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Abstract: A modified Mundell-Fleming-Dornbusch model that allows analysis of the effect of changes in long-run output expectations on the exchange rate is used to simulate the evolution of the euro-dollar exchange rate. This is done under two assumptions regarding monetary policy: constant-money-supply policy and constant-price policy. We show the dynamics that follow a change in long-run output expectations under both scenarios. Actual data is used to calculate the change in expectations that is needed to match the euro-dollar experience. The quantitative results show that a small change in long-run output expectations is enough to generate the recent euro-dollar dynamics. (JEL F31, F41)

Keywords: euro, dollar, exchange rates, expectations.

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1. Introduction

From its launch in January 1999 to December 2000, the euro lost about 25 percent of its value against the US dollar. This is an interesting phenomenon not only because it is a large depreciation but also because the European currency was, in fact, expected to appreciate with respect to the dollar.

We can compare this recent episode to the January 1992 - December 1993 experience in Europe. During that period three European currencies, the British pound, the Italian lira and the Spanish peseta, suffered several speculative attacks and experienced a loss of value with respect to the German Mark of 11%, 27% and 26%, respectively. This precipitated the breakdown of the European Monetary System\(^1\) (EMS), the UK and Italy abandoned the EMS and the system had to be redesigned in order to avoid further attacks on the Spanish peseta and other European currencies. Although the 92-93 episode was labeled as a crisis, we do not think that the present euro-depreciation is one. But its loss of value is as large as it was for the troubled currencies of the EMU and that makes the current euro-dollar experience a crisis-like situation worth looking at.

What makes the early euro-dollar experience even more interesting is the fact that it was completely unexpected. The Bank of International Settlements wrote in their 70\(^{th}\) Annual Report: “the Euro’s weakness throughout the period confounded earlier general expectations that it would trend upwards.” For the IMF\(^2\), the Euro’s performance “defied market expectations”.

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\(^1\) The EMS was a semi-floating exchange regime characterized by an upper and a lower bound, the flotation bands.

According to surveys conducted by Consensus Forecasts, at its birth in January 1999, the Euro was expected to appreciate about 4.5 percent with respect to the dollar. This error in expectations is well documented by Corsetti (2000). A quick summary of his argument is that exchange rate expectations were off target because growth rate expectations were wrong. The US economy was not expected to grow as fast as it did. In particular, Consensus Forecast projected 2.3% and 3.4% growth rates in 1999 and 2000, respectively. *The Economist* forecasted 2.0% in 1999 and 3.4% in the year 2000. Actually, the US economy grew by 4.2% in 1999 and 5% in 2000.

On the other hand, Consensus Forecast expected the Euro-area to grow by 2.3% and 2.9% during the same period; *The Economist* forecasted growth rates of 2.2% and 3%. In reality the Euro-area’s growth rates were 2.4% in 1999 and 3.3% in 2000.

Corsetti suggests that the higher-than-expected US growth not only explains why exchange rates expectations were mistaken but also why the Euro depreciated so much. However, he is not completely convinced about this last argument and finally remarks, “it is hard to provide a convincing interpretation of the recent evolution of the Euro”.

Corsetti also stresses an interpretation of the appreciation of the dollar based on the evolution of domestic demand in the US. According to him, “as internal demand grows faster than supply, the real exchange needs to appreciate […]. At the same time, the current account of the booming country goes into a deficit”.

Our goal is to follow up on Corsetti’s work and to find out how convincing the simplest model with exchange rate dynamics can be. To the best of our knowledge, there has been no article that tries to simulate the euro-dollar experience using an exchange rate model and actual data. We think that this is an interesting exercise not only because it

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3 These were the one-year exchange rate expectations from Consensus Forecasts in January 1999.
provides a formal explanation for this somewhat extraordinary evolution of these two currencies, but also because it provides a counterpart to another view, supported by Paul Krugman⁴ and Willem Duisenberg⁵, President of the European Central Bank (ECB), which argues that the euro-dollar experience is a case of market irrationality and that the evolution of the euro did not reflect the values of the fundamentals (growth, inflation and interest rates).

Simply put, the question that we want to answer is: can we generate the appreciation dynamics of the dollar with respect to the euro using the Mundell-Fleming-Dornbusch (MFD) model of exchange rates and a shock in long-run output expectations? And, if so, how large does this shock have to be to generate these dynamics?

It has been argued that the shock in long-run output expectations during this period is the result of unprecedented improvements in Information Technology. Alan Greenspan, chairman of the Federal Reserve Board, has expressed in several occasions that “although it still is possible to argue that the evident increase in productivity growth is ephemeral, I find such arguments hard to believe, and I suspect that most in this audience would agree.”⁶ So, the sometimes referred as “New Economy” has generated an increase in productivity that goes beyond the short run. This permanent productivity change with implications for long-run output has become particularly obvious by the end of the century. In Alan Greenspan’s own words, “four or five years into this expansion, in the middle of the 1990’s, it was unclear whether, going forward, this cycle would differ significantly from many others that have characterized post-World War II America. More

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recently, however, it has become increasingly difficult to deny that something profoundly different from the typical postwar business cycle has emerged.\textsuperscript{7}

Our quantitative results show that a small change in long-run output expectations is enough to generate the recent euro-dollar dynamics.

The paper is structured as follows. The next section presents a version of the MFD model that allows analysis of the effect of changes in long-run output expectations on the exchange rate. This is done under two assumptions regarding monetary policy: constant-money-supply policy and constant-price policy. Once the model is developed, we show the dynamics that follow a change in long-run output expectations under both scenarios. We then proceed to perform a simulation of these dynamics using parameters estimated from actual data to calculate the change in expectations that is needed to match the euro-dollar experience. The last section summarizes our conclusions.

2. Mundell-Fleming-Dornbusch Model

In this section we develop a version of the MFD model that allows us to analyze output and exchange rate dynamics after a change in long-run output expectations. The choice of model is based on tractability. Although the MFD model has no micro foundations, there are micro-founded versions that generate similar results\textsuperscript{8}.

First we present the general framework. The system has two state (i.e., non-jumping) variables, output and the price level. The real exchange rate is the control (i.e., jumping) variable, and the nominal exchange rate, which can also jump, is a function of

\textsuperscript{7} Remarks by Chairman Alan Greenspan, January 13, 2000.
\textsuperscript{8} See Obstfeld and Rogoff (1995).
the previous three variables. We derive the law of motion for output, real exchange rate and nominal exchange rate and their steady state values. Then we proceed to analyze the effects of a change in long-run output expectations under two different assumptions regarding monetary policy: a constant-money-supply policy and a constant-price policy. Finally, we discuss the main differences between the two cases.

Our framework follows closely Obstfeld and Rogoff’s (1995) analysis of the Dornbusch (1976) model. The model considers a small open economy that faces an exogenous world interest rate \( i^* \), which is assumed constant. The domestic monetary equilibrium is given by a standard LM curve, \( M_t^d \) / \( P_t = f(Y_t, i_t) \), of the following form:

\[
\begin{align*}
\Delta m_t - \Delta p_t &= -\eta_i + \phi y_t \\

\end{align*}
\]

where \( m_t \equiv \log M_t \) is the log of the nominal money supply, \( p_t \equiv \log P_t \) is the log of the domestic price level and \( y_t \equiv \log Y_t \) is the log of domestic output. Equation (1) establishes the traditional inverse relation between money demand and the interest rate, and the positive relation between money demand and output\(^9\). The parameter \( \eta \) is the interest rate elasticity of money demand and \( \phi \) is the income elasticity of money demand.

Under the assumptions of perfect foresight, open capital markets and perfect substitutability between foreign and domestic assets, uncovered interest parity must hold:

\[
\begin{align*}
i_t &= i^*_t + \epsilon_{t+1} - \epsilon_t \\

\end{align*}
\]

\(^9\) A micro-founded money demand function can be derived using money in the utility function or money in advance assumptions as in Stockman (1980) and Lucas (1982), respectively.
where $i_t = \log(1 + i_t)$ is the domestic interest rate; $i^*_t = \log(1 + i^*_{t+1})$ is the world interest rate, and $e_t = \log E_t$ is the log of the nominal exchange rate. Equation (2) states that, given a positive (negative) difference in the interest rate, people must expect an appreciation (depreciation) to hold assets from both countries.

Let $y_t^d$ denote home output demand. To clear the goods market, we follow the standard assumption that output is demand-determined. In addition, we assume that home output is an increasing function of the real exchange rate,

$$y_t^d = \bar{y} + \delta(e_t + p^*_t - p_t - \bar{q}), \quad \delta > 0 \tag{3}$$

where $\bar{y}$ is the expectation of the long-run level of output and $\delta$ the elasticity of output demand to real depreciation. If we define $q$ as the real exchange rate,

$$q_t \equiv e_t + p^*_t - p_t \tag{4}$$

then $\bar{q}$ represents the equilibrium exchange rate consistent with full employment.

As expressed in Obstfeld and Rogoff (1996), there are different ways to justify the inclusion of the real exchange rate in equation (3). The most intuitive one is that a real depreciation stimulates foreign demand and, hence, raises output. A similar way to

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10 Equation (2) is derived from \((1+ i_t) = (1+ i^*_t) \exp[E_{t+1}] / E_t\) (2). Taking logs, (2) would be approximately equal to $i_t = i^*_t + \exp[e_{t+1}] - e_t$, the difference accounted by Jensen’s inequality: $\log(\exp[e_{t+1}]) > \exp[\log(E_{t+1})]$.  
11 Obstfeld and Rogoff’s (1995) monopolistic competition model provides micro foundations for these assumptions.  
12 See Obstfeld and Rogoff (1996) for a discussion on this assumption and Obstfeld and Rogoff (2000) for a discussion on the evidence.
interpret equation (3) is that output is determined by domestic and foreign demand and that both are stimulated by potential output. In addition, foreign demand is an increasing function of the real exchange rate.

The original Dornbusch (1976) model has not generally been used to explore the effects of changes in expected long-run output $\overline{y}$. Since our main motivation is precisely to understand how the exchange rate responds to changes in growth expectations, output dynamics become a relevant part of our analysis. To capture these dynamics we follow Blanchard’s (1981) specification of output adjustment over time, which states that output converges to output demand according to:

$$\Delta y_{t+1} = \gamma \left( y^d_t - y_t \right), 0 \leq \gamma \leq 1$$

(5)

where the parameter $\gamma$ measures the response of output to deviations from domestic demand.

By substituting $y^d_t$, equation (3), in expression (5), we rewrite output dynamics as:

$$\Delta y_{t+1} = -\gamma (y_t - \overline{y}) + \gamma \delta \left( e_t + p_t^* - p_t - \overline{q} \right)$$

(6)

where output grows whenever demand is stimulated, either by an increase in long-run output expectations or by a real depreciation.

Additionally, we assume that output cannot immediately jump in response to an expectations update. Instead, it starts to grow to its new equilibrium. In other words, in
the short run output is fixed. The intuition behind this assumption is that in the short run, due to investment costs or time-to-build, output cannot react instantaneously.

Notice that, under this assumption, an increase in long-run output expectations, \( \bar{y}' > \bar{y} \), generates a crowding out of foreign demand due to an increase in domestic demand.

To capture the stylized fact that prices tend to adjust more slowly than exchange rates, the model assumes that the price level \( p \) is fixed in the short run and adjusts slowly to changes in excess demand or supply. Price dynamics follow an inflation-expectations-augmented Phillips curve:

\[
p_{t+1} - p_t = \psi \left( y^d_t - \bar{y} \right) + \hat{p}_{t+1} - \hat{p}_t
\]

where \( \hat{p}_t \) is the price level that would have prevailed if markets had cleared, in other words, the equilibrium price level,

\[
\hat{p}_t = e_t + p_t^* - \bar{q}
\]

The first term in the right-hand side of equation (7) is the effect on inflation of deviations from long-run output or excess demand and the parameter \( \psi \) is the inverse of the sacrifice ratio. The second part of equation (7) accounts for inflation expectations due to “fundamentals”. This last term is the inflation “needed to keep up with expected inflation or productivity growth; that is, the movement in prices that would be needed to

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13 Obstfeld and Rogoff (1996), pages 607 and 611.
keep \( y = \bar{y} \) if the market output were in equilibrium.\(^{14}\) Simply put, this last term is the expected rate of inflation in the expectations-augmented Phillips curve.

Substituting (8) in (7) and imposing the assumptions that the foreign price level \( p^* \) and the equilibrium real exchange rate \( \bar{q} \) are constant, we obtain

\[
p_{t+1} - p_t = \psi \left( y^*_t - y \right) + e_{t+1} - e_t
\]

Equation (9) has the same intuition as equation (7), but in (9) expected inflation is captured by expected depreciation since it leads to higher prices for tradable goods and thus higher inflation.

To solve the model, we combine equations (4), (9) and the simplifying assumption \( p^* = p^*_{t+1} = 0 \) to obtain the law of motion for the real exchange rate \( q \).

\[
\Delta q_{t+1} = q_{t+1} - q_t = -\psi \left( y_t - \bar{y} \right)
\]

The intuition behind equation (10) is that the real exchange rate converges to its steady state level as output reaches its own equilibrium. In other words, as output grows there is less crowding out of foreign demand so the real exchange rate depreciates.

Plugging (2) and (4) into (1) and assuming \( i^* = 0 \) we obtain the law of motion for the nominal exchange rate,

\(^{14}\) See Mussa (1984) for further discussion.
As can be seen in equation (11), the dynamic behavior of the nominal exchange rate is quite complicated. It depends on the current nominal exchange rate, the current real exchange rate and the current level of output. The intuition behind equation (11) is easier to see when we compute the steady-state nominal exchange rate and, later on, when we use it to derive its saddle-path under our two different policy assumptions.

The conditions $\Delta q_{t+1} = 0$ and $\Delta e_{t+1} = 0$, and $\Delta y_{t+1} = 0$ determine the steady state values of output, real and nominal exchange rates:

$$y = \bar{y}$$  \hspace{1cm}  (6')

$$q = \bar{q}$$  \hspace{1cm}  (10')

$$e = m + \bar{q} - \phi \bar{y}$$  \hspace{1cm}  (11')

In the steady state, both output and the real exchange rate are equal to their long-run levels $\bar{y}$ and $\bar{q}$.

Whether the nominal exchange rate returns to its initial value depends on our assumption regarding monetary policy. Under a constant-money-supply policy, the exchange rate stays permanently appreciated after an increase in long-run output. Conversely, other things equal, increases in the money supply generate a permanent depreciation. The following sections analyze these points in detail.
2.a. Unexpected Increase in Output under a Constant-Money-Supply Policy.

We first consider a change in long-run output expectations from $\bar{y}$ to $\bar{y}'$ under a constant-money-supply policy. In the new steady state, the nominal exchange rate appreciates and prices fall. The equilibrium real exchange rate, which is exogenous in this model, does not change. So, the new steady state is characterized by:

$$
y = \bar{y}'$$

$$
q = \bar{q}
$$

$$
e = m + \bar{q} - \phi \bar{y}'
$$

$$
p = m - \phi \bar{y}'.
$$

Combining the law of motion for the real exchange rate $q_t$, and the nominal exchange rate $e_t$, equations (10) and (11) respectively, the law of motion of output, equation (6), and the constant-money-supply assumption, we obtain the saddle-path equation for the nominal exchange rate $e_t$. Appendix 1 provides a detailed derivation of this result.

$$
e_t = m - \phi \bar{y} + \bar{q} + \frac{(1 - \phi \delta)}{1 + \psi \eta} (q_t - \bar{q}) + \frac{1}{(1 + \eta)} \frac{(\phi + \psi \eta)}{1 + \psi \eta} \sum_{s=1}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^{s-1} (\phi \Delta y_{t+s})
$$

Notice that, for a given path of output, the evolution of the nominal exchange rate depends on $(1 - \phi \delta)$. If $(1 - \phi \delta) > 0$, the exchange rate overshoots its long-run level in
response to changes in the money supply or the expected level of long-run output. The first three terms in the right-hand side of equation (12) are the equilibrium level of the nominal exchange rate. The last two terms measure the extent of the overshooting, for $(1-\phi\delta) > 0$, and are zero in equilibrium.

For $(1 - \phi\delta) > 0$, the initial appreciation of the home currency is followed by a depreciation to a new equilibrium nominal exchange rate that is lower than its initial value, so there is a permanent appreciation of the home currency.

Given the constant-money-supply assumption, the equilibrium in the money market is reached through a change in the price level. That is, given that higher output increases money demand and that the equilibrium interest rate is determined by the exogenous world interest rate, constant money supply requires the price level to fall. Consequently, real money balances increase to offset the rise in money demand. This allows the money market to equilibrate at the initial interest rate. Hence, the new equilibrium is characterized by a lower (i.e., appreciated) nominal exchange rate, a lower price level, the same real exchange rate, the same interest rate and the same money supply.

2.b. Unexpected Increase in Output Under a Constant-Price Policy.

The constant-money-supply policy assumption may seem unreasonable given the typical central banker’s concerns about price stability. It is more realistic to assume that the central bank is concerned about inflation rather than keeping the money supply
constant. So we now consider the exercise of an unanticipated change in long-run output expectations from $\bar{y}$ to $\bar{y}'$ under a constant-price monetary policy.

Let us recall our initial equations for the money market equilibrium, uncovered interest parity (with $i^*_t = 0$), and the expectations-augmented Phillips curve:

\[
m_t - p_t = -\eta i_t + \phi y_t \quad (1)
\]
\[
i_t = e_{t+1} - e_t \quad (2')
\]
\[
p_{t+1} - p_t = \psi\left(y_t - \bar{y}\right) + e_{t+1} - e_t \quad (9)
\]

If we combine (2’) and (9) and we assume that the monetary authority keeps prices constant, then the interest rate is set according to:

\[
i_t = -\psi(y_t - \bar{y}) \quad (13)
\]

Equation (13) shows an inverse relation between deviations from potential output and the interest rate. As output is expected to increase, there is downward pressure on prices. To prevent prices from falling the central bank wants to create a path of decreasing interest rates. However, to accomplish this path, the central bank has to tighten monetary policy at the time of the expectations update. Generally speaking, by increasing the interest rate today, the central bank is “buying” interest rate cuts to offset the downward pressure on prices in the future.
We can also express this policy in terms of money supply. Substituting (13) into (1), money supply is an increasing function of current output and a decreasing function of future output expectations:

\[ m_t = (\eta \psi + \phi) y_t - \eta \psi \bar{y} \quad (1') \]

Once again, the intuition is that, by cutting money supply at the time of the expectations update, the central bank is “buying” loose monetary policy for the future so it can keep prices constant.

The law of motion for the nominal exchange rate is obtained by substituting (1’) and (6) into (11):

\[ \Delta e_{t+1} = \frac{e_t}{\eta} - \frac{(1-\phi \delta) q_t}{\eta} - \frac{(\eta \psi + \phi)(y_t - \bar{y})}{\eta} - \frac{\phi}{\eta \gamma} \Delta y_{t+1} \quad (14) \]

Under the assumption that the monetary authority keeps prices constant, the nominal and the real exchange rates are the same, \( e_t = q_t \). Therefore, the law of motion for the nominal exchange rate (14) can be rewritten as:

\[ \Delta e_{t+1} = \Delta q_{t+1} = \frac{\phi \delta (q_t - \bar{q})}{\eta} - \frac{(\eta \psi + \phi)(y_t - \bar{y})}{\eta} - \frac{\phi}{\eta \gamma} \Delta y_{t+1} \quad (14') \]

The intuition behind equation (14’) is that as output grows and approaches its long-run level, there is less crowding out of foreign demand, so there is less need for real or nominal exchange rate appreciation.
The saddle path, derived in detail in Appendix 2, is given by

\[ e_t = \bar{q} + \psi \sum_{s=t}^{\infty} (y_t - \bar{y}) \]  \hspace{1cm} (15)

Once again, the intuition is that as output approaches its long-run level, there is no need for real or nominal exchange rate appreciation.

The steady state values for the exchange rate (nominal and real) and output are given by the solution to \( \Delta e_{t+1} = 0, \Delta y_{t+1} = 0 \). That is:

\[ e = \bar{q} \]  \hspace{1cm} (16)

So in the new equilibrium the exchange rate returns to its initial level. As in the constant-money-supply case, the initial appreciation of the home currency is also followed by a depreciation. However, the new equilibrium nominal exchange rate is equal to its initial value, so there is no permanent appreciation of the home currency.

Given the constant-price assumption, the equilibrium in the money market is reached through a change in the money supply. That is, given that higher output increases money demand and that the equilibrium interest rate is determined by the exogenous world interest rate, a constant price level requires the money supply to rise. Consequently, real money balances increase to offset the rise in money demand. This way, the money market reaches its new equilibrium at the initial interest rate. Hence, the new equilibrium is characterized by the same nominal exchange rate, the same price level, the same real exchange rate, the same interest rate, and a higher money supply.
2.c. Constant-Money-Supply vs. Constant Prices:

Our two monetary policy assumptions generate an initial appreciation of the home currency followed by a depreciation, so they both generate overshooting dynamics. In the constant-money-supply case, the nominal exchange rate is permanently lower (i.e., appreciated) in equilibrium while in the constant-price case it reverts to its initial value.

The main difference between the two cases is the behavior of the money market. While in the constant-money-supply case the money market is equilibrated through a decrease in the price level, in the constant-price case the equilibrium is reached because of an increase in the money supply.

3. Quantitative Analysis

In the previous section we develop a version of the MFD model that allows for changes in expected long-run output. We use this model to show that a positive shock in long-run output expectations leads to an appreciation of the home currency under both a constant-money-supply policy and a constant-price policy.

In both cases the initial change in the nominal exchange rate is equal to the initial change in the real exchange rate because we assume that the price level is fixed in the short run. As we discussed in the previous section, a change in long-run output expectations generates a real appreciation because we assume that output is also slow to adjust to its new equilibrium. In other words, we do not allow output to jump in response to a change in long-run output expectations, so the increase in domestic demand that
higher long-run output expectations generate is offset by a decrease in foreign demand (i.e., an increase in the trade balance deficit). This crowding out of foreign demand takes the form of a real appreciation that also generates an equivalent nominal appreciation in the short run.

Even though the focus of our research is on the exchange rate impact of a change in long-run output expectations, the impact on the trade balance is a relevant part of our analysis. Figures 1 and 2 show the recent evolution of the US trade balance deficit, using quarterly data. Figure 1 shows the evolution of the trade balance deficit as a percentage of GDP. The increase in the trade deficit started in 1998 and continued during 1999 and 2000. At the beginning of 1999 the trade balance deficit was 2 percent of GDP and about 3.5 percent of GDP at the end of 2000. Figure 2 shows the same data in logarithms. Here we can see that the trade balance deficit increased about 62 percent during the period of interest. This evidence on the increase in the US trade balance deficit supports our model’s implication that higher domestic demand generated a crowding out of foreign demand.

The goal of our paper is to provide a quantitative idea of how large the change in long-run output expectations has to be to generate an initial appreciation that matches the recent euro-dollar experience. To do this, we evaluate our model with a variety of parameter values and compute the change in output expectations that generates an initial appreciation of 25 percent, which is the extent of the appreciation of the dollar with respect to the euro. For each combination of parameters we provide the percentage change in long-run output and the dynamics that follow that change under our two different policy assumptions.
It is easy to find in the literature some “consensus” estimates of three of the five parameters that we need, the inverse of the sacrifice ratio $\psi$, the income elasticity of money demand $\phi$, and the interest rate elasticity of money demand $\eta$.

The textbook number for the inverse of the sacrifice ratio $\psi$, which comes from Gordon and King’s (1982) paper, is $\frac{1}{5}$\textsuperscript{15} However, there are a variety of estimates in the literature that we will also consider in our exercise. Gordon (1997) provides an estimate of $\frac{1}{3}$ and Staiger, Stock and Watson’s (1996) paper reports small, but statistically significant, numbers in the range $\frac{1}{8}$ to $\frac{1}{4}$.

For the income elasticity of money demand $\phi$ and the interest rate elasticity of money demand $\eta$ we find a large consensus in the literature represented by Hoffman and Rasche (1991), Lucas (1988), Mankiw and Summers (1986) and Stock and Watson (1993). The estimates in all these articles are 1 and 0.1 for the income and interest rate elasticities, respectively.

Since we have not been able to find a number for the elasticity of output demand to real depreciation $\delta$, we perform a variety of estimations using different data sets and time periods with an ordinary least squares (OLS) regression for the logarithm of US real GDP on the logarithm of the US real exchange rate.

Table 1 shows the OLS results for the elasticity of output demand to real depreciation $\delta$. The first row presents an estimate based on quarterly data from the IMF’s International Financial Statistics from 80/Q1 to 00/Q4. The estimate in the second row uses yearly data from 1975 to 1992. Real GDP comes from the Penn World Table

\textsuperscript{15} See Mankiw 2000, page 371.
data set and the real exchange rate comes from the IMF’s International Financial Statistics. We have also performed these regressions with the World Bank’s World Development Indicators data set and using different measures of output and real exchange rate obtaining similar results.

Table 1

<table>
<thead>
<tr>
<th></th>
<th>δ</th>
<th>Std. Error</th>
<th>Adj. R2</th>
<th># Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>IFS:</td>
<td>.40</td>
<td>***</td>
<td>.14</td>
<td>.07</td>
</tr>
<tr>
<td>80/Q1-00/Q4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>.14</td>
<td>.07</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PWT and IFS:</td>
<td>.49</td>
<td>*</td>
<td>.27</td>
<td>.12</td>
</tr>
<tr>
<td>1975-1992</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(***): significant at a 1% confidence level, (*) significant at a 10% confidence level.

Finally, we need values for the parameter γ, which measures the response of output to deviations from domestic demand. We do not think that there is a sensible way of estimating this parameter so we consider all its range of values, 0 ≤ γ ≤ 1. However, the parameter γ only affects the speed of adjustment to the new equilibrium. For low values of γ the adjustment is slow and faster for higher values. The fastest adjustment takes place for intermediate values between .4 and .6. Since the focus of this paper is more in the initial change in long-run output expectations than in the dynamics that follow that change we simply assume a value of γ = 1 in our simulation. Although this is an arbitrary assumption, it implies no loss of generality.

Table 2 summarizes the set of parameter values that we use to perform our simulation.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\psi$</th>
<th>$\phi$</th>
<th>$\eta$</th>
<th>$\delta$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>$\frac{1}{8}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}$</td>
<td>1</td>
<td>.1</td>
<td>.40, .49</td>
<td>$\gamma = 1$</td>
</tr>
</tbody>
</table>

3.a. Constant-Money-Supply Case:

To perform our simulation we need some initial values for the different variables so, in the initial equilibrium, we assume:

$$y_0 = \bar{y} = 0$$

$$q_0 = \bar{q} = 0$$

These initial values and equations (6) and (10) imply that both output and the real exchange rate are constant in equilibrium. That is:

$$\Delta y_0 = 0$$

$$\Delta q_0 = 0$$

We also assume that, at first, the domestic price level is zero:

$$p_0 = 0$$
Money supply, the foreign price level and the long-run equilibrium real exchange rate are exogenous, constant and zero:

\[ m = 0, \forall t \]
\[ p^* = 0, \forall t \]
\[ \bar{q} = 0, \forall t \]

The previous assumptions and equations (4) and (11) imply that, initially, the nominal exchange rate is equal to the real exchange rate and constant. That is:

\[ e_0 = q_0 = \bar{q} = 0 \]
\[ \Delta e_0 = 0 \]

The assumption that output is fixed in the short run can be interpreted as:

\[ \Delta y_1 = 0 \]

where period 1 is the period when long-run output expectations are updated. With this assumption, our initial values and equation (6) we can calculate the change of long-run output expectations consistent with a 25 percent appreciation of the dollar as:

\[ \bar{y}' - \bar{y} = \bar{y}' = -\delta(q_t - \bar{q}) = -\delta(q_t) = -\delta(-.25) = \delta(.25) \] (17)
where $q_t$ is the real exchange rate after the expectations update. $q_t$ is also equal to the nominal exchange rate after the expectations update thanks to the assumption that the price level is fixed in the short run. Hence, the initial real appreciation that is necessary to offset the effect of higher domestic demand caused by the expectations update is equal to the initial nominal appreciation that we want to explain. That is:

$$e_t - e_0 = q_t - q_0 = -0.25$$

Equation (17) shows how the change in long-run output expectations is a function of the elasticity of output demand to real depreciation $\delta$ and the magnitude of the appreciation that we want to explain, in our case 25 percent.

We evaluate equation (17) using our two estimates for $\delta$, .4 and .49, and show the results in Table 3. Each cell reports the percentage change in long-run output expectations that generates a 25 percent appreciation of the dollar.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>10%</th>
<th>12%</th>
</tr>
</thead>
<tbody>
<tr>
<td>.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.49</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The numbers in Table 3 show how an expectations update of 10 to 12 percent of long-run output is enough to explain an initial appreciation of 25 percent. These numbers also give the new equilibrium nominal exchange rate. Using equation (11’) and the steady
state conditions of the other variables, the new equilibrium nominal exchange rate is given by:

\[ e^* = \bar{q} + m - \phi \bar{y}' \]  

(18)

So, under our assumptions for the values of \( \bar{q} \) and \( m \) and taking the value of the income elasticity of money demand \( \phi = 1 \), the change in the equilibrium nominal exchange rate is equal to the change in long-run output. In our case, the nominal exchange rate appreciates between 10 to 12 percent in the new equilibrium. It is clear from equation (18) that an equilibrium nominal appreciation critically depends on the constant-money-supply assumption.

We can compute the dynamics for output and the real exchange rate that follow the change in long-run output expectations using equations (6) and (10). We only need these two equations because the dynamics for output and the real exchange rate do not depend on the nominal exchange rate. On the other hand, the dynamics of the nominal exchange rate depend on output, the real exchange rate and the nominal exchange rate. So, once we have the dynamics for output and the real exchange rate, we can get the dynamics for the nominal exchange rate using the saddle path for the nominal exchange rate, equation (12).

To compute the dynamics of output and the real exchange rate we proceed as follows:

1) We take a 25% real appreciation as our starting point.
2) Using equation (17) we compute the change in long-run output expectations that is consistent with a 25% real appreciation.

3) We do not allow output to change in the period when expectations are updated so \( \Delta y_1 = 0 \). This implies \( y_2 = y_1 \).

4) Using equation (10) we compute the change in the real exchange rate, \( \Delta q_1 \), which allows us to compute \( q_2 \).

5) Using equation (6) we compute the change in output, \( \Delta y_2 \), which allows us to compute \( y_3 \).

6) We keep iterating equations (6) and (10) until the new equilibrium is reached. We consider that the new equilibrium is reached when both output and the real exchange rate reach their equilibrium levels up to the first three decimal numbers.

Once we have the values of output and the real exchange rate from the first period to the new equilibrium, we can use the saddle path for the nominal exchange rate, equation (12), to compute the values of the nominal exchange rate.

Table 4 shows the results of computing these dynamics using all the different parameter values that are reported in Table 2. For each combination of parameter values we report the number of years it takes for the new equilibrium to be reached and the average growth rate of output during those years.
The average growth rate of output has to be interpreted as additional to the growth rate that had already been expected. For instance, if people expected the US to grow at an average growth rate of 2% for the following 50 years, they update their expectations to an average growth rate of 2.2%, assuming $\delta = .40$ and $\psi = \frac{1}{4}$.

The results are very similar for the two values of $\delta$ and they react to different values of the inverse of the sacrifice ratio in the same way. A higher sacrifice ratio increases the time it takes to reach the new equilibrium and, for a given increase in long-run output, decreases the average growth rate.

However, most of the adjustment takes place during the first years after the change in expectations. We exemplify this point with a more detailed description of the
dynamics in the case when $\delta = .40$ and $\psi = \frac{1}{4}$. In this case the new equilibrium is reached in 48 years with an average growth rate of .2% per year. However, the average growth rate during the first ten years is .7% per year, .4% during the first 20 years, and .3% during the first 30 years.

Most of the adjustment is concentrated at the beginning. In fact, after 20 years output is at .088, which is very close to the equilibrium level of .1. The real exchange rate after 20 years is -.02 and the nominal exchange rate is -.11, both close to their equilibrium levels of 0 and -.1, respectively.

Figure 3 shows the evolution of output, the real exchange rate and the nominal exchange rate. The nominal exchange rate comes from equation (12) and the values of output and the real exchange rate from equations (6) and (10) following steps 1) to 6). Clearly, the adjustment to the new equilibrium is faster for all variables during the initial periods.

3.b. **Constant-Price Case:**

We perform the simulation for the constant-price case using the same initial values as in the constant-money-supply case with the difference that, in the present case, the price level that is constant over time and money supply is allowed to change. That is:

\[ m_0 = 0 \]

\[ p_0 = p = 0, \forall t \]
As discussed in the theory section, the only differences between these two cases are that, in the constant-price case:

1) The nominal and the real exchange rate are the same. They have the same evolution and converge to their initial value of zero.

2) The equilibrium in the money market is reached through a change in the money supply.

As in the previous case, output dynamics determine the real exchange dynamics, which, in this case, are also the nominal exchange rate dynamics. The driving force behind these dynamics is the crowding out effect that domestic demand has on foreign demand. This effect depends critically on the assumption that output cannot jump in the period when expectations are updated, so the increase in domestic demand generated by higher long-run output expectations is offset by an equivalent decrease in foreign demand. Such decrease in foreign demand takes the form of a real exchange rate appreciation of the dollar.

To compute output and exchange rate dynamics we can use the law of motion for output, equation (6), and the new law of motion for the exchange rate, equation (14). These new exchange rate dynamics incorporate the constant-price monetary policy assumption, equation (1’), and the fact that under constant prices the real and the nominal exchange rate are the same. Following the same steps 1) to 6) described in the constant-money-supply case and equations (6) and (14), we compute output and exchange rate dynamics for the constant-price case.

Another way to generate the exchange rate dynamics is to use the new saddle-path equation for the exchange rate, equation (15), and the path of output that we calculate...
with equations (6) and (14). The results are the same showing that the model is internally consistent.

Since the driving force behind output and exchange rate dynamics is the same as in the constant-money case, it is not surprising that the results are also the same. The evolution of output and the real exchange rate is exactly the same as in the previous case. And, since nominal and real exchange rates are equivalent under the constant-price policy, the dynamics of the nominal exchange rate are now the same as the dynamics we previously reported for the real exchange rate. Once again, depending on the value of the elasticity of output to real depreciation $\delta$, the change in long-run output expectations that generates an initial appreciation of the dollar of a 25 percent is between 10 and 12 percent. The effect of changing the combination of parameters on the evolution of output is the same as reported in Table 4.

Figure 4 shows the evolution of output and the exchange rate (real and nominal) after the change in long-run output expectations assuming $\delta = .40$ and $\psi = \frac{1}{4}$.

3.c. **Constant-Money-Supply vs. Constant-Price:**

As we discussed before, the main difference between the constant-money case and the constant-price case is the behavior of the money market. Since we already have the values of output, the real exchange rate, and the nominal exchange rate under both assumptions, we can also compute the evolution of the money supply, prices, and the interest rate in both cases. We do so assuming $\delta = .40$ and $\psi = \frac{1}{4}$. 
Figure 5 shows the evolution of the interest rate under our two different monetary policy assumptions calculated from the interest rate parity condition, equation (2). In both cases the interest rate initially jumps to a higher-than-equilibrium level and then converges to its original value. The initial jump is due to the depreciation path that follows the appreciation of the dollar generated by the expectations update and is also given by the interest rate parity condition, equation (2). Although the equilibrium is the same in both cases, in the constant-money-supply case the initial increase in the interest rate is smaller. This happens because in this case the nominal exchange rate does not depreciate as much as in the constant-price case, where nominal depreciation has to completely offset any pressure on the price level.

Figure 6 shows the evolution of the price level in the constant-money-supply case calculated from the expectations-augmented Phillips curve, equation (9). In the period when expectations are updated, the price level does not change because it is assumed that it is fixed in the short run. As output and the nominal exchange rate converge to their new equilibrium levels, the interest rate also converges to its initial value. So, given the assumption that the money supply is constant and that money demand increases with output, the increase in real money balances that is needed to equilibrate the money market is achieved through a fall in the price level. In the new equilibrium, the price level is 10 percent lower than in the initial equilibrium.

Finally, Figure 7 shows the evolution of money supply under the constant-price assumption calculated from the monetary policy rule, equation (1’). The intuition is very similar to the previous case. As output and the nominal exchange rate converge to their new equilibrium levels, the interest rate also converges to its initial value. So, given the
assumption that the price level is constant and that money demand increases with output, the increase in real money balances that is needed to equilibrate the money market is achieved through an increase in the money supply. In the new equilibrium, the money supply is 10 percent higher than in the initial equilibrium.

4. Conclusions

A modified version of the Mundell-Fleming-Dornbusch model allows us to analyze the exchange rate impact of an increase in long-run output expectations. We simulate this model with parameter values estimated from actual data and show that an increase in long-run output expectations of between 10 and 12 percent is enough to generate a 25 percent appreciation of the dollar with respect to the new European currency. Depending on parameter values, this increase in long-run output expectations is equivalent to between .1 and .5 percent of higher-than-expected growth during the years that follow the expectations update.

These results suggest that the recent euro-dollar experience does not call for assumptions of irrationality since it can be explained with a standard and plausible model.
Appendix 1

To derive the saddle path equation for the exchange rate, we first derive the short-term dynamics for $q$ by substituting the output dynamics, equation (6), into equation (10) to obtain

$$q_{t+1} - q = (1 - \delta \psi)(q_t - q) + \frac{\psi \Delta y_{t+1}}{\gamma} \quad (A1.1)$$

To obtain a similar equation for $e$, we substitute (6) into (11) and we subtract $q$:

$$e_t - q = \frac{\eta}{(1 + \eta)}(e_{t+1} - q) + \frac{1}{(1 + \eta)}(1 - \phi \delta)(q_t - q) + \frac{1}{(1 + \eta)}(m_t - \phi \bar{y}) + \frac{\phi}{\gamma(1 + \eta)} \Delta y_{t+1} \quad (A1.2)$$

Iterating and imposing the no-speculative-bubble condition, $\lim_{T \to \infty} \left( \frac{\eta}{1 + \eta} \right)^T e_{t+T} = 0$ we obtain:

$$e_t - q = \frac{1}{1 + \eta} \sum_{s=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^s (q_t - q) + \frac{1}{1 + \eta} \sum_{s=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^s \left( m_t - \phi \bar{y} + \frac{\phi}{\gamma} \Delta y_{t+1} \right) \quad (A1.3)$$

Given our constant-money-supply assumption, we can re-rewrite the previous equation as:

$$e_t - q = m - \phi \bar{y} + \frac{1}{1 + \eta} (1 - \phi \delta) \sum_{s=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^s (q_t - q) + \frac{1}{1 + \eta} \sum_{s=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^s \left( \phi \bar{y} + \frac{\phi}{\gamma} \Delta y_{t+1} \right) \quad (A1.4)$$

Substituting equation (A1.1) into (A1.4) we obtain the saddle-path equation, equation 12 in the text:

$$e_t = m - \phi \bar{y} + q + \frac{(1 - \phi \delta)}{1 + \delta \psi \eta} (q_t - q) + \frac{1}{(1 + \eta)} \left( \frac{\phi + \psi \eta}{1 + \delta \psi \eta} \sum_{s=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^s \right) (\phi \Delta y_{t+1}) \quad (A1.5)$$
Appendix 2

Recall the law of motion for $e_t$ given in equation (14):

$$\Delta e_{t+1} = \frac{\phi\delta(q_t - \bar{q})}{\eta} - \frac{(\eta\psi + \phi)(y_t - \bar{y})}{\eta} - \frac{\phi}{\gamma}\Delta y_{t+1}$$  

(14)

we can re-write (14) as:

$$\eta e_{t+1} - \eta e_t = \phi\delta(q_t - \bar{q}) - (\eta\psi + \phi)(y_t - \bar{y}) - \frac{\phi}{\gamma}\Delta y_{t+1}$$  

(A2.1)

given $e_t = q_t$, we obtain:

$$e_t(\eta + \phi\delta) = \eta e_{t+1} + \phi\delta q_t + (\eta\psi + \phi)(y_t - \bar{y}) + \frac{\phi}{\gamma}\Delta y_{t+1}$$  

(A2.2)

Substituting the law of motion of output $\Delta y_{t+1} = -\gamma(y_t^d - \bar{y}) + \delta\gamma(q_t - \bar{q})$ into equation (A2.2) we obtain:

$$e_t(\eta + \phi\delta) = \eta e_{t+1} + \phi\delta q_t + (\eta\psi + \phi)(y_t - \bar{y}) - \phi(y_t - \bar{y}) + \delta\phi(q_t - \bar{q})$$  

(A2.3)

Substituting again for $e_t = q_t$, we can re-write (A2.3) as:

$$e_t(\eta + \phi\delta) = \eta e_{t+1} + (\eta\psi + \phi)(y_t - \bar{y})$$  

(A2.4)

Subtracting $\bar{q}$ from (A2.4):

$$e_t - \bar{q} = e_{t+1} - \bar{q} + \psi(y_t - \bar{y})$$  

(A2.5)

Iterating (A2.5):

$$e_t - \bar{q} = e_{t+T} - \bar{q} + \psi\sum_{s=t}^{\infty}(y_t - \bar{y})$$  

(A2.6)

In equilibrium $e_{t+T} = \bar{q}$, thus we obtain the saddle path equation for $e_t$, equation (15) in the text:
\[ e_t = \bar{q} + \psi \sum_{s=t}^{\infty} (y_s - \bar{y}) \]  \hspace{1cm} (A2.7)

Alternatively, equation (A2.7) can be derived from equation (13) and (2'):

\[ i_t = -\psi(y_t - \bar{y}) = e_{t+1} - e_t = q_{t+1} - q_t \]  \hspace{1cm} (A2.8)

\[ e_t = e_{t+1} + \psi(y_t - \bar{y}) \]  \hspace{1cm} (A2.9)

Once again, subtracting \( \bar{q} \) and iterating we obtain equation (A2.7):

\[ e_t = \bar{q} + \psi \sum_{s=t}^{\infty} (y_s - \bar{y}) \]  \hspace{1cm} (A2.7)
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Figure 1

Balance of Goods and Services
(as a % of GDP)

Figure 2

Log* of the Trade Balance Deficit
as a % of GDP

*Chart shows minus log of the trade balance deficit
Figure 3

Evolution of Output, Real and Nominal Exchange Rate
Constant-Money-Supply Case

Figure 4

Evolution of Output, Real and Nominal Exchange Rate
Constant-Price Case
Figure 5

Evolution of the Interest Rate

- $r$ constant $p$
- $r$ constant $m$

Figure 6

Evolution of the Price Level
Constant-Money-Supply Case
Figure 7

Evolution of the Money Supply
Constant-Price Case

Years

-0.020
0.000
0.020
0.040
0.060
0.080
0.100
0.120

0 4  7 10 13 16 19 22 25 28 31 34 37 40 43 46

m