries, the number of emergency shipments, and for special packaging.

This paper studies the interaction between cost accounting systems and pricing problems in a setting where a monopolist sells a base product and related support services to cus-

\(^1\) See Brem and Narayanan (2000) for a complete description of the customer profitability measurement system at Owens and Minor.

tomers whose preference for support services are known only to them. The paper considers two pricing mechanisms—Activity-Based Pricing (ABP) and traditional pricing, and two cost-accounting systems—Activity-Based Costing (ABC) and traditional costing. Under traditional pricing, only the base product is priced while support services are provided free because detailed cost-driver volume information on the consumption of support services by each customer is unavailable. Under ABP, customers pay for both base product and support services because detailed cost-driver information on the volume of support services consumed by each customer is available. Likewise, under traditional costing, the firm knows only the distribution of the cost-driver rates for the base product and support services, while under ABC the firm knows the actual cost-driver rates for the base product and support services. This paper compares the equilibrium quantities of the base product and support services sold, the information rent paid to the customers, and the expected surplus to the monopolist under all four combinations of cost-driver volume and cost-driver rate information.

The paper shows that ABP is valuable to the seller even when the cost of providing support services is entirely fixed and there is no uncertainty in costs. This is because contracting on the quantity of support services helps reduce the information rent paid to the buyer. When the cost of providing support services is variable, apart from the savings in information rent paid to the buyer, the seller is also able to mitigate the free-rider problem in the consumption of services. Thus, the paper shows that ABP helps reduce moral hazard and adverse selection problems for the supplier and increases its ability to engage in price discrimination. We show that firms would adopt ABP when their customer base is very diverse and the variable costs of providing customer support is high.

Firms adopt ABC when their priors on cost-driver rates under traditional costing are very diffuse. These benefits of ABP and ABC have to be traded off against the information technology systems costs of implementing ABP and ABC. The paper also shows that cost-driver rate information and cost-driver volume information are complements since one is more valuable in the presence of the other.

Section 2 reviews the extant literature on customer profitability measurement, activity-
based costing, product bundling, and mechanism design. Section 3 presents the model. In
Section 4, we analyze several scenarios. First, a setting where the seller can observe the buyer’s
type, the quantity of goods, and the quantity of support services consumed by the buyer. This
scenario, the first-best case, serves as a useful benchmark for other scenarios. Second, a setting
labeled traditional pricing where the seller can observe only the quantity of the base product sold
to the buyer, and not the buyer’s type or consumption of support services. Finally, we consider
a setting labeled Activity-Based Pricing where the seller can contract on the consumption of
both the base product and support services by the buyer but not the buyer’s type. Both
pricing schemes are analyzed under both traditional cost accounting and activity-based cost
accounting. The section compares traditional costing and pricing with activity-based costing
and pricing, and derives conditions under which a firm might consider adopting activity-based
systems. The section quantifies the incremental value of cost-driver volume versus cost-driver
rate information. Implications for the supply chain as a whole are also examined. Section 5
concludes the paper.

2 Literature Review

This paper is related to Larsen and Narayanan (2001) who study activity-based pricing in a
duopoly. Larsen and Narayanan study the impact of one firm adopting ABP on the rival firm’s
prices and profits. They show conditions under which one firm adopting ABP results in the
other firm also adopting ABP. They find that when both firms competing in a duopoly market
adopt ABP, both firms benefit. Surprisingly, customers benefit as well. These benefits are
realized because ABP induces a better alignment of customers with firms. The firm with the
lower cost of providing customer support attracts high users of customer support and the firm
with the higher cost of providing customer support attracts the low users of customer support.
Thus, ABP serves as a screening device in Larsen and Narayanan. The insights of this paper
are more applicable when one firm has market/monopoly power while Larsen and Narayanan
is more relevant in a market with price competition. Moreover, Larsen and Narayanan do not
consider free-rider problems in the consumption of support services. Hence, we can think of
this paper as addressing the long-term effects of ABP where customers can adjust the quantity of support service that they can consume, while Larsen and Narayanan models the short-term effects of ABP where customers can only switch suppliers.

Price discrimination in the form of non-linear prices is not allowed in Larsen and Narayanan. When customers can engage in commodity arbitrage (that is, buy the product from one supplier and resell the product to another customer to avail of quantity discounts), the only feasible pricing mechanism is linear pricing. Thus, the model in Narayanan and Larsen is more descriptive of commodity products. Non-linear pricing, as modeled in this paper, is more descriptive of settings where the customer cannot resell the product to other customers. All services and manufactured products that involve customization, special packaging or shipping, and significant freight costs would be examples of products that are not prone to commodity arbitrage. The additional contribution of this paper over Larsen and Narayanan is to show that ABP can be used to mitigate control problems in supply chains and to help firms with market power to engage in better price discrimination.

This paper is also related to the economics literature on bundling, which identifies more efficient extraction of consumer surplus as the main reason for bundling (see Stigler (1968), Adams and Yellen (1976), Schmalensee (1984), and McAfee, McMillan, and Whinston (1989)). When a firm has to charge one price to all customers, variability in customer valuations reduces the seller’s ability to extract consumer surplus. Thus, bundling, which serves as a tool to reduce heterogeneity in valuations, helps a monopolist increase profits as long as the valuations for the products being bundled are not perfectly positively correlated.

In Section 4.2 of the paper we study traditional pricing – a form of bundled pricing where support services are provided for free. In section 4.3 of the paper we study ABP where the base product and support services are both priced separately. The motivation for pure bundling in this paper is not the reduction in the variation in customer valuations of products but the inability to price support services separately in the absence of cost-driver (activity) volume information at the customer level to facilitate contracting. Basing the marginal price for the base product on the level of the support service being consumed helps the firm to engage
in better price discrimination. Thus, the motivation for ABP in this paper is again not the reduction in variation in customer valuations, but better price discrimination.

This paper is also related to the mechanism design literature in economics and accounting. See Baron and Myerson (1982) for a detailed treatment of mechanism design in the context of regulating a monopolist with unknown costs, and Tirole (1988) for a textbook treatment of mechanism design in the context of second-degree price discrimination. We extend this literature by examining the value of an additional signal to the monopolist for price discrimination when the monopolist sells a base product and additional support services. In the accounting literature, see Kirby, Reichelstein, Sen, and Paik (1991) and Melumad, Mookherjee, and Reichelstein(1992) for applications of mechanism design concepts to study delegation and incentives. Our paper contributes to this literature by addressing the question—“What is the value of an additional signal for mechanism design?” We show that the principal uses the additional signal to reduce the information rent paid to the agent and to mitigate the agent’s moral hazard problem.

This paper also makes a contribution to the literature on activity-based costing.3 In the practitioner literature, Cooper and Kaplan (1988 and 1991) describe ABC and Activity-Based Management (ABM). While there are many uses of ABC information, such as process improvement and new product development, this paper focuses on using ABC information for pricing decisions. Banker and Potter (1993) compare the value of ABC for pricing decisions in a monopoly with its value in a duopoly. They find that sometimes, ABC may not be valuable in a duopoly setting. Banker and Potter, in contrast to this paper, study product profitability rather than customer profitability. Moreover, the strategic interaction of the firm, in their paper, is with a rival firm rather than with its customers. Banker and Hughes (1994) also study the role of ABC information in pricing decisions by a monopolist. They find that activity-based unit costs are relevant for pricing decisions if support activity capacities and pricing decisions are taken simultaneously. However, they do not model a strategic interaction with customers.

3 This paper is also related to the empirical literature on CPM. In an empirical study, Foster and Gupta (1998) study how customer satisfaction metrics are related to customer profitability. Niraj, Gupta, and Narasimhan (1999) study customer profitability measurement in a supply chain. Narayanan and Sarkar (2000) show that customer-mix decisions change when a firm adopts a CPM system as a part of an ABC system.
and do not consider pricing support activities separately. That is, they model traditional pricing decisions based on ABC information (scenario 3 in this paper), but do not model ABP where support activities are separately priced. However, the biggest difference between this paper and the prior literature on ABC and ABM is the use of ABC and ABP (a form of ABM) for control purposes rather than for decision-making.

3 The Model

We model a monopolist who sells to customers a base product and additional support services. Customers differ in their valuation of support service and their type is indexed by the variable $k$. Let $q(k)$ be the quantity of the base product and $s(k)$ the quantity of support service bought by a customer of type $k$. Let the tariff charged by the monopolist be $T(q(k), s(k), k)$. The utility for customer type $k$ from consuming quantity $q$ of the base product and quantity $s$ of support services is given by $\phi(q, s, k) = \alpha_1 q - \alpha_2 q^2 + \alpha_3 ks - \alpha_4 s^2 + \alpha_5 q s - T$. We assume that reservation utility is 0 for all types of customers. Without loss of generality, we set $\alpha_2 = \alpha_4 = 1$. We assume $\alpha_1, \alpha_3 > 0$ and $0 < |\alpha_5| < 2$. The quadratic expression for customer utility captures the diminishing marginal utility for both the base product and support services. If $\alpha_5 > 0$, then the base product and support service are complements, while they are substitutes if $\alpha_5 < 0$.

Support services provided to customers can be complements or substitutes for the base product. For example, if the service is number of shipments in a month, going from two shipments a month to four shipments could be more valuable to a customer who buys large quantities of the base product than to a customer who buys only small quantities. However, the service of emergency express shipments of the base product might be more valuable to a small customer than to a large customer who might maintain his own inventory of the base product. In the former case, the base product and support services are complements, and in the latter case, they are substitutes.

$k$ is assumed to be distributed with density function $f(k)$, distribution function $F(k)$, and

\[ k \text{ is assumed to be distributed with density function } f(k), \text{ distribution function } F(k), \text{ and} \]

\[ \alpha_1 q - \alpha_2 q^2 + \alpha_3 ks - \alpha_4 s^2 + \alpha_5 q s = \frac{\alpha_1}{\alpha_2} \tilde{q} - q^2 + \frac{\alpha_3}{\sqrt{\alpha_4}} \tilde{s} - s^2 + \alpha_5 \frac{\tilde{q} \tilde{s}}{\sqrt{\alpha_4}}, \text{ where } \tilde{q} = q \sqrt{\alpha_2} \text{ and } \tilde{s} = s \sqrt{\alpha_4}. \]
support \([k, \infty)\). We assume that the inverse hazard rate function \(h(k) \equiv \frac{1-F(k)}{f(k)}\) is decreasing in \(k\). That is, \(h'(k) \equiv \frac{\partial h(k)}{\partial k} < 0\). This property is satisfied by many distributions and is widely used in the mechanism design literature. For example, Tirole (1988, pp.156) says, “This property is satisfied by many distributions, including the uniform, the normal, the Pareto, the logistic, the exponential, and any distribution with nondecreasing density.” We denote the profits of the monopolist from customer of type \(k\) by \(\Pi(k)\) and revenues (tariffs) as \(T(q(k), s(k), k)\). Let the per-unit cost of providing support services be \(c_s\), the per-unit cost of manufacturing the base product be \(c_q\), and the total fixed cost be \(c_f\) with \(c_q, c_s, c_f \geq 0\). The total cost of manufacturing the base product and providing support services is \(\eta(k) = c_f + c_q q(k) + c_s s(k) + \epsilon\), where \(c_q, c_s, c_f, \) and \(\epsilon\) are random variables with joint density function \(g(c_q, c_s, c_f, \epsilon)\). Let the expected values of \(c_q, c_s, c_f, \) and \(\epsilon\) be \(\mu_q, \mu_s, \mu_f, \) and \(\mu_\epsilon\), and their variances be \(\sigma_q^2, \sigma_s^2, \sigma_f^2, \) and \(\sigma_\epsilon^2\), respectively. Let the covariance between \(c_q \) and \(c_s\) be \(\sigma_{q,s}\). When we want to take the expectation of a function, \(R(c_q, c_s, c_f, \epsilon)\) with respect to \(c_q, c_s, c_f, \) and \(\epsilon\), we will write \(\mathbb{E}(R) \equiv \int \int \int R(\tilde{c}_q, \tilde{c}_s, \tilde{c}_f, \tilde{\epsilon})g(\tilde{c}_q, \tilde{c}_s, \tilde{c}_f, \tilde{\epsilon})d\tilde{c}_qd\tilde{c}_sd\tilde{c}_fd\tilde{\epsilon}\). Likewise, when we want to take expectation of a function \(G(k)\) with respect to \(k\), we will write \(\mathbb{E}(G) \equiv \int \int \int \mathbb{E}(R) f(k)dk\). Let the profitability of customer of type \(k\) be \(\Pi(k) = T(k) - \eta(k)\).

We consider two cost accounting systems—traditional and activity-based. Under ABC, the firm observes the specific realizations of cost-driver rates \(c_q, c_s, \) and \(c_f\) before making its pricing decisions. Under the traditional system the firm does not. Instead, it has to base its decisions on its knowledge of the joint distribution of \(c_q, c_s, \) and \(c_f\). However, the finer cost information under ABC is not free. We assume that it costs the firm \(\omega_t \) to implement ABC.

Three comments about the cost systems are in order. One, it is conceivable that a firm has a very good understanding of its cost structure and does not need an ABC system. In the limit, if \(\sigma_q^2 = \sigma_s^2 = \sigma_f^2 = \sigma_{q,s} = 0\), the ABC system does not provide any new cost-driver rate information. Two, if a firm knew total costs but not the break up of base-product costs and support related costs, we would expect, \(\sigma_{q,s} < 0\). Three, it is conceivable that a firm might have pretty good priors on \(c_q, c_s, \) or \(c_f\), in which case we would expect their respective variances to be small. Thus, the variance-covariance structure of the various costs captures the accuracy of
the traditional cost system and the potential benefits of the ABC system.

We also consider two pricing systems—traditional and activity-based. Under traditional pricing, only the base product is priced. Support services are provided for free to the extent demanded by the customer. Under activity-based pricing, support services are also priced. ABP is facilitated by an accounting system that measures the cost-driver volume for each customer. This cost-driver volume is used for contracting (i.e., pricing). We assume that it costs $\omega_v$ to implement the accounting system that provides contractible cost-driver volume information that is necessary for activity-based pricing. Observe that the monopolist charges a customer who consumes $q$ units of the base product, $s$ units of support service, and is type $k$, tariff $T(q,s,k)$. This allows for the possibility that the tariff is a function of the level of support service supplied to the customer and the customer’s type. However, in the absence of ABP, the tariff is independent of the level of support services supplied to the customer and $T(q,s,k) = T(q,k)$, $\forall s$. Likewise, if the customer type is not observable then $T(q,s,k) = T(q,s)$, $\forall k$. Finally, if neither the customer type nor the extent of customer support is observable to the firm, then $T(q,s,k) = T(q)$, $\forall s, k$.

We have chosen to present $s$ as the quantity of support service provided to the customer. We can also choose to interpret $s$ as the quality or type of service provided. Consider a business executive’s family purchasing Digital Subscriber Line service (a high-speed internet connection) from an internet service provider. The executive might use the DSL to connect to her corporate network at work. Her husband might use the DSL for videoconferencing, while her children use it for interactive gaming service. If the internet service provider had the ability to track what the DSL line is being used for, it can bill incrementally for each type of service. Cisco recently introduced NetFlow, a Cisco software feature, to do precisely this. An article in Packet Magazine (Third Quarter 2000) titled “Pricing for Profitability” has the following to say about the value of NetFlow:

Service providers have typically dodged the limitations of legacy billing infrastructures by offering flat-rate pricing—thus foregoing the profitability that can be realized from closely aligning prices with the actual value of service. To price for profitabil-
ity, service providers need the network to tell them what’s going on. They need the ability to aggregate massive amounts of data and correlate it with rating information. In short, they need usage-based accounting and billing solutions to meet market demands.

“The current billing situation can be compared to a clothing company that doesn’t have the means to distinguish between Oxford and plain dress shirts,” says Kurt Dahm, Senior Marketing Manager in the Communications Software group at Cisco. Not only will the company have a hard time giving customers exactly what they want, but it’s forced to charge the same price for all shirts, even though the shirts have different values.

Thus, what we have called Activity-Based Pricing, Cisco has called usage-based accounting and billing. Interpreting, $s$ as the type of service provided, we can apply the insights of our model to this setting.

### 4 Analysis

In this section, we analyze five different scenarios. In scenario 1, the firm knows its customer’s type $k$, and can observe $q$ and $s$. It also knows the realization of cost-driver rates before choosing $T(q(k), s(k), k)$. We term this the first-best case where there are no moral hazard or adverse selection problems and no cost-driver-rate uncertainty. All endogenous variables are subscripted $f$ to denote the first-best case.

The following table characterizes the settings of the next four scenarios. The subscript used to denote the endogenous variables in that scenario is given in parenthesis next to the scenario number.

<table>
<thead>
<tr>
<th>Setting</th>
<th>Traditional Costing</th>
<th>Activity-Based Costing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional Pricing</td>
<td>Scenario 2 (t)</td>
<td>Scenario 3 (r)</td>
</tr>
<tr>
<td>Activity-Based Pricing</td>
<td>Scenario 4 (v)</td>
<td>Scenario 5 (a)</td>
</tr>
</tbody>
</table>

Table 1: Settings Analyzed
is also unobservable in scenarios 2 through 5. Under traditional pricing, the firm can contract only on $q$, while $s$ is provided for free to the extent demanded by each customer. The unobservability of $k$ leads to an adverse selection problem, while the unobservability of $s$ leads to a moral hazard problem. We consider two cost accounting scenarios. Under traditional costing, the firm must choose $T(q)$ without observing the realization of cost-driver rate data. Scenario 2 is thus traditional costing and pricing. In scenario 3, the firm has access to the realization of cost-driver rate (ABC) information while choosing $T(q)$. Under ABP, the firm observes all cost-driver volume information and observes $q$ as well as $s$. In this setting, there is only an adverse selection problem. In scenario 5, the firm observes the realization of $c_q$, $c_s$, $c_f$, and $\epsilon$ while it does not in scenario 4.

4.1 First-best Case—Scenario 1

The firm solves the following program:

**Program 1**

$$\max_{T(q(k), s(k), q(k), s(k))} \int (T(q(k), s(k), k) - c_qq(k) - c_s(s(k) - c_f - \epsilon)f(k)dk$$

Subject to:

$$\phi(q(k), s(k), k) - T(q(k), s(k), k) \geq 0, \forall k$$

The firm seeks to maximize its profits (tariffs less costs) subject to the customers’ participation constraints.

**Proposition 1**

i Support provided to customer of type $k$, $s_f(k) = \frac{\alpha_1\alpha_5 - \alpha_3c_q - 2c_s + 2\alpha_3k}{4 - \alpha_5}$.

ii Base good provided to customer of type $k$, $q_f(k) = \frac{2\alpha_1 - 2c_q + \alpha_5(\alpha_3k - c_s)}{4 - \alpha_5}$.

iii $T_f((q_f(k), s_f(k), k) = \phi_f((q_f(k), s_f(k), k)$.

**All Proofs are in Appendix II**
Observe that $s_f$ is increasing in $k$. That is, customers who value support services more, consume more of it. $q_f$ is increasing in $k$ if, and only if, $\alpha_5 > 0$. Thus, if the base product and support services are complements, then as $k$ increases, the consumption of $q$ increases along with the increase in consumption of $s$. However, if the base product and support services are substitutes, then as $k$ increases, the consumption of support services increases and consequently the consumption of the base product decreases.

4.2 Traditional Pricing

This sub-section analyzes scenarios 2 and 3 where the firm can contract only on $q$ and not on $s$. Moreover, $k$ is not known to the firm. The firm can proceed along one of two equivalent ways. The firm can announce a menu of contracts characterized by $T(k)$, $q(k)$, and $s(k)$. Customers can reveal $k$ and self-select the contract meant for them. Alternatively, the firm can announce $T(q)$ and delegate the choice of $q(k)$ and $s(k)$ to consumers. We formulate the firm’s program and derive both mechanisms below.

4.2.1 Traditional Pricing and Traditional Costing—Scenario 2

The firm solves the following program:

\[
\text{Program 2} \quad \max_{T(q(k)), q(k), s(k)} \int \mathbb{E}(T(q(k)) - c_q q(k) - c_s s(k) - c_f - \epsilon) f(k) dk
\]

Subject to:

\[
\phi(q(k), s(k), k) - T(q(k)) \geq 0, \forall k
\]

\[
\phi(q(k), s(k), k) - T(q(k)) \geq \phi(q(\tilde{k}), \tilde{s}, k) - T(q(\tilde{k})), \forall k, \tilde{k}, \tilde{s}
\]

The firm maximizes its expected profits where the expectations are taken over all cost realizations and types of customers.
4.2.2 Traditional Pricing and Activity-Based Costing—Scenario 3

The firm solves the following program:

**Program 3**

$$\max_{T(q(k)), q(k), s(k)} \int (T(q(k)) - c_q q(k) - c_s s(k) - c_f - \epsilon - \omega_r) f(k) dk$$

Subject to:

$$\phi(q(k), s(k), k) - T(q(k)) \geq 0, \forall k$$

$$\phi(q(k), s(k), k) - T(q(k)) \geq \phi(q(\tilde{k}), \tilde{s}, k) - T(q(\tilde{k})), \forall k, \tilde{k}, \tilde{s}$$

The firm maximizes its expected profits for the actual realized values of the cost related random variables, where the expectation is taken across customer types.

4.2.3 Equilibrium under Traditional Pricing

In both programs 2 and 3, the firm offers a tariff $T(q)$. A customer of type $k$ will purchase $q(k)$ units of the base product and $s(k)$ units of support services. The first inequality in both programs represents the participation constraints for all types while the second inequality represents the incentive compatibility constraints, which require that each type $k$ should only pick the bundle $T(q(k)), q(k)$ meant for it, and should only consume support services $s(k)$ that it is meant to consume.

To solve these programs, we will use the standard mechanism design approach. Accordingly, to ensure that each type picks the bundle meant for them, information rents would have to be paid for all but the worst types. Let the information rent paid to type $k$ be $\iota(k)$. The following lemma characterizes the solutions to programs 2 and 3 given above. Recall that subscript $t$ denotes values of endogenous variables under scenario 2, while subscript $r$ denotes the values of endogenous variables under scenario 3.

**Lemma 1**
i. Support provided to customer of type \( k \), \( s_t(k) = \frac{2\alpha_3(\alpha_1-\mu_k)-\alpha_2^2(\alpha_2 h(k)+\mu_s)+4\alpha_3 k}{2(4-\alpha_4^2)} \) and \( s_r(k) = \frac{2\alpha_2(\alpha_1-c_q)-\alpha_2^2(\alpha_2 h(k)+c_s)+4\alpha_3 k}{2(4-\alpha_4^2)} \).

ii. Base good provided to customer of type \( k \), \( q_t(k) = \frac{2(\alpha_1-\mu_k)+\alpha_3(\alpha_3 h(k)-\mu_s)}{4-\alpha_5^2} \) and \( q_r(k) = \frac{2(\alpha_1-c_q)+\alpha_3(\alpha_3 h(k)-\mu_s)}{4-\alpha_5^2} \).

iii. The information rent paid to customer of type \( k \), \( \eta_t(k) = \int_k^k \alpha_3(s_t(\tilde{k}))d\tilde{k} \) and \( \eta_r(k) = \int_k^k \alpha_3(s_t(\tilde{k}))d\tilde{k} \).

iv. Tariff charged \( T_t(k) = \phi(q_t(k), s_t(k), k) - \eta_t(k) = T_t(q_t^{-1}(q)) = T_t(q) \) and \( T_r(k) = \phi(q_r(k), s_r(k), k) - \eta_r(k) = T_r(q_r^{-1}(q)) = T_r(q) \).

Proposition 2

i. \( \frac{dq_t}{dk} > 0, \frac{dq_r}{dk} > 0, \frac{dq_t}{dq_s} < 0, \frac{dq_r}{dq_s} < 0, \frac{ds_t}{dq} < 0, \frac{ds_t}{dq} < 0 \) and \( \frac{ds_r}{dq} < 0 \) if, and only if, \( \alpha_5 > 0 \).

ii. \( \frac{dq_t}{dq_s} < 0, \frac{dq_r}{dq_s} < 0, \frac{ds_t}{dk} > 0, \frac{ds_t}{dk} > 0, \frac{ds_r}{dq_s} < 0, \frac{ds_r}{dq_s} < 0 \) and \( \frac{ds_t}{dq_s} < 0 \).

Observe that the type \( k \) earns zero information rents and is held to its reservation utility. All types \( k > \bar{k} \) earn positive information rents. As \( k \) increases, customers consume more support services. When support services and the base product are complements (substitutes), as \( k \) increases, the customers buy more (less) of the base product. The quantity of the base product sold and the quantity of support services sold are decreasing in their expected marginal costs and decreasing (increasing) in the expected marginal cost of their complement (substitute). We can compute \( T(k) \), by setting it equal to the utility of each customer less their respective information rent. Since \( q(k) \) is strictly monotonic in \( k \), we can invert from \( q(k) \) to \( k \). Hence, we can characterize \( T \) as a function of \( q \) as well. It is a well-known result in the mechanism design literature that communication and delegation are equivalent. Here, the customers can reveal \( k \) and pick the bundle, \( T(k), q(k) \), and \( s(k) \) (communication approach), or the firm can announce \( T(q) \) and decentralize the choice of \( q \) and \( s \) to its customers. Both approaches are equivalent.
It is instructive to compare the equilibria characterized above with the equilibrium in the first-best case. In the first-best case, the firm has no moral hazard problem in the consumption of support services, and customers have no private information.

**Proposition 3**

\[ i \ E_c(s_t - s_f) = E_c(s_r - s_f) = \frac{\mu_s}{2} - \frac{\alpha^2 \alpha h(k)}{2(4-\alpha^2)} \]

\[ ii \ E_c(q_t - q_f) = E_c(q_r - q_f) = -\frac{\alpha \alpha h(k)}{4-\alpha^2} \]

Comparing the expected consumption of support services for each customer under traditional costing with that under the first-best case, we see that there are two factors that determine the difference between them. Since support services are not priced (that is \( T \) is not dependent of \( s \)), there is a free-rider problem under traditional pricing and customers consume more support services relative to the first-best case. As the expected unit cost of support services \( \mu_s \) increases, the free-rider problem gets worse. If \( \mu_s = 0 \), then there is no free-rider problem as it costs the firm nothing to provide all the support that is required by the customer. That would be the case when all support related costs are fixed. The other factor is the distortion in the consumption of support services induced by the firm to prevent the higher \( k \) types from choosing the bundle meant for the lower \( k \) types. The firm accomplishes this by reducing the support services meant for the lower \( k \) types. Observe that for the highest type \( \bar{k} \), \( h(\bar{k}) = 0 \), and there is no distortion induced. Also note that the distortions, relative to the first-best case, on account of adverse selection and moral hazard go in opposite directions.

It might seem counter-intuitive that the firm can reduce the supply of support service to its customers when it cannot even observe how much support service they consume. The way this reduction in supply is accomplished is subtle. The firm reduces (increases) the supply of the base product when the base product and support services are complements (substitutes). This is the intuition behind Proposition 3(ii).

From Proposition 3 we see that \( E_c(s_t) = E_c(s_r) \) and \( E_c(q_t) = E_c(q_r) \). Hence, \( E_c(t_t - t_r) = 0 \). Since there is no difference in expected information rent or the consumption of the base product or support services between Scenario 2 and Scenario 3 for any customer, it might be tempting
to conclude that cost-driver rate information has no value. As the following proposition shows, this conclusion is incorrect.

**Proposition 4** Under traditional pricing, the expected value of cost-driver rate information

\[ \mathbb{E}_k \mathbb{E}_c (\Pi_r(k) - \Pi_t(k)) = \frac{(4\sigma_q^2 + 4\alpha_5\sigma_{q,s} + \alpha_5^2\sigma_s^2)}{16 - 4\sigma_k^2} - \omega_r. \]

Gross of the measurement cost \( \omega_r \), the firm earns strictly greater profits under ABC costing than under traditional costing. Although the expected consumption of the base product and support services is the same under both traditional and activity-based costing, the actual consumption is correlated with the cost realization under ABC. Thus, customers consume more of the base product and support services when \( c_q \) and \( c_s \) are low, respectively. The firm achieves this fine tuning of consumption under ABC by choosing \( T(q) \) appropriately, based on the realization of \( c_q \) and \( c_s \), while the pricing under traditional costing is based on \( \mu_q \) and \( \mu_c \). Thus, the value of ABC information comes from better pricing. The following proposition analyzes properties of the optimal non-linear pricing schedule.

**Proposition 5**

i. \( \frac{dT_t}{dq_t} = \mathbb{E}_c (\frac{d^2T_r}{dq_t^2}) = 2(4 - \alpha_5^2) \frac{h'(k)}{1-h'(k)} < 0 \)

ii. \( \frac{dT_t}{dq_t} \) is increasing in \( \mu_q \) and \( \mu_s \) but decreasing in \( k \).

iii. \( \frac{dT_r}{dq_r} \) is increasing in \( c_q \) and \( c_s \) but decreasing in \( k \).

Proposition 5(i) shows that the price of the marginal unit of the base product \( \frac{dT_t(q_t)}{dq_t} \) is decreasing in the quantity of the base product. That is, the firm offers quantity discounts. This result is independent of whether the base product and support services are complements or substitutes. The traditional reasons offered for quantity discounts are diminishing marginal costs or lower costs to serve larger volumes. Here, we find that even though the firm has constant marginal costs and the cost to serve might be higher for higher volumes (when the base product and support services are complements), the firm still offers volume discounts. The firm offers volume discounts because the customer’s exhibits diminishing marginal utility
for the base product. The extent of volume discounts is affected by the firm’s adverse selection problem. If, and only if, $\frac{d^2h(k)}{dk^2} < 0$, the firm increases the volume discount as the customer’s type increases.

Propositions 5(ii) and 5(iii) show how the marginal price charged by the firm is affected by various parameters. Thus, when the marginal cost of the base product $\mu_q$ increases, we know from Proposition 2 that the quantity of the base product supplied decreases. This decrease is accomplished by an increase in the marginal price charged to customers. Proposition 5 also makes it clear that under ABC the firm can respond to the actual realized value of $c_q$ and $c_s$ when setting prices, while under traditional costing prices are set based on $\mu_q$ and $\mu_s$.

However, ABC information does not come for free. There is a cost of $\omega_r$ to implement ABC. Although this paper considers only a one-period model, an ABC system once installed will probably give valid cost signals for several periods till the production environment of the firm changes. Likewise, a firm that does not adopt ABC can use the realized aggregate values of cost-driver volumes at the end of each period to update its priors on the distribution of cost-driver rates. Appendix I considers a two-period model. The firm can choose an ABC system or a traditional costing system at the beginning of the first period. If it adopts an ABC system, it enjoys the benefits of finer cost information for both periods. If the firm adopts a traditional costing system, it updates its priors on cost-driver rates at the end of the first period based on the total cost-driver volume information that it observes at the firm level ($\mathcal{E}_k(q(k))$ and $\mathcal{E}_k(s(k))$). To make the updating process tractable, cost-driver rates are assumed to be distributed multivariate normal in Appendix I.

The main insight from Appendix I is that the marginal price in the second period is increasing in the first period’s total cost realization, even though the total cost includes fixed cost $c_f$. The firm’s second period pricing decisions are influenced by first period total costs because the first period’s total cost realization is informative about the marginal costs of the second period. The appendix also shows that the benefits of ABC are strictly less in the second period compared to the first period. Moreover, firms are less likely to adopt ABC if $\sigma_e$ and $\sigma_f$ are low. Thus, the one-period model may overstate the value of ABC by overlooking the firm’s
ability to learn more about its cost cost-driver rates by observing the prior period’s total cost realization and aggregate cost-driver volume information. This is particularly true if the firm has tight priors on its cost-driver rates and its total cost is not subject to much random noise.

4.3 Activity-Based Pricing

This subsection considers the case where the firm can contract on both $q$ and $s$. We present two programs— one under traditional costing and one under activity-based costing.

4.3.1 Activity-Based Pricing and Traditional Costing—Scenario 4

The firm solves the following program:

Program 4

$$\max_{T(q(k), s(k)), q(k), s(k)} \int (E_c(T(k) - c_q q(k) - c_s s(k) - c_f - \epsilon - \omega_r - \omega_v) f(k) dk$$

Subject to:

$$\phi(q(k), s(k), k) - T(k) \geq 0, \forall k$$

$$\phi(q(k), s(k), k) - T(k) \geq \phi(q(\tilde{k}), s(\tilde{k}), k) - T(\tilde{k}), \forall k, \tilde{k}$$

4.3.2 Activity-Based Pricing and Activity-Based Costing—Scenario 5

The firm solves the following program:

Program 5

$$\max_{T(q(k), s(k)), q(k), s(k)} \int (T(k) - c_q q(k) - c_s s(k) - c_f - \epsilon - \omega_r - \omega_v) f(k) dk$$

Subject to:

$$\phi(q(k), s(k), k) - T(k) \geq 0, \forall k$$

$$\phi(q(k), s(k), k) - T(k) \geq \phi(q(\tilde{k}), s(\tilde{k}), k) - T(\tilde{k}), \forall k, \tilde{k}$$

In both programs 4 and 5, the firm offers a bundle $T(k), q(k), s(k)$. A customer of type $k$ will purchase $q(k)$ units of the base product and $s(k)$ units of support services. In both programs, the first constraint is the participation constraint for all types while the second constraint is the incentive compatibility constraint that each type $k$ should only pick the bundle $T(k), q(k), s(k)$ meant for it.

### 4.3.3 Equilibrium under Activity-Based Pricing

The following lemma characterizes the solution to the programs given above. Recall that we subscript endogenous variables with $v$ for the activity-based pricing under traditional costing, and with $a$ for activity-based pricing under activity-based costing.

**Lemma 2**

1. **Support provided to customer of type $k$,**
   
   
   $$
   s_v(k) = \frac{\alpha_2(\alpha_1 - \mu_q - 2(\alpha_3(h(k) - k) + c_s)}{(4 - \alpha_5^2)}
   $$

   
   And

   
   $$
   s_a(k) = \frac{\alpha_2(\alpha_1 - c_q - 2(\alpha_3(h(k) - k) + c_s)}{(4 - \alpha_5^2)}
   $$

2. **Base good provided to customer of type $k$,**

   $$
   q_v(k) = \frac{2(\alpha_1 - \mu_q + \alpha_2(\alpha_3(k - h(k)) - c_s)}{(4 - \alpha_5^2)}
   $$

   And

   $$
   q_a(k) = \frac{2(\alpha_1 - c_q + \alpha_2(\alpha_3(k - h(k)) - c_s)}{(4 - \alpha_5^2)}
   $$

3. **The information rent paid to customer of type $k$,**

   $$
   \nu_v(k) = \int_k^{\tilde{k}} \alpha_3(s_v(\tilde{k}))d\tilde{k}
   $$

   And

   $$
   \nu_a(k) = \int_k^{\tilde{k}} \alpha_3(s_a(\tilde{k}))d\tilde{k}
   $$

4. **Tariff charged $T_v(k) = \phi(q_v(k), s_v(k), k)$ and $T_a(k) = \phi(q_a(k), s_a(k), k)$ and $\nu_a(k) = \nu_v(k) - \nu_a(k)$.**

Note that all the comparative statics results in Proposition 2 for traditional pricing hold for ABP as well with the subscripts $v$ and $a$ substituted for $t$ and $r$, respectively. Note also that the lemma above characterizes the mechanism involving communication of $k$. An equivalent mechanism where the firm announces $T(q, s)$ and customers are delegated the choice of $q(k)$ and $s(k)$ can also be solved for.\(^5\) The following Proposition compares $q$ and $s$ under ABP and the first-best case.

---

\(^5\)To observe this, note that $s_a(k)$ is monotone increasing in $k$ and is thus invertible. We have two differential equations from the customer optimization $\frac{dT}{dk} = \alpha_1 - 2q + \alpha_5 s$ and $\frac{ds}{dk} = \alpha_3 k - 2s + \alpha_5 q$. From these two equations, $T(q, s) = \int_0^s \alpha_3 s_a^{-1}(\tilde{s})d\tilde{s} - s^2 + \alpha_5 qs + \alpha_1 q - q^2 + \theta$. The constant of integration $\theta$, can be solved for from the fact that the lowest type $k$ earns no information rent and $T_a(k) = \phi(q_a(k), s_a(k), k)$. 18
Proposition 6

\[ i \quad \mathcal{E}_c(s_a - s_f) = \mathcal{E}_c(s_v - s_f) = \frac{-\alpha_3 h(k)}{2(4 - \alpha_5^2)} < 0 \]

\[ ii \quad \mathcal{E}_c(q_a - q_f) = \mathcal{E}_c(q_v - q_f) = -\frac{\alpha_5 \alpha_3 h(k)}{4 - \alpha_5^2} < 0 \text{ if, and only if, } \alpha_5 > 0. \]

Since \( h(k) = 0 \) only for \( k = \overline{k} \), \( \mathcal{E}_c(s_a) = \mathcal{E}_c(s_v) = \mathcal{E}_c(s_f) = \mathcal{E}_c(q_v) = \mathcal{E}_c(q_f) \), and \( \mathcal{E}_c(q_a) = \mathcal{E}_c(q_f) \) only for the highest type. Thus, even though both \( q \) and \( s \) are observable and contractible for the firm, it has to pay information rents on account of unobservability of \( k \).

To lower the information rent paid to the higher types of \( k \), the firm chooses to distort the \( q \) and \( s \) consumed by lower types of \( k \). Note that this distortion, in contrast to the distortion characterized in Proposition 3, is independent of any cost variables. This independence is because the free-rider problem in the consumption of support services has been solved by the observability of \( s \). We next compare traditional pricing with ABP.

Proposition 7

\[ i \quad \mathcal{E}_c(s_a - s_r) = \mathcal{E}_c(s_v - s_t) = -\frac{h(k) \alpha_3 + \mu_s}{2} < 0 \]

\[ ii \quad \mathcal{E}_c(q_a - q_r) = \mathcal{E}_c(q_v - q_t) = 0 \]

\[ iii \quad \mathcal{E}_c(\iota_a - \iota_r) = \mathcal{E}_c(\iota_v - \iota_t) = \int_{\underline{k}}^{\overline{k}} -\alpha_3 \frac{h(k) \alpha_3 + \mu_s}{2} dk < 0 \]

Thus, compared to traditional pricing, when ABP is used, the firm allows each type of customer to consume strictly less support services. This reduction is on account of two factors. There is no longer any free-rider problem in the consumption of support services by the customer. The higher the \( \mu_s \), the greater the savings from ABP on account of reduction in the free-rider problem. There is also reduction in support services supplied on account of reduction of information rent paid to the agent. The higher the value of \( \alpha_3 \), the higher is this reduction. There is, however, in expectation no change in the expected quantity of the base product that is supplied to the customer. The information rent paid to customers is strictly less under ABP than traditional pricing for all customers except the lowest type for whom there is no information rent under either accounting system. The information rent is also equal to customer
surplus (customer utility less tariffs paid to the firm). Thus, customer surplus is strictly less under ABP for all types of customers other than the lowest type, who earns no surplus under either accounting system.

We next turn to the question of which type of customer is the most profitable for the firm under the various scenarios.

**Proposition 8**

i Under traditional pricing, expected customer profitability is decreasing in \(k\) if, and only if, \(\mu_s > \frac{h(k)(1-h'(k))\alpha_3\alpha_5^2}{4-\alpha_5^2}\).

ii Under ABP, expected customer profitability is increasing in customer types since \(E_c\left(\frac{d\Pi_a(k)}{dk}\right) = E_c\left(\frac{d\Pi_v(k)}{dk}\right) = \frac{2h(k)(1-h'(k))\alpha_3^2}{4-\alpha_5^2} > 0\).

iii By switching to ABP from traditional pricing, profitability of customers increases more for the higher type customers since \(E_c\left(\frac{d\Pi_a(k)-\Pi_r(k)}{dk}\right) = E_c\left(\frac{d\Pi_v(k)-\Pi_t(k)}{dk}\right) = \frac{\alpha_3}{2} (h(k)\alpha_3(1-h'(k)) + \mu_s) > 0\).

Under traditional pricing, support services are provided for free. Hence, we might expect customers with a high value of \(k\), who consume a lot of support services, will be highly unprofitable for the firm. However, customers with a high value of \(k\), who do consume more support services for free may actually be more profitable. Since support services are worth more to customers with a high value of \(k\), the firm is able to charge them more even after paying them information rents to reveal their true type. However, if \(\mu_s\) is too high, customers with a high value of \(k\) become more unprofitable because the moral hazard problem dominates. Under ABP there is no moral hazard problem and hence, customers who value customer support more (high values of \(k\)) are relatively more profitable under ABP.

While the expected increase in customer profitability in switching from traditional pricing to ABP, gross of implementation costs \(\omega\), is non-negative, customer profitability increases more for higher \(k\) types. The firm has a bigger free-rider problem in the consumption of support services and pays more information rent to customers who value customer support highly (high values of \(k\)). Thus, it is in dealing with these customers that the firm gains more under ABP.
4.4 Cost-driver Rate Versus Cost-driver Volume Information

In this subsection, we will evaluate the value of the cost-driver rate and volume information for the firm and the supply chain as a whole. We define supply chain surplus as the sum of the expected firm profits and the expected surplus of all its customers. Thus, supply chain surplus $\chi = \mathcal{E}_k \mathcal{E}_c ((\Pi(k) + \iota(k)))$.

**Proposition 9**

i. Under ABP, the expected value of cost-driver rate (ABC) information for the firm $\mathcal{E}_k (\mathcal{E}_c (\Pi_a(k) - \Pi_v(k))) = \frac{(\sigma^2 + \alpha_2 \sigma_{q_a} + \sigma^2)}{4 - \alpha^2} \omega_r$. This also equals the expected surplus for the supply chain $\chi_a - \chi_v$.

ii. Under traditional pricing, the expected value of cost-driver rate (ABC) information $\mathcal{E}_k (\mathcal{E}_c (\Pi_r(k) - \Pi_t(k))) = \mathcal{E}_k (\mathcal{E}_c (\Pi_a(k) - \Pi_v(k))) \frac{\alpha^2}{4}$. This also equals the expected surplus for the supply chain $\chi_r - \chi_t$.

In a result similar to Proposition 4, we have now established that gross of the measurement cost $\omega_r$, the firm earns strictly greater profits under ABC than traditional costing when it engages in ABP. Proposition 9(ii) shows that ABC is more valuable for a firm engaging in ABP than it is for a firm practicing traditional pricing. Equivalently, the cost-driver volume information that is necessary for ABP is more valuable for a firm that already has ABC information. That is, the cost-driver rate and cost-driver volume information are complements and one is more valuable in the presence of the other. Under both traditional pricing and ABP, a firm with ABC allows customers to consume more support service when $c_s$ is low, and allows customers to consume less support service when $c_s$ is high. The firm adjusts the level of customer support that it allows its customers to consume through the pricing schedule that it announces. Under traditional pricing, however, the firm is constrained to make its pricing only a function of $q$, while under ABP the price is a function of both $q$ and $s$. Thus, the firm is able to make more use of the cost-driver rate information provided by ABC under ABP than under traditional pricing.
The cost-driver rate information is more valuable under both traditional and activity-based pricing when the firm’s priors on its cost-driver rates is not very tight (i.e. when $\sigma_q$ and $\sigma_s$ are high). The impact of an increase in the covariance between the variable costs of the base product and support services $\sigma_{q,s}$ on the desirability of ABP for the firm depends on whether the base product and support services are substitutes or complements. When the base product and support services are complements (substitutes), as $\sigma_{q,s}$ becomes large, the pricing errors in the traditional cost system get exacerbated (reduced). When $\sigma_{q,s}$ is small and perhaps negative, the pricing errors on the base product and support services tend to cancel out(magnify) each other, provided the two are complements (substitutes). This suggests that all else being equal, ABP is less valuable when $\sigma_{q,s}$ is very small (large), as would be the case when it is very negative (positive) and the base product and support services are complements (substitutes).

We know from Propositions 3 and 6 that ABC leaves the expected consumption of support services and base product, and the information rent paid to each customer unchanged. Thus, it does not affect customer surplus at all and all the benefits from ABC accrue to the firm. We next turn to the value of cost-driver volume information.

**Proposition 10**

- **i** Under traditional costing, the expected value of cost-driver volume information (ABP) $\mathcal{E}_k(\mathcal{E}_c(\pi_v(k) - \pi_t(k))) = \int_k \frac{(\alpha_3h(k))^2 + 2h(k)\alpha_3\mu_s + \mu_s^2}{4} f(k) dk - \omega_v$.

- **ii** Under ABC, the expected value of cost-driver volume information (ABP) $\mathcal{E}_k(\mathcal{E}_c(\pi_a(k) - \pi_r(k))) = \mathcal{E}_k(\mathcal{E}_c(\pi_v(k) - \pi_t(k))) + \sigma_s^2$.

- **iii** The value of cost-driver volume information (ABP) for the supply chain under traditional costing $\chi_v - \chi_t = \mathcal{E}_k(\mu_v^2 - (\alpha_3h(k))^2) - \omega_v$ and under ABC $\chi_a - \chi_r = \mathcal{E}_k(\mu_v^2 + \sigma_s^2 - (\alpha_3h(k))^2) - \omega_v$.

Gross of the measurement cost, $\omega_v$, cost-driver volume information has strictly positive value under both ABC and traditional costing. The value of the cost-driver volume information derives from its use in ABP. This value is clearly increasing in $\mu_s$, the expected marginal cost of providing customer support service. This is because ABP mitigates the moral hazard
problem in the consumption of support services. Hence, the greater the cost of providing support services, the greater the value of ABP. As seen in Proposition 9(ii), the cost-driver volume information is more valuable for a firm that already has cost-driver rate information. Moreover, this incremental value of ABP under ABC relative to its value under traditional costing is increasing in $\sigma_s$. That is, the more diffuse the firm’s priors are under traditional costing about its cost to serve its customers, the greater the incremental value of ABP under ABC relative to traditional costing.

The impact of ABP on supply chain surplus is ambiguous. The reduction in moral hazard helps the supply chain. More efficient price discrimination by the firm under ABP results in the distortion in consumption of support services by customers. This distortion hurts customer and supply chain surplus. The net effect of the moral hazard reduction and increased price discrimination on supply chain surplus could go in either direction.

To obtain greater insight into the value of ABP, we consider the case where $k$ has a uniform distribution.

**Proposition 11** When $k$ is distributed uniform $[\mu_k - \sigma_k, \mu_k + \sigma_k]$

i. Under traditional costing, the expected value of cost-driver volume information (ABP) $E_k(E_c(\pi_v(k) - \pi_t(k))) = \frac{3\sigma^2 + 3\mu^2 + 6\mu\alpha\sigma_k + 4\alpha^2\sigma^2_k}{12} - \omega_v$.

ii. Under ABC, the expected value of cost-driver volume information (ABP) $E_k(E_c(\pi_a(k) - \pi_r(k))) = E_k(E_c(\pi_v(k) - \pi_t(k))) + \frac{\sigma^2_s}{4}$.

iii. $\frac{d(E(\Pi_a) - E(\Pi_r))}{d\sigma_k} = \frac{d(E(\Pi_v) - E(\Pi_t))}{d\sigma_k} = \frac{\alpha_3\mu_k}{2} + \frac{2\alpha^2_k\sigma_k}{3} > 0$.

iv. $\frac{d(E(\Pi_a) - E(\Pi_r))}{d\mu_k} = \frac{d(E(\Pi_v) - E(\Pi_t))}{d\mu_k} = 0$.

v. The value of cost-driver volume information (ABP) for the supply chain under traditional costing $\chi_v - \chi_t = \frac{\mu^2}{4} - \left(\frac{\alpha_3\sigma_k}{3}\right)^2 - \omega_v$ and under ABC $\chi_a - \chi_r = \frac{\mu^2}{4} - \left(\frac{\alpha_3\sigma_k}{3}\right)^2 + \frac{\sigma^2_s}{4} - \omega_v$.

From Proposition 11 we can see that in the case of the uniform distribution, the value of ABP for the firm under both ABC and traditional costing is increasing in the diversity of
customer types (as measured by $\sigma_k$), but is not affected by the average preference for support services ($\mu_k$). The value of ABP for the supply chain however is decreasing in the diversity of customer types. Thus, if $\sigma_k$ is high, ABP becomes very attractive to the firm but ends up hurting the overall supply chain.

5 Conclusions

Although the concepts of CPM and ABP are not new, they appear to have become more popular recently because they are IT intensive, and IT costs of implementing them have declined dramatically in the last few years. Firms that have implemented Customer Relationship Management and Enterprise Resource Planning database systems find it easy to implement ABP because they already have the cost-driver volume data.

In this paper, we show how and when activity-based costing and activity-based pricing are useful in contracting with customers who buy a base product and support services. As might be expected, ABC helps the firm price its products and services better. Perhaps less apparent, ABP also helps reduce the information rent paid to customers and their free riding on support services. Moreover, after the adoption of ABP, high users of customer support become the most profitable customers for the firm. We show that ABP is more useful to the firm when the costs associated with support services are more variable, and when the “types” of customers the firm deals with are highly diverse. ABC is more useful when the firm’s priors about its costs are not very tight. The firm has to tradeoff these benefits of ABP and ABC against their respective implementation costs.

We show that it is the cost-driver volume information, rather than the cost-driver rate information that helps the firm improve its price discrimination and mitigate customer moral hazard. Cost-driver rate information on the other hand, is useful for selecting the appropriate quantity of the base product and support services through an appropriate choice of prices. However, the cost-driver rate information and cost-driver volume information are complements and the marginal value of one increases with the availability of the other.

It has been shown in prior literature that the firm’s ability to engage in price discrimination
is reduced, although not eliminated, in the presence of competition.\textsuperscript{6} Moreover, the firm's ability to mitigate its customer's moral hazard problem and its ability to price its products appropriately would be valuable even in a competitive setting. Thus, we would expect many of the insights from this paper to hold in competitive settings as well. What is an open and interesting research question is how well a firm can use prices in a competitive market setting to infer information about its own costs without investing in an expensive ABC information to estimate its cost-driver rates. We leave that question for future research.

\textsuperscript{6}See Stole(1995).
Appendix I: Two-Period Model

In this appendix, we consider a two-period model of a firm that has a traditional costing system and uses traditional pricing. We will use the subscripts 1 and 2 to indicate the first or second period, respectively. We assume costs are normally distributed. In the second period, the firm uses its observation of total realized cost in the first period and its observation of total first period cost-driver volume information $E_k(q_1(k))$ and $E_k(s_1(k))$ to update its priors on the distribution of its costs. Let $c = \begin{pmatrix} c_f \\ c_q \\ c_s \\ \epsilon \end{pmatrix}$ be distributed multi-variate normal, $N(\mu, \Sigma)$ where

$$\mu = \begin{pmatrix} \mu_f \\ \mu_q \\ \mu_s \\ \mu_\epsilon \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} \sigma_f^2 & 0 & 0 & 0 \\ 0 & \sigma_q^2 & 0 & 0 \\ 0 & 0 & \sigma_s^2 & 0 \\ 0 & 0 & 0 & \sigma_\epsilon^2 \end{pmatrix}.$$ 

Since total cost $E_k(\eta_1(k)) = c_f + c_q E_k(q_1(k)) + c_s E_k(s_1(k)) + \epsilon$, we know that $E_k(\eta_1(k))$ is also normally distributed. Let

$$\eta_\Sigma = \begin{pmatrix} \sigma_{f\eta} \\ \sigma_{q\eta} \\ \sigma_{s\eta} \\ \sigma_{\epsilon\eta} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{E_k(q_1(k))\sigma_q^2}}{\sigma_f^2} \\ \frac{\sqrt{E_k(s_1(k))\sigma_s^2}}{\sigma_q^2} \\ \frac{\sigma_k^2}{\sigma_s^2} + \frac{\sigma_\epsilon^2}{\sigma_f^2} \end{pmatrix},$$

$$\mu_\eta = \mu_f + \mu_q E_k(q_1(k)) + \mu_s E_k(s_1(k)) + \mu_\epsilon \quad \text{and} \quad \sigma_\eta^2 = \sigma_f^2 + (E_k(q_1(k)))^2 \sigma_q^2 + (E_k(s_1(k)))^2 \sigma_s^2 + \sigma_\epsilon^2.$$ 

Let the realization of total cost in the first period $\eta_1(k)$, be $\varphi$. Then, conditional on $\varphi$, the distribution of $c$ for the second period is multivariate normal $N(\bar{\mu}, \bar{\Sigma})$ where

$$\bar{\mu} = \mu + \frac{(\varphi - \mu_\eta)}{\sigma_\eta^2} \eta_\Sigma \quad \text{and} \quad \bar{\Sigma} = \Sigma - \frac{1}{\sigma_\eta^2} \eta_\Sigma \eta_\Sigma^T.$$ 

Lemma 3 

In the second period,

i support provided to customer of type $k$,

$$s_2(k) = \frac{2\alpha_5(\alpha_1 - (\mu_q + \frac{(\varphi - \mu_\eta)}{\sigma_q^2}\sigma_q \eta)) - \alpha_3^2(\alpha_3 h(k) + (\mu_s + \frac{(\varphi - \mu_\eta)}{\sigma_s^2}\sigma_s \eta) + 4\alpha_3 k}{2(4 - \alpha_3^2)}$$

ii base good provided to customer of type $k$,

$$q_2(k) = \frac{2(\alpha_1 - (\mu_q + \frac{(\varphi - \mu_\eta)}{\sigma_q^2}\sigma_q \eta)) + \alpha_5(\alpha_3 (k - h(k)) - (\mu_s + \frac{(\varphi - \mu_\eta)}{\sigma_s^2}\sigma_s \eta))}{4 - \alpha_3^2}$$
iii the information rent paid to customer of type \( k \), \( \nu_2(k) = \int_\mathbb{K} \alpha_3 s_2(\tilde{k}) d\tilde{k} \)

iv tariff charged \( T_2(k) = \phi(q_2(k), s_2(k), k) - \nu_2(k) = T_2(q_2^{-1}(q)) = T_2(q) \)

v the marginal price \( \frac{dT_2}{dq_2} \) is increasing in the realization of the first period total cost \( \varphi \), that is \( \frac{d^2T_2}{dq_2d\varphi} = \frac{-2k'(k)2\sigma_{qn} + \alpha_5 \sigma_q n}{1 - k'(k)\sigma_n^2} > 0 \)

Observe that the firm uses the realized value of first period total cost (even though total cost includes fixed cost) for second period pricing. This is because first period total cost is informative about marginal costs in the second period. The extent to which the second period quantity of support service and base product is affected by first period total cost realization \( \varphi \) is a function of \( \sigma_{sn}, \sigma_{qn}, \mu_q, \) and \( \sigma_n^2 \) which, in turn, are functions of first period cost-driver volumes \( \mathcal{E}_k(q_1(k)) \) and \( \mathcal{E}_k(s_1(k)) \).

**Proposition 12** Under traditional pricing, the expected value of cost-driver rate (ABC) information

i in the first period is \( \mathcal{E}_k \mathcal{E}_c(\Pi_{r1}(k) - \Pi_{t1}(k)) = \frac{(4\sigma_q^2 + \alpha^2 \sigma_s^2)}{16 - 4\alpha^2} - \omega_r \)

ii in the second period, to a firm that did not adopt ABC in the first period, is \( \mathcal{E}_k \mathcal{E}_c(\Pi_{r2}(k) - \Pi_{t2}(k)) = \frac{(4\sigma_q^2 + \alpha^2 \sigma_s^2)}{16 - 4\alpha^2} - \omega_r - \frac{(4\mathcal{E}_k(q_2^2(k))\sigma_q^4 + 4\alpha_5 \mathcal{E}_k(s_2(k))\sigma_q^2 \sigma_s^2 + \alpha_5 \mathcal{E}_k(s_2^2(k))\sigma_s^4)}{\sigma_q^2(16 - 4\alpha^2)} \)

iii is strictly greater in the first period than in the second period

Proposition 12 shows that the value of ABC information is strictly less in the second period compared to the first period. This is because in the second period, the firm can base its pricing decision on first period cost-driver volume information and total costs. Some of the benefits of ABC have thus been partially offset by the information produced by the traditional cost system in the first period.
Appendix II: Proofs

Proof of Proposition 1 The participation constraint needs to hold for each type \( k \). Suppose to the contrary that the constraint does not bind for some types. Let the types for which the participation constraint does not bind in equilibrium be the set \( K' \). Consider an alternate mechanism where the firm maintains \( q(k) \) and \( s(k) \) as before, but increases \( T(q(k), s(k), k) \) so that the participation constraint binds for all types \( k \in K' \). The principal’s objective function clearly increases as \( T \) increases. Thus, a mechanism where participation constraint does not bind in equilibrium for a set of types cannot be an equilibrium.

Thus, we can convert the firm’s program into an unconstrained optimization program as follows:

\[
\max_{q(k), s(k)} \int \left( \phi(q(k), s(k), k) - c_q q(k) - c_s s(k) - c_f - \epsilon \right) f(k) dk
\]

This program can be optimized pointwise. Furthermore, \( \frac{\delta^2 \phi}{\delta q^2} = \frac{\delta^2 \phi}{\delta s^2} = -2 \) and \( \frac{\delta^2 \phi}{\delta q \delta s} = \left( \frac{\delta^2 \phi}{\delta q \delta s} \right)^2 = 4 - \alpha_2^2 > 0 \). Hence, first-order conditions are necessary and sufficient for a unique global maximum. Solving \( \frac{\delta \phi}{\delta q} = c_q \) and \( \frac{\delta \phi}{\delta s} = c_s \), and using the subscript \( f \) to denote equilibrium values in the first-best case, we have \( s_f = \frac{\alpha_1 q - \alpha_5 q - 2c_q + 2\alpha_3 k}{4 - \alpha_5^2} \) and \( q_f = \frac{2\alpha_1 - 2c_q + \alpha_5 (\alpha_3 k - c_s)}{4 - \alpha_5^2} \). We get \( \phi_f(k) \) by substituting \( s_f \) and \( q_f \) for \( s \) and \( q \), respectively. We get \( T_f(k) \) by setting \( T_f(k) = \phi_f(k) \) because the participation constraint binds for all types.

Proof of Lemma 1 We begin by solving Program 2 first. We show that the participation constraint needs to be satisfied only for the lowest type of customer. Consider the participation constraint for the lowest type \( \underline{k} \):

\[
\phi(q(k), s(k), \underline{k}) - T(q(\underline{k})) \geq 0
\]

(1)

If Equation 1 is satisfied, a type \( k \) customer \( (k > \underline{k}) \) also realizes a non-zero surplus, because they can always choose the type \( \underline{k} \)’s bundle and obtain utility of

\[
\alpha_1 q(\underline{k}) - q(\underline{k})^2 + \alpha_3 ks(\underline{k}) - s(\underline{k})^2 + \alpha_5 q(\underline{k}) s(\underline{k}) - T(\underline{k})
\]

\[
\geq \alpha_1 q(\underline{k}) - q(\underline{k})^2 + \alpha_3 ks(\underline{k}) - s(\underline{k})^2 + \alpha_5 q(\underline{k}) s(\underline{k}) - T(\underline{k}) \geq 0
\]
Hence, if the participation constraint is satisfied for the lowest type, it is automatically satisfied for all other types.

Observe that \( \frac{d^2 \phi}{ds^2} = -2 \). Hence, setting \( \frac{d\phi}{ds} = 0 \), we get \( s = \frac{\alpha_5 + \alpha_2 q}{2} \). Substituting into \( \phi \), we get

\[
\phi(q(k), k) = \frac{\alpha_2^2 k^2 + 2\alpha_3 \alpha_5 k q(k) + q(k)(4\alpha_1 - (4 - \alpha_5^2)q(k))}{4}
\]  

(2)

The incentive compatibility constraints,

\[
\phi(q(k), s(k), k) - T(q(k)) \geq \phi(q(\tilde{k}), \tilde{s}, k) - T(q(\tilde{k})), \forall k, \tilde{k}, \tilde{s}
\]

become

\[
\phi(q(k), k) - T(q(k)) \geq \phi(q(\tilde{k}), k) - T(q(\tilde{k})), \forall k, \tilde{k}
\]

We can replace the incentive compatibility constraints with the following first-order condition.

\[
\frac{d\phi}{dq} - \frac{dT}{dq} = 0
\]  

(3)

It remains to be verified that the second-order conditions are satisfied locally and globally and we shall do so later.

To derive the optimal \( q(k) \) and \( s(k) \), we write the utility of customer type \( k \) as \( \iota(k) \). From the incentive compatibility constraint,

\[
\iota(k) \equiv \phi(q(k), s(k), k) - T(q(k)) = \max_{\tilde{k}} \phi(q(\tilde{k}), s(\tilde{k}), k) - T(q(\tilde{k}))
\]

From the envelope theorem, \( \frac{d\iota}{dk} = \frac{\delta \iota}{\delta k} \) since \( \frac{\delta \iota}{\delta q} = \frac{\delta \iota}{\delta s} = 0 \). Thus,

\[
\frac{d\iota}{dk} = \alpha_3 s(k)
\]  

(4)

Integrating Equation 4, we can express the utility of type \( k \) customers as

\[
\iota(k) = \int_{k}^{k} \alpha_3 s(\tilde{k})d\tilde{k} + \iota(\underline{k}) = \int_{k}^{k} \alpha_3 s(\tilde{k})d\tilde{k}
\]
The last equality follows from the participation constraint binding for the lowest type. Since
\( T(q(k)) = \phi(q(k), s(k), k) - \iota(k) \), the firm’s objective function can be rewritten as

\[
\mathcal{E}_k(\mathcal{E}_c(\Pi(k))) = \int_k^q (\phi(q(k), s(k), k) - \int_k^q \alpha_3 s(\tilde{k}) d\tilde{k} - \mathcal{E}_c((c_q)q(k) + (c_s)s(k) + (c_f + \epsilon)) f(k) dk
\]

Integrating by parts yields

\[
\int_k^q ((\phi(q(k), s(k), k) - \mathcal{E}_c(c_q q(k) + c_s s(k) + c_f + \epsilon)) f(k) - \alpha_3 s(k)(1 - F(k)) dk
\]

Substituting for \( s \) into Equation 5, we get
\[
\mathcal{E}_k(\mathcal{E}_c(\Pi(k))) = \int_k^q ((\phi(q(k), k) - \mathcal{E}_c(c_q q(k) + c_s s(k) + c_f + \epsilon)) f(k) - \alpha_3 s(k)(1 - F(k)) dk.
\]

The maximization of \( \mathcal{E}_k(\mathcal{E}_c(\Pi(k))) \) with respect to \( q(k) \) requires that the term under the integral be optimized pointwise for all \( k \). Hence, we get
\[
\alpha_1 - 2q + \alpha_5 \frac{\alpha_3 k + \alpha_5 q}{2} = \frac{\mu_q + \mu_s \frac{\alpha_5}{2} + \alpha_3 \frac{1 - F(k)}{f(k)} - \frac{\alpha_5}{2}}{2}
\]
\[
\Rightarrow q_t(k) = \frac{2(\alpha_1 - \mu_q) + \alpha_5 (\alpha_3 (k - h(k)) - \mu_s)}{4 - \alpha_5^2}
\]
\[
\Rightarrow s_t(k) = \frac{\alpha_3 k + \alpha_5 q_t(k)}{2} = \frac{2\alpha_5 (\alpha_1 - \mu_q) - \alpha_5^2 (\alpha_3 h(k) + \mu_s) + 4\alpha_3 k}{2(4 - \alpha_5^2)}
\]
\[
\Rightarrow \iota_t(k) = \int_k^q \alpha_3 s_t(\tilde{k}) d\tilde{k} = \int_k^q \frac{\alpha_3 (2\alpha_5 (\alpha_1 - \mu_q) - \alpha_5^2 (\alpha_3 h(\tilde{k}) + \mu_s) + 4\alpha_3 \tilde{k})}{4 - \alpha_5^2} d\tilde{k}
\]

Since \( \iota(k) = \phi - T \), we can write
\[
T(k) = \phi(q_t(k), s_t(k), k) - \iota_t(k). \quad \frac{d \iota_t(k)}{dk} = \frac{-\alpha_5 \alpha_3 (k'(k) - \mu_s)}{4 - \alpha_5^2}
\]

Since \( \frac{dh}{dk} < 0 \) by assumption, \( \frac{d \iota_t(k)}{dk} \) is monotonic in \( k \). \( \frac{d \iota_t(k)}{dk} > 0 \) if \( \alpha_5 > 0 \) and \( \frac{d \iota_t(k)}{dk} < 0 \) if \( \alpha_5 < 0 \). Since \( q(k) \) is strictly monotonic in \( k \), we can invert from \( q(k) \) to \( k \). Hence, we can characterize \( T \) as a function of \( q \) as well. Customers can reveal \( k \) and pick the bundle, \( T(k) \), \( q(k) \), and \( s(k) \) (communication approach) or the firm can announce \( T(q) \) and decentralize the choice of \( q \) and \( s \) to its customers. Both approaches are equivalent.

Now, we verify the local and global satisfaction of second-order conditions. Let \( \iota(k, \tilde{k}) \) denote the utility of customer of type \( k \) if he consumes the quantity of a consumer of type \( \tilde{k} \).

\[
\iota(k, \tilde{k}) = \alpha_1 q(\tilde{k}) - q(\tilde{k})^2 + \alpha_3 k s(q(\tilde{k})) - s(q(\tilde{k}))^2 + \alpha_5 q(\tilde{k}) s(q(\tilde{k})) - T(q(\tilde{k}))
\]
\[
\Rightarrow \iota(k, \tilde{k}) = \frac{\alpha_5^2 k^2 + 2\alpha_3 \alpha_5 q(\tilde{k}) + q(\tilde{k})(4\alpha_1 - (4 - \alpha_5^2)\alpha_5 \tilde{k}) - T(q(\tilde{k}))}{4}
\]
The first-order condition is for all \( k \), \( \frac{\delta \iota(k,k)}{\delta k} = 0 \). Differentiating the first-order condition with respect to \( k \) gives \( \frac{\delta^2 \iota(k,k)}{\delta k \delta k} = 0 \). Thus, the local second-order condition is equal to \( \frac{\delta^2 \iota(k,k)}{\delta k \delta k} \geq 0 \). However, \( \frac{\delta^2 \iota(k,k)}{\delta k \delta k} = 2\alpha_3\alpha_5 \frac{d\mathbf{q}(k)}{dk} \) when \( k \) is characterized in Lemma 1 and we know from Proposition 2(i) that \( q(k) \) is strictly increasing. Therefore, \( q(k) \) is strictly increasing.

To check the global second-order condition, suppose that \( \iota(k_1,k_2) > \iota(k_1,k_1) \) for some \( k_1 \) and \( k_2 \). This implies that \( \int_{k_1}^{k_2} \frac{d\mathbf{q}(x)}{dk} \, dx > 0 \). Suppose \( k_2 > k_1 \). Because \( \frac{\delta^2 \iota(k,k)}{\delta k \delta k} \geq 0 \) we have \( \frac{d\mathbf{q}(k_1)}{dk} \leq \frac{d\mathbf{q}(x)}{dk} = 0 \) for \( k \geq k_1 \), where we use the first-order condition. We thus have a contradiction. Similarly for \( k_1 > k_2 \).

The solution method for Program 3 is analogous. The solution differs in that for all the endogenous variables the expected cost variables have to be replaced with actual realized values of the cost variables.

Proof of Proposition 2 Proposition 2 follows directly from Lemma 1.

Proof of Proposition 3 Proposition 3 follows from subtracting the expected values of \( s_f \) and \( q_f \) characterized in Lemma 1 from the values of \( s_t \) and \( q_t \) characterized in Proposition 2, where the expectation is taken with respect to cost uncertainty.

Proof of Proposition 4 Proposition 4 follows from substituting \( s_t, q_t, \iota_t, s_r, q_r, \) and \( \iota_r \) derived in Lemma 1 into the objective functions of Programs 2 and 3 and then subtracting \( E_k(\mathcal{E}_c(\Pi_t(k})) \) from \( E_k(\mathcal{E}_c(\Pi_r(k))) \).

Proof of Proposition 5 We know from Equation 3 that \( \frac{d\phi}{dq} = \frac{dT_t}{dq} \). Differentiating Equation 2 with respect to \( q \), we get

\[
\frac{dT_t}{dq} = \frac{d\phi(q)}{dq} = 2\alpha_3\alpha_5 k + 4\alpha_1 - 2(4 - \alpha_5^2)q(k)
\]  

(6)
increasing in $k$ and is hence invertible. Hence
\[
\frac{dT_t}{dq} = 2\alpha_3\alpha_5q_t^{-1}(q(k)) + 4\alpha_1 - 2(4 - \alpha_5^2)q_t(k) \tag{7}
\]
\[
\frac{d^2T_t}{dq^2} = -2(4 - \alpha_5^2) + 2\alpha_3\alpha_5\frac{dq_t^{-1}(q(k))}{dq} \tag{8}
\]
\[
\frac{d^2T_t}{dq^2} = -2(4 - \alpha_5^2) + 2\alpha_3\alpha_5\frac{dk}{dq(k)} \tag{9}
\]

We know from Lemma 1 that
\[q_t(k) = \frac{2(\alpha_1 - \mu_q) + \alpha_5(\alpha_3(k - h(k)) - \mu_s)}{4 - \alpha_5^2} \]
\[\Rightarrow \frac{dq_t(k)}{dk} = \frac{\alpha_3\alpha_5}{4 - \alpha_5^2} \frac{d(k - h(k))}{dk} \]
\[\Rightarrow \frac{d^2T_t}{dq^2} = 2(4 - \alpha_5^2)^{-1}\frac{h'(k)}{1 - h'(k)} < 0 \]

The proof for $E_c(\frac{d^2T_t}{dq^2})$ is similar.

We next prove Proposition 5(ii).
\[
\frac{d^2T_t}{dq^2}\frac{d\mu_q}{d\mu_s} = \frac{d^2T_t}{dq^2}\frac{dq_t}{d\mu_q} \tag{10}
\]

We know from Proposition 2(ii) that $\frac{dq_t}{d\mu_q} < 0$. Hence, $\frac{d^2T_t}{dq^2}\frac{dq_t}{d\mu_q} > 0$. Similarly, $\frac{d^2T_r}{dq^2}\frac{dq_r}{d\mu_s}$ is increasing in $\mu_s$ but decreasing in $k$, and $\frac{d^2T_t}{dq^2}\frac{dq_t}{d\mu_q}$ is increasing in $c_q$ and $c_s$ but decreasing in $k$.

Proof of Lemma 2 First, we solve Program 5. As in the proof of Lemma 1, the participation constraint needs to bind only for the lowest type of customer. We replace the incentive compatibility constraints with two first-order local conditions.
\[
\frac{d\phi}{dq} \frac{dT}{dq} = 0
\]
\[
\frac{d\phi}{ds} \frac{dT}{ds} = 0
\]

We take $E_c(E_c(\Pi(k)))$ as characterized in Equation 5 and maximize with respect to the schedules $s(k)$ and $q(k)$.

From pointwise maximization, we get
\[
\alpha_1 - 2q + \alpha_5s = c_q \tag{10}
\]
\[
\alpha_3k - 2s + \alpha_5q = c_s + \alpha_3 \frac{1 - F(k)}{f(k)} \tag{11}
\]

32
Solving Equations 10 and 11 simultaneously we get

\[ s_a(k) = \frac{\alpha_5(\alpha_1 - c_q) - 2(\alpha_3(h(k) - k) + c_s)}{(4 - \alpha_5^2)} \]

\[ q_a(k) = \frac{2(\alpha_1 - c_q) + \alpha_5(\alpha_3(h(k) - k) + c_s)}{4 - \alpha_5^2} \]

\[ \Rightarrow \iota_a(k) = \int_k^\kappa \alpha_3 s_a(\tilde{k}) d\tilde{k} = \int_k^\kappa \frac{\alpha_3(\alpha_5(\alpha_1 - c_q) - 2(\alpha_3(h(\tilde{k}) - \tilde{k}) + c_s))}{(4 - \alpha_5^2)} d\tilde{k} \]

\[ T_a(k) = \phi(q_a(k), s_a(k), k) - \iota_a(k) \text{ since } \iota(k) = \phi - T \]

We skip the proofs for the local and global second-order conditions being satisfied as the proof is analogous to that in the Proof of Lemma 1.

The solution method to program 4 is analogous. The only difference in the solution is that the actual cost realizations are replaced with expected values of the cost variables.

**Proof of Proposition 6** Proposition 6 follows from subtracting the values of \( s_t \) and \( q_t \) characterized in Proposition 1 from the values of \( s_a \) and \( q_a \) characterized in Lemma 2.

**Proof of Proposition 7** Proposition 7 follows from subtracting the expected values of \( s_a, q_a, \) and \( \iota_a \) characterized in Lemma 2 from the values of \( s_t, q_t, \) and \( \iota_t \) characterized in Lemma 1 where the expectation is taken with respect to cost uncertainty.

**Proof of Proposition 8** First, we write \( \Pi_t(k) = T_t(k) - (c_q q_t(k) + c_s s_t(k) + c_f + \epsilon) \). Next, we substitute for \( T_t, q_t, \) and \( s_t \) from Lemma 1. We then take expectations with respect to \( c \). Finally, we take the derivative with respect to \( k \) to get \( E_c(\frac{d\Pi_t(k)}{dk}) = \frac{\alpha_3(h(k)(1-h'(k))\alpha_5^2)}{2(4 - \alpha_5^2)} - \frac{\alpha_3\mu_s}{2} \).

Similarly for \( \Pi_v(k) \). Hence Proposition 8(i) follows.

To prove Proposition 8(ii), we write \( \Pi_a(k) = T_a(k) - (c_q q_a(k) + c_s s_a(k) + c_f + \epsilon) \). Next, we substitute for \( T_a, q_a, \) and \( s_a \) from Lemma 2. We then take expectations with respect to \( c \). Finally, we take the derivative with respect to \( k \) to get \( \frac{2h(k)(1-h'(k))\alpha_5^2}{4 - \alpha_5^2} \). Similarly for \( \Pi_v(k) \). Hence Proposition 8(ii) follows.

Proposition 8(iii) follows from Propositions 8(i) and (ii).

**Proof of Proposition 9** We can write \( \Pi_a(k) = T_a(k) - (c_q q_a(k) + c_s s_a(k) + c_f + \epsilon) \). Substituting for \( T_a, q_a, s_a, \) and \( \iota_a \) from Lemma 2 and taking expectations with respect to cost variables
and then \( k \) we get \( \mathcal{E}_k(\mathcal{E}_c(\Pi_a(k))) \). We can similarly write down \( \mathcal{E}_k(\mathcal{E}_c(\Pi_v(k))) \). Subtracting the two, we get the desired result. \( \chi_a - \chi_v \) follows by adding the expected difference in information rents \( \mathcal{E}_k(\upsilon_a(k)) - \mathcal{E}_k(\upsilon_v(k)) \).

Proposition 9(ii) follows similarly by substituting endogenous variables from Lemma 1.

**Proof of Proposition 10** First, we write \( \pi_v(k) = T_v(k) - (c_q q_v(k) + c_s s_v(k) + c_f + \epsilon) \) and \( \pi_t(k) = T_t(k) - (c_q q_t(k) + c_s s_t(k) + c_f + \epsilon) \). We substitute for \( T_v, q_v, \) and \( s_v \) from Lemma 2 and for \( T_t, q_t, \) and \( s_t \) from Lemma 1. Next, we take expectations with respect to the cost variables to get

\[
\mathcal{E}_c(\Pi_v(k) - \Pi_t(k)) = \mathcal{E}_c(\upsilon_t(k) - \upsilon_v(k)) + \frac{\mu_s^2 - (h(k) \alpha_3)^2}{4} 
\]

\[ (12) \]

\[ \Rightarrow \mathcal{E}_c(\Pi_v(k) - \Pi_t(k)) = \int_k^\mathcal{K} \frac{\alpha_3^2 h(\tilde{k}) + \alpha_3 \mu_s}{2} d\tilde{k} + \frac{\mu_s^2 - (h(k) \alpha_3)^2}{4} 
\]

\[ (13) \]

\[ \Rightarrow \mathcal{E}_k(\mathcal{E}_c(\Pi_v(k) - \Pi_t(k))) = \int_k^\mathcal{K} (\int_k^\mathcal{K} \frac{\alpha_3^2 h(\tilde{k}) + \alpha_3 \mu_s}{2} d\tilde{k} + \frac{\mu_s^2 - (h(\tilde{k}) \alpha_3)^2}{4}) f(\tilde{k}) d\tilde{k} 
\]

\[ (14) \]

Where Equation 13 follows from substituting for \( \upsilon_v \) and \( \upsilon_t \) from Lemma 2 and Lemma 1, respectively. Using integration by parts, the right-hand side of Equation 14 can be simplified as follows

\[
\int_k^\mathcal{K} (-1 - F(\tilde{k})) \int_k^\mathcal{K} \frac{\alpha_3^2 h(\tilde{k}) + \alpha_3 \mu_s}{2} d\tilde{k} + (1 - F(\tilde{k})) \frac{\alpha_3^2 h(\tilde{k}) + \alpha_3 \mu_s}{2} - \frac{(h(\tilde{k}) \alpha_3)^2}{4} f(\tilde{k}) d\tilde{k} + \frac{\mu_s^2}{4}
\]

The first term under the outside integral vanishes. Since \( \frac{1 - F(\tilde{k})}{f(\tilde{k})} = h(\tilde{k}) \), we can further simplify the expression as

\[
\int_k^\mathcal{K} \left( \frac{(\alpha_3 h(\tilde{k}))^2}{4} + \frac{h(\tilde{k}) \alpha_3 \mu_s}{2} + \frac{\mu_s^2}{4} \right) f(\tilde{k}) d\tilde{k}
\]

The expression is strictly positive since each of the terms under the integral are positive.

Through a similar process it can be shown that

\[
\mathcal{E}_k(\mathcal{E}_c(\Pi_a(k) - \Pi_v(k))) = \int_k^\mathcal{K} \left( \frac{(\alpha_3 h(\tilde{k}))^2}{4} + \frac{h(\tilde{k}) \alpha_3 \mu_s}{2} + \frac{\mu_s^2 + \sigma_s^2}{4} \right) f(\tilde{k}) d\tilde{k}
\]

\[ \Rightarrow \mathcal{E}_k(\mathcal{E}_c(\Pi_a(k) - \Pi_v(k))) = \mathcal{E}_k(\mathcal{E}_c(\Pi_v(k) - \Pi_t(k))) + \frac{\sigma_s^2}{4} \]

34
Proposition 10(iii) can be proved as follows. \( \chi_v - \chi_t = \mathcal{E}_k(\mathcal{E}_c(\Pi_v(k) + \iota_v(k) - \Pi_t(k) - \iota_t(k))) \). 
\( \Pi_v(k) = T_v(k) - (c_q q_v(k) + c_s s_v(k) + c_f + \epsilon) \) and \( \Pi_t(k) = T_t(k) - (c_q q_t(k) + c_s s_t(k) + c_f + \epsilon) \). We can substitute for \( \iota_v(k), T_v(k), q_v(k), s_v(k), \iota_t(k), T_t(k), q_t(k), \) and \( s_t(k) \) from Lemmas 2 and 1. We then take expectations with respect to the cost variables to derive \( \chi_v - \chi_t \). Likewise, for \( \chi_a - \chi_r \).

**Proof of Proposition 11** When \( k \) is distributed uniform \([\mu_k - \sigma_k, \mu_k + \sigma_k]\), \( f(k) = \frac{1}{2\sigma_k} \) and \( h(k) = \mu_k + \sigma_k - k \).

Proposition 11(i) follows from proposition 10(i) after substituting for \( f(k) \) and \( h(k) \). Proposition 11(ii) follows from proposition 10(ii) after substituting for \( f(k) \) and \( h(k) \). Propositions 11(iii) follows from taking derivative with respect to \( \sigma_k \) from Propositions 11(ii) and 11(i). While Propositions 11(iv) follows from taking derivative with respect to \( \mu_k \) from Propositions 11(ii) and 11(i), Proposition 11(v) follows from Proposition 10(iii) after substituting for \( h(k) \).

**Proof of Lemma 3** The Proof of Lemma 3(i) through Lemma 3(ii) is exactly similar to the proof of Lemma 1. To prove Lemma 3(v), see that \( \frac{d^2T_2}{dq_2d\varphi} = \frac{d^2T_2}{dq_2d\varphi} - \frac{d^2T_2}{dq_2d\varphi} \). We know from Proposition 5 that \( \frac{d^2T_2}{dq_2d\varphi} = 2(4 - \alpha_3^2) \frac{h'(k)}{1 - h'(k)} < 0 \). We can compute \( \frac{dq_2}{d\varphi} \) from differentiating \( q_2 \) characterized in Lemma 3.

**Proof of Proposition 12** The proof of Proposition 12(i) is analogous to proof of Proposition 4, keeping in mind that by assumption in Appendix I, for the first period, \( \sigma_{q,s} = 0 \). For the second period, we use the variance-covariance matrix \( \Sigma \).

Proposition 12(iii) follows because \( \frac{(4\mathcal{E}_k(q_2(k))\sigma_q^2 + 4\alpha_2\mathcal{E}_k(s_1(k))q_1(k))\sigma_q^2 + \alpha_2^2\mathcal{E}_k(s_1^2(k))\sigma_q^2}{\sigma_h} \) is a perfect square and hence positive.
References


