Financial Regulation in a Quantitative Model of the Modern Banking System*

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Abstract

This paper builds a quantitative general equilibrium model with commercial banks and shadow banks to study the unintended consequences of capital requirements. In particular, we investigate how the shadow banking system responds to capital regulation for traditional banks. A key feature of our model are defaultable bank liabilities that provide liquidity services to households. In case of default, commercial bank debt is fully insured and thus provides full liquidity services. In contrast, shadow banks are only randomly bailed out. Thus, shadow banks’ liquidity services also depend on their default rate. Commercial banks are subject to a capital requirement. Tightening the requirement from the status quo, leads households to substitute shadow bank liquidity for commercial bank liquidity and therefore to more shadow banking activity in the economy. But this relationship is non-monotonic due to an endogenous leverage constraint on shadow banks that limits their ability to deliver liquidity services. The basic trade-off of a higher requirement is between bank liquidity provision and stability. Calibrating the model to data from the Financial Accounts of the U.S., the optimal capital requirement is around 15%.

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1 Introduction

Unintended consequences are a challenging aspect of financial sector regulation. Consider for example Regulation Q that was introduced after the Great Depression. The cap on interest rates supposed to prevent excessive competition for deposit funds that was viewed to have weakened the banking system. As long as interest rates remained low, savers had little incentives to pull their funds out of the traditional banking system. But as soon interest rates rose, depositors looked for alternatives and the competition for their savings generated one: money market mutual funds (\cite{1}). This and numerous other examples\(^1\) highlight possible side-effects of regulation that oftentimes affect not only traditional banks (that are typically targeted) but also their competitors – shadow banks.\(^2\)

In this paper, we study and quantify the effects of capital requirements in a general equilibrium model that features regulated (commercial banks) and unregulated (shadow banks) financial institutions. Tightening the capital requirement on commercial banks can shift activity to shadow banks, and thereby potentially increase the fragility of the entire financial system. Calibrating the model to aggregate data from the Flow of Funds, we find that higher capital requirements shift activity away from the traditional banks (though not monotonically), increasing shadow bank fragility and leverage. The system overall however becomes safer. Welfare is maximized at a capital requirement of 15%.

We derive this results in an endowment economy with households, regulated (commercial) banks, unregulated (shadow) banks, and a regulator.\(^3\) The general equilibrium environment

\(^1\)Asset-backed commercial paper conduits are another example for entities that emerged arguably as a response to regulation, more precisely capital regulation (see \cite{2}).

\(^2\)We define shadow banks as financial institutions that share features of depository institutions, either by providing liquidity services such as money market mutual funds or by providing credit either directly (e.g. finance companies) or indirectly (e.g. asset-backed security issuers). At the same time, they are not subject to the same regulatory supervision as traditional banks.

\(^3\)Figure 1 provides an overview of the model.
is key to analyze the potential side effects of new policies. The main features of our model are heterogeneous banks that make productive investments, have the option to default, and differ in their ability to guarantee the safety of their liabilities, as well as household preferences for safe and liquid assets in the form of bank debt whose value to households depends on their safety. While commercial bank debt is insured, shadow bank debt is not. When banks default, a fraction of the remaining bank value is lost to either the insurance fund or the investor.

Two trees produce a stochastic endowment. Households can only access one of the trees. Shadow banks and commercial banks compete over shares and intermediate access to the second tree. Matching the model to aggregate data from the Flow of Funds as well as the Federal Reserve, Moody’s, and the FDIC, we find an optimal capital requirement that is higher compared to current regulation. The optimal requirement finds the welfare maximizing balance between a reduction in liquidity services and the increase in the safety of the financial sector and consumption.

Key to this result are households’ preference for liquidity and the model’s forces that pin down the relative size of the two banking sectors. To fix ideas we can think about shadow bank debt as money market mutual fund shares and bank deposits. From the perspective of investors, money market shares are generally uninsured and therefore exposed to default risk. Investors take this into account, implying that shadow banks face an endogenous leverage constraint. That is, the price at which they issue debt takes into account the default probability implied by their leverage choice. In contrast, commercial bank debt is insured and therefore safe. The guarantee on their debt gives commercial banks an advantage in producing liquidity services over shadow banks. However, as long as both assets have different characteristics (e.g. are imperfect substitutes, or with risk-aversion have different risk-profiles, etc) shadow banks coexist with commercial banks in equilibrium. Their relative
size is mainly determined by the value of their liquidity services.

Liquidity production is a central function of the financial sector. Households aggregate liquidity services of both intermediaries in a CES function, where the elasticity determines whether households care more about a specific mix between money market shares and deposits or just the overall quantity of safe and liquid assets. The weight on money market shares in households’ utility depends on the default probability of shadow banks since they cannot guarantee the safety on their liabilities. In other words, when shadow banks’ default probability is high, their money market shares produce less liquidity services. In contrast, deposit insurance guarantees the value of liquidity service from commercial bank debt.

Both shadow banks and commercial banks overproduce liquidity. The failure rate of shadow banks affects households’ valuation of shadow bank debt. In essence, the riskier shadow banks are, the lower the liquidity services that households can derive from shadow bank debt. Shadow banks do not internalize this effect on the liquidity value of their liabilities for households, which causes them to issue too much debt. Deposit insurance insulates depositors from commercial bank failure. Thus, commercial banks choose leverage without taking into account the associated default risk.

We find that higher capital requirements on regulated banks shifts intermediation activity from regulated to unregulated banks (though not necessarily monotonically), lowers overall financial fragility and liquidity services, leads to more equity issuances, and higher market values of banks. A higher capital requirement forces commercial banks to lower their leverage, effectively reducing their ability to produce liquidity services holding prices fixed. At the same time, they become safer, reducing expected deadweight losses associated with their failure. The reduction of liquidity production by commercial banks increases the attractiveness of all types of bank liabilities as well as the value of banking in general. However the degree to which deposits can be substituted for with shadow bank liabilities is limited because of
the endogenous leverage constraint and decreasing marginal value of liquidity services. Thus liquidity services are generally decreasing in the capital requirement. The increased value of the intermediated assets as well as the reduction in funding costs due to the increased value of liquidity services relaxes the endogenous leverage constraint of shadow banks and thereby increases their default risk.

Quantitatively, our results depend on households preferences for liquidity, i.e. the parameter that governs how much households dislike variations in the liquidity service to consumption ratio, the weight on liquidity services in the utility, and finally the degree of substitutability between shadow and commercial bank debt. Studies on the stability of money demand (e.g. ? ) suggests that liquidity services and consumption are complements implying that the first parameter value should be larger than 1. We infer the other two parameters from shadow bank value weighted market leverage (debt over market value of assets gives 0.65) using Compustat data and the share of shadow bank activity (30%) as estimated by ?. The higher the degree in the curvature, particularly combined with a low degree in substitutability, the lower the scope for shadow banks to make up for fall in liquidity production. Such a parametrization can actually imply a rebounding of the commercial bank share when capital requirements exceed a value of 20-25%. Intuitively, households care a lot about the mix of money market shares and deposits and demand relatively more deposits when their share falls by too much. A higher weight on liquidity services in households’ utility increases the value of liquidity services overall and therefore all banks’ incentives to overproduce liquidity services. Thus, the improvement in welfare due to higher capital requirements is larger the higher the weight on liquidity services.

The simulated dynamic model shows that a greater capital requirement for commercial banks reduces the volatility of asset prices, liquidity provision, and consumption. The reduction in the volatility of consumption is another source of welfare gains to households from
increased capital requirements.

The dynamic model also features procyclical shadow banking activity: positive aggregate shocks make shadow banks safer and therefore their debt more attractive. This increases the share of shadow banking activity in good states of the world.

We match the model to aggregate data from the Flow of Funds on financial positions of U.S. households and financial institutions, interest rates from FRED, as well as Compustat data for publicly traded shadow banks. We also use industry reports to obtain estimates for default and recovery rates of financial intermediaries. It’s intuitive to think of commercial bank debt (mostly deposits) and shadow bank debt (mostly money market mutual fund shares, commercial paper, repo, etc) as being slightly different securities. The elasticity of substitution parameter is pinned down by the relative size of commercial banks and shadow banks.

Related Literature

The financial crisis has highlighted the relevance of banking sector. Naturally, research in this area is burgeoning, revamping banking theory in general (e.g. ? , ?) and shedding light on the role of shadow banks (e.g.? and ?). ?, ?

The model is structured as follows. Section 2 develops the intuition of our results in a two-period model. Section 3 briefly lays out the dynamic model, describes its parametrization and results.

2 Main mechanism in a two period model

This section demonstrates the main mechanisms of our dynamic general equilibrium model in a two-period model. We first describe the two-period model and the forces that determine the optimal capital requirement. For a quick overview of the model see Figure 1. We discuss
the key assumptions of the model in a separate section.

The basic structure of the model is as follows. Households maximize utility from consuming goods and liquidity services. The economy receives two endowments of goods: one that households directly receive as income and one that households can only access through financial intermediaries. Two types of intermediaries, C-banks and S-banks, can perform the intermediation. They issue short-term debt and equity to households to fund the intermediated asset. The short term debt of both banks provides households with liquidity services.

Both type of banks can declare bankruptcy and default on their debt. However, the debt of C-banks is riskfree to households since the government provides deposit insurance for C-banks. In return, C-banks are subject to capital regulation. S-banks, on the other hand, are not subject to regulation that limits their leverage. Their debt is risky for households since debt of defaulting S-banks only pays off a fraction of the face value. S-banks take into account the effect of their leverage choice on the expected payoff of their debt, and hence endogenously choose to limit their leverage.

2.1 Description of the model

Endowments There are two periods. Households receive an income of $Y_0$ at date 0 and $Y_1$ at date 1. Further, the intermediated asset is in unit supply and pays off $Z$ at date 1. $Y_0$ is known at date 0 whereas $Y_1$ and $Z$ are stochastic. In particular, trees $Z$ and $Y_1$ have two realizations $G = \text{good}$ and $B = \text{bad}$. Banks are exposed to idiosyncratic valuation shocks $\rho^j_i$ that are uniformly distributed on the interval $[\rho^j, \overline{\rho}^j]$ for bank types $j = S, C$, respectively. The shocks $\rho^j_i$ are independent of the endowment processes $Y_1$ and $Z$. 
Figure 1: Model Overview
Shadow banks  There is a unit mass of $S$-banks. $S$-bank $i$ chooses $A_i^S$ shares of the $Z$-tree at the beginning of period 0. The shares trade at a market price of $p_0$. The tree has a stochastic payoff of $Z$ in period 1. To fund their investment, $S$-banks issue short term debt $B_i^S$ that trades at the price $q_i^S$. All trades occur in 0, so all prices must be 0 in $t = 1$. At the beginning of period 1, $S-$banks face idiosyncratic valuation risks $\rho_i^S$ that are proportional to their assets, with $\rho_i^S \sim F_\rho^S$, i.i.d. across banks. After a large realization of $\rho_i^S$, $S-$bank $i$ may find it optimal to declare bankruptcy. In case of a bankruptcy, the bank’s equity is wiped out, and its assets are seized by its creditors. In case the bank does not default, it returns the debt it ows and pays out dividends to households.

The problem for shadow bank $i$ is

$$V_i^S = \max_{A_i^S, B_i^S} q(A_i^S, B_i^S)B_i^S - p_0A_i^S + E_0 \left[ M \left( \max \{0, Z A_i^S - B_i^S - \rho_i^S A_i^S \} \right) \right].$$

Since $\rho_i^S$ is uniformly distributed, the probability that bank $i$ with assets $A_i^S$ and debt $B_i^S$ stays in business is given by the probability that $\rho_i^S \leq Z - \frac{B_i^S}{A_i^S}$, i.e.

$$F_\rho^S \left( Z - \frac{B_i^S}{A_i^S} \right) = \frac{Z - \frac{B_i^S}{A_i^S} - \rho^S}{\rho^S - \rho^S}.$$  

The bank problem has constant returns to scale in $A^S$, allowing for direct aggregation. Since there is a unit mass of $S-$ banks, we can directly interpret $F^S$ as the mass of banks that do not fail. To further simplify the problem, we exploit the homogeneity in the scale of the bank $A^S$ by defining leverage per asset $b_i^S = B_i^S/A_i^S$.

Dropping the $i-$subscript, we rewrite the problem as

$$V^S = \max_{A^S \geq 0, b^S} A^S v^S(b^S)$$
where\(^{4}\)

\[ v^S(b^S) = q^S(b^S)b^S - p_0 + E_0 \left[ M \left( F^S_\rho \left( Z - b^S - \rho^-_S \right) \right) \right]. \] (2)

In the above expression, \(\rho^-_S\) is the expected value of \(\rho^S_i\) conditional on \(S\)-bank’s survival and a realization of the random variable \(Z\). We define \(\rho^+_S\) as the expected value of \(\rho\) conditional on \(S\)-bank’s failure. Investors who hold debt issued by \(S\)-banks that go bankrupt recover a fraction of each bond’s value

\[ r^S = (1 - \xi_S) \frac{A^S(Z - \rho^+_S)}{B^S} = (1 - \xi_S) \frac{Z - \rho^+_S}{b^S}. \] (3)

The date-0 dividend payments are equal to the initial equity required by the bank,

\[ D^S_0 = A^S(p_0 - q^Sb^S). \]

and the date-1 dividends are the payout amounts of the non-bankrupt banks

\[ D^S_1 = F^S A^S(Z - b^S - \rho^-_S). \]

**Commercial banks** The problem of \(C\)-banks is similar to the \(S\)-bank problem with one important exception. \(C\)-banks are subject to an exogenous leverage constraint and do not take into account price effects due to their default risk. For each unit of debt, commercial banks need to pay fee \(\kappa\) to fund the insurance. Defining \(b^C = B^C/A^C\), we can write the problem for a \(C\)-bank as

\[ V^C = \max_{A^C,b^C} A^C v^C(b^C) \]

subject to

\[ b^C \leq (1 - \theta)p_0, \]

where

\[ v^C(b^C) = (q^C - \kappa) b^C - p_0 + E_0 \left[ M \left( F^C_\rho \left( Z - b^C - \rho^-_C \right) \right) \right]. \] (4)

\(^{4}\)We describe the derivation in more detail in the appendix section A.1.
The term $\rho_C$ in the expression above is defined analogously to the $S-$ banks.

Households who invest in C-bank debt always receive the full face value of the bonds. The difference between the face value and the recovery value of the bonds for failing C-banks is provided by the government (deposit insurance). The insurance fund recovers a fraction of the value of the debt of bankrupt C-banks, per bond issued:

$$r^C = (1 - \xi_C) \frac{A^C(Z - \rho_C)}{B^C} = (1 - \xi_C) \frac{Z - \rho_C}{B^C}.$$ 

As $S-$banks, $C-$banks payout $D^C_0$ and $D^C_1$ in period 0 and 1, respectively.

**Households** In period 0, households have power utility over consumption $C_0$. In period 1, they have power utility over consumption $C_1$ and liquidity services $H$ relative to consumption. Thus utility is

$$U(C_0, C_1, N^S, N^C) = \begin{cases} 
\frac{C_1^{1-\gamma}}{1-\gamma} + \beta \left( \frac{C_1^{1-\gamma}}{1-\gamma} + \psi \frac{(H/C_1)^{1-\eta}}{1-\eta} \right) & \text{for } \eta > 1 \\
\frac{C_0^{1-\gamma}}{1-\gamma} + \beta \left( \frac{C_0^{1-\gamma}}{1-\gamma} + \psi \log \left( \frac{H}{C_1} \right) \right) & \text{for } \eta = 1,
\end{cases}$$

with $\gamma > 1$ and $\eta \geq 1$.

Liquidity is a composite good (consisting of $N^S$ and $N^C$) that is produced in equilibrium by both types of intermediaries

$$H(N^S, N^C) = \begin{cases} 
\left[ (1 - \nu) \left( N^S \right)^{\alpha} + \nu \left( N^C \right)^{\alpha} \right]^{1/\alpha} & \text{for } \alpha \neq 1 \\
\left( N^S \right)^{(1-\nu)} \left( N^C \right)^{\nu} & \text{for } \alpha = 1,
\end{cases}$$

where $\alpha$ parameterizes the elasticity of substitution between liquidity services of both types of banks. Weight $\nu$ determines the quality of liquidity services provided by C-banks relative to S-banks. The weight $\nu$ depends on the fraction of defaulting S-banks $F^S$, i.e. $\nu = 1/(1 + (F^S)^{\tilde{\nu}})$, and consequently

$$1 - \nu = \frac{(F^S)^{\tilde{\nu}}}{1 + (F^S)^{\tilde{\nu}}}.$$
Households maximize utility in equation (5) by choosing consumption $C_0$, $C_1$, and liquidity services from each bank $N^S$, $N^C$, subject to the budget constraints

$$C_0 = Y_0 - \sum_j D^j_0 - \sum_j q^j N^j,$$

$$C_1 = Y_1 + \sum_j D^j_1 + N^C + N^S \left( F^S + (1 - F^S) r^S \right) - T.$$

In period 1, households have to pay a lump-sum tax of $T$ that the government raises to make whole the depositors of failed C-banks. Households are the sole owners of all banks and therefore receive the dividend payments both at date 0 and date 1.

**Government** The government uses the deposit insurance fund to reimburse depositors of failed C-banks. The government covers the deficit by imposing a lump-sum tax on households in period 1. Therefore the amount of tax funds the government raises is

$$T = B^C \left[ (1 - F^C)(1 - r^C) - \kappa \right].$$

**Markets** Asset market clearing in period zero requires

$$1 = A^S + A^C,$$

$$N^S = B^S,$$

$$N^C = B^C.$$

This implies for the goods market in period 1

$$C_1 = Y_1 + Z - \sum_j A^j \left[ E^\rho (\rho^j) + \xi_j (1 - F^j) (Z - \rho^j) \right].$$

### 2.2 Discussion of assumptions

The following paragraphs discuss key assumptions of the model.
Banks’ Role as Intermediaries  In the model, banks are special because they provide liquidity (discussed below) and intermediate assets. The role of banks as intermediaries can be derived from first principles in numerous ways. For example, in models with asymmetric information between borrowers and lenders, lenders with access to a cheaper screening or monitoring technology than other lenders (regular households) become banks (see ? for many other examples).

The Role of Banks as Liquidity Providers  In this model, households value bank debt because it is liquid and safe, an interpretation of bank debt in ? and in ?. The notion of safe and liquid assets includes bank deposits, money market fund shares, commercial paper, repos, short-term interbank loans, Treasuries, agency and municipal debt, securitized debt, and high-grade financial sector corporate debt. Aside from the government, commercial banks and shadow banks are the most important providers of these securities. The savings glut hypothesis articulated in ? and other recent work (e.g. ?, ?, ?, and ?) rests on the notion that there exists a demand for safe and liquid securities. Economic agents demanding these assets are for example households that hold deposits for transaction or liquidity reasons as well as corporations, institutional investors, and high net worth individuals that carry large cash-balances and seek safe and liquid investment vehicles with higher yields than deposits, such as money market mutual funds. Commercial banks provide mostly deposits, but also issue money market fund shares, repos, and commercial paper. Some of these securities that commercial banks hold (most notably deposits) are explicitly insured through deposit insurance. Others, such as money market fund shares and commercial papers are

\[5\] The idea to view banks as liquidity provider goes back to ?. Other work has built upon this idea (e.g. ?)

\[6\] Historically, money market mutual funds (a type of shadow bank) emerged precisely to satisfy demand for safe and liquid assets when Regulation Q imposed a ceiling on deposit rates.
indirectly insured due to government guarantees.\footnote{A number of empirical papers presents evidence for market expectations for government guarantees on U.S. banks (see for instance \textit{?}, \textit{?}, and \textit{?}).} Shadow banks generally do not benefit from government guarantees. Nevertheless outside of recessions and banking crisis, money market mutual funds shares and collateralized short term funding sources such as repo are considered safe and liquid.

In the model, liquidity services are generated through debt issued by shadow banks and commercial banks. Commercial bank debt represents all commercial bank liabilities precisely because the demand for safe and liquid assets goes beyond merely deposits. We apply the same idea to shadow bank debt with one notable difference: the value of shadow bank liabilities depends on the likelihood at which shadow banks default.

**Liquidity services in households preference** We capture the idea that bank liabilities provide liquidity services with our utility specification. The households in our model represent a blend of different agents with demands for different types of safe and liquid assets (deposits, money market mutual fund shares, and so forth) provided by all financial institutions. This is why our utility specification aggregates the liquidity services of both bank types.

Commercial bank debt always provides liquidity services no matter their default probability. This is different for shadow banks as the value from their liquidity service depends on their probability of default. This captures the idea that shadow bank debt is only safe as long as shadow banks are safe, that is, not too many of them go bankrupt.

The demand for liquidity services is captured with a money-in-the-utility function specification. Since \textit{?} money-in-the-utility specifications have been used to capture the benefits from money-like-securities for households in macroeconomic models, \textit{?} proved the functional equivalence of models with money-in-the-utility and models with transaction or liquidity
costs. The specific functional form is a version of \( ? \) and \( ? \). We further assumed \((\eta \geq 1)\) complementarity between consumption and liquidity. This is intuitive as higher consumption levels go along with higher transaction volumes and higher savings demand.

### 2.3 Equilibrium Characterization

**Households** The stochastic discount factor \( M_1 \) equals

\[
M = \beta E_0 \left[ \frac{C_0}{C_1} \left( 1 - \psi \left( \frac{H}{C_1} \right)^{1-\eta} \right) \right].
\]

Its first term is the standard discount factor implied by log utility, while the second term reflects the adjustment to the discount factor resulting from the complementarity between liquidity services and consumption.

\[
MRS_C^C = \nu \frac{U_1}{U_2} \left( \frac{H}{N_C} \right)^{1-\alpha} \quad \text{and} \quad MRS_S^S = (1 - \nu) \frac{U_1}{U_2} \left( \frac{H}{N_S} \right)^{1-\alpha}
\]
denote the marginal rates of substitution between consumption and liquidity services by \( C \)-banks and \( S \)-banks respectively.

The solution to the HH’s problem is characterized by the two FOCs for deposits, which are also the HH’s intertemporal Euler equations. The FOC for C-bank deposits \( N_C^C \) is

\[
q_C^C = MRS_C^C + E_0 [M].
\]  

The first term on the RHS of equation (7) is the marginal utility from consuming the payoff of 1 per C-bank deposit bond purchased at time 0. \( MRS_C^C \) is the marginal utility from liquidity services.

The FOC for S-bank deposits \( N_S^S \) is

\[
q_S^S = MRS_S^S + E_0 [M \left( F_S^S + (1 - F_S^S) r_S^S \right)].
\]

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\(^8\)In \( \eta \), the money and deposit utility parameter relates to the bank debt-consumption elasticity \( \eta \) in the following way: \( \sigma_q = 2 - \eta \).
The payoff of one deposit bond issued by S-banks depends on the fraction of surviving banks $F^S$. For surviving banks, the payoff is simply 1 as for C-bank deposits. For failing banks, it is given by the recovery value per bond $r^S$. The second term, the marginal utility from liquidity services, is analogous to C-banks.

**Bank Portfolio Choices** The optimal choice of asset purchases $A^j$ for each bank type $j = C, S$ imply

$$v^j(b^j) = 0,$$

for any $A^j > 0$. We can think of condition (9) as determining the relative size of the $j$-bank sector. To see this, we show in appendix A that these conditions can be rewritten as

$$p_0 - (q^C - \kappa)b^C = E_0 \left( M \frac{\sqrt{12}}{2} \sigma^C \left( F^C \right)^2 \right), \quad (10)$$

$$p_0 - q^S b^S = E_0 \left( M \frac{\sqrt{12}}{2} \sigma^S \left( F^S \right)^2 \right). \quad (11)$$

where $\sigma^j$ is the standard deviation of $\rho^j$.

The FOC for leverage per unit of assets $b^C$ is

$$(v^C)'(b^C) = 0.$$  

In appendix A, we show that this condition can be expressed as

$$q^C = \lambda^C + E_0 \left( M F^C \right). \quad (12)$$

This implies that the Lagrange multiplier on C-banks’ leverage constraint is always positive as long as the marginal value of C-bank liquidity is positive.

The positive multiplier implies a binding constraint, i.e.

$$b^C = (1 - \theta)p_0.$$
As for C-banks, S-bank leverage per unit of assets is determined through S-banks’ FOC for $b^S$

$$(v^S)'(b^S) = 0,$$

which can be written as

$$q^S + b^S \frac{\partial q^S(b^S)}{\partial b^S} = E_0 \left( M_1 F^S \right).$$

Comparing the FOC for C-bank leverage in (12) to the S-bank FOC in (13) above reveals the fundamental difference between the two types of banks. While C-banks are subject to a constant leverage constraint that in equilibrium is always binding, S-banks choose to limit their leverage because they internalize the effect of their leverage choice on the price of their deposits, $q^S$.

Increasing leverage $b^S$ will decrease the survival probability $F^S$ and hence lower the value of S-bank deposits to HH, implying $\frac{\partial q^S(b^S)}{\partial b^S} < 0$, as can be seen in equation (8).

When choosing leverage, we assume that shadow banks take into account that their default risk is priced. In this sense, each shadow bank acts as a monopolist for its own debt. But it does not internalize how its leverage choice and default risk affects the value of liquidity services for households. This is intuitive, as shadow bank bond prices are sensitive to the specific default risk of the issuer. But they also move with changes in aggregate liquidity conditions, which are caused by actions of all shadow banks, but not by any individual bank. Thus, it is best to think of the changes in the value of aggregate liquidity services as an externality arising in general equilibrium.

The following proposition that we prove in the appendix states the resulting endogenous leverage choice of S-banks:

**Proposition.** Leverage per units of assets $b^S$ of S-banks is

$$b^S = \frac{E_0 \left( M_1 MRS^S \right)}{E_0 (M_1)} \sqrt{12 \sigma^S} \xi^S.$$
For any reasonable parameter combinations, the above equilibrium choices of asset purchases and leverage for both types of banks imply that $C-$banks have a dominant position in the intermediated asset ($A^C > A^S$). The intuitive reason for this equilibrium outcome is as follows.

First, C-banks’ debt is insured and therefore C-banks do not internalize the effect of their leverage choice on the price of their debt. Since the marginal liquidity benefit is always positive, C-banks always exhaust their leverage constraint. S-banks, however, do internalize the increase in their default risk. Hence, if C-banks and S-banks are fundamentally equally risky, S-banks choose lower leverage\(^9\). Required initial equity for C-banks is $p_0 - q^Cb^C$. Leverage $b^C$ is a constant fraction of the asset price by the collateral constraint and higher than that of S-banks if $\theta$ is sufficiently small. In equilibrium, the bond price $q^C$ adjusts in order for required initial equity to equal the expected dividend. For reasonable parameter combinations, this in turn means that the marginal benefit of C-bank debt to households must be lower than that of S-bank debt. If debt is further sufficiently substitutable, this means that C-banks must hold a greater share of the intermediated asset in equilibrium\(^{10}\).

**Size of the Banking Sectors & Procyclical Shadow Banking Activity** The exact split between both types of banks depends on several parameters, particularly the elasticity of substitution between both kinds of liquidity $\alpha$.

The relative size of both banking sectors is determined in equilibrium by the marginal benefit of opening each bank in this period and receiving dividends next period. Mathemat-

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\(^9\)This statement of course depends on the parameters of the model, in particular the value of $\theta$, i.e. the tightness of C-banks’ leverage constraint. For any values close to the capital requirements of commercial banks, we found this statement to be true.

\(^{10}\)Even for equal shares of asset holdings ($A^S = A^C$), C-banks will produce $N^C > N^S$ due to their higher leverage. If both types of debt are close to being complements, the higher leverage of C-banks by itself is sufficient to create the lower marginal benefit. Thus the C-bank share is increasing in the elasticity parameter $\alpha$. 

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ically, this means that the holdings of the intermediated asset by both types of banks, $A^C$ and $A^S$, and thus the relative size of both banking sectors, are jointly determined by the FOCs (10) and (11).

\[ \text{Figure 2: Equilibrium Determination of } A^C \text{ and } A^S \text{ for } \alpha = 1/3 \]

Left-hand side (required equity at time 0, blue line) and right-hand side (expected dividend at time 1, red line) of banks’ first-order condition for asset holdings ($A^j$), while imposing market clearing $1 = A^C + A^S$ and holding fixed all other variables.

Figure (2) shows the LHS and RHS of both equations graphically for the calibrated model, depending on the current state of the economy. We first numerically compute the equilibrium values of all variables. Then we vary the share of S-banks and C-banks, $A^S$ and $A^C$, while holding all other variables fixed (and imposing market clearing $A^S + A^C = 1$). The blue lines (i.e. the equity funding costs) trace the value of the LHS of the first-order conditions (10) and (11) as we vary the shares, $p_0 - q^j b^j$, for $j = C, S$, respectively. Holding constant $p_0$ and $b^j$, the only source of variation is through the bond price $q^j$. Since total debt issued by each type is given by asset share times leverage per unit of assets, $N^j = b^j A^j$, the marginal
benefit of deposits of each type changes as we vary the asset shares, as can be seen from the pricing equations for the $q^j$, (7) and (8). In particular, as we increase the share of type $j$ holding total provided liquidity constant, the marginal liquidity benefit of type $j$’s deposits will decline (for any $\alpha < 1$), and therefore type $j$’s bond price will also decline. This means that the equity required to purchase the bank’s initial asset position becomes larger for the same face amount of debt issued.

An opposing effect is that, as long as liquidity services provided by both types of debt are imperfect substitutes, the marginal benefit derived from each type’s debt is also affected by the composition of the total debt. When we increase the share of C-banks, we decrease the share of S-banks due to market clearing. Consequently the composition of liquidity services becomes more unequal and the amount of services derived from total debt issued by both banks declines, which leads to a general increase in the marginal benefit of both kinds of liquidity. Both effects, the pure effect of an increase in $A^C$ and the equilibrium effect through the implied decrease in $A^S$ can be seen in left panel of (2). Lowering $A^S$ from 0.5 to about 0.15 (= raising $A^C$ from 0.5 to 0.85) causes a decrease in the marginal benefit of C-bank debt, which in turn lowers the bond price $q^C$ and therefore raises the required initial equity of C-banks (the blue line in the graph). By lowering $A^S$ any further, the composition of debt becomes so unequal that the marginal benefit of any liquidity rises again, causing both bond prices (also $q^C$) to increase again. Hence the required equity of C-banks bends backwards, yielding the overall non-monotonic shape.

The effect on the RHS of equation (11) is depicted by the red lines and quantitatively smaller. The discounted expected dividend at time 1 (per unit of assets) is only indirectly affected by a variation in asset shares through the households’ discount factor which depends on consumption $C_1$. As the share is shifted towards C-banks, consumption $C_1$ decreases since S-banks have lower leverage and cause lower bankruptcy-induced consumption losses to
households. This decrease in $C_1$ raises the discount factor, leading to a higher present value of the expected dividend.

The share of shadow banking activity also depends on the economic state. In a boom (solid line), shadow banks can issue debt at a higher price because they are less likely to fail next period. Moreover, because the default probability is lower, the expected dividend payment is larger. This means that shadow banking activity is pro-cyclical.

The overall take-away from the left panel of figure (2) is that the FOC of C-banks is satisfied at two points, the actual equilibrium share of $A^C = 0.80$ ($A^S = 0.20$) and an even higher share of $A^C = 0.95$. The unique equilibrium split is then determined by the FOC of S-banks, depicted in the right panel of the figure. The forces determining the shapes of required equity (blue line) and expected dividend (red line) are the same as for C-banks. With respect to required initial equity, both the isolated effect of lowering $A^S$ on the marginal benefit of S-bank debt, the equilibrium effect through debt composition work in the same direction. Thus the required initial equity of S-banks is strictly decreasing as we lower $A^S$. The unique intersection with the expected discounted dividend of S-banks is at $A^S = 0.20$.

The split between the two types in equilibrium will also depend on other parameters of the model, most importantly the relative quality of C-bank debt compared to S-bank (parameterized by $\nu$).
Figure (3) shows the equilibrium split for the identical parameters as in figure (2), but with close to perfectly substitutable debt at $\alpha = 0.7$. We can see that for this parameter combination the marginal benefits of liquidity of both types are less responsive to changes in the relative scale, and only at $A^S$ very close to zero does the marginal benefit of S-bank debt increase sufficiently to make required equity equal to expected dividend. Thus as the elasticity of substitution approaches infinity, S-banks’ share goes to zero.

Moreover, the figure also shows that the model features procyclical shadow banking activity. During booms the aggregate shadow banking failure rate is low, leading to a higher value of shadow banking liquidity services and thus a higher value of shadow banks. Given a $A^S$, the required initial equity to open up a shadow bank is lower, leading to a higher equilibrium
share of shadow banking activity.

**Welfare-maximizing Capital Requirement**  The two-period model delivers a qualitative welfare result with respect to the optimal capital requirement for C-banks, $\theta$. The general conditions for this result to obtain are

C1. C-banks are sufficiently risky such that there exists a range for low values of $\theta$ for which some C-banks default, i.e. $F^C < 1$.

C2. S-banks are at least as risky as C-banks; in other words, the standard deviation of their idiosyncratic shocks $\sigma^S$ is at least as large as that of C-banks.

C3. Households derive a strictly positive utility benefit from liquidity services ($\psi > 0$), and the liquidity services provided by C-banks are at least as good as those of S-banks.

Under these fairly general conditions there exists a trade-off in the model that leads to a unique utility maximum in $\theta$. Figure (4) illustrates the trade-off: increasing the capital requirement leads to an increase in numeraire consumption (top right graph), as fewer C-banks default due to lower leverage and hence bankruptcy losses become smaller. At the same time, decreasing C-bank leverage through tighter capital requirements lowers the amount of liquidity services provided to households (bottom left), as a greater share of the intermediated asset is shifted to S-banks. The non-monotonicity in liquidity services occurs at the point where the commercial bank share rebounds. Since total utility is a weighted sum of both components, household utility is maximized when C-banks have become completely safe ($F^C = 1$) and no further increase in consumption is possible (but liquidity decreases further with higher $\theta$).
3 Dynamic Model

This section presents the quantitative general equilibrium model. The main structure is similar to the two-period.

3.1 Model Description

Agents and Environment  Time is discrete and infinite. The agent, intermediary, and endowment ownership structure is identical to the two-period model. The two types of
intermediaries finance $Z-$tree investments by issuing equity and debt to households. They have limited liability, i.e., they can choose to declare bankruptcy.

**S-Banks** There is a unit mass of $S$-banks, indexed by $i$. $S$-bank $i$ holds $A^S_{t,i}$ shares of the $Z$-tree at the beginning of period $t$. The shares trade at a market price of $p_t$. To fund their investment, $S$-banks can issue short term debt. The debt of $S$-bank $i$ trades at the price $q^S_{t,i}$. At the beginning of the period, $S$-bank $i$ has $B^S_{t,i}$ bonds outstanding.

There is an idiosyncratic component to the payoff of $Z$-shares when held by bank $i$. Specifically, the payoff per share held by bank $i$ is $Z_t$. At the beginning of each period, $S$-banks can decide to declare bankruptcy. Each period, they face idiosyncratic valuation risks $\rho^S_{t,i}$ that are proportional to its assets. $\rho^S_{t,i}$ is an idiosyncratic loss with $\rho^S_{t,i} \sim F^S_{\rho}$, i.i.d. across banks and time. In case of a bankruptcy, banks’ equity is wiped out, and their assets are seized by their creditors.

Since the idiosyncratic valuation shocks are uncorrelated over time, it is convenient to write bank $i$’s optimization problem after the bankruptcy decision, net of the idiosyncratic payoff. Conditional on asset holdings $A^S_{t,i}$ and debt $B^S_{t,i}$, all banks have the same value and face identical problems:

$$V^S(B^S_t, A^S_t, Z_t) = \max_{A^S_{t+1,i}, B^S_{t+1,i}} (Z_t + p_t - \rho^S_{t,i}) - B^S_{t,i} + q^S_t B^S_{t+1,i} - p_tA^S_{t+1,i}$$

$$+ E_t \left[ M_{t,t+1} V^S(A^S_{t+1,i}, B^S_{t+1,i}, Z_{t+1}, \rho^S_{t+1,i}) \right].$$

As in the two period model, the problem is homogenous of degree one and identical for each firm. We can omit subscripts $i$ and define leverage $b^S_t = \frac{B^S_t}{A^S_t}$ and asset growth $a^S_{t+1} = \frac{A^S_{t+1}}{A^S_t}$.
and

\[ v^S(b^S_t, Z_t) = \frac{V^S(A^S_{t,i}, B^S_{t,i}, Z_t)}{A^S_{t,i}} = \max_{a^S_{t+1}, b^S_{t+1}} Z_t + p_t - b^S_t + a^S_{t+1} \left( q^S_t b^S_{t+1} - p_t + E_t \left[ M_{t,t+1} \hat{v}^S(b^S_{t+1}, Z_{t+1}) \right] \right). \]

Using \( \rho^S_{t,i} \)'s independence of \( Z_t \) the continuation value \( \hat{v}^S(b^S_{t+1}, Z_{t+1}) \) is derived from

\[
\hat{V}^S(A^S_{t,i}, B^S_{t,i}, Z_t, \rho^S_{t,i}) = \max\{0, V(A^S_{t,i}, B^S_{t,i}, Z_t) - \rho^S_{t,i} A^S_{t,i}\}
\]

\[
= 1_{\{\rho^S_{t,i} \leq V(A^S_{t,i}, B^S_{t,i}, Z_t)\}} \left( V(A^S_{t,i}, B^S_{t,i}, Z_t) - \rho^S_{t,i} A^S_{t,i} \right).
\]

Taking the expectation with respect to \( \rho \) of the last line gives

\[
\hat{V}(A^S_t, B^S_t, Z_t) = F^S_\rho \left( \frac{V(A^S_t, B^S_t, Z_t)}{A^S_t} \right) V(A^S_t, B^S_t, Z_t) - A^S_t \int_\rho V(A^S_t, B^S_t, Z_t) \frac{dF_\rho(\rho)}{A^S_t}.
\]

Since we take expectations over the idiosyncratic valuation shock, the problem is identical for each bank and so

\[
\hat{v}^S(b^S_t, Z_t) = \frac{\hat{V}^S(A^S_{t,i}, B^S_{t,i}, Z_t)}{A^S_{t,i}}.
\]

**C-Banks** There is a unit mass of C-banks. C-banks are different from S-banks in two ways: (i) they issue short-term debt that is insured and risk free from the perspective of creditors, and (ii) they are subject to regulatory capital requirements. The pay an insurance fee of \( \kappa \) for each bond they issue. The problem of all C banks is identical, thus we can drop the subscript.

We can write the problem of C-bank \( i \) as

\[
v^C(b^C_t, Z_t) = \frac{V^C(A^C_t, B^C_t, Z_t)}{A^C_t} = \max_{a^C_{t+1}, b^C_{t+1}} Z_t + p_t - b^C_t + a^C_{t+1} \left( q^C_t b^C_{t+1} - p_t + E_t \left[ M_{t,t+1} \hat{v}^C(b^C_{t+1}, Z_{t+1}) \right] \right),
\]

subject to

\[
(1 - \theta)E_t[p_{t+1}] \geq b^C_t.
\]
The continuation value $E_t \left[ M_{t,t+1} \tilde{v}^C(b^C_t, Z_{t+1}) \right]$ is analogous to the continuation value of shadow banks.

**Bankruptcy** The idiosyncratic asset valuation shock is realized just before the period starts.

If a $S$-bank declares bankruptcy, its equity becomes worthless, and creditors seize all of the banks assets. Before defining the recovery value, it is useful to define the expectations of idiosyncratic bank losses conditional on survival or bankruptcy as

$$
\rho^-(\nu^j(b^j_t)) = E_p \left[ \rho^j_{t,i} | \rho^j_{t,i} < \nu^j(b^j_t) \right]
$$

for surviving banks, and

$$
\rho^+(\nu^j(b^j_t)) = E_p \left[ \rho^j_{t,i} | \rho^j_{t,i} > \nu^j(b^j_t) \right]
$$

for failing banks, for $j = S, C$ respectively.

Banks that go bankrupt do no pay a dividend and their equity becomes worthless upon bankruptcy. After their restructuring has completed, the bankrupt banks are replaced by new banks who face the same forward-looking portfolio problem as existing banks. The recovery amount per bond issued is hence

$$
u^j(b^j_t) = (1 - \xi^j) \frac{Z_t + p_t - \rho^+(\nu^j(b^j_t))}{b^j_t},
$$

with a fraction $\xi^j$ lost in the bankruptcy proceedings. After the bankruptcy proceedings are completed, and new shadow bank is set up to replace the bankrupt one. This bank sells its equity to new owners, and is otherwise identical to an existing shadow bank with zero assets and liabilities.

If a $C$-bank declares bankruptcy, its equity becomes worthless as well. The bank is then taken over by the government that uses lump sum taxes and revenues from deposit insurance,
\( \kappa B_{t+1}^C \), to pay out the bank’s creditors in full. That is, lump sum taxes are defined as

\[
T_t = (1 - F^C(b_t^C))(1 - r^C(b_t^C))B_t^C - \kappa B_{t+1}^C.
\]

The average dividend conditional on survival for \( S \)-banks

\[
d_t^S = \left( Z_t + p_t - \rho^- (v^S(b_t^S)) \right) - b_t^S + a_{t+1}^S (q_t^S b_t^S - p_t),
\]

and for \( C \)-banks

\[
d_t^C = \left( Z_t + p_t - \rho^- (v^C(b_t^C)) \right) - b_t^C + a_{t+1}^C (q_t^C b_t^C - p_t).
\]

**Households** Households derive utility from the consumption \( C_t \) of the fruit of both trees. Households hold a portfolio of all securities that both types of intermediaries issue. In particular, they buy equity shares of both types of intermediaries, \( S_j^i \), that trade at price of \( p_j^i \), for \( j = S, C \) respectively. They further buy the short terms bonds both types issue, \( N_j^i \), trading at prices \( q_j^i \), for \( j = S, C \).

Households consume the liquidity services provided by the short term debt they hold at the beginning of the period. This reflects that the liquidity services accrue at the time when the deposits from last period are redeemed. Let \( N_t^j = \int_0^1 N_{t,i}^j \, di \), for \( j = S, C \). Then the total liquidity services produced are

\[
H(N_t^S, N_t^C),
\]

and household utility in period \( t \) is

\[
U(C_t, H(N_t^S, N_t^C)).
\]

We specify utility as

\[
U(C_t, H(N_t^S, N_t^C)) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma} + \psi \left( \frac{H(N_t^S, N_t^C) / C_t}{1 - \eta} \right)^{1-\eta},
\]

28
with

\[ H \left( N_t^S, N_t^C \right) = \left[ \Lambda_{S,t} \left( N_t^S \right) \alpha + \Lambda_{C,t} \left( N_t^C \right) \alpha \right]^{1/\alpha}. \]

The elasticity of substitution between the two types of bank liabilities is \(1/(1 - \alpha)\).

We define the weights on the liquidity services of deposits of commercial banks \(\Lambda_{C,t}\) and shadow banks \(\Lambda_{S,t}\) as follows:

\[
\Lambda_{C,t} = \frac{1}{1 + F_s^p \left( v^S \left( b_t^S \right) \right)^\nu},
\]

\[
\Lambda_{S,t} = \frac{F_s^p \left( v^S \left( b_t^S \right) \right)^\nu}{1 + F_s^p \left( v^S \left( b_t^S \right) \right)^\nu},
\]

with \(\nu > 0\). The liquidity productivity of shadow banks is lower than that of commercial banks. The discount depends on the fraction of surviving shadow banks. If both bank types are equally safe, that is \(F_s^p \left( v^S \left( b_t^S \right) \right) = 1\), the weights amount each to \(1/2\).

Denoting household wealth at the beginning of the period by \(W_t\), the complete intertemporal problem of households is

\[
V^H(W_t, Y_t) = \max_{C_t, S_t^S, N_t^C} U(C_t, H \left( N_t^S, N_t^C \right)) + \beta E_t [V(W_{t+1}, Y_{t+1})]
\]

subject to

\[
W_t + Y_t - T_t = C_t + \sum_{j=S,C} p_{t,j}^i S_{t,j}^i + \sum_{j=S,C} q_{t,j}^i N_{t,j}^i
\]

\[
W_{t+1} = \sum_{j=S,C} F_{\rho}^j \left( v^j \left( b_{t+1}^j \right) \right) \left( D_t^j + p_{t+1}^j \right) S_{t,j}^i
\]

\[
+ N_t^S \left[ \pi_B + \left( 1 - \pi_B \right) \left( F_s^p \left( v^S \left( b_{t+1}^S \right) \right) + \left( 1 - F_s^p \left( v^S \left( b_{t+1}^S \right) \right) \right) r^S \left( b_t^S \right) \right) \right]
\]

\[
+ N_t^C.
\]

The budget constraint in equation (15) shows that households spend their wealth and income on consumption and purchases of equity and debt of both types of intermediaries. The securities issued are the same for all banks, independent of the previous bankruptcy status.
The equity purchases for banks that have gone through bankruptcy at the beginning of period \( t \) can be understood as initial equity offerings for these banks, while the purchases of equity of surviving banks are in a secondary market. However, since both new and surviving banks hold identical portfolios, their securities have the same price and there is no need to distinguish primary and secondary markets.\footnote{It is possible to show that the price to an equity claim of bank types \( j \), \( p_t^j \), is equal to the value of that bank’s security portfolio, \( A_{t+1}^j p_t - q_t^j B_{t+1}^j \).}

**Market Clearing** Asset markets

\[
A_{t+1}^S + A_{t+1}^C = 1 \\
B_t^S = N_t^S \\
B_t^C = N_t^C \\
S_t^S = 1 \\
S_t^C = 1. 
\]

Resource constraint

\[
C_t = Y_t + Z_t - \mu - \sum_{j=S,C} \xi^j (Z_t + p_t - \rho_{ij} A_t^j (1 - F_{ij}^j (v^j (b_t^j)))) .
\]

**3.2 Equilibrium Conditions**

**Household** The household’s first-order conditions for purchases of bank equity are, for \( j = S, C \),

\[
p_t^j = E_t \left[ M_{t,t+1} F_{t}^j (v^j (b_{t+1}^j)) (D_{t+1}^j + p_{t+1}^j) \right] ,
\]

where we have defined the stochastic discount factor

\[
M_{t,t+1} = \beta \frac{U_1 (C_{t+1}, H_{t+1})}{U_1 (C_t, H_t)} .
\]
We further define the intratemporal marginal rate of substitution between consumption and liquidity services

\[ Q_t = \frac{U_2(C_t, H_t)}{U_1(C_t, H_t)}. \]

Then the first-order conditions for purchases of bonds of either type of bank are

\[ q^C_t = Q_t \Lambda_{C,t} \left( \frac{H_t}{N^C_t} \right)^{1-\alpha} + E_t [M_{t,t+1}], \quad (17) \]

\[ q^S_t = Q_t \Lambda_{S,t} \left( \frac{H_t}{N^S_t} \right)^{1-\alpha} + E_t \left[ M_{t,t+1} \left[ F^S_{\rho} \left( v^S(b^S_{t+1}) \right) + (1 - F^S_{\rho} \left( v^S(b^S_{t+1}) \right)) \tilde{r}^S(b^S_{t+1}) \right] \right]. \quad (18) \]

The payoff of commercial bank bonds is 1, whereas the payoff of shadow bank bonds depends on their default probability and recovery value. The first terms in each expression represent the marginal benefit of liquidity services to households.

**Banks**  
S-banks are subject to an endogenous borrowing constraint. Each S-bank is a monopolist for its own debt, and hence internalizes the effect of supplying additional bonds on the bond price.

Specifically, each S-bank views the price of its debt as a function of its supply of bonds

\[ q^S_t = q(b^S_{t+1}) \]  
that is determined by households’ first order condition in equation 18.

It follows that the FOC of S-banks for leverage is

\[ q(b^S_{t+1}) + b^S_{t+1} q'(b^S_{t+1}) = E_t [M_{t,t+1} F^S_{\rho} \left( v^S(b^S_{t+1}, Z_{t+1}) \right) ]. \quad (19) \]

The partial derivative \( q'(b^S_{t+1}) \) can be obtained directly from households’ FOC for purchases of shadow bank debt. In the appendix we show that differentiating equation (18) yields

\[ q'(b^S_{t+1}) = -E_t \left\{ M_{t,t+1} \left[ f_{\rho} \left( v \left( b^S_{t+1} \right) \right) \left( 1 - r \left( b^S_{t+1} \right) \right) \left( 1 - F^S_{\rho} \left( v \left( b^S_{t+1} \right) \right) \right) \right] \frac{F^S_{\rho} \left( v \left( b^S_{t+1} \right) \right)}{b^S_{t+1}} + \frac{1 - \xi^S}{b^S_{t+1}} f_{\rho} \left( v \left( b^S_{t+1} \right) \right) \left( v \left( b^S_{t+1} \right) - \rho_{t+1}^S \right) \right\}. \quad (20) \]
The RHS is strictly negative, implying that the price of shadow bank debt is decreasing in shadow bank leverage $b_{t+1}^S$. The first term on the RHS is the loss for lenders from a marginal increase in the probability of default. The second term reflects that the recovery value per bond in case of bankruptcy is marginally decreased if the shadow bank issues more debt. The third term is positive and captures that the conditional expectation of the idiosyncratic losses of bankrupt firms decreases as leverage increases.

The debt price of commercial banks is independent of their leverage choice. Therefore the FOC of $C$-banks for leverage is

$$q_t^C - \kappa = \frac{\lambda_t^C}{a_t^{C+1}} + E_t \left[ M_{t,t+1} F_{\rho} \left( v_t^C(b_{t+1}^C, Z_{t+1}) \right) \right].$$

with $\lambda_t^C$ being the Lagrange multiplier on the leverage constraint. This FOC and the household FOC for purchases of commercial bank debt (17) jointly imply that the Lagrange multiplier is positive and hence the $C-$bank leverage constraint is binding\(^{12}\), i.e.

$$(1 - \theta)E_t[p_{t+1}] = b_t^C.$$  

### 3.3 Parametrization

We match the model to quarterly data.

The stochastic process for the $Y$-tree is a AR(1) in logs

$$\log(Y_{t+1}) = (1 - \rho_Y)\log(\mu_Y) + \rho_Y\log(Y_t) + \epsilon_t^Y,$$

where $\epsilon_t^Y$ is i.i.d. with mean zero and volatility $\sigma_Y$. To capture the correlation of asset payoffs with fundamental income shocks, we model the payoff of the intermediated asset as

$$Z_t = \phi^Y Y_t \exp(\epsilon_t^Z),$$

\(^{12}\)The constraint is binding if the marginal liquidity benefit from commercial bank debt is positive and the deposit insurance fee is not too large. This is the case for all relevant parameter combinations.
where $\epsilon_t^Z$ is i.i.d. with mean zero and volatility $\sigma^Z$, independent of $\epsilon_t^Y$. This structure of the shocks implies that $Z_t$ inherits all stochastic properties of aggregate income $Y_t$, and is subject to an additional temporary shock that reflects risks specific to intermediated assets, such as credit risk.

We quantify the model with data from the Flow of Funds\textsuperscript{13}, Compustat, and NIPA. We use quarterly data from 1999 (after the passage of the Gramm-Leach-Bliley Act that revoked parts of the Glass-Steagall Act) until the second quarter of 2015. We choose depository institutions as data counterparts for $C-$banks and shadow bank institutions as data counterparts for $S-$banks. Shadow banks are defined on their asset side as security broker and dealer, finance companies, asset-backed security issuers and so on and on their liability side as money market mutual funds.

For the parameters of the shock process we use Flow of Funds tables S.1 and S.6 to obtain a time series for financial sector value added and GDP which we deflate and express in GDP per capita using NIPA data. Normalizing the mean of real per capita GDP growth process to $\mu^Y = 0.5$ we can derive $\sigma^Y$ and $\rho^Y$ from the observed volatility and autocorrelation. Similarly, using the observed volatility in the real per capital financial sector value added time series we find a value for $\sigma^Z$.

\textsuperscript{13}The Flow of Funds tables are organized according to institutions and instruments. We focus on the balance sheet information on institutions. This is important, as we want to take into account all bank and shadow-bank positions when we quantify the model.
The key parameters are those that determine households’ utility for liquidity services. These are the elasticity of substitution of HH ($\alpha = 1 - 1/\text{elasticity}$) between the two liquidity services provided by the two types of banking sector liabilities, the weight on liquidity services $\psi$, and the curvature parameter $\eta$.

It’s intuitive to think of commercial bank debt (mostly deposits, but also commercial paper, repo etc) and shadow bank debt (mostly money market mutual fund shares, commercial paper, repo, etc) as being slightly different securities. The elasticity of substitution parameter is pinned down by the relative size of commercial banks relative to shadow banks. We use the estimate developed by ? to set the share of shadow banking intermediation. In that paper, ? used data from the Flow of Funds to carefully trace back how much shadow banking sector funding the real economy received. Since many shadow banks fund each other and...
not necessarily real activity the actual share of shadow activity is much lower (around 30\%) than what one would expect given the total asset size of the financial sector. We use 30\% as a target to parametrize \( \alpha \).

The utility weight on liquidity services determines its scale. Since shadow banks face an endogenous leverage constraint that is loosened when the overall level of liquidity services is higher, we use the average market leverage of shadow banks from Compustat. That is, we compute the value weighted market leverage of publicly traded shadow banks\(^\text{14}\) as total debt over the market value of assets weighted by the relative market value of each institution and average across time and banks. The result is a market value weighted leverage ratio of 0.65.

The last key parameter is \( \eta \) that determines the curvature of the liquidity preferences relative to consumption. As such, it determines whether consumption and liquidity services are complements or substitutes, as well as the volatility of the consumption liquidity service ratio. We choose a fairly conservative value of 1.5 that implies that consumption and liquidity services are complements and still allows for fairly high variability in their ratio.

We set \( \kappa \), the deposit insurance fee to 0.0006168. This is in the range of quarterly FDIC assessment rates. The parameter \( \xi^S \) (loss in bankruptcy) is set to match an average recovery value of 37\% percent from Moody’s report.\(^\text{15}\) The parameter \( \xi^C \) is set reflect the higher recovery value (80\%) under FDIC bankruptcy resolutions. We set \( \sigma^C_\rho \) such that the default probability of bank \( C \) equals that of the data. The FDIC publishes\(^\text{16}\) estimates for average annual default probabilities of bank holding companies (0.4175 percent) for the 2006 – 2008 period\(^\text{17}\). We set \( \sigma^C_\rho \) such that the annual probability of default equals 4.2\%. We set \( \sigma^S_\rho \) of

\(^{14}\)We define shadow banks as all institutions with SIC code 6111, 6141, 6153, 6159, 6162, 6163, 6172, 6211, and 6798.

\(^{15}\)We use Moody’s 1984-2004. Exhibit 9 in the report presents the recovery rates of defaulted bond for financial institutions. We use the mean for financial institutions over all bonds and preferred stocks.

\(^{16}\)We take the average default probability from table 1 on the FDIC website.

\(^{17}\)That is \( \frac{0.13 + 3.21}{8} = 0.4175 \)
shadow banks idiosyncratic risk such that we match an annual default probability of 6% as reported by Moody’s.

3.4 Results

We solve the dynamic model using second-order approximation around the model’s deterministic steady state. We then simulate the model for many periods and compute moments of the simulated series, for different values of the capital requirement $\theta$. Table (2) reports first and second moments for capital requirements ranging from 10% to 20%.

As for the two-period model, there is a unique maximum in aggregate welfare. The welfare maximum for the quantitative model occurs in the interval around 15 percent. The main forces leading to this result are the same as in the simple model. Increasing $\theta$ makes the debt of both types of banks safer, thus reducing bankruptcy losses and increasing aggregate consumption. At the same time, a higher level of $\theta$ restricts the amount of liabilities and thus liquidity commercial banks can produce for each unit of assets.

**Changing the capital requirement** Rows 3 of table (2) shows that an increase in the capital requirement shifts a greater fraction of the intermediated asset to shadow banks. Restricting the liquidity production by commercial banks increases the marginal benefit of liquidity to households. Shadow banks take advantage of the lower funding costs by increasing their demand for intermediated assets. Similar to the two-period model, the effect is non-monotonic at values of $\theta$ larger than 20%.

Since shadow banks have a higher marginal valuation of the asset than commercial banks, the price of the $Z$-asset rises by 7% when the capital requirement is increased from 10% to 15%. As the S-bank share continues to rise and shadow banks can only increase their liquidity production by demanding more of the intermediated assets, the price of the $Z$-asset increases
<table>
<thead>
<tr>
<th></th>
<th>$\theta = 0.1$</th>
<th>$\theta = 0.15$</th>
<th>$\theta = 0.20$</th>
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<tr>
<td></td>
<td>mean</td>
<td>stdev</td>
<td>mean</td>
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<tr>
<td>Exogenous Variables</td>
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<td>0.50</td>
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<td>Asset payoff</td>
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<td>0.002</td>
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<td>Intermediated asset share &amp; price</td>
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<td></td>
<td></td>
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<tr>
<td>S-bank share</td>
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<td>0.003</td>
<td>0.40</td>
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<td>Asset price</td>
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<td>Bank debt &amp; prices</td>
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<tr>
<td>S-bank debt</td>
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<td>0.032</td>
<td>1.75</td>
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<td>S debt price</td>
<td>0.999</td>
<td>0.038</td>
<td>0.998</td>
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<tr>
<td>Consumption &amp; welfare</td>
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<td></td>
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<tr>
<td>Liquidity</td>
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<td>0.029</td>
<td>2.56</td>
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<tr>
<td>Consumption</td>
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<td>0.014</td>
<td>0.54</td>
</tr>
<tr>
<td>Welfare$^a$</td>
<td>3%</td>
<td>-23.82%</td>
<td>2.76%</td>
</tr>
</tbody>
</table>

The table unconditional means and standard deviations of the main outcome variables from a 1,000 period simulation of four different models with different capital requirements.

$^a$: Welfare is the percentage change of mean and volatility of the household value function relative to the benchmark model with $\theta = 0.1$. 

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further. This effect illustrates that the amount of total liquidity strongly depends on the market value of the intermediated asset. The $\theta = 15\%$ economy produces more liquidity services (1.91) than the $\theta = 10\%$ economy (1.67). This is because the value of the asset backing the banks’ liabilities increases so much that it more than compensates for the fact that commercial banks can now only turn 85% of their asset value into liquidity-producing deposits. The large increase in the asset price allows shadow banks to increase their deposits by 25% (from 1.4 to 1.75), even though their leverage increased by only 2.3% (unreported in table) and their asset share only increases by 14%.

Both types of banks become safer as in the two-period model when $\theta$ is raised. Bankruptcy losses approach zero around a capital requirement of 13%. Further increases in the requirement only reduce liquidity, but do not yield any additional consumption benefit. Why then is the welfare maximum at a level around 15%, where liquidity provision is already decreasing in $\theta$? The reason is that in the dynamic model increasing the requirement yields the additional benefit of lower consumption and liquidity volatility. For the risk-averse household, this leads to a further utility gain, albeit a small one. However, the effect of the capital requirement on second moments of consumption and liquidity is an important difference between the two-period and the dynamic model.

**Dynamic responses** The lower volatility of the shadow banking share and liquidity services in the economies with higher capital requirements can be seen clearly in figure (5).
Figure 5: Impulse responses for different levels of $\theta$

Top left: Consumption, Top right: Utility from liquidity services, Bottom left: Share of S-banks, Bottom right: Price of Z-asset.

The dynamic responses of consumption and the price of the intermediated asset to an aggregate income and payoff shock is reduced for higher levels of $\theta$. This is intuitive as commercial banks who still hold most of the intermediated asset at $\theta = 0.20$ are less levered and therefore less prone to default and therefore to cause deadweight losses.

4 Conclusion

This paper proposes a novel model to study the consequences of higher bank capital requirements for the economy. The optimal level of capital regulation trades-off a reduction in liquidity services against an increase in the safety of the banking system and consumption. Increasing the capital requirement for regulated banks leads to more intermediation activity by the shadow banking system and thus a higher valuation for the intermediated asset be-
cause households’ substitute shadow bank liquidity for commercial bank liquidity. This effect is non-monotonic as shadow banks’ endogenous borrowing constraint restricts their ability to provide liquidity services. Moreover, a higher capital requirement makes both bank types safer: they affect commercial banks directly through a mandated reduction in leverage while they make shadow banks indirectly safer through their effect on the intermediated asset valuation.
References


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A Appendix

A.1 Two-period model

Preliminaries Since \( \rho_i^j \) is uniformly distributed, the probability that bank \( i \) with assets \( A_i^j \) and debt \( B_i^j \) stays in business is given by the probability that \( \rho_i^j \leq Z - \frac{B_i^j}{A_i^j} \), i.e.

\[
F_{\rho_i^j} \left( Z - \frac{B_i^j}{A_i^j} \right) = \frac{Z - b_i^j - \rho_i^j}{\bar{\rho}^j - \underline{\rho}^j}.
\] (21)

Furthermore, \( \rho_j^- \) is the expected value of \( \rho_i^j \) conditional on \( j \)-bank’s survival and a realization of the random variable \( Z \):

\[
\rho_j^- = E[\rho^j | \rho^j < Z - b^j] = \frac{1}{2} (Z - b^j + \rho^j).
\] (22)

The expected value of \( \rho \) conditional on \( j \)-bank’s failure is

\[
\rho_j^+ = E[\rho^j | \rho^j > Z - b^j] = \frac{1}{2} (Z - b^j + \bar{\rho}^j).
\] (23)

First-order conditions with respect to \( A^j \) The first-order condition for holdings of the intermediated asset for both types of banks is

\[
v^j(b^j) = 0,
\]

which implies

\[
q^j b^j - p_0 = E_0 \left[ M \left( F_{\rho_i^j}^j (Z - b^s - \rho_j^-) \right) \right]
\]

using the expressions for \( v^j(b^j) \) defined in equations (4) and (2) for \( j = C, S \), respectively.

We can now substitute the expressions for \( F_{\rho_i^j}^j \) given in equation (21) and for \( \rho_j^- \) given in equation (22) into the condition above, which yields

\[
q^j b^j - p_0 = E_0 \left[ \frac{M}{2(\bar{\rho}^j - \underline{\rho}^j)} (Z - b^j - \rho^j)^2 \right].
\]
Using once more the definition of $F^j_\rho$ this becomes
\[ q^j b^j - p_0 = E_0 \left[ \frac{M(\bar{\rho}^j - \rho^j)}{2} (F^j_\rho)^2 \right]. \]

Recognizing that the standard deviation of a uniform random variable with support $[\rho^j, \bar{\rho}^j]$ is given by
\[ \sigma^j = \frac{\bar{\rho}^j - \rho^j}{\sqrt{12}} \]
and plugging in gives the result in equations (10) and (11), respectively.

**First-order condition for $b^C$** Starting with the definition of $v^C(b^C)$ in equation (4), and substituting the expressions for $F^C_\rho$ and $\rho^C_-$ given in equations (21) and (22), we get
\[ v^C(b^C) = (q^C - \kappa)b^C - p_0 - E_0 \left[ \frac{M}{2(\bar{\rho}^C - \rho^C)} (Z - b^C - \rho^C)^2 \right]. \]

Taking the derivative with respect to $b^C$ (including the leverage constraint $b^C \leq (1 - \theta)p_0$) then yields
\[ q^C - \kappa = \lambda^C + E_0 \left[ M \left( \frac{Z - b^C - \rho^j}{\bar{\rho}^j - \rho^j} \right) \right], \]
which gives the result in equation (12).

**First-order condition for $b^S$** To derive the FOC of the S-bank, it is helpful to first rewrite several expressions. As for the C-bank, we can write the continuation value as
\[ F^S(Z - b^S - \rho^S) = \frac{1}{2(\bar{\rho}^S - \rho^S)} (Z - b^S - \rho^S)^2. \]

Furthermore, noting that $Z - \rho^S = \frac{1}{2}(Z + b^S - \bar{\rho}^S)$, we get for the recovery value on the bonds of bankrupt S-banks
\[ (1 - F^S) r^S = \frac{1 - \xi_S}{2(\bar{\rho}^S - \rho^S)} \frac{(b^S)^2 - (Z - \bar{\rho}^S)^2}{b^S}. \]
Using these expressions, we can rewrite HH’s FOC for $N^S$ (see equation 8) as

$$q^S = E_0 \left\{ M_1 \frac{1}{(p^S - \rho^S)} \left[ Z - b^S - \rho^S + 0.5(1 - \xi^S) \left( \frac{(b^S)^2 - (Z - \rho^S)^2}{b^S} + MRS^S \right) \right] \right\}.$$

Taking the derivative of this equation with respect to $b^S$

$$\frac{\partial q^S(b^S)}{\partial b^S} = -E_0 \left\{ M_1 \left( \frac{\xi^S}{p^S - \rho^S} + (1 - F^S) \frac{r^S}{b^S} \right) \right\}. \quad (24)$$

We can now derive the S-bank’s FOC with respect to its leverage ratio $b^S$

$$q^S + b^S \frac{\partial q^S(b^S)}{\partial b^S} = E_0 \left[ M_1 \frac{Z - b^S - \rho^S}{p^S - \rho^S} \right].$$

Plugging in from equation (24), and noting that the last term on the right-hand side equal the survival probability $F^S$

$$q^S - E_0 M_1 \left[ \frac{b^S \xi^S}{p^S - \rho^S} + (1 - F^S)r^S \right] = E_0 M_1 F^S,$$

which can be rearranged to yield

$$q^S - E_0 M_1 \left[ F^S + (1 - F^S)r^S \right] = E_0 M_1 \frac{1}{p^S - \rho^S} \xi^S b^S.$$

Using the HH’s FOC for shadow bank debt (8), we can substitute for the difference on the LHS

$$E_0 \left[ M_1 (1 - \nu) \frac{U_1}{U_2} \left( \frac{H}{N^S} \right)^{1-a} \right] = E_0 \left[ M_1 \frac{1}{p^S - \rho^S} \xi^S b^S \right],$$

and solve for $b^S$, which yields the expression in equation (14).

**Equations and Solution Method**  The equilibrium of the economy needs to be computed numerically. The equilibrium can be reduced to a system of four nonlinear equations in four unknowns. We numerically find a unique solution for every parameter combination we have tried. The four variables are $(p_0, A^S, C_1, b^S)$. Using the variables, we can first compute the C-bank leverage ratio as

$$b^C = (1 - \theta)p_0$$
Using the market clearing conditions \( N^S = b^S A^S, N^C = A^C b^C (1 - (1 - F^C)(1 - r^C) + \kappa) \) and \( A^C = 1 - A^S \), we can get the two bond prices \((q^S, q^C)\) from the HH FOCs for deposits. Then the four equations are

\[
\begin{align*}
p_0 &= (q^C - \kappa)b^C + E_0 \left( M_1 \frac{1}{4\sigma_2 \sqrt{3}} (Z - b^C - \rho^C)^2 \right) \\
p_0 &= q^S b^S + E_0 \left( M_1 \frac{1}{4\sigma_2 \sqrt{3}} (Z - b^S - \rho^S)^2 \right) \\
C_1 &= Y_1 + \sum_j D_1^j + N^C + N^S (F^S + (1 - F^S)r^s) \\
b^S &= \frac{E_0 (M_1 \text{MRS}^S) \sqrt{12 \sigma^S}}{E_0 (M_1) \xi^S}.
\end{align*}
\]

A.2 Steady state

\[
H = \left[ \frac{F_{p}^{S\nu}}{1 + F_{p}^{S\nu}} (N^S)^\alpha + \frac{1}{1 + F_{p}^{S\nu}} (N^C)^\alpha \right]^{1/\alpha} \tag{25}
\]

\[
U_1(C, H) = C^{-\gamma} - \psi H^{1-\eta} C^{n-2} \tag{26}
\]

\[
U_2(C, H) = \psi H_{t}^{1-\eta} C^{n-1} \tag{27}
\]

\[
\text{MRS}_S = \left( \frac{F_{p}^{S\nu}}{1 + F_{p}^{S\nu}} \right) \frac{U_2}{U_1} \left( \frac{H}{N^S} \right)^{1-\alpha} \tag{28}
\]

\[
\text{MRS}_C = \left( \frac{1}{1 + F_{p}^{S\nu}} \right) \left( \frac{U_2}{U_1} \right) \left( \frac{H}{N^C} \right)^{1-\alpha} \tag{29}
\]

\[
F^j(v^j) = \Phi \left( \frac{v^j - \mu_{p,j}}{\sigma_{p,j}} \right) \tag{30}
\]

\[
f^j(v^j) = \phi \left( \frac{v^j - \mu_{p,j}}{\sigma_{p,j}} \right) / \sigma_{p,j} \tag{31}
\]

\[
M = \beta \tag{32}
\]
\[ W + Y - T = C_t + \sum_{j=S,C} p^j + \sum_{j=S,C} q^j N^j \quad (33) \]
\[ W = \sum_{j=S,C} \left( (F^j_\rho \left( D^j + p^j - \mu_{\rho,j} A^j \right) + \sigma^2_{\rho,j} f^j (v^j) A^j) \right) \]
\[ N^C_t + N^S \left[ F^S_\rho (v^S) + \tilde{r}^S \right] \quad (34) \]
\[ q^C = MRS_C + \beta \quad (35) \]
\[ q^S = MRS_S + \beta \left( F^S_\rho + \tilde{r}^S \right) \quad (36) \]
\[ p^j = \beta \left( F^j_\rho \left( D^j + p^j - \mu_{\rho,j} A^j \right) + \sigma^2_{\rho,j} f^j (v^j) A^j \right), \text{ for } j = S, C \quad (37) \]
\[ v^S = Z - b^S \left( 1 - q^S \right) + \beta \hat{v}^S \quad (38) \]
\[ \hat{v}^S = F^S_\rho \left( v^S - \mu_{\rho,S} \right) + \sigma^2_{\rho,S} f^S (v^S) \quad (39) \]
\[ B^S = b^S A^S \quad (40) \]
\[ D^S = d^S A^S \quad (41) \]
\[ d^S = Z - b^S \left( 1 - q^S \right) \quad (42) \]
\[ q + b^S q' = \beta F^S_\rho \quad (43) \]
\[ b^S = (1 - \xi^S) (Z + p - v^S) + \frac{MRS_S}{\beta f^S} \quad (44) \]
\[ p - b^S q = \beta \hat{v}^S \quad (45) \]
\[ \tilde{r}^S = \frac{(1 - \xi^S)}{b^S} \left( (1 - F^S_\rho) (Z + p - \mu_{\rho,S}) - \sigma^2_{\rho,S} f^S \right) \quad (46) \]

This is the recovery value weighted by the probability of default \((1 - F^S) r^S\)
\[ v^C = Z - b^C \left( 1 - (q^C - \kappa) \right) + \beta \hat{v}^C \] (47)

\[ \hat{v}^C = F^C \left( v^C - \mu_{\rho,C} \right) + \sigma^2_{\rho,C} f^C \left( v^C \right) \] (48)

\[ B^C = b^C A^C \] (49)

\[ D^C = d^C A^C \] (50)

\[ d^C = Z - b^C \left( 1 - (q^C - \kappa) \right) \] (51)

\[ p = \frac{\beta \hat{v}^C}{1 - (1 - \theta)(q^C - \kappa)} \] (52)

\[ b^C = (1 - \theta)p \] (53)

\[ \lambda^C = q^C - \kappa - \beta F^C \] (54)

\[ \tilde{r}^C = \frac{1 - \xi^C}{b^C} \left( (1 - F^C_{\rho}) (Z + p - \mu_{\rho,C}) - \sigma^2_{\rho,C} f^C \right) \] (55)

This is the recovery value weighted by the probability of default: \( (1 - F^j) r^j \)

\[ A^S + A^C = 1 \] (56)

\[ B^S = N^S \] (57)

\[ B^C = N^C \] (58)

\[ T = (1 - F^C - \tilde{r}^C - \kappa) B^C \] (59)

\[ C = Y + Z - \sum_{j=S,C} A^j \left[ \mu_{\rho,j} + \xi^j \left( (Z + p - \mu_{\rho,j}) (1 - F^j_{\rho}) - \sigma^2_{\rho,j} f^j \right) \right]. \] (60)
A.3 Equilibrium Condition Dynamic Model

\[ H(N^S_t, N^C_t) = \left[ \frac{F^S_{\rho} (v^S (b^S_t))^{\nu}}{1 + F^S_{\rho} (v^S (b^S_t))^{\nu}} (N^S_t)^{\alpha} + \frac{1}{1 + F^S_{\rho} (v^S (b^S_t))^{\nu}} (N^C_t)^{\alpha} \right]^{1/\alpha} \] (61)

\[ U_1(C_t, H_t) = C_t^{-\gamma} - \psi H_t^{-\eta} C_t^{-\eta-2} \] (62)

\[ U_2(C_t, H_t) = \psi H_t^{-\eta} C_t^{-\eta-1} \] (63)

\[ MRS_{S,t} = \left( \frac{F^S_{\rho} (v^S (b^S_t))^{\nu}}{1 + F^S_{\rho} (v^S (b^S_t))^{\nu}} \right) \left( \frac{U_2(C_t, H_t)}{U_1(C_t, H_t)} \right) \left( \frac{H_t}{N^S_t} \right)^{1-\alpha} \] (64)

\[ MRS_{C,t} = \left( \frac{1}{1 + F^S_{\rho} (v^S (b^S_t))^{\nu}} \right) \left( \frac{U_2(C_t, H_t)}{U_1(C_t, H_t)} \right) \left( \frac{H_t}{N^C_t} \right)^{1-\alpha} \] (65)

\[ V^H(W_t, Y_t) = \max_{C_t, N^S_t, N^C_t, S^t, S^C_t} U(C_t, H(N^S_t, N^C_t)) + \beta E_t [V(W_{t+1}, Y_{t+1})] \] (66)

\[ F^j (v^j) = \Phi \left( \frac{v^j - \mu_{\rho,j}}{\sigma_{\rho,j}} \right) \] (67)

\[ f^j (v^j) = \phi \left( \frac{v^j - \mu_{\rho,j}}{\sigma_{\rho,j}} \right) / \sigma_{\rho,j} \] (68)

\[ M_{t,t+1} = \beta \frac{U_1(C_{t+1}, H_{t+1})}{U_1(C_t, H_t)} \] (69)

\[ W_t + Y_t - T_t = C_t + \sum_{j=S,C} p^j_t S^j_t + \sum_{j=S,C} q^j_t N^j_t \] (70)

\[ W_{t+1} = \sum_{j=S,C} \left( (F^j_{\rho} (v^j (b^j_{t+1})) (D^j_{t+1} + p^j_{t+1} - \mu_{\rho,j} A^j_t) + \sigma^2_{\rho,j} f^j (v^j (b^j_{t+1})) A^j_t) S^j_t \right) \]

\[ + N^S_t (F^S_{\rho} (v^S (b^S_{t+1})) + \tilde{r}^S (b^S_{t+1})) \]

\[ + N^C_t. \] (71)

\[ q^C_t = MRS_{C,t} + E_t [M_{t,t+1}] \] (72)

\[ q^S_t = MRS_{S,t} + E_t [M_{t,t+1} (F^S_{\rho} (v^S (b^S_{t+1})) + \tilde{r}^S (b^S_{t+1}))] \] (73)
\[ p^j_t = \mathbb{E}_t \left[ M_{t,t+1} \left( F^j_{\rho} (v^j(b^j_{t+1})) \left( D^j_{t+1} + p^j_{t+1} - \mu_{\rho,j} A^j_t \right) + \sigma^2_{\rho,j} f^j (v^j(b^j_{t+1})) A^j_t \right) \right], \text{ for } j = S, C \]  

(74)

\[ v^S(b^S_t, Z_t) = \max_{a^S_{t+1}, b^S_{t+1}} Z_t + p_t - b^S_t + a^S_{t+1} (q^S b^S_{t+1} - p_t + \mathbb{E}_t \left[ M_{t,t+1} \hat{v}^S(b^S_{t+1}, Z_{t+1}) \right]) \cdot \]  

(75)

\[ \hat{v}^S(b^S_t, Z_t) = F^S_1 \left( v^S(b^S_t, Z_t) - \mu_{\rho,S} \right) + \sigma^2_{\rho,S} f^S (v^S) \]  

(76)

\[ B^S_t = b^S_t A^S_t \]  

(77)

\[ D^S_t = d^S_t A^S_{t-1} \]  

(78)

\[ d^S_t = (Z_t + p_t) - b^S_t + a^S_{t+1} (q^S b^S_{t+1} - p_t) \]  

(79)

\[ A^S_t = a^S_t A^S_{t-1} \]  

(80)

\[ q(b^S_{t+1}) + b^S_{t+1} q'(b^S_{t+1}) = \mathbb{E}_t \left[ M_{t,t+1} F^S_1 \left( v^S (b^S_t, Z_{t+1}) \right) \right] \]  

(81)

\[ q'(b^S_{t+1}) = - \mathbb{E}_t \left\{ M_{t,t+1} \left[ f^S (v (b^S_{t+1})) + \frac{f^S (v (b^S_{t+1}))}{b^S_{t+1}} \left( 1 - \xi^S \right) (v (b^S_{t+1}) - (Z_{t+1} + p_{t+1})) \right] \right\} \]  

(82)

\[ + \frac{1}{b^S_{t+1}} r^S(b^S_{t+1}) \right\} \right\} \]  

(83)

\[ p_t - b^S_t q(b^S_t) = \mathbb{E}_t \left[ M_{t,t+1} \hat{v}^S(b^S_{t+1}, Z_{t+1}) \right] \]  

(84)

\[ r^S(b^S_t) = \frac{(1 - \xi^S)}{b^S_t} \left( (1 - F^S_1 (v^S(b^S_t))) (Z_t + p_t) - \mu_{\rho,S} \right) - \sigma^2_{\rho,S} f^S (v^S(b^S_t)) \]  

(85)

This is the recovery value weighted by the probability of default \((1 - F^S) r^S\).

\[ v^C(b^C_t, Z_t) = \max_{a^C_{t+1}, b^C_{t+1}} Z_t + p_t - b^C_t + a^C_{t+1} \left( (q^C_t - \kappa) b^C_{t+1} - p_t + \mathbb{E}_t \left[ M_{t,t+1} \hat{v}^C(b^C_{t+1}, Z_{t+1}) \right] \right) \cdot \]  

(86)

\[ \hat{v}^C(b^C_t, Z_t) = F^C_1 v^C(b^C_t) - F^C_1 \mu_{\rho,C} + \sigma^2_{\rho,C} f^C \left( v^C(b^C_t) \right) \]  

(87)

\[ b^C_t = B^C_t / A^C_t \]  

(88)
\[ D_t^C = a_t^C A_{t-1}^C \]  

(89)

\[ d_t^C = Z_t + p_t - b_t^C + a_{t+1}^C ( (q_t^C - \kappa) b_{t+1}^C - p_t) \]  

(90)

\[ q_t^C - \kappa = \frac{\lambda_t^C}{a_{t+1}^C} + E_t [M_{t,t+1} F_{\rho} (v_t^C (b_{t+1}^C, Z_{t+1}))] \]  

(91)

\[ p_t - b_{t+1}^C ( q_t^C - \kappa) = E_t [M_{t,t+1} a_t^C (b_{t+1}^C, Z_{t+1})] \]  

(92)

\[ \tilde{r}^C (b_t^C) = \frac{(1 - \xi_t^C)}{b_t^C} \left( (1 - F_F (v_t^C (b_t^C))) (Z_t + p_t - \mu_{p_t,c}) - \sigma_{p,t,c}^2 f^C (v_t^C (b_t^C)) \right) \]  

(93)

This is the recovery value weighted by the probability of default: \((1 - F^j) r^j\)

\[ (1 - \theta) E_t [p_{t+1}] = b_t^C \]  

(94)

\[ A_t^C = a_t^C A_{t-1}^C \]  

(95)

\[ A_{t+1}^S + A_{t+1}^C = 1 \]  

(96)

\[ B_t^S = N_t^S \]  

(97)

\[ B_t^C = N_t^C \]  

(98)

\[ T_t = \left( \left(1 - F^C (b_t^C) \right) - \tilde{r}_t^C (b_t^C) \right) B_t^C + \pi_B \left( \left(1 - F^S (b_t^S) \right) - \tilde{r}_t^S (b_t^S) \right) B_t^S - \kappa B_{t+1}^C \]  

(99)

\[ C_t = Y_t + Z_t - \mu_p - \sum_{j=S,C} (Z_t + p_t - \mu_{p,j}) \left(1 - F_{\rho}^j (v_j^t (b_t^j)) \right) - \sigma_{p,j,c}^2 f^j (v_j^t) A_t^j. \]  

(100)

Exogenous Variables

\[ Z_t = \phi^Z Y_t \exp(\sigma^Z \varepsilon_t^Z) \]  

(101)

\[ \log(Y_{t+1}) = (1 - \rho_Y) \log(\mu_Y) + \rho_Y \log(Y_t) + \sigma_Y \varepsilon_{t+1}^Y \]  

(102)

\[ Y_t = \exp(\log(Y_t)) \]  

(103)