Mental Accounting and Small Windfalls: Evidence from an Online Grocer

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We study the effect of small windfalls on consumer spending decisions by examining the purchasing behavior of a sample of online grocery shoppers over the course of a year. We compare the purchases customers make when redeeming a $10-off coupon they received from their online grocer with the purchases the same customers make when shopping without a coupon. The standard permanent income or lifecycle theory of consumption predicts that grocery spending will be unaffected by the use of a $10-off coupon, while a simple mental accounting framework predicts that such a coupon will increase spending on groceries. Controlling for customer fixed effects and other relevant variables, we find that grocery spending increases by $1.59 with the use of a $10-off coupon. In addition, even though the receipt of a $10-off coupon does not correspond to a meaningful increase in wealth, the extra spending associated with the redemption of such a coupon is focused on “marginal” grocery items, or grocery items that a customer does not typically buy.

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I. INTRODUCTION

In the course of daily life, people occasionally receive small windfalls. Every so often we are handed a gift certificate for five dollars off a meal at our favorite local restaurant, find a ten dollar bill on the street, or win twenty dollars in an impromptu game of poker. According to the standard permanent income or lifecycle theory of consumption (Friedman, 1957; Modigliani and Brumberg, 1954), these types of small windfalls should have no noticeable effect on spending decisions because such windfalls constitute meaningless changes to lifetime wealth. However, if you have ever been the recipient of a small windfall, you may remember thinking about ways to put this unexpected cash to use buying items you might not have otherwise purchased. This kind of behavior can be interpreted as an example of “mental accounting” (Thaler and Shefrin, 1981). In this paper, we present evidence supporting some of the implications of a theory of mental accounting in the domain of online grocery shopping.

Thaler and Shefrin have argued that people create mental accounting systems, similar to the way organizations create accounting systems, to organize and manage their spending (Thaler and Shefrin, 1981; Thaler, 1985; Shefrin and Thaler, 1988; Thaler, 1990; Thaler, 1999). According to this theory, rather than optimizing consumption choices over a life-long horizon, people make many spending decisions over considerably shorter time horizons using “mental accounts” in order to manage their self-control problems. We refer to this behavior as budgeting. By budgeting before opportunities for consumption arise, people are better able to avoid the temptation to spend their money as carelessly as they otherwise would, and this helps some people reach their savings goals.

\footnote{The “standard” permanent income or lifecycle theory refers to the certainty-equivalent version.}
unanticipated small windfall does not prevent a consumer from achieving her budgeted savings goals, so consumers respond to small windfalls by spending them immediately.

It has been demonstrated in the laboratory that people spend more out of unexpected income than out of anticipated income, a finding which is consistent with our interpretation of mental accounting (Arkes et al., 1994). To extend the study of the effect of small windfalls on spending decisions beyond the laboratory setting, we use a novel data set from an online grocer containing individual-level information about grocery purchases over the course of a year. This data set includes information about the decisions made by thousands of consumers both when they redeem coupons of a certain type for $10 off their online grocery orders and when they order groceries without any such discount.

A $10-off coupon of the type examined in this paper can be sent by a first-time patron of the online grocer we collaborated with to any other person she likes. We argue that the date on which a customer receives such a $10-off coupon is exogenous from the point of view of that customer. Under this assumption, we can estimate the effect of a $10-off coupon on grocery spending by comparing each customer’s orders with coupons to her orders without coupons. When we regress spending for a grocery order on an indicator variable for whether or not the order involved a $10-off coupon, we find that coupon use increases spending by $1.59, controlling for customer fixed effects and other factors. We also find evidence that these spending increases are particularly focused on “marginal” grocery items, which we define as items that a customer does not typically purchase.

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2 In this paper, we use the term “spending” to denote the total price of the groceries in a customer’s order, ignoring the effects of taxes, delivery fees, and coupons on the customer’s out-of-pocket expenses.
Our data set allows us to examine the predictions of mental accounting in the field rather than in the laboratory. Furthermore, because the data set is from the online grocery domain, we can infer that a $10 windfall is an inconsequential sum in the context of the overall wealth of the consumers studied. In order to be included in our sample, consumers must be able to afford both internet access and the fees associated with ordering groceries for delivery. Another advantage of our data is that it allows us to use a within-subject design to study small windfalls, which, unlike large windfalls, are not predicted to induce meaningful wealth effects according to standard economic theory. Finally, the data come from a domain where individuals choose from a wide range of goods, allowing us to examine the types of purchases customers make after receiving a small windfall.

The rest of this paper is organized as follows. Section II reviews the relevant literature and outlines a model that formalizes our hypotheses about windfall spending. In Section III we describe our data set and regression specification. We present our results in Section IV. Section V concludes.

II. RELEVANT LITERATURE AND THEORETICAL FRAMEWORK

A. Past Research on Marginal Propensities to Consume out of Income

Several previous studies conducted with field data have examined the spending behavior of people who receive windfall income, although the windfalls examined in past research have been considerably larger than those analyzed in this paper. Bodkin (1959) analyzed the spending decisions of veterans who received surprise, one-time, lump-sum life insurance dividend payments averaging $250 in 1950 ($2,115 adjusted for inflation).\(^3\) He found that families in his sample had a higher propensity to consume out of windfall

income than out of regular income. However, when Kreinin (1961) examined the spending decisions of Israelis who received unexpected, lump-sum Holocaust restitution payments, he found that families had a considerably lower propensity to consume out of windfall income than out of regular income. Landsberger (1966) argued that Kreinin and Bodkin obtained these seemingly contradictory results because the two data sets they analyzed had different average windfall sizes. Using Kreinin’s data set, Landsberger reconciled Kreinin and Bodkin’s findings with the observation that the propensity to consume out of windfall income decreases as the size of the windfall increases, and he estimated a marginal propensity to consume out of windfall income greater than one (but not significantly different from one) for the smallest windfalls in his sample.4

Another set of empirical studies has analyzed the response of consumption to anticipated changes in income rather than unanticipated wealth shocks. These studies have typically examined the effects of fiscal policy changes such as tax rebates or increases in social security benefits that were announced prior to their implementation. Thus, consumers knew about an upcoming change in income well before it occurred. According to the standard permanent income or lifecycle theory, changes in consumption should not coincide with anticipated changes in income but should instead coincide with the announcement of an income change, and some studies find evidence consistent with this prediction (see Hsieh, 2003, for example). However, other studies detect excess sensitivity and reject the permanent income or lifecycle null (Poterba, 1988; Wilcox, 4 Keeler, James, and Abdel-Ghany (1985) similarly find that the propensity to consume out of windfalls declines with the size of the windfall in the Consumer Expenditure Survey.
In a paper that specifically addresses the implications of mental accounting for consumption decisions, Baker, Nagel, and Wurgler (2006) study people’s propensity to consume out of dividends. Using cross-sectional variation in dividend receipts, they find a strong response of consumption to dividends, controlling for total stock returns. This evidence is consistent with mental accounting and inconsistent with standard economic models, which predict that only total returns (not the decomposition of returns into dividends and capital gains) should affect consumption.

Laboratory studies have also found evidence consistent with the predictions of mental accounting. For example, Arkes et al. (1994) demonstrated that unexpected small windfalls ($3 to $5) are more likely to be spent on gambling or at a basketball game than anticipated windfalls of the same size. This result is consistent with the idea that consumers who engage in mental accounting will spend unanticipated windfalls immediately because doing so will not prevent them from conforming to the budgets they have set for themselves.

Heilman et al. (2002) conducted another study of windfall spending that is closely related to ours. They examined the effect of one-dollar coupons for spaghetti sauce, laundry detergent, cereal, and paper towels on the behavior of grocery shoppers. The authors found that targeted one-dollar coupons increased the number of unplanned purchases made by consumers, the total number of purchases made by consumers, the amount of unplanned spending by consumers, and the total amount spent by consumers. In addition, these coupons increased consumer spending on treats and on complementary

\[5\] For a more thorough review of the literature on excess sensitivity, see Browning and Lusardi (1996).
goods. In a large observational data set of consumer grocery spending over time, they found patterns consistent with the results of their field experiment. This study offers suggestive evidence about the mental accounting effects of coupons on spending decisions. However, the results of this study may be due to substitution effects induced by category-specific coupons, which change the relative prices of goods. This explanation is supported by the increased spending the authors observe on goods that are complements to the discounted groceries.

According to the permanent income or lifecycle theory, households maximize their net utility over all future consumption, implying under standard assumptions that the consumption of any windfall wealth is spread across all future years of life (Friedman, 1957; Modigliani and Brumberg, 1954). However, the literature reviewed above suggests that this prediction is not consistent with empirical evidence. A mental accounting framework may better describe the types of behavior people exhibit when they receive a windfall.

B. Mental Accounting – A Theoretical Framework

In this subsection, we outline a simple model of mental accounting and discuss the past research that supports the assumptions of our model. We then highlight two implications of our model of mental accounting, which we can test using our online grocery data set.

As a basis for our model of mental accounting, we postulate that in order to mitigate their self-control problems, people set savings goals. Furthermore, people arrange to be penalized for failing to achieve their savings goals, and the presence of such goals prevents them from spending too much money when they receive income. Here is
a stylized story to illustrate the intuition behind our model. Consider Robert, a maître d’.

Each Monday, he arrives at his restaurant and finds out exactly how much money he earned the previous week (an uncertain number that depends on the pooled value of wait staff tips), and he is given a paycheck for his work. Robert knows that if he does not make a plan to constrain himself before receiving his paycheck, he will inevitably use it to splurge at his favorite pastry shop on Monday evening, leaving him with very little cash for the rest of the week. To avoid this fate, Robert sets a budget for himself every Sunday. Based on his expected weekly earnings, Robert decides how much of his paycheck he will set aside for the second half of the week. To enforce this savings goal, Robert may simply make a promise to himself, which will be costly to break because of the ensuing guilt.\(^6\) Alternatively, Robert may use an external commitment device so that it will be costly for him to break his budget. For example, he may ask his sister to deposit his paycheck in her bank account and give him a fraction of it for use during the first half of the week.\(^7\) An implication of this story is that if Robert receives a small windfall early in the week he will spend it immediately on items he might not feel able to afford otherwise because doing so will be enjoyable and will not interfere with his ability to meet his savings goal.

\(^6\) The idea that self-control failures are accompanied by guilt costs is supported by the finding that self-control failures are responsible for a major category of guilt episodes (Baumeister et al., 1995; Baumeister and Exline, 1999; Dahl et al., 2003).

\(^7\) The pay schedule in some New York State school districts provides an example of an external commitment device. Teachers who only work during the ten-month school year are given a choice between two payment contracts: (a) they may receive 1/10\(^{th}\) of their yearly salary during each of the months when they teach or (b) they may receive less than 1/10\(^{th}\) of their yearly salary during the first nine months of the year when they teach and the remainder of their salary during the tenth month. The second option, although economically inferior due to foregone interest, is often preferred because it helps teachers ensure that they will have enough money on hand to cover their summer expenses. An explanation of how these different contracts work is available at [http://www.nps.k12.va.us/bf/paycalc/](http://www.nps.k12.va.us/bf/paycalc/)
The first crucial assumption of the mental accounting framework illustrated by this story is that people suffer from self-control problems. There is evidence from a variety of decision-making domains that people exhibit more impatience when making decisions that will take effect in the short run than when making decisions that will take effect in the long run. In our model of mental accounting, we capture this dynamic inconsistency in agents’ preferences by assuming that agents have quasi-hyperbolic time discount functions (Laibson, 1997; O’Donoghue and Rabin, 1999), which lead them to maximize the discounted sum of expected utility flows using the discount function

$$\left\{1, \beta \delta, \beta \delta^2, \beta \delta^3, \ldots \right\}.$$ 

Here, $\delta < 1$ is the standard exponential discount factor, and $\beta << 1$ is an additional factor that uniformly downweights utility flows from all periods beyond the present. An agent with this discount function has a relatively high discount rate in the short run and a relatively low discount rate in the long run.

Another crucial assumption of our mental accounting framework is that people are aware of their self-control problems and willing to take actions that restrict the options of their future selves in order to increase their long-term net utility. The existence of Christmas clubs and addiction treatment centers, which charge people for help with their self-control problems, suggests that people do indeed have a desire for commitment devices. Ashraf et al. (2006) provide further evidence that supports this assumption. In a field experiment, the authors find that people are interested enough in constraining their future selves that many (28.4%) are willing to take up a commitment.

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savings product that restricts their ability to access their savings but offers the same interest rate as other savings accounts without this commitment feature.9

A final assumption on which our theory of mental accounting rests is that people are able to constrain their future selves by creating budgets – a form of goal setting. When people can arrange to have their savings goals enforced by others, by using a commitment savings product for example, this assumption seems reasonable. However, there is also evidence that the mere act of goal setting effectively imposes constraints on people’s future behavior. Locke and Latham (1990) review nearly 400 studies of goal setting, which find overall that when people have specific goals (for example, saving a fixed amount of money each week) they improve their performance at whatever their goal applies to, even when achieving the goal is the only reward they have to look forward to. In our model, agents have the ability to set savings goals and penalize themselves if they fail to meet those goals. Specifically, an agent in our model can set a goal to save \( S \) dollars, and if she fails to reach that goal, she will pay a non-monetary cost that is equivalent to a reduction in her current consumption of \( \kappa \) dollars for every dollar of undersaving relative to \( S \), where \( \kappa \in (0,1) \).10 This cost may represent the psychic cost of guilt for failing to meet a goal or the physical cost of obtaining money that has been previously stored in a deliberately inaccessible place.

Now that we have explained the crucial assumptions of our model, we turn to a formal description of the model’s setup. The model has three periods. For concreteness, we can think of the three periods as unfolding over the course of a week, but the model

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10 We have chosen to model this cost as non-monetary, but it can also be modeled as a monetary cost (or a combination of monetary and non-monetary costs) without altering the main conclusions of the model.
may also apply to other timescales. At \( t = 0 \) (the beginning of the week), the agent chooses her savings target \( S \). At \( t = 1 \) (the first half of the week), the agent’s wealth \( W \) is drawn from the uniform distribution on \([W, \tilde{W}]\), where \( W > 0 \). The agent divides her wealth at \( t = 1 \) between current consumption \( X_1 \) and savings. At \( t = 2 \) (the second half of the week), the agent spends her savings on consumption \( X_2 = S - X_1 \). For simplicity, we set the interest rate to zero, let \( \delta = 1 \), and give the agent logarithmic per-period utility.

The \( t = 0 \) agent’s objective function is the expectation of

\[
\beta \cdot \log(X_1 - \kappa \cdot \max(0, S - X_2)) + \beta \cdot \log(X_2).
\]

Notice that both the \( t = 1 \) and the \( t = 2 \) utility flows from consumption are discounted by the factor \( \beta \), since neither of these flows are experienced in the present moment from the perspective of the \( t = 0 \) agent. The expression \( \kappa \cdot \max(0, S - X_2) \) captures the non-monetary cost incurred by the agent when she saves less than her savings target \( S \).

The \( t = 0 \) agent is sophisticated – she recognizes that at \( t = 1 \) she will maximize the objective function

\[
\log(X_1 - \kappa \cdot \max(0, S - X_2)) + \beta \cdot \log(X_2)
\]

subject to the constraint \( X_1 + X_2 \leq W \). Here, the \( t = 1 \) utility flow from consumption is no longer discounted by the factor \( \beta \). As a matter of terminology, we adopt the convention that \( X_1 \), not \( \left(X_1 - \kappa \cdot \max(0, S - X_2)\right) \), is called “first-period consumption.”

We now describe the outcome of our mental accounting model (see Appendices A and B for a more detailed analysis). First, the \( t = 0 \) agent chooses \( S \) to be in the range

\[
\left[\frac{\beta}{1 + \beta} W, \frac{\beta}{1 + \beta - \kappa} \tilde{W}\right].
\]

Here, the \( t = 0 \) agent is using her savings target as a partial commitment device to influence the spending decisions of her \( t = 1 \) self. Given this choice of \( S \), there
is a range of values for $W$ where the $t = 1$ agent consumes $X_1 = W - S$ and saves exactly $X_2 = S$. Intuitively, the $t = 1$ agent does not want to decrease $X_2$ below $S$ because of the incremental penalty $\kappa$ that would be incurred, yet she does not want to increase $X_2$ because her quasi-hyperbolic time discount function implies that she places a high value on $X_1$. Thus, for the range of values of $W$ where the $t = 1$ agent saves exactly $X_2 = S$, we have $\frac{dx_2}{dw} = 1$. In other words, if $W$ is expected to be in this range, the marginal propensity to consume out of small windfalls is one.

In the context of online grocery shopping, we treat the $10$-off discount coupon as a small windfall. We expect the propensity to consume out of it to be higher than would be predicted by the standard permanent income or lifecycle theory, and we expect some of the incremental expenditure to be devoted to online groceries. Thus, the first hypothesis we test is that a $10$-off coupon will induce customers to increase their online grocery spending.

To generate our second hypothesis, we extend our model to allow for multiple consumption goods, reinterpreting $X_1$ as the number of dollars allocated to a range of goods consumed at $t = 1$ (see Appendix C for a formal description of this extension). If $X_1$ increases because the agent receives a windfall, the agent’s budget set for $t = 1$ consumption expands, and the constraint that consumption of a particular good must be non-negative may cease to bind. Therefore, our second hypothesis is that some of the incremental expenditure induced by a $10$-off discount coupon will be allocated to goods that customers would not consume in the absence of a coupon.

It is important to note that our hypotheses are inconsistent with the standard permanent income or lifecycle theory of consumption. According to the standard model,
a windfall of $10 off a grocery order should have no appreciable effect on the spending decisions of customers. Such a windfall does not create meaningful wealth effects, and it does not create substitution effects since the relative prices of groceries are unchanged.

III. DATA SET AND EMPIRICAL STRATEGY

A. Online Grocery Business Model

The online grocer we collaborated with operates in North America and serves urban customers. Its customers place orders by visiting a website where they may tour virtual supermarket aisles or search for specific products as they make decisions, one by one, about what items to add to their online shopping carts. Returning customers have easy access to the lists of items they purchased on their previous shopping trips to facilitate repeat purchases. Customers can schedule a delivery in the near term or many days in advance. During the period studied, the grocer charged a delivery fee for all orders. In addition, customers were required to spend a minimum dollar amount on each order.

B. Online Grocery Data Set

We obtained a novel panel data set from the aforementioned online grocery company containing information about the orders placed by all of the company’s customers between January 1, 2005 and December 31, 2005. The online grocery company provided a record of each item in each order as well as the price each customer paid for each item, the date of each order, the date of each order’s delivery, and the customer who placed each order. In addition, if a discount coupon was used during an order, we were given information about the type of coupon the customer used and the size of the discount he or she received. If a customer modified his or her order, we were
told how many times order modifications were made, as well as the first and last dates when the customer modified his or her shopping basket. All customer accounts in our data set are labeled by anonymous, unique ID numbers, and all customer ID numbers are accompanied by the date when a customer first placed an online grocery order. Our online grocery collaborator also provided us with detailed information about the items available for purchase through its website, including their category and brand.

We restrict our analysis to customers who made use of a particular $10-off discount coupon sometime between January 1, 2005 and December 31, 2005. New patrons of the online grocer in 2005 were allowed to send one of these coupons to an e-mail address of their choice, excluding their own. We assume that the timing of the receipt of such a coupon is exogenous from the recipient’s point of view, since customers have little if any control over when they will receive this coupon.

In total, between January 1, 2005 and December 31, 2005, there were 2,889 customers who used a $10-off discount coupon of the type described above. We eliminate each customer’s first order of the year, any orders that made use of other kinds of discount coupons, spending outliers (top 1%), outliers in the number of visits made to the grocer’s website during an order (top 1%), and orders by customers who

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11 In our regression analyses, we control for the amount of time that has elapsed since a customer’s previous order. We eliminate each customer’s first order of the year because we are unable to calculate this variable for these observations. By dropping these orders, we eliminate 2,889 data points.
12 We eliminate orders involving all other types of discount coupons for two reasons. First, we are concerned that many of these coupons impose conditions on customers when redeemed that may induce atypical shopping behavior. For example, some coupons expire quickly, some impose a higher than usual minimum spending requirement, and some are only redeemable for certain types of groceries. Second, many of these coupons are not awarded at random but are instead offered to customers when they exhibit certain purchasing patterns. We address potential biases resulting from our exclusion of these coupons when we present our results (see Section IV.C). By dropping these orders, we eliminate 7,736 data points.
13 We eliminate spending outliers and orders involving an unusually large number of visits to the grocer’s website so that these observations do not exert undue influence on the results of our regression analyses. We drop orders that are outliers relative to the entire universe of online grocery orders from 2005, not relative to the data set that only includes customers who redeemed a $10-off coupon in 2005. This
never shopped in 2005 without redeeming a coupon.\footnote{We eliminate orders placed by customers who never shopped in 2005 without redeeming a coupon because such customers may be different from the population of customers who shopped both when in possession of a coupon and when no coupon was available. By dropping these orders, we eliminate 696 data points.} We are left with 34,410 grocery orders, an average of 11.9 per customer. The average dollar size of an order in this sample is $150.23, and the average grocery order consists of 59 items. Of the orders in our data set, 3,110 (approximately 9\%) involve the redemption of a $10-off coupon. For additional summary statistics, see Table 1.

\begin{table}[h]
\centering
\begin{tabular}{lcc}
\hline
 & Mean & Standard Deviation \\
\hline
Spending & 150.23 & 57.47 \\
Number of Groceries & 59.38 & 23.16 \\
Number of Web Visits for Order & 3.88 & 2.86 \\
Days btw First and Last Web Visits for Order & 7.54 & 16.87 \\
Days Since Last Delivery & 17.69 & 21.20 \\
\hline
\end{tabular}
\caption{GROCERY ORDER SUMMARY STATISTICS}
\end{table}

This table reports grocery order summary statistics describing our primary data set.

Throughout the year, a relatively constant proportion of orders placed by the customers in our sample involved the redemption of a $10-off discount coupon. Figure 1 presents a graph over time of the fraction of orders placed that involved the use of such a coupon. Table 2 shows summary statistics about the percentage of a customer’s 2005 orders that involved coupon redemptions. The summary statistics presented in this table suggest that online grocery customers did not find ways to send themselves $10-off discount coupons, as nearly all customers in our data set redeemed just one such coupon in 2005.

\begin{figure}[h]
\centering
\caption{This figure shows the seven-day moving average of the proportion of orders involving $10-off coupon redemptions in our primary data set.}
\end{figure}

procedure eliminates 2,058 data points. Our results do not rely on the elimination of these outliers. In fact, including outliers in the data set strengthens our results considerably.
This table reports coupon use summary statistics from our primary data set. For each customer, we calculate the percentage of orders involving a coupon redemption and the number of orders involving a coupon redemption. We then present the distributions of these statistics across customers (Customers = 2,889, Coupons = 3,110, Orders = 34,410).

C. Regression Specification

To study the effect of coupon redemptions on spending in our online grocery data set, we use the following regression specification:

$$ spending_{it} = \alpha_i + \gamma \cdot \text{coupon\_used}_{it} + \theta X_{it} + \epsilon_{it} $$  

where $spending_{it}$ is the number of dollars spent by customer $i$ for order $t$ or the logarithm of one plus the number of dollars spent by customer $i$ for order $t$, $\alpha_i$ is an
unobserved customer-specific effect, \textit{coupon\_used}_{it} is a dummy variable that takes a value of one when an order involves the redemption of a $10-off coupon and a value of zero otherwise, \( X_{it} \) is a vector of other variables (including interactions of some control variables with \textit{coupon\_used}_{it} ), and \( \varepsilon_{it} \) is the error term. We estimate the equation using a fixed-effects regression and cluster standard errors by customer. Under our assumptions about the timing of coupon receipt, our estimates of the coefficient \( \gamma \) give the effect of coupon redemption on spending.

**IV. RESULTS**

**A. Do Customers Spend More When Redeeming a $10-Off Discount Coupon?**

In Table 3 we present the results of regressions estimating the relationship between the amount a customer spends on groceries and whether or not she redeems a $10-off discount coupon of the type described in Section III.B. In these regressions and in subsequent regressions, the explanatory variables include a coupon redemption dummy, the number of times the customer visited the online grocer’s website in the course of placing an order, the number of days between the first and last visit the customer made to the grocer’s website in the course of placing an order, an interaction between the coupon redemption dummy and the number of website visits during an order, an interaction between the coupon redemption dummy and the days between the first and last visits to the grocer’s website during an order, the number of days since a customer last received a grocery delivery as well as the square and cube of this term, the number of days between when the customer’s order was placed and when it was delivered, the number of orders placed by the customer year to date, dummies for the day of the week when the order was placed, dummies for the day of the week when the order was
delivered, dummies for each week in 2005, and customer fixed effects. The two variables that are interacted with the coupon redemption dummy were normalized before being included in these regressions.

The coefficient estimate on the coupon redemption dummy in regression (2) of Table 3 indicates that holding all else constant, the dollar size of a grocery order increases by approximately 1.3 percent when a customer redeems a $10-off discount coupon. Regression (1) indicates that this effect corresponds to $1.59 in additional spending. The results presented in Table 3 support the hypothesis that customers spend small windfalls when they are obtained rather than dividing their use of this additional wealth over the course of a lifetime. It is worth noting that if the number of trips a customer makes to modify her grocery order online is one standard deviation below its mean value of 3.88, the effect of redeeming a coupon on spending is increased by 1.5 percentage points. This pattern may be due to the fact that the fewer times a customer visits her online grocery basket, the higher the odds are that she makes the majority of her purchasing decisions while accounting for her coupon. However, the coefficient on the interaction between our coupon dummy and the variable indicating how many times a customer returned to her online grocery basket is identified off of the cross section in our data set rather than within person, so this result may simply be due to customer-level heterogeneity in shopping habits that is correlated with heterogeneity in customer responsiveness to coupons.
Table 3
THE EFFECT OF COUPONS ON SPENDING: MAIN RESULTS

<table>
<thead>
<tr>
<th></th>
<th>Spending in Dollars (1)</th>
<th>Log(1+Spending in Dollars) (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coupon Used</strong></td>
<td>1.59**</td>
<td>0.0129***</td>
</tr>
<tr>
<td></td>
<td>(0.76)</td>
<td>(0.0050)</td>
</tr>
<tr>
<td><strong>Number of Web Visits for Order (Standardized)</strong></td>
<td>7.57***</td>
<td>0.0515</td>
</tr>
<tr>
<td></td>
<td>(0.37)</td>
<td>(0.0023)</td>
</tr>
<tr>
<td><strong>Days btw First and Last Web Visits for Order (Standardized)</strong></td>
<td>-2.24***</td>
<td>-0.0164***</td>
</tr>
<tr>
<td></td>
<td>(0.41)</td>
<td>(0.0031)</td>
</tr>
<tr>
<td><strong>Coupon Used x Number Web Visits</strong></td>
<td>-2.13**</td>
<td>-0.0152***</td>
</tr>
<tr>
<td></td>
<td>(0.70)</td>
<td>(0.0044)</td>
</tr>
<tr>
<td><strong>Coupon Used x Days btw First and Last Web Visits</strong></td>
<td>0.62</td>
<td>0.0050</td>
</tr>
<tr>
<td></td>
<td>(0.67)</td>
<td>(0.0047)</td>
</tr>
<tr>
<td><strong>Days Since Last Delivery</strong></td>
<td>0.85***</td>
<td>0.0056***</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.0004)</td>
</tr>
<tr>
<td><strong>(Days Since Last Delivery)</strong></td>
<td>-0.82***</td>
<td>-0.0055***</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td><strong>(Days Since Last Delivery)</strong></td>
<td>0.20***</td>
<td>0.0014**</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td><strong>Days btw Order and Delivery</strong></td>
<td>0.32*</td>
<td>0.0014</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.0012)</td>
</tr>
<tr>
<td><strong>Days Since First Order with Grocer</strong></td>
<td>0.07**</td>
<td>0.0005**</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td><strong>Orders Year to Date</strong></td>
<td>-0.05</td>
<td>-0.0004</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td><strong>Day of the Week Order Placed Dummies</strong></td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Day of the Week Order Delivered Dummies</strong></td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Week of the Year Dummies</strong></td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Customer Fixed Effects</strong></td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>34,410</td>
<td>34,410</td>
</tr>
<tr>
<td><strong>Customers</strong></td>
<td>2,889</td>
<td>2,889</td>
</tr>
<tr>
<td><strong>Coupons</strong></td>
<td>3,110</td>
<td>3,110</td>
</tr>
<tr>
<td><strong>Overall R²</strong></td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Columns (1) and (2) report OLS coefficients from regressions of customer spending and the logarithm of one plus spending on a dummy indicating whether an order involved the redemption of a $10-off discount coupon, controlling for the other variables listed. Standard errors (in parentheses) are clustered by customer. *, **, and *** denote significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

B. Do Customers Increase Their Spending on “Marginal” Goods When Redeeming a $10-Off Coupon?

Our model suggests that when redeeming a $10-off coupon, online grocery shoppers will purchase “marginal” groceries, or items that they would not purchase otherwise. If individuals have heterogeneous preferences, one way to test this hypothesis
empirically is to examine whether people redeeming coupons spend more money than usual on items they never purchased before and will never purchase again in our data set.\textsuperscript{15} In Table 4 we present the results of two regressions estimating the relationship between coupon redemption and the amount a customer spends on groceries that were not included in her other orders. On average, customers spend $39.24 per order on groceries they have not purchased before and will not purchase again in our data set. The coefficient estimate on the coupon redemption dummy in regression (4) of Table 4 indicates that holding all else constant, spending on these groceries increases by approximately 4.9 percent when a customer redeems a $10-off coupon. Regression (3) indicates that this effect corresponds to $1.56 in additional spending on these groceries. These results are consistent with our hypothesis that people purchase “marginal” items when they receive a $10 windfall. It is particularly interesting to note that when a customer redeems a $10-off coupon, $1.56 of her additional $1.59 in overall spending is devoted to groceries that are not included in her other orders.

\textsuperscript{15} When we calculate how much money customers spend during an order on groceries they have not ordered before and will not order again, our data set does not include customers’ first orders of 2005, orders involving the redemption of other coupons, or orders that were eliminated because they were spending or web visit outliers. In creating this “marginal spending” variable, we intend to capture spending on groceries that a customer would not purchase under typical ordering conditions, so our calculations rely only on orders in our trimmed, final data set.
In order to paint a clearer picture of the types of items that absorb the additional $1.59 in grocery spending associated with the redemption a $10-off coupon, we examine how redeeming a coupon affects spending on each of the 112 grocery categories in our data set. Groceries in our data set have all been classified by our online grocer into one
of 112 categories (e.g., Frozen Vegetables, Cream, Cosmetics, Cookies, etc.). We run 112 regressions in which the outcome variable in a given regression is spending on one category of groceries and 112 regressions in which the outcome variable in a given regression is the logarithm of one plus spending on one category of groceries. The primary predictor in all of these regressions is a coupon redemption dummy, and the same controls are included as in regressions (1) through (4). For each set of 112 regressions, Table 5 lists the five categories with the largest coefficient estimates for the coupon redemption dummy and the five categories with the smallest coefficient estimates for the coupon redemption dummy. Casual inspection suggests that the grocery categories with the largest spending increases are relatively luxurious. For example, the three categories of groceries that appear in both top five lists are Produce-Fruits, Produce-Vegetables, and Seafood-Frozen. However, these results are merely suggestive.
Table 5
THE EFFECT OF COUPONS ON SPENDING AT THE GROCERY CATEGORY LEVEL, SORTED BY EFFECT SIZE

<table>
<thead>
<tr>
<th>Category Name</th>
<th>Coefficient on Coupon Use Dummy</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRODUCE-FRUITS</td>
<td>0.29**</td>
<td>0.12</td>
</tr>
<tr>
<td>MEAT-FRESH</td>
<td>0.22</td>
<td>0.19</td>
</tr>
<tr>
<td>PRODUCE-VEGETABLES</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td>SEAFOOD-FROZEN</td>
<td>0.14*</td>
<td>0.07</td>
</tr>
<tr>
<td>MEAT-FROZEN</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>PASTA/GRAINS</td>
<td>-0.11*</td>
<td>0.06</td>
</tr>
<tr>
<td>PAPER</td>
<td>-0.11</td>
<td>0.12</td>
</tr>
<tr>
<td>BABY HEALTH</td>
<td>-0.12</td>
<td>0.09</td>
</tr>
<tr>
<td>HOUSEHOLD CLEANERS</td>
<td>-0.16**</td>
<td>0.06</td>
</tr>
<tr>
<td>BABY FOOD</td>
<td>-0.24***</td>
<td>0.09</td>
</tr>
</tbody>
</table>

** Log(1+Spending) Regressions **

<table>
<thead>
<tr>
<th>Category Name</th>
<th>Coefficient on Coupon Use Dummy</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEAFOOD-FROZEN</td>
<td>0.0324***</td>
<td>0.0137</td>
</tr>
<tr>
<td>PRODUCE-FRUITS</td>
<td>0.0251*</td>
<td>0.0144</td>
</tr>
<tr>
<td>LAUNDRY CARE</td>
<td>0.0246</td>
<td>0.0173</td>
</tr>
<tr>
<td>PRODUCE-VEGETABLES</td>
<td>0.0179</td>
<td>0.0154</td>
</tr>
<tr>
<td>SEAFOOD-FRESH</td>
<td>0.0177</td>
<td>0.0137</td>
</tr>
<tr>
<td>DRIED BREAD</td>
<td>-0.0205</td>
<td>0.0156</td>
</tr>
<tr>
<td>FROZEN SNACKS/APPETIZERS</td>
<td>-0.0218**</td>
<td>0.0110</td>
</tr>
<tr>
<td>PAPER</td>
<td>-0.0226</td>
<td>0.0192</td>
</tr>
<tr>
<td>HOUSEHOLD CLEANERS</td>
<td>-0.0321***</td>
<td>0.0139</td>
</tr>
<tr>
<td>DISH CARE</td>
<td>-0.0325</td>
<td>0.0330</td>
</tr>
</tbody>
</table>

For each grocery category, we performed a regression of customer spending on the category and a regression of the logarithm of one plus customer spending on the category on a dummy indicating whether an order involved the redemption of a $10-off discount coupon, controlling for the other variables listed in Regressions (1) through (4). We then sorted each set of 112 regressions according to the size of the coefficient on the coupon dummy variable. This table reports the top five and bottom five categories from each set of 112 regressions, as well as the associated coupon dummy coefficient estimates and standard errors. Standard errors are clustered by customer. *, **, and *** denote significance at the 10 percent, 5 percent, and 1 percent levels, respectively.
C. Robustness of Results

The first robustness issue we address is a potential feedback problem in our primary regression analyses. We have estimated the effect of coupon redemptions on grocery spending using a regression with customer fixed effects. The consistency of our estimates relies on the “strict exogeneity” assumption – that the error term in equation (1) (see Section III.C) has an expectation of zero conditional on the unobserved, customer-specific effect and the right-hand side variables for all of the customer’s orders. Mathematically, this assumption can be expressed as:

\[ E(\epsilon_i \mid \alpha_i, coupon_{used_i}, \ldots, coupon_{used_{1T}}, X_{i1}, \ldots, X_{iT}) = 0. \]

However, this assumption may be invalid because of feedback effects in some of the variables in \( X_i \). For instance, if customer \( i \) places a large grocery order because of a high realization of \( \epsilon_i \), she may not need to return to the online grocer in the near future. Therefore, \( \epsilon_i \) may be correlated with the \( t + 1 \) values of the variables days since last delivery, days since last delivery squared, days since last delivery cubed, and days since first order with grocer. Under some assumptions, the inconsistency due to the violation of strict exogeneity is less severe for panel data sets with a large time series dimension. Because our data set has a relatively large time series dimension, we have presented fixed effects regression results despite the potential feedback problem. However, we can also conduct our analysis under the less restrictive assumption of “sequential exogeneity”:

\[ E(\epsilon_i \mid \alpha_i, coupon_{used_i}, \ldots, coupon_{used_{1T}}, X_{i1}, \ldots, X_{iT}) = 0. \]

This assumption may hold even in the presence of the feedback effects discussed above. Instead of using a fixed effects regression to estimate equation (1), we estimate the equation in first differences,
\[ \Delta \text{Spending}_{it} = \gamma \cdot \Delta \text{coupon}_\text{used}_{it} + \theta \Delta X_{it} + \Delta \epsilon_{it}, \]  

(2)

using a pooled OLS regression. We use the first lags of the variables with potential feedback problems as instruments for the first differences of these variables, and the standard errors are clustered by customer. The estimates of \( \gamma \) from these first-difference regressions that correspond to the fixed effects regressions (1)-(4) are still statistically significant (although the coefficient corresponding to regression (1) is only significant at the 10% level), and they are slightly larger in magnitude.\(^{16}\)

The second issue we address is the implication of dropping orders from our data set when they involved the redemption of coupons besides the $10-off coupons we are studying. As discussed in Section III.B, many of these other types of coupons could only be redeemed on orders that met certain requirements. For example, one common condition for coupon redemption was that the size of a customer’s order exceed a minimum dollar threshold (the minimum dollar threshold for using such coupons was higher than the threshold that applied to all other orders). The $10-off coupons we are studying had no such elevated minimum spending requirement. In order to avoid confounding the interpretation of our results, our data set does not include any orders involving the redemption of coupons other than the $10-off coupons. Of course, it is possible that eliminating these observations biased our results in favor of supporting the mental accounting hypothesis by removing large orders that did not involve $10-off coupons from our data set. To check the robustness of our results, we restore the orders that involved other types of coupons to our data set, and we treat them as if they were not associated with any type of coupon. When we repeat our analysis of the impact of a $10-

\(^{16}\) Our discussion of the concepts and techniques in this paragraph is derived entirely from Wooldridge (2002).
off coupon on total spending with this altered data set, our main results in Regressions (1) and (2) are actually strengthened, both in terms of statistical significance and effect size.

The third issue we discuss is the implication of the reduced cost of ordering groceries for delivery that is induced by the receipt of a $10-off coupon. Although the $10-off coupon we are studying does not change the relative prices of groceries available from the online grocer, it does reduce the price per order of having groceries delivered, which is a potential concern. Customers may respond to the reduced price per order by increasing the frequency of their orders from the online grocer. Of course, we would expect an increase in ordering frequency to decrease the dollar size of individual grocery orders. If a customer purchases the same total number of groceries but distributes those groceries across more orders, her orders will become smaller. Similarly, if a customer increasingly uses online grocery shopping as a substitute for trips to purchase a few items at, say, a small convenience market, additional online orders are likely to be smaller in size. This potential bias should reduce the likelihood of finding evidence consistent with the mental accounting hypothesis.

D. Alternative Interpretations

The first alternative explanation for our findings that we address is the possibility that there are certain times when a customer is better able to plan her future food consumption and also more likely to redeem a $10-off coupon. When customers are in this “planning mode,” they may have larger grocery orders and longer lags between grocery orders, and they may be more prone to redeem a $10-off coupon. In order to test the plausibility of this explanation, we run two regressions, which are presented in Table 6. In regression (5), the outcome variable is the number of days between the current
online grocery delivery and the previous delivery, and in (6) it is the logarithm of this value. The explanatory variables are an indicator for whether a $10-off coupon was used on the previous grocery order, an indicator for whether a $10-off coupon was used on the current grocery order, and a subset of the control variables from the previous regressions. The coefficient on the indicator for whether a $10-off coupon was used on the previous grocery order is positive but not statistically significant. Thus, coupon redemption appears to result in larger grocery orders without significantly reducing the rate at which customers return to the online grocer for their next order. This result neither confirms nor rules out the proposed alternative explanation. However, in order to be viable, the “planning mode” explanation must also rationalize the evidence that coupon redemption is associated with increased spending on particular types of grocery items. Spending increases are often focused on perishable foods (see Table 5), and it is not clear that planning for the future should increase purchases of foods that are probably intended for relatively immediate consumption.
Modified versions of the permanent income or lifecycle theory provide another potential interpretation of our results. Although our results are inconsistent with the standard theory, adding liquidity constraints to the standard model can give agents a high propensity to consume out of windfalls (Zeldes, 1989; Deaton, 1991; Deaton, 1992). Judging from the demographic characteristics of online grocery shoppers, it does not seem likely that the consumers in our data set are liquidity constrained, but we cannot rule out this possibility or related explanations for our findings.

It is also important to note that although our results are consistent with a model in which consumers engage in mental accounting, there are other behavioral models that
might also predict the spending patterns observed in our data. For example, if people experience a positive emotional response towards the online grocer when they receive a coupon and therefore want to engage in reciprocity, they may substitute away from spending money with the online grocer’s competitors and increase their spending with the online grocer. This explanation could also account for our finding that people seem to buy more “marginal” goods when redeeming a coupon. However, if reciprocity explained our findings, it would seem that the receipt of a coupon should cause spending increases not only on orders associated with a coupon redemption but on future orders as well, and we find no evidence that this is the case. Another potential behavioral explanation for our findings is that the receipt of a coupon simply induces happiness in consumers, which causes them to spend money more freely. While past research on the impact of emotions on spending suggests that sadness increases spending relative to a baseline state (Lerner, Small, and Loewenstein, 2004), the impact of positive emotions on spending is not well understood. Thus, it is not clear whether this explanation, where happiness increases spending, can plausibly account for our findings.

V. CONCLUSION

In this paper, we present evidence indicating that the redemption of a $10-off coupon increases an individual’s spending in the domain of online groceries, as predicted by the mental accounting framework of Thaler and Shefrin (1981). We also find evidence, consistent with our formalization of mental accounting, that the increase in spending stimulated by the redemption of a $10-off coupon is focused on groceries that customers would not purchase in the absence of such a coupon (“marginal” goods).
REFERENCES


APPENDIX A: Analysis of the Model of Mental Accounting

This Appendix describes the outcome of the model of mental accounting outlined in Section II.B. A proof of the claims articulated here is given in Appendix B.

Throughout our analysis, we assume that $\frac{\beta}{1+\beta-\kappa} \overline{W} < \frac{1}{\kappa} \overline{W}$ and $\overline{W} > \frac{1+\beta}{1+\beta-\kappa} \overline{W}$. The first of these two assumptions ensures that the agent’s objective functions are defined for the values of $S$ under consideration.\(^{17}\) The second ensures that the $t=0$ agent cannot choose a value of $S$ such that the $t=1$ agent will save exactly $S$ for all possible realizations of wealth $W$.\(^{18}\)

The $t=0$ agent optimally chooses $S$ to be in one of the following intervals:

\[
\left(\frac{\beta}{1+\beta} \overline{W}, \frac{\beta}{1+\beta} \overline{W} \right), \left(\frac{\beta}{1+\beta-\kappa} \overline{W}, \frac{\beta}{1+\beta} \overline{W} \right), \text{ or } \left[\frac{\beta}{1+\beta} \overline{W}, \frac{\beta}{1+\beta-\kappa} \overline{W} \right],
\]

which we label case 1, case 2, and case 3, respectively. The $t=1$ agent chooses first-period consumption $X_1$ as follows. If $W < \frac{1+\beta-\kappa}{\beta} S$, which occurs in cases 2 and 3, the agent consumes

\[
X_1 = \frac{1}{1+\beta} W - \frac{\rho \kappa}{(1+\beta)/(1-\kappa)} (W - S).
\]

Here, the agent’s saving falls below her budget target $S$, but the agent’s desire to increase first-period consumption $X_1$ is tempered by the cost $\kappa$ of marginal dollars spent on first-period consumption. For $W \in \left[\frac{1+\beta-\kappa}{\beta} S, \frac{1+\beta}{\beta} S \right)$, which occurs in all three cases, consumption is $X_1 = W - S$ and $X_2 = S$. In this range of values for $W$, the agent’s saving is exactly equal to the budget target $S$. Finally, if

\(^{17}\) Without this inequality, some values of $S$ that we discuss would be infeasible in the sense that the agent’s objective functions would require evaluating the logarithm of a non-positive number. The agent would never choose such values of $S$, so this assumption is not essential to the qualitative properties of the model. However, the assumption simplifies the exposition of the model.

\(^{18}\) Although we formally analyze the case where $\overline{W} > \frac{1+\beta}{1+\beta-\kappa} \overline{W}$, the case where $\overline{W} \leq \frac{1+\beta}{1+\beta-\kappa} \overline{W}$ delivers similar predictions.
\( W \geq \frac{1}{1+\beta} S \), which occurs in cases 1 and 2, the agent chooses \( X_1 = \frac{1}{1+\beta} W \). For these values of \( W \), the agent’s saving exceeds the budget target \( S \).

Thus, when \( W \in \left[ \frac{1}{1+\beta} - S, \frac{1}{1+\beta} S \right] \), the agent’s marginal propensity to consume out of wealth is \( \frac{dX_1}{dW} = 1 \), while the marginal propensity to consume is \( \frac{dX_1}{dW} = \frac{1}{1+\beta} < 1 \) when \( W \geq \frac{1}{1+\beta} S \). This implies that if the agent expects wealth \( W \) to be in the interval \( \left[ \frac{1}{1+\beta} - S, \frac{1}{1+\beta} S \right] \), her propensity to consume out of positive shocks to wealth is decreasing in the size of the shock, consistent with empirical evidence (Landsberger, 1966), and her propensity to consume out of small shocks equals one.
APPENDIX B: Proof of Claims Regarding the Model with One Consumption Good

We first solve the problem of the \( t = 1 \) agent, given the budget target \( S \) chosen by the \( t = 0 \) agent. We substitute \( W - X \) for \( X \), and note that the \( t = 1 \) agent’s problem is concave in \( X \). Now, we find the conditions under which the \( t = 1 \) agent will elect to save strictly less than the budget target \( S \). If she were to choose \( X_1 = W - S \) and \( X_2 = S \), the net marginal benefit from increasing \( X_1 \) and decreasing \( X_2 \) would be \( \frac{1 - \kappa}{W - S} - \frac{\beta}{S} \), which is positive for \( W < \frac{1 + \beta - \kappa}{\beta} S \). For these values of \( W \), the first-order condition gives

\[
X_1 = \frac{1 - \kappa}{1 + \beta} W - \frac{\beta \kappa}{(1 + \beta)(1 - \kappa)} (W - S)
\]

and therefore

\[
X_2 = \frac{\beta}{1 + \beta} W + \frac{\beta \kappa}{(1 + \beta)(1 - \kappa)} (W - S).
\]

Next, we find the conditions under which the \( t = 1 \) agent will elect to save strictly more than the budget target \( S \). If she were to choose \( X_1 = W - S \) and \( X_2 = S \), the net marginal benefit from decreasing \( X_1 \) and increasing \( X_2 \) would be \( \frac{S W \beta}{\kappa} - \frac{1}{S} \), which is positive for \( W > \frac{1 + \beta}{\beta} B \). For these values of \( W \), the first-order condition gives

\[
X_1 = \frac{1 + \kappa}{1 + \beta} W
\]

and therefore

\[
X_2 = \frac{\beta}{1 + \beta} W. \quad \text{For } W \in \left[\frac{1 + \beta - \kappa}{\beta} S, \frac{1 + \beta}{\beta} S\right], \text{we have } X_1 = W - S \text{ and } X_2 = S.
\]

We now solve the problem of the \( t = 0 \) agent, who anticipates the behavior of the \( t = 1 \) agent. The \( t = 0 \) agent will choose \( S \) in one of the three intervals \( \left[\frac{\beta}{1 + \beta} W, \frac{\beta(1 - \kappa)}{1 + \beta - \kappa} W\right], \left[\frac{\beta}{1 + \beta - \kappa} W, \frac{\beta}{1 + \beta} W\right], \text{ or } \left[\frac{\beta}{1 + \beta} W, \frac{\beta(1 - \kappa)}{1 + \beta - \kappa} W\right] \). To see this, note that choosing \( S < \frac{\beta}{1 + \beta} W \) is equivalent to choosing \( S = \frac{\beta}{1 + \beta} W \), since it results in the same consumption choices in all states of nature. Furthermore, we will see below that choosing \( S = \frac{\beta}{1 + \beta} W \) is strictly suboptimal, so we can disregard the possibility of choosing \( S < \frac{\beta}{1 + \beta} W \). Also note that choosing \( S > \frac{\beta}{1 + \beta - \kappa} \frac{W}{\kappa} \) results in the same values of \( X_1 \) and \( X_2 \) as choosing \( S = \frac{\beta}{1 + \beta - \kappa} W \). In addition, the choice \( S > \frac{\beta}{1 + \beta - \kappa} W \) is strictly dominated by the choice \( S = \frac{\beta}{1 + \beta - \kappa} W \) because it increases the amount by which saving falls below the budget target, thereby imposing a cost that is equivalent to a reduction in first-period consumption at a rate of \( \kappa \).

Define \( U(S) \) to be the \( t = 0 \) agent’s objective as a function of \( S \). The function \( U \) is continuous, differentiable, and piecewise twice-differentiable. We analyze the function \( U \) for values of \( S \) in each of the three intervals given above.

For \( S \in \left[\frac{\beta}{1 + \beta} W, \frac{\beta(1 - \kappa)}{1 + \beta - \kappa} W\right], \) we have (ignoring the factor \( \frac{\beta}{1 + \beta - \kappa} W \))

\[
U(S) = \int_{\frac{\beta}{1 + \beta} W}^{\frac{\beta(1 - \kappa)}{1 + \beta - \kappa} W} \left[ \log(W - S) + \log(S) \right] dW + \int_{\frac{\beta}{1 + \beta} W}^{\frac{\beta}{1 + \beta - \kappa} W} \left[ \log\left(\frac{1}{1 + \beta} W\right) + \log\left(\frac{\beta}{1 + \beta} W\right) \right] dW,
\]

\[
\frac{dU}{ds} = -\left[ \log\left(\frac{1}{1 + \beta} S\right) - \log(W - S) \right] + \frac{1 + \beta}{\beta} S - \frac{W}{S}, \quad \text{and}
\]

\[
\frac{d^2U}{ds^2} = -\frac{1}{S} - \frac{1}{W - S} + \frac{W}{S^2}.
\]
Note that \( \frac{d^2U}{ds^2} \geq 0 \) for \( S \leq \frac{1}{2}W \), and that \( \frac{d^2U}{ds^2} < 0 \) for \( S > \frac{1}{2}W \). We have \( \frac{d^2U}{ds^2} |_{S=\frac{\beta}{1+\beta-k}W} > 0 \) and \( \frac{dU}{ds} |_{S=\frac{\beta}{1+\beta-k}W} = 0 \), so the optimal \( S \) is greater than \( \frac{\beta}{1+\beta-k}W \).

For \( S \in \left[ \frac{\beta}{1+\beta-k}W, \frac{\beta}{1+\beta-k}W \right] \), we have (ignoring the factor \( \frac{\beta}{W-W} \))

\[
U(S) = \int \left[ \log \left( \frac{1}{W}W + \frac{k}{1+\beta} (W - S) \right) + \log \left( \frac{\beta}{(1+\beta)(1-k)}W - \frac{\beta}{(1+\beta)(1-k)} S \right) \right] dW \\
+ \int \left[ \log(W-S) + \log(S) \right] dW + \int \left[ \log \left( \frac{1}{1+\beta}W \right) + \log \left( \frac{\beta}{1+\beta} W \right) \right] dW,
\]

\[
\frac{dU}{ds} = -2\kappa \left[ \log \left( \frac{1+\beta}{\beta} (1-k) W \right) - \log \left( \frac{1}{k} W - S \right) \right] + \log(1-k) + \frac{k}{\beta} \quad \text{and} \quad \frac{d^2U}{ds^2} = 0.
\]

Finally, for \( S \in \left[ \frac{\beta}{1+\beta-k}W, \frac{\beta}{1+\beta-k}W \right] \), we have (ignoring the factor \( \frac{\beta}{W-W} \))

\[
U(S) = \int \left[ \log \left( \frac{1}{W}W + \frac{k}{1+\beta} (W - S) \right) + \log \left( \frac{\beta}{(1+\beta)(1-k)}W - \frac{\beta}{(1+\beta)(1-k)} S \right) \right] dW \\
+ \int \left[ \log(W-S) + \log(S) \right] dW,
\]

\[
\frac{dU}{ds} = -2\kappa \left[ \log \left( \frac{1+\beta}{\beta} (1-k) W \right) - \log \left( \frac{1}{k} W - S \right) \right] - \left[ \log \left( \frac{W}{W-S} \right) - \log \left( \frac{1}{k} W - S \right) \right] + \frac{W-S}{W} - \frac{1+\beta-k}{\beta}, \quad \text{and} \quad \frac{d^2U}{ds^2} = \frac{1}{S} \left( 1 - 2\kappa - \frac{W-S}{W} \right) + \frac{1}{W-S} - \frac{W}{W-S}.
\]

It can be verified that \( \frac{d^2U}{ds^2} < 0 \).

To complete our analysis of the function \( U \), we evaluate its first derivative at the upper endpoints of the three intervals.

\[
\left. \frac{dU}{ds} \right|_{S=\frac{\beta}{1+\beta-k}W} = \log(1-k) + \frac{k}{\beta}
\]

\[
\left. \frac{dU}{ds} \right|_{S=\frac{\beta}{1+\beta-k}W} = -2\kappa \left[ \log \left( \frac{1+\beta}{\beta} W \right) - \log \left( \frac{1}{k} W - \frac{\beta}{1+\beta} W \right) \right] + \log(1-k) + \frac{k}{\beta}
\]

\[
\left. \frac{dU}{ds} \right|_{S=\frac{\beta}{1+\beta-k}W} = -2\kappa \left[ \log \left( \frac{1+\beta}{\beta} (1-k) W \right) - \log \left( \frac{1}{k} W - \frac{\beta}{1+\beta} W \right) \right] < 0
\]

We conclude that when \( \log(1-k) + \frac{k}{\beta} < 0 \), the optimal \( S \) is in the interval \( \left[ \frac{\beta}{1+\beta-k}W, \frac{\beta}{1+\beta-k}W \right] \). When \( \log(1-k) + \frac{k}{\beta} \geq 0 \) and \( -2\kappa \left[ \log \left( \frac{1+\beta}{\beta} W \right) - \log \left( \frac{1}{k} W - \frac{\beta}{1+\beta} W \right) \right] + \log(1-k) + \frac{k}{\beta} < 0 \), the optimal \( S \) is in the interval \( \left[ \frac{\beta}{1+\beta-k}W, \frac{\beta}{1+\beta-k}W \right] \). And when \( -2\kappa \left[ \log \left( \frac{1+\beta}{\beta} W \right) - \log \left( \frac{1}{k} W - \frac{\beta}{1+\beta} W \right) \right] + \log(1-k) + \frac{k}{\beta} \geq 0 \), the optimal \( S \) is in the interval \( \left[ \frac{\beta}{1+\beta-k}W, \frac{\beta}{1+\beta-k}W \right] \).
APPENDIX C: Extension of the Model to the Case of Multiple Goods

In this Appendix, we extend the model of mental accounting outlined in Section II.B so that the agent chooses among multiple consumption goods. Otherwise, the setup of the model is unchanged. The $t = 0$ agent’s objective function is the expectation of

$$\beta \cdot \log(G(Z_{11}, Z_{21}, \ldots, Z_{j1}) - \kappa \cdot \max\left(0, S - \sum_{j=1}^{J} P_j Z_{j2}\right)) + \beta \cdot \log(G(Z_{12}, Z_{22}, \ldots, Z_{j2})).$$

The $t = 1$ agent chooses $Z_{11}, Z_{21}, \ldots, Z_{j1}$ and has the objective function

$$\log\left(G(Z_{11}, Z_{21}, \ldots, Z_{j1}) - \kappa \cdot \max\left(0, S - \sum_{j=1}^{J} P_j Z_{j2}\right)\right) + \beta \cdot \log(G(Z_{12}, Z_{22}, \ldots, Z_{j2})).$$

Finally, the $t = 2$ agent chooses $Z_{12}, Z_{22}, \ldots, Z_{j2}$ and has the objective function

$$\log(G(Z_{12}, Z_{22}, \ldots, Z_{j2})).$$

The constraints are $\sum_{j=1}^{J} P_j Z_{j1} + \sum_{j=1}^{J} P_j Z_{j2} \leq W$ and $Z_{jk} \geq 0$ for $j = 1, \ldots, J$ and $k = 1, 2$. Here, $P_j > 0$ represents the price of good $j$, and $Z_{jk}$ represents the amount of good $j$ consumed at time $k$. We assume the function $G$ is continuously differentiable, strictly positive, strictly increasing, and strictly quasiconcave.

Solving backwards, the $t = 2$ agent maximizes $G(Z_{12}, Z_{22}, \ldots, Z_{j2})$ subject to the constraints $\sum_{j=1}^{J} P_j Z_{j2} \leq X_2$ and $Z_{j2} \geq 0$ for $j = 1, \ldots, J$. Here, $X_2 = W - \sum_{j=1}^{J} P_j Z_{j1}$ is the number of dollars that the $t = 1$ agent allocates for second period expenditure. Let $F(X_2)$ denote the value function of this maximization problem.\(^\text{19}\) We assume that $F$ is a concave function of $X_2$. Then the $t = 1$ agent’s problem is to choose $Z_{11}, Z_{21}, \ldots, Z_{j1}$ and $X_2$ to maximize

$$\log(G(Z_{11}, Z_{21}, \ldots, Z_{j1}) - \kappa \cdot \max(0, S - X_2)) + \beta \cdot \log(F(X_2))$$

\(^{19}\)Our notation suppresses the prices as arguments of the value function.
subject to \( \sum_{j=1}^{J} P_j Z_{j1} + X_2 \leq W \) and \( Z_{j1} \geq 0 \) for \( j = 1, \ldots, J \). Inspection of the first order conditions with respect to \( Z_{11}, Z_{21}, \ldots, Z_{j1} \) reveals that the \( t = 1 \) agent’s problem can be rewritten as a two-stage problem. First, the \( t = 1 \) agent chooses period one and two expenditure levels \( X_1 \) and \( X_2 \) to maximize

\[
\log(F(X_1) - \kappa \cdot \max(0, S - X_2)) + \beta \cdot \log(F(X_2))
\]

subject to \( X_1 + X_2 \leq W \). Second, the \( t = 1 \) agent chooses \( Z_{11}, Z_{21}, \ldots, Z_{j1} \) to maximize

\[
G(Z_{11}, Z_{21}, \ldots, Z_{j1}) \text{ subject to } \sum_{j=1}^{J} P_j Z_{j1} \leq X_1 \text{ and } Z_{j1} \geq 0 \text{ for } j = 1, \ldots, J.
\]

The first-stage problem is similar to the \( t = 1 \) agent’s problem in the single-good model, and many properties carry over. In particular, there is a range of values for \( W \) where the marginal propensity to consume out of incremental wealth is one. The second-stage problem determines how incremental first-period expenditure is allocated among goods. As \( X_1 \) increases, there may be some goods for which the constraint \( Z_{j1} \geq 0 \) ceases to bind. In this case, the \( t = 1 \) agent’s incremental consumption bundle includes goods she would not otherwise consume.

Many discussions of mental accounting argue that people create mental accounts for different types of goods in addition to mental accounts for different time periods. For example, people may create an account for housing expenses and an account for entertainment expenses. Our model lacks this feature, but we speculate that a further variant on our model could incorporate it. If consumption of a good at \( t = 1 \) generates utility flows both at \( t = 1 \) and at \( t = 2 \), the \( t = 1 \) agent’s allocation of first-period expenditure among goods may be suboptimal from the perspective of the \( t = 0 \) agent, giving the \( t = 0 \) agent a rationale to set budget targets for different types of goods.