Courts of Law and Unforeseen Contingencies∗

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Abstract. We study a contracting model with unforeseen contingencies in which the court is an active player. Ex-ante, the contracting parties cannot include the risky unforeseen contingencies in the contract they draw up. Ex-post the court observes whether an unforeseen contingency occurred, and decides whether to void or uphold the contract. If the contract is voided by the court, the parties can renegotiate a new agreement ex-post.

There are two effects of a court that voids more contracts. The parties’ incentives to undertake relationship-specific investment are reduced, while the parties enjoy greater insurance against the unforeseen contingencies which the ex-ante contract cannot take into account.

In this context, we are able to characterize fully the optimal decision rule for the court. The behavior of the optimal court is determined by the tradeoff between the need for incentives and the gains from insurance that voiding in some circumstances offers to the agents.

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1. Introduction

1.1. Motivation

Courts regularly intervene in contracts at the behest of one of the contracting parties to void, or otherwise modify, an agreement the parties have agreed to. One justification for courts overriding voluntary agreements is to insure the parties against changes in the environment between the time the agreement was made and the time when it is to be consummated. Changes in the environment can lead to changes in the costs and benefits to the parties involved that expose them to risks which they prefer to avoid.\(^1\) The possibility of renegotiation protects the parties from carrying out Pareto-dominated transactions, but not from the fluctuations in utility that stem from the uncertainty in the underlying environment.

If the parties foresee all relevant contingencies and agree on the optimal transactions given them, these can be included in the contract, thus providing protection from these risks. Both common sense and court decisions suggest that such foresight is unreasonable, however. Regardless of the parties’ experience and care in designing their contract, there will always be residual risk they face due to “unforeseen contingencies”.

There is considerable ambiguity about the meaning of unforeseen contingencies, we will discuss the term briefly before proceeding further. We take the position that parties can perfectly foresee the possibility of various contingencies, but are unable to describe the circumstances in sufficient detail to include all relevant contingencies in their contract.\(^2\) When the contracting parties understand that they are unaware of all significant potential events, the question arises as to how these parties can protect themselves against the risks they face when committing to a necessarily incomplete contract.

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\(^1\)See Kaplow and Shavell (2000, Section 4) for a general discussion of incomplete contracts and enforcement.

\(^2\)Al-Najjar, Anderlini, and Felli (2002) provide a formal model that fits this view of what an unforeseen contingency is. Concerning the possible meanings of the term unforeseen contingencies that have been discussed in previous literature see also Tirole (1999) and the survey by Dekel, Lipman, and Rustichini (1998).
In this paper, we take the view that although contracting parties are unable to identify all relevant contingencies ex-ante, it may be clear both to the parties and outsiders that the circumstances at the time the contract calls upon one of the parties to act differ materially from those envisioned at the time the contract was written. In this event, a court can make such a determination, and void the contract in order to provide insurance the parties arguably desire, but cannot effect on their own. A primary goal of this paper is to model formally the effects of *ex ante* unforeseen events that might be recognized as such *ex post*.

A court that voids contracts in this way may provide desirable insurance, but not without cost. A central benefit of a contract is a guarantee that parties will receive a return for investments that have specific value in their relationship. Without a guarantee, an individual has a diminished incentive to invest, because he may obtain only a portion of the benefits stemming from investment under an ex-post (re-)negotiated outcome. Courts that void contracts to provide insurance do so at the cost of reducing the ability to provide incentives for an efficient level of ex-ante investment.

We develop and analyze a model of a buyer and seller who contract in an environment that includes an active court whose role is to determine which contracts to void and which to uphold. In our model, a court can coordinate and guide contracting parties by means of precedents that shape the contracting parties' expectations about future rulings. We assume that the court maximizes ex-ante expected gains from contracting, and characterize the optimal policy, which is to void contracts in events which are deemed ex-post to have been low probability ex-ante, but which impose a high level of risk on the contracting parties.

In the simple set-up that we analyze the interests of all participants are aligned. Ex-ante, the objective function of the court is not in conflict with the expected utility

\[3^{3}\text{Throughout the rest of the paper we use the terms uphold and enforce (a contract) in a completely interchangeable way. Therefore our court does not engage in \text{“gap-filling” in that it only rules on whether a contract should be voided or not, rather than attempt to impose new terms on the parties. This seems to be the predominant view of how actual courts behave and of how they should in fact behave (Kull 1991).}
of either of the trading parties. This, in turn, implies that the parties could attempt to replicate the behavior of the optimal court using private means. However, this will only be true in the simplified set-up that we deal with here. For example, if there is any asymmetric information between the contracting parties informational externalities would arise and this conclusion would not necessarily be true.\footnote{In Anderlini, Felli, and Postlewaite (2003) we analyze a model similar to that analyzed in this paper, but where there is an informational externality between different types of contracting parties.}

1.2. Relation to the Literature

The seminal works on incomplete contracts by Grossman and Hart (Grossman and Hart 1986) and Hart and Moore (Hart and Moore 1990) took as given the existence of contingencies that may occur after the signing of a contract, but that cannot be described at the time the parties contract. The inability to describe all relevant contingencies, and make contract terms a function of them, affects agents’ incentives. When contracts are incomplete, the contracting parties may find it optimal to renegotiate the terms of trade in the event certain contingencies arise. Agents whose investments are sunk at this time will not receive the full benefits of those investments; this holdup problem leads to inefficient initial investments. In summary, incomplete contracts may make it impossible to avoid inefficient outcomes.

A number of papers have shown that the amount of inefficiency, however, is not fixed. Grossman and Hart (1986) and Hart and Moore (1990) show that the ownership structure of physical assets can affect investment incentives, and hence, efficiency; Bernheim and Whinston (1998) show that if it is impossible to contract over some part of a relationship, it may be optimal to be less specific than is possible in other parts of that relationship; Aghion and Tirole (1997) and Rajan and Zingales (1998) show that the distribution of authority and power in a firm can affect efficiency when complete contracts are impossible.

Both the original work illustrating how incomplete contracts can precipitate inefficiency, and the subsequent work demonstrating how institutional design can ameliorate that inefficiency, essentially ignore the role of a court in adjudicating and enforc-
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The inefficiencies analyzed in the papers discussed above might be diminished by a court that can ameliorate them through various forms of intervention. Stated more strongly, the work on incomplete contracts is "partial equilibrium," analyzing a subset of agents’ behavior taking as fixed the behavior of agents outside the model (the courts), without investigating whether the assumed fixed behavior of the outside agents is in fact optimal. Maskin and Tirole (1999) make this point most forcefully by showing that in a standard incomplete contracting model, the existence of indescribable (unforeseen) contingencies does not affect the set of payoff outcomes that can be achieved through contracting, if one allows a court with large discretionary authority. This is shown by exhibiting a mechanism capable of generating as equilibrium any payoffs that could be achieved with complete contracts. This mechanism does not mean that incompleteness of contracts is irrelevant, however. Maskin and Tirole invest courts with substantially more authority than we see in practice, and the plausibility of the mechanism they exhibit is open to discussion.

Our paper incorporates an active court whose authority lies between that in the traditional literature (simply enforcing contracts that are written) and that implicit in Maskin and Tirole. We provide a detailed specification of indescribable contingencies, including the information available to a court at the time performance is called for. The contracts that parties write differ from those they would write if courts did nothing more than passively enforce the contracts that are written. Despite the inclusion of a more active court, the basic message of the incomplete contract literature remains: contracts will still be incomplete, and the incompleteness causes inefficiency.

There is a relatively large literature on the effect of the rules courts use on the actions of those governed by the rules. For example, there is a substantial body of analysis comparing the incentive effects of strict liability with the incentive effects of a negligence rule in tort theory, and comparisons of different remedies for breach

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5 A "minimal" court is assumed to exist to force parties of a contract to perform according to the contract as originally written.

in contract theory.\footnote{See Kaplow and Shavell (2000) for a discussion of these literatures.} Our analysis differs from this work in two ways. First, these literatures focus largely on particular rules that are used in practice, and compare the incentive effects of those rules in different environments. In contrast, we consider a richer set of rules, with courts optimizing across that set; our framework admits more easily the formulation of alternative rules to those already in existence. The second difference is that earlier work is typically concerned with comparisons between qualitatively different rules, while our court must make quantitative decisions, such as the threshold for which unforeseen contingencies will change the court’s decision of whether or not to void the contract.\footnote{See Kaplow and Shavell (2000) and Kaplow (2000).}

A major benefit of formally incorporating the court is that it allows a richer analysis of contracting. In addition, it provides the structure for a serious examination of what precisely a court might do. In this paper, we restrict attention to particularly simple rules a court can follow, namely to determine the circumstances under which a contract will be voided.

1.3. Court Practices

We discussed in the previous subsection the relation of our work to previous literature. Before proceeding to our formal model, it is useful to also discuss the relation between our work and actual court practice to illustrate that courts insure contracting parties along the lines we argue is optimal. They will discharge a party’s obligation to perform under a contract based on the emergence of risks that were not foreseen at the time the contract was entered into under some conditions. There are several categories of intervening events that might be the basis for excusing performance, two of which are similar to the unforeseen contingencies that are the focus of this paper. The first is impracticability of performance; this occurs when unanticipated events subsequent to contracting make the promised performance extremely burdensome economically. The second category is termed frustration of purpose. One view of the frustration doctrine is that it will “... excuse performance where performance remains possible,
but the value of the performance to at least one of the parties and the basic reason recognized by both parties for entering into the contract have been destroyed by a supervening and unforeseen event."\(^9\)

The court intervention proposed in this paper that voids contracts under some circumstances can be considered to be of either of these two types. Performance is clearly not impossible, since ultimately the contracted transaction is consummated; the voiding of the contract serves only to relieve one or the other of the parties from an abnormally negative consequence resulting from supervening events. Frustration of purpose has been applied in a manner very similar to that proposed in this paper. Small risks will not be cause for voiding the contract, but sufficiently large risks will be.

“\(\text{It is, of course, the very essence of contract that it is directed at the elimination of some risks for each party in exchange for others. Each receives the certainty of price, quantity, and time, and assumes the risk of changing market prices, superior opportunity, or added costs. It is implicit in the doctrine of impossibility (and the companion rule of \text{“frustration of purpose”}) that certain risks are so unusual and have such severe consequences that they must have been beyond the scope of the assignment of risks inherent in the contract, that is, beyond the agreement made by the parties. To require performance in that case would be to grant the promisee an advantage for which he could not be said to have bargained in making the contract. ... The question is, given the commercial circumstances in which the parties dealt: Was the contingency which developed one which the parties could reasonably be thought to have foreseen as a real possibility which could affect performance? Was it one of that variety of risks which the parties were tacitly assigning to the promisor by their}

failure to provide for it explicitly? If it was, performance will be required. If it could not be so considered, performance is excused.\(^\text{10}\)

This case is not an isolated instance. Williston on Contracts (Rev. ed., 1938) a standard reference to the interpretation of contracts has this to say:

“The important question is whether an unanticipated circumstance has made performance of the promise vitally different from what should reasonably have been within the contemplation of both parties when they entered into the contract. If so, the risk should not fairly be thrown upon the promisor.”

We focus on the case in which the events that alter the costs and benefits to the parties of performance as specified in the contract were unforeseeable, but courts have typically taken a less stringent attitude of the circumstances in which supervening events could warrant excusing performance. Specifically, it is not necessary that a supervening event be literally unforeseeable, but rather, that it was in fact unforeseen; this is illustrated by the following two cases.

“The question we answer here is not whether the destruction of the forest’s regenerative capacity should have been considered at the time of contracting but, rather, whether it was considered. There is nothing in the contract nor in the parties’ dealings to suggest that the parties ever presumed more than a continuance of the conditions necessary to give purpose to a selective cut contract. In short, the contract did not address

the conditions that arose; hence, further performance under the contract is excused.”

“... it would be untenable to conclude that the parties intended that the [plaintiffs] should assume the risk of an adverse tax ruling simply because such a ruling was, in a sense, ‘foreseeable’ and because the contract did not expressly excuse performance in the event of its occurrence.”

1.4. Outline

The plan of the rest of the paper is as follows. In Section 2 below we describe the model of buyers and sellers in full detail, and we comment on the assumptions we make. We characterize in Section 3 the equilibrium contract that the parties to the trade will choose for general court’s decision rules while in Section 4 we present the main result of the paper: the characterization of the optimal decision rule for the court. Section 5 briefly concludes the paper. For ease of exposition we have relegated all proofs to an Appendix.

2. The Model

As mentioned in the introduction, we are interested in courts that have a role in trading off parties’ incentive to invest with their desire for insurance in the event of

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12West Los Angeles Institute for Cancer Research, Appellant, v. Ward Mayer et al., Appellees, No. 19551; United States Court of Appeals for the Ninth Circuit; 366 F.2d 220; 1966 U.S. App. LEXIS 5088. In August 1951, Ward Mayer and his wife and son contracted to sell the business to the West Los Angeles Institute for Cancer Research, a tax-exempt entity. The transaction was patterned after the sale and leaseback agreements previously approved by the IRS. The IRS rejected the tax premises upon which the transaction was based, and the Mayers sued to recover the property. The district court granted the relief sought on the ground that the sale and leaseback arrangement was frustrated by the revenue ruling.
unforeseen contingencies. To investigate this tradeoff, we consider a simple buyer and seller model.

For insurance to have any benefit, at least one of the parties must be risk averse; we assume a risk neutral buyer and risk averse seller. The buyer and seller trade a widget; the risk they face is that the cost and benefit of the widget are uncertain at the time they contract. The uncertainty about costs and benefits captures the idea that there is a “normal” cost and benefit, $c_N$ and $v_N$, but that both parties are aware that there is a possibility that an unforeseen contingency could give rise to costs and benefits either taking on high values, $c_H$ and $v_H$, or taking on low values, $c_L$ and $v_L$.\(^{13}\) For simplicity we assume that the gains from trade are constant, that is

$$\Delta = v_H - c_H = v_N - c_N = v_L - c_L.$$  

This assumption is made for tractability. Our results would not qualitatively change if the costs and benefits were not perfectly correlated or if the magnitude of the gains from trade were variable. We assume that

$$c_H \geq c_N \geq c_L.$$  

Hence, it is efficient to trade whether the costs and benefits are normal, high or low.

We assume that the buyer has all the bargaining power ex-ante when a contract is proposed. In other words, the equilibrium contract is the result of a take-it-or-leave-it offer from the buyer to the seller. Ex-post, in some instances, renegotiation will take place. We assume that the seller has all the bargaining power in the ex-post renegotiation: if renegotiation occurs, the seller makes a take-it-or-leave-it offer to the buyer. The assumption that both ex-ante and ex-post, one or the other of the parties has all the bargaining power is for expositional ease; none of our results depends qualitatively on bargaining power being absolute for one or the other. Our results would not hold, however, if the buyer has all the bargaining power ex-post.

\(^{13}\)One might think of unexpected changes in the real value of the currency, for example.
A central issue in this paper is how unforeseen contingencies are modelled, and we will discuss verbally our approach before describing the formal modelling. We assume that ex-post, the court, as well as the parties can recognize some events that are out of the ordinary. For example, all parties recognize and agree that the events of 9/11 were, in some sense unforeseen. However, it is likely that for every possible unfolding of events, one could claim that there is some unforeseen component, so excusing performance whenever there has been an unforeseen event cannot be a useful rule. We assume that the court can “categorize” events ex-post in the following sense. For any given realized event, the court will understand that if performance were excused in that instance, consistency (i.e., following precedent) would lead it to excuse performance in similar circumstances in the future. Assuming that the court can categorize events ex-post essentially means that the court understands the consequences of excusing performance in the present contractual arrangement on future contracting parties, if the court wishes to be consistent.

In addition to the court’s categorizing events, we assume that the court, implicitly or explicitly, assigns a probability to the category of events that are similar to the events at hand. That is, the court understands that if it desires to be consistent, excusing performance in the present contract will result in excusing performance in future contracts with the probability the court assigns to the category of events similar to the case before it. The basic notion then, is that courts make decisions at the ex-post stage, but understand that, based on the court’s decision, future contracting parties will make inferences about the probability that performance will be excused.

We formalize these ideas next. We let \( \Omega \) denote the states of the world with an elementary state denoted by \( \omega \). A subset of the states will be deemed “exceptional”, meaning that, ex-post, they will be deemed to have been unforeseen (henceforth, we will use the terms exceptional and unforeseen interchangeably); we denote this set of exceptional states by \( \Theta \). We will refer to a state as “normal” if it is not exceptional; for a normal state, the cost of the widget to the seller is \( c_N \) while the value of the widget to the buyer is \( v_N = c_N + \Delta \). An exceptional state \( \omega \in \Theta \) corresponds to the realization (by the parties and the court) that an unforeseen contingency has
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arisen. The court assesses the likelihood of the category of state like \( \omega \), and assigns the category a probability; we denote by \( \theta(\omega) \) this probability. We assume that the unforeseen contingencies are such that it is equally likely that the court assigns any probability in \([0, \frac{1}{2}]\) to the category containing an unforeseen contingency.\(^{14}\)

Our aim is to model a court that trades off the diminished incentive effects resulting from voiding contracts with the insurance gains such voiding generates. Categorization of an unforeseen event \( \omega \), and assigning that category a probability, allows the court to measure the incentive costs of excusing performance. What remains is a specification of the information the court would need to gauge the insurance benefits of voiding. There cannot be a role for a court that excuses performance if the court can precisely observe the payoffs to the parties; in such a world, the parties could simply specify a contract price for any change in payoffs resulting from unforeseen contingencies, thereby providing full insurance within the contract itself. Thus, a necessary condition for a court to have a role that includes excusing performance in some unforeseen events, but not in all, is that the court must have some idea of the magnitude of the effect of the unforeseen contingency on payoffs, but not observe precisely (and hence condition on) those payoffs. We model the court’s information in the simplest way to capture this: we will assume that, given an observed unforeseen contingency, the court knows the expected cost and the variance of costs, but not the precise realization. Specifically, for an exceptional state, \( \omega \in \Theta \), we denote the cost of the widget to the seller \( c_L(\omega) \) or \( c_H(\omega) \) (and hence, from the assumption that the gains from trade are constant, the buyer’s valuation is respectively \( v_L(\omega) = c_L(\omega) + \Delta \) or \( v_H(\omega) = c_H(\omega) + \Delta \)), and we assume that the expected cost is \( c_N \).\(^{15}\) We represent the cost of the widget to the seller as a binary random variable \( \kappa \) which takes one of the two values \( c_L \) and \( c_H \): \( \kappa \in \{c_L, c_H\} \). We assume \( \kappa = c_H \) with probability

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\(^{14}\)The reason for assigning an upper bound of \( 1/2 \) to the probability of unforeseen contingencies is a purely aesthetic one. Given that we refer to the states in the complement of \( \Theta \) as “normal,” this upper bound implies that the probability of the normal states is always at least as large as that of the exceptional ones.

\(^{15}\)The assumption that average cost in an exceptional state is constant is purely for pedagogical purposes. Although the calculations would be more complicated without this assumption, our results would not be qualitatively affected.
$q_H$ and $c_L$ with the complementary probability $1 - q_H$. We assume that whether the cost increases or decreases in an exceptional state is independent of $\omega$ and $\theta$, but that the size of the change in cost is not independent of $\theta$. Specifically, we assume that the difference between $c_L$ and $c_N$ is a function of $\theta$, $f(\theta)$. This will allow us to consider, for example, exceptional events that have small probability but which lead to large deviations from normal cost.

If we denote by $g(\theta)$ the difference between $c_H$ and $c_N$ for a given $\theta$, we have that

$$c_H = c_N + g(\theta), \quad c_L = c_N - f(\theta)$$

where we assume that

$$q_H g(\theta) - (1 - q_H) f(\theta) = 0$$

We also take $g(1/2) = f(1/2) = 0$ and $\lim_{\theta \to 0} g(\theta) = \lim_{\theta \to 0} f(\theta) = \infty$. Thus, for any given $\theta$, the expected cost is the same as the normal cost, and for $\theta = 1/2$ there is no risk associated with whether costs increase or decrease. This risk increases (in the sense that the variance of costs increases) as $\theta$ decreases. Finally we make the following technical assumption that we will use below

$$\lim_{\theta \to 0} \theta g(\theta) = \lim_{\theta \to 0} - \theta^2 g'(\theta) > 0$$

which stipulates that the riskiness of unforeseen contingencies increases sufficiently fast as $\theta$ goes to 0, so that the impact of the riskiness does not vanish in the limit.

To summarize, the parties face a risk at the time they contract that, as a consequence of an unforeseen contingency, the cost and value of the widget will be abnormally high or low at the time production and delivery are to take place. Ex-post, unforeseen contingencies will be recognized by both the court and the contracting parties. The court will assign a likelihood to the category of events similar to an unforeseen contingency, should one arise, and it will know the variance of costs associated with the unforeseen contingency, but not the actual payoffs to the parties. We assume that the parties cannot contract on $\theta$, the likelihood the court assigns to the
category associated with an unforeseen contingency. They can only rely on the court to be protected against the uncertainty associated with unforeseen contingencies (if this is what the court finds optimal to do).

This risk can be avoided by not contracting ex-ante, and simply contracting after the state is realized. So that there is a benefit to contracting ex-ante, we assume that the buyer can undertake an ex-ante, non-contractible, investment $e \in [0, 1]$ at a cost $\psi(e)$, where we assume that $\psi$ is convex, $\psi'(0) = 0$ and $\lim_{e \to 1} \psi'(e) = +\infty$. A buyer’s investment of $e$ increases the value to him of the widget of an amount $eR$. Consequently, if the buyer chooses the level of relationship-specific investment $e$ his value of the widget is $eR + \Delta + c_i$, where $i \in \{L, N, H\}$.

Since the buyer is risk-neutral, he maximizes expected profit, minus the convex cost of investment as above. The risk-averse seller maximizes the expected value of a strictly increasing, strictly concave and bounded above utility function $V(\cdot)$ such that $V'(\cdot) > 0$, $V''(\cdot) < 0$ and $V'''(\cdot) > 0$.\(^\dagger\)

The timing of the model can be specified as follows. The parties form beliefs about the court’s rule for enforcing or excusing performance, based on the court’s past record (that is, based on the precedents). Negotiation then takes place between the contracting parties. Recall that the buyer has all the bargaining power at this stage, hence negotiation is a simple take-it-or-leave-it offer of a contract from the buyer to the seller. A contract may specify an ex-ante transfer; if it does, the transfer is made immediately after a contract is agreed upon.\(^\dagger\) After the negotiation of an ex-ante contract the buyer chooses the level of specific investment $e$ that increases the value of the widget to him by $eR$.

\(^\dagger\)We use $V'(\cdot)$, $V''(\cdot)$ and $V'''(\cdot)$ to denote respectively the first, second and third derivative of the seller’s utility function $V(\cdot)$. Recall that $V'''(\cdot) > 0$ is a necessary condition for $V(\cdot)$ to exhibit decreasing absolute risk-aversion.

\(^\dagger\)Notice that if the transfer were “refundable” if the contract is voided, then we could simply incorporate it in the trade price that the contract specifies. Hence a non-refundable ex-ante transfer like the one we consider allows for a richer set of possible contracts.

With respect to the actual behavior of courts, it is argued that when courts determine that contracts should not be enforced as written, “...parties will be permitted to walk away from their bargain, without damages for reliance or restitution for benefits conferred” (Kull 1991).
The state of nature $\omega$ is then realized and is observed by both parties to the contract. Moreover, if $\omega \in \Theta$ we also assume that the parties to a contract observe the exact value of the cost $c_i$, $i \in \{L, H\}$. Should the court become involved, it categorizes the unforeseen contingency, and assigns a likelihood $\theta$. Either party can bring the other side to court, and if this occurs, the court is assumed to mandate or excuse performance consistent with past rulings.

In the case in which the court decides to void the existing contract, renegotiation takes place between the buyer and the seller. Renegotiation is modelled as a take-it-or-leave-it offer from the seller to the buyer of a price at which to trade. When renegotiation occurs, following the court’s decision to void the contract, the parties’ outside options are represented by the payoffs associated with no trade. These payoffs are normalized to zero.

Finally, trade occurs according to the terms of the original contract, if the court decides to enforce it, or according to the terms of the renegotiated agreement, if the court decides to void the original ex-ante contract.

### 3. The Optimal Ex-ante Contract

Given our assumptions above the parties to a contract can only specify in an ex-ante contract a constant price at which to trade $p$, and an ex-ante transfer from the buyer to the seller $t$. If the parties decide to draw up such an ex-ante contract, it is then left to the court to determine whether or not to protect them against the possibly very large risk associated with the unforeseen contingencies $\Theta$.

We identify the optimal court’s ruling solving the model backwards from the last stage. We begin with the renegotiation that follows the court’s decision to void the contract. Denote $\hat{e}$ the given level of investment chosen by the buyer. Since the seller has all the bargaining power at the renegotiation stage he will receive all the gains from trade available to the parties; these of course total $\hat{e} R + \Delta$.

Consider now the court’s decision if one of the two parties brings the other to court. In the event of an unforeseen contingency, the court assigns a likelihood to that category of contingency. Hence, we can specify the court’s decision rule without
loss of generality to be such that if \( \omega \in \Theta \) the court enforces all contracts when \( \theta \in \mathcal{E} \) where \( \mathcal{E} \subseteq [0, 1/2] \). In other words when the probability of the unforeseen contingency is within predetermined bounds, the court provides the parties with insurance by voiding the existing contract.

As we will see in Lemma 1 below, it will never be optimal for the court to void a contract if it observes \( \omega \notin \Theta \). However, the general specification of the court’s decision rule must allow for this possibility. Without loss of generality again, we specify the court’s decision rule as follows. If \( \omega \notin \Theta \) the court enforces the contract only when \( \theta \in \mathcal{N} \), where \( \mathcal{N} \subseteq [0, 1/2] \). In other words the court may void the parties’ contract when \( \omega \notin \Theta \). Moreover this decision to void may, in principle, depend on the value of \( \theta \) which the court assigns.

The court determines \( \mathcal{E} \) and \( \mathcal{N} \) prior to the parties’ negotiation of the ex-ante contract. The parties infer the court’s decision rule from precedents when they decide which ex-ante contract to draw up.

Before we analyze the parties’ negotiation of the ex-ante contract, we need to specify the seller’s and buyer’s outside options if the ex-ante negotiation breaks down. Notice that even in the absence of an ex-ante contract the parties can still trade the widget ex-post. As we mentioned above, in any ex-post negotiation the seller has all the bargaining power. Hence, in any ex-post agreement he appropriates all the gains from trade and receives utility \( V(\bar{e} R + \Delta) \), where \( \bar{e} \) is the level of specific investment chosen by the buyer in the absence of any ex-ante contract. The buyer receives a zero share of the gains from trade.

Notice that the advantage for the parties to trade ex-post is that they do not face any uncertainty, and therefore the seller is provided with full insurance. However, since the returns to the buyer from his ex-ante investment are zero, he will choose an investment level such that \( \psi'(\bar{e}) = 0 \). In other words, when trade takes place ex-post because there is no ex-ante contract the buyer has no incentive to invest: \( \bar{e} = 0 \). We can then conclude that, in the absence of an ex-ante contract the buyer’s

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18 Of course, \( \mathcal{E} \) is assumed to be a Lebesgue-measurable set.
19 Of course, \( \mathcal{N} \) is also assumed to be a Lebesgue-measurable set.
payoff is 0 while the seller’s level of utility is $V(\Delta)$. The seller is fully insured but no relationship-specific investment is undertaken by the buyer. The buyer’s outside option when the ex-ante contract is negotiated is 0, while the seller’s outside option is $V(\Delta)$.

Next, we turn to the parties’ negotiation of the ex-ante contract. Recall that ex-ante the buyer makes a take-it-or-leave-it offer to the seller of a contract $(p,t)$. Given the court’s decision rule, and a level of investment $\hat{e}$, the seller’s expected utility associated with $(p,t)$ can now be written as follows.

$$V^*(p,t) = \int_{E} \left[ q_H V(p + t - c_H) + (1 - q_H) V(p + t - c_L) \right] 2 d\theta + \int_{\left[\left[0, \frac{1}{2}\right] \setminus \mathcal{E} \right]} \theta V(\hat{e}R + \Delta + t) 2 d\theta + \int_{N} (1 - \theta) V(p + t - c_N) 2 d\theta + \int_{\left[\left[0, \frac{1}{2}\right] \setminus \mathcal{N} \right]} (1 - \theta) V(\hat{e}R + \Delta + t) 2 d\theta$$

Notice that the first integral in (4) refers to the case in which $\omega \in \Theta$ and the contract is upheld by the court. The second integral in (4) captures those cases in which $\omega \in \Theta$ and the court voids the ex-ante contract, while the last two integrals in (4) cover the cases in which $\omega \notin \Theta$, both when the contract is upheld and when it is voided.

Using (2) above and taking again as given the court’s decision rule and a level of investment $\hat{e}$, the buyer’s expected profit associated with $(p,t)$ can be computed as follows.

$$B^*(p,t) = \int_{N} (1 - \theta) [\hat{e}R + \Delta + c_N - p] 2 d\theta + \int_{E} \theta [\hat{e}R + \Delta + c_N - p] 2 d\theta - t - \psi(\hat{e})$$

Notice that if we set $\bar{\theta}_N = \int_{N} (1 - \theta) 2 d\theta$ and $\bar{\theta}_E = \int_{E} \theta 2 d\theta$, the payoffs in (4) and
(5) can be rewritten more simply as

\[
V^*(p, t) = \int E \left[ q_H V(p + t - c_H) + (1 - q_H) V(p + t - c_L) \right] 2 \, d\theta + \bar{\theta}_N V(p + t - c_N) + (1 - \bar{\theta}_N - \bar{\theta}_E) V(\hat{e}R + \Delta + t)
\]

and

\[
B^*(p, t) = (\bar{\theta}_N + \bar{\theta}_E) \left[ \hat{e}R + \Delta + c_N - p \right] - t - \psi(\hat{e})
\]

From (7) it is immediate that given \((p, t)\) and the court’s decision rule the buyer will select a level of relationship-specific investment \(\hat{e}\) such that

\[
\psi'(\hat{e}) = (\bar{\theta}_N + \bar{\theta}_E) R
\]

We can now state the buyer’s optimization problem for choosing an ex-ante contract. Given the court’s decision rule, the buyer’s take-it-or-leave-it offer to the seller is the solution, if it exists, to the following problem.

\[
\max_{p, t} \left( \bar{\theta}_N + \bar{\theta}_E \right) \left[ \hat{e}R + \Delta + c_N - p \right] - t - \psi(\hat{e})
\]

\[
\text{s.t. } V^*(p, t) \geq V(\Delta) \quad B^*(p, t) \geq 0 \quad \psi'(\hat{e}) = (\bar{\theta}_N + \bar{\theta}_E) R
\]

where the first two constraints guarantee that it is optimal for both the seller and the buyer to sign an ex-ante contract rather than to trade ex-post. If the feasible set of problem (9) is in fact empty, then no ex-ante contract will be signed and trade will take place ex-post. However, when the court’s decision rule is chosen so as to maximize the parties’ welfare an ex-ante contract will be signed.

**Remark 1:** For some specifications of the court’s decision rule the feasible set of problem (9) is clearly not empty.
For example, suppose that the court never voids the contract if \( \omega \not\in \Theta \), that is, \( \mathcal{N} = [0, 1/2] \). Suppose further that the court always voids the contract if \( \omega \in \Theta \) so that \( \mathcal{E} = \emptyset \). In this case, the agents do not face any uninsurable risk from unforeseen contingencies, and can take advantage of a fixed price for the case \( \omega \not\in \Theta \) so that the buyer will undertake a positive amount of relationship-specific investment. It is clear that in this case there is an ex-ante contract that is preferred to no contract by both the buyer and seller.

Finally, notice that when an ex-ante contract is preferred to trading ex-post it is immediate by standard arguments that the solution to problem (9) is in fact unique.

Notice that if the court’s decision rule is such that \( (\bar{\theta}_N + \bar{\theta}_E) = 0 \) we obtain a trivial special case. When \( (\bar{\theta}_N + \bar{\theta}_E) = 0 \) the court voids the contract for every \( \omega \). Therefore, in this case the expected profit of the buyer is 0 and the expected utility of the seller is \( V(\Delta) \), whatever the contract \( (p, t) \). In this case, since both parties are indifferent, we assume that they prefer to implement the same outcome by having no contract at all.

Our characterization of the optimal contract \( (p^*, t^*) \) given the court’s decision rule can now be summarized as follows.

**Proposition 1**: Let a decision rule for the court be given, and assume that it is such that it is optimal for the parties to draw up an ex-ante contract. Let the optimal ex-ante contract be denoted by \( (p^*, t^*) \). Then \( p^* \) and \( t^* \) satisfy

\[
\bar{\theta}_N V'(p^* + t^* - c_N) + \\
+ \int_{\mathcal{E}} \theta [q_H V'(p^* + t^* - c_H) + (1 - q_H) V''(p^* + t^* - c_L)] \, d\theta = \]

\[
(\bar{\theta}_N + \bar{\theta}_E) V'(\bar{e}R + \Delta + t^*)
\]

and hence

\[
p^* - c_N \geq \bar{e} R + \Delta
\]
Moreover the transfer $t^*$ is such that

$$V^*(p^*, t^*) = V(\Delta)$$

(12)

The fact that (12) of Proposition 1 must hold is a simple consequence of the fact that the seller’s expected utility is increasing in $t$, while the buyer’s expected profit is a decreasing function of $t$.

The intuition behind (10) and (11) of Proposition 1 is not hard to explain. Recall that the uninsurable risk embodied in the variation between $c_L$ and $c_H$ has mean zero. Moreover, in those states in which the contract is renegotiated, the seller necessarily gets a pay-off (on top of the transfer $t$) of $\hat{e} R + \Delta$. The price $p^*$ is chosen so as to provide the seller with the optimal partial insurance against the fluctuations of cost between $c_L$ and $c_H$. Of course, this means equating the seller’s expected marginal utility in these two eventualities with the seller’s marginal utility that he achieves when the contract is voided by the court. Since the seller’s marginal utility is a convex function this implies that the price $p^*$ minus the average of $c_L$ and $c_H$ must be above $\hat{e} R + \Delta$.

4. The Court’s Optimal Decision Rule

We are now equipped with the characterization (Proposition 1 above) of the optimal contract $(p^*, t^*)$ given an arbitrary decision rule for the court. This is enough to proceed to characterize the court’s optimal decision rule.

Recall that our court is a “Stackelberg leader.” Through precedents, its decision rule is effectively announced to the parties. Taking into account the effect of its choice of rule on the parties’ behavior, the court then acts so as to maximize their welfare. From Proposition 1 we know that, as a result of the fact that the buyer makes a take-it-or-leave-it offer of an ex-ante contract to the seller, the seller’s expected utility will be $V(\Delta)$, regardless of the court’s decision rule. Therefore, the court’s decision rule can be characterized as the solution to the problem of maximizing the buyer’s expected profit subject to appropriate constraints.
The court’s maximization problem can be written as follows. Choose two Lebesgue-measurable sets $E$ and $N$ so as to solve

$$
\max_{N, E} \left( \bar{\theta}_N + \bar{\theta}_E \right) \left[ \hat{e}R + \Delta + c_N - p^* \right] - t^* - \psi(\hat{e})
$$

s.t. \quad V^*(p^*, t^*) \geq V(\Delta) \tag{13}

where $B^*(p^*, t^*) \geq 0$

$\psi'(\hat{e}) = (\bar{\theta}_N + \bar{\theta}_E) \ R$

where $(p^*, t^*)$ is the optimal ex-ante contract characterized in Proposition 1 above.\(^{20}\)

We begin with two partial characterizations of the court’s optimal decision rule. Our first claim asserts that, provided a solution to problem (13) exists, it will be such that the court never voids the parties’ ex-ante contract when $\omega \notin \Theta$. In other words, it is never optimal for the court to void the contract if the contingency that occurs is not an unforeseen one.

**Lemma 1:** It is optimal for the court to enforce the contract whenever $\omega \notin \Theta$. More formally, assume that a solution to problem (13) exists. Then any solution to this problem will satisfy

$$
N = [0, 1/2]
$$

up to a set of $\theta$s of Lebesgue-measure zero.

The intuition behind Lemma 1 is very simple to outline. The court’s decision to void the contract provides the parties with insurance against unforeseen contingencies. If a contingency $\omega \notin \Theta$ that is not unforeseen arises, it is optimal for the court to enhance the buyer’s incentives to undertake the relationship-specific investment by enforcing the ex-ante contract.

We now turn to a further partial characterization of the court’s optimal decision rule. We are concerned with the “shape” of the court’s optimal decision rule for those $\omega$ that are in $\Theta$. We first show that this part of the court’s optimal decision rule

\(^{20}\)Obviously, the contract $(p^*, t^*)$ depends on the pair of sets $(N, E)$ that describe the court’s decision rule. We suppress this from the notation since doing so does not cause any ambiguity.
rule consists of a *threshold* level $\theta^*$. The court will void the ex-ante contract when $\theta < \theta^*$ is observed, and will uphold the ex-ante contract otherwise.

**Lemma 2:** Assume that a solution to problem (13) exists. Then any solution to this problem has the feature that there exists a $\theta^* \in [0, 1/2]$ such that the court will enforce the ex-ante contract if $\omega \in \Theta$ and $\theta \geq \theta^*$ and will void it if $\theta < \theta^*$. That is, $\mathcal{E} = [\theta^*, 1/2]$, up to a set of $\theta$s of Lebesgue-measure zero.

The intuition behind this second partial characterization of the optimal court decision rule can be described as follows. The court is trading off the insurance it provides to the parties when it voids the contract with the decrease in incentives to invest that results from voiding. Incentives are adversely affected because when the court voids, at the margin, the buyer will not receive a full return from his investment. Hence, the higher the probability that the court voids, the lower is his incentive to invest. This negative effect on investment depends only on the probability that the court will void the contract. On the other hand, the value of the insurance to the parties from voiding is greater when $\theta$ is smaller, since, by assumption, the spread between $c_L$ and $c_H$ becomes higher as $\theta$ becomes smaller. Hence, whatever decrease in incentives is accepted, the optimal thing for the court to do is to void for the smallest values of $\theta$. In other words, whatever the overall probability that the court voids the ex-ante contract, the set of values of $\theta$ for which the contract is in fact voided must take the “threshold” form described in Lemma 2.

We have now all the elements to complete the characterization of the court’s optimal decision rule.

According to Lemma 1 the optimal court’s policy is to enforce every contract if $\omega \notin \Theta$, while, according to Lemma 2, if an unforeseen contingency occurs so that $\omega \in \Theta$ it is optimal for the court to use a threshold $\theta^*$ and void the contract if $\theta < \theta^*$ and uphold it if $\theta \geq \theta^*$. We summarize our characterization of the court’s optimal decision rule in Proposition 2 below. Aside from incorporating the content of Lemmas 1 and 2, Proposition 2 asserts that an optimal decision rule for the court
does in fact exist, that it is unique up to a set of $\theta$s of Lebesgue measure zero, and that the threshold $\theta^*$ used by the court is interior in the sense that $0 < \theta^* < 1/2$.

**Proposition 2:** An optimal decision rule for the court exists and it is unique up to a set of $\theta$s of Lebesgue-measure zero.

The court’s unique optimal decision rule can be described as follows. The court upholds the ex-ante contract if a contingency $\omega \notin \Theta$ occurs. On the other hand, if an unforeseen contingency arises, $\omega \in \Theta$, the court voids the contract if $\theta < \theta^*$ and upholds the contract if $\theta \geq \theta^*$, where $\theta^*$ is a threshold value in the open interval $(0, 1/2)$.

We have already outlined the intuition behind part of the characterization of the court’s optimal decision rule presented in Proposition 2. To understand why the threshold $\theta^*$ used by the court cannot be either 0 or 1/2 it is enough to refer back to the specification of the risk that the unforeseen contingencies entails that we described in Section 2 above. Recall that, essentially, as $\theta$ approaches 1/2 the unforeseen contingencies entail a negligible amount of risk for the contracting parties. As $\theta$ approaches 1/2, both $c_L$ and $c_H$ approach $c_N$. Therefore, as $\theta$ becomes larger the benefits of voiding the ex-ante contract shrink to zero as the value of the insurance that voiding provides shrinks to zero. On the other hand, the costs of voiding the ex-ante contract do not vanish as $\theta^*$ approaches 1/2. Indeed, the marginal cost (in terms of diminished incentives for the buyer to undertake relationship-specific investment) of increasing $\theta^*$ (the threshold for voiding the ex-ante contract) increases with the size of $\theta^*$. Therefore it will be optimal to enforce the ex-ante contract when a $\omega \in \Theta$ and a $\theta$ sufficiently close to 1/2 is observed.

Consider now the nature of the risk associated with the unforeseen contingencies for small $\theta$, approaching 0. As we specified in Section 2 above, the difference between $c_L$ and $c_H$ becomes unboundedly large as $\theta$ approaches zero. Therefore, in this case the tradeoff between incentives and insurance is exactly reversed from the case of $\theta$ approaching 1/2 that we have just described. As $\theta$ approaches 0, the gain in incentives from upholding the ex-ante contract is bounded above (it can never exceed $R$) while
upholding the ex-ante contract becomes more and more costly as the parties are faced with an ever increasing amount of uninsurable risk.

We conclude this section with an observation that is a direct consequence of Proposition 2. Recall that it is always possible for the court to adopt a decision rule that will make both contracting parties indifferent between drawing up an ex-ante contract and having no contract at all. This is the trivial case in which the court’s decision rule is to void the ex-ante contract for every state \( \omega \in \Omega \). Since the court’s optimal decision rule characterized in Proposition 2 is \textit{not} to void the ex-ante contract for every possible \( \omega \in \Omega \) it immediately follows that an ex-ante contract is strictly preferable to having no contract at all. Indeed, the buyer’s expected profit, given the court’s optimal decision rule, is strictly above 0 (the expected profit of the buyer when the parties do not draw up an ex-ante contract at all).

Before we turn to some more general concluding remarks, we discuss two features of our model that may be generalized.

We have assumed that the likelihood of an unforeseen contingency \( \theta \) also determines its degree of risk (see equation (1) above). This is a polar case that simplifies the analysis considerably. However, the qualitative nature of our results would be completely unaffected if we considered the more general case in which the likelihood of the unforeseen contingency is imperfectly correlated with its riskiness. In this case the optimal rule for the court would take into account the likelihood parameter \( \theta \) only as an imperfect signal of the associated risk. As above, the optimal court rule would then be determined as the one yielding an optimal trade-off between the parties’ need for insurance and incentives.

Finally, we have only considered the case in which the court does \textit{not} observe even the “direction” of the change in the widget’s cost and value. An alternative specification of the model would enable the court to distinguish between the two cases of a negative and a positive change (still without being able to condition its actions on the actual cost and value of course). In this case, clearly the court could use different thresholds (according to whether the cost and value has increased or decreased) in its decision to void or uphold the contract. Again, the qualitative nature of our results
would be unaffected by this change: the court would still void contracts when \( \theta \) is low and hence risk is high, and uphold contracts when \( \theta \) is high and risk is low. In the case in which an increase and a decrease in cost are equally likely, the comparison between the two thresholds used by the court is in fact entirely pinned down by our assumption that the seller’s absolute risk-aversion is decreasing. The court will void more often in the case in which the cost increases since this impacts the seller’s utility proportionately more and hence the need for insurance is higher.

5. Concluding Remarks

We have taken a particularly simple specification of the court’s strategy set and of its preferences. We will discuss each of these and how it relates to our analysis above.

There is a sense in which any restrictions (except for strictly physical ones) on the court’s strategy set take us back into a “partial equilibrium” approach. If there are restrictions on the court’s strategy set, who put them there if the model is truly a closed one? This paper is but one step in the direction of a model that is truly closed in this sense. \(^{21}\)

Once we take the view that some external considerations must be taken as given, it is easy to see why our modelling choice of a “simple” strategy set for the court is plausible. Courts typically face a large pool of possible disputes, and have very little prior specific knowledge about each case. It is clearly efficient to develop court procedures that are “detail free” wherever possible in the sense of being robust to even large variations in the parameters characterizing the situations to which they apply. Our courts that can only void or uphold contracts rather than dictate new terms of trade are a simple way to capture some of these considerations.

The restricted strategy space for the court that we have worked with in this paper can also be interpreted as a crude way to model the effects a richer domain for the

\(^{21}\)In a different context — the design of a “legal system” for society as a whole — Mailath, Morris, and Postlewaite (2000) explore a model in which all “laws” are cheap-talk. They find that the role of the legal system in this case is limited to selecting among the multiple equilibria of the game determined by the physical description of the environment. See also the discussion in Schwartz and Watson (2000).
preferences of the court. In particular it is clear that in a dynamic world courts must care about the reputation they accumulate about their rulings. In the static analysis above, precedents are assumed to be equivalent to the court announcing to the parties the rule that it will use in case of a dispute. In a richer dynamic model this would be substituted by the reputation that the court has. At this point the rationale for simple behavior becomes, again, apparent. In practice, simple rules will have greater “penetration” as the reputation of the court among the pool of (possibly simple-minded) contracting parties who might take their disputes before the court.

One might ask whether courts are necessary to insure against unforeseen contingencies. Should it not be possible for the parties to specify within the contract the nature of the events in which performance is to be excused? It is possible, and in fact common, for parties to specify within a contract that performance is to be excused in particular circumstances, for example, a force majeure clause. A contract provision that excuses one or both parties from part or all of their obligations in the event of war, natural disaster, or some other event outside the parties’ control. An example of such a clause found on the web is as follows.

Neither party shall be liable in damages or have the right to terminate this Agreement for any delay or default in performing hereunder if such delay or default is caused by conditions beyond its control including, but not limited to Acts of God, Government restrictions (including the denial or cancellation of any export or other necessary license), wars, insurrections and/or any other cause beyond the reasonable control of the party whose performance is affected.  

Such a clause would not eliminate the need for an analysis as in this paper. A half inch of snow is certainly outside the parties’ control, and it might slow down slightly a truck making a contractually agreed upon delivery, but a court would rule against a party seeking to be excused from performing on account of the snow. Unless courts

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implicitly or explicitly set a threshold for excusing performance, any party that would like a contract voided can always find some event that technically falls within the force majeure clause. A court that must determine whether a contractual disputes that center on a force majeure clause.

Our court increases efficiency by excusing performance in circumstances that are deemed to have been unforeseen, and further, that expose at least one of the parties to risk of substantial magnitude. The court, of course, is acting after all uncertainty has been resolved (even though the court does not know the realization). Hence, excusing performance at that date is simply a transfer from one party to the second. The increases in efficiency that stem from voiding a contract are a consequence of superior risk-sharing between future contracting parties. It follows, then, that it is not simply the voiding or enforcement of a contract that determines the efficiency gains, but the expectations induced in future contracting parties due to the court’s decision. If the court determines that performance is to be excused, there is still substantial scope for the court to affect expectations through its written decision. Future parties’ expectations will be quite different following narrowly written decisions than following broadly written decisions.

References


Appendix

Proof of Proposition 1: Consider the first order conditions associated with problem (9). After elementary manipulations we obtain that the following must hold.

\[ \tilde{\theta}_N V'(p^* + t^* - c_N) + \int_{\mathcal{E}} \theta [q_H V'(p^* + t^* - c_H) + (1 - q_H) V'(p^* + t^* - c_L)] 2 d\theta = (\tilde{\theta}_N + \tilde{\theta}_E) V'(\hat{e}R + \Delta + t^*) \]  

(A.1)

which of course proves (10).

The convexity of the seller’s marginal utility, \( V'''(\cdot) > 0 \), and (2) above imply that

\[ q_H V'(p^* + t^* - c_H) + (1 - q_H) V'(p^* + t^* - c_L) > V'(p^* + t^* - c_N). \]  

(A.2)

Substituting (A.2) into (A.1) yields

\[ V'(\hat{e}R + \Delta + t^*) > V'(p^* + t^* - c_N) \]

which together with the fact that \( V'''(\cdot) < 0 \) implies (11).

Finally, the fact that (12) must hold follows from the fact that the seller’s expected utility \( V^*(p, t) \) is monotonic increasing in \( t \) while the buyer’s expected surplus \( B^*(p, t) \) is monotonic decreasing in \( t \).

Proof of Lemma 1: We consider two possible cases. Either \( \mathcal{E} \) has measure zero, or it has positive measure. We will show that in both cases if \( \tilde{\theta}_N \) is below its upper bound of \( 3/4 \), the value of the maximand in problem (13) can be improved without violating any of the constraints by increasing the value of \( \tilde{\theta}_N \).

We start with the case in which \( \mathcal{E} \) has measure zero. In this case (A.1) simplifies to read

\[ V'(p^* + t^* - c_N) = V'(\hat{e}R + \Delta + t^*) \]  

(A.3)
and therefore we know that $p^* - c_N = \hat{e}R + \Delta$. This observation, together with the envelope theorem directly yield that the derivative of the Lagrangean of problem (13) with respect to $\bar{\theta}_N$ is equal to

$$\frac{(1 - \bar{\theta}_N) R^2}{\psi''(\hat{e})}$$

which is of course strictly greater than zero.

Now consider the case in which $E$ has positive measure. In this case, we can consider a marginal increase in $\bar{\theta}_N$ together with a (multiplicative) shift in the set $E$ so that $\bar{\theta}_N + \Delta$ remains constant.

In other words, let every point in $E$ be multiplied by a constant $k$ (less than or equal to one), and consider changes in $k$ that satisfy $dk/d\bar{\theta}_N = -1/\bar{\theta}_E$. Along this constraint, the derivative of the Lagrangean of problem (13) with respect to $\bar{\theta}_N$ is equal to

$$\frac{1}{V'(\hat{e}R + \Delta + t^*)} \int_E \theta \left\{ V(p^* + t^* - c_N) - [qH V(p^* + t^* - c_H) + (1 - qH) V(p^* + t^* - c_L)] \right\}^2 d\theta$$

The concavity of $V(\cdot)$, together with (2) obviously implies that this expression is strictly positive. This is enough to prove our claim.

**Proof of Lemma 2:** We proceed by contradiction. Assume there is a solution to the court’s problem (13) with the following features. The set $E$ has positive Lebesgue measure and there exists a $\theta^* \in [0, 1/2]$ such that $E = [\theta^*, 1/2]$ up to a set of $\theta$s of Lebesgue-measure zero. We will show that in this case there exists a new decision rule for the court that uses a set $E'$ such that a $\theta^*$ as above exists and such that, for a given contract $(p^*, t^*)$ as in Lemma 1, the buyer’s expected profit is unchanged while the expected utility of the seller has increased. Therefore if we switch the court’s decision rule from $E$ to $E'$ the expected profit of the buyer after the change in the optimal contract has been taken into account must increase. This will clearly suffice to prove our claim.

Let the contract $(p^*, t^*)$ be given and let $\theta'$ be such that

$$\bar{\theta}_{E'} = \int_{\theta'}^1 \theta 2 d\theta = \int_{\theta}^1 \theta 2 d\theta = \bar{\theta}_E$$  \hspace{1cm} (A.4)

The new decision rule for the court is simply defined as $E' = [\theta', 1/2]$.

Notice next that using (A.4) and (8) it is immediate that the buyer’s choice of investment is unchanged after the change in the court’s decision rule. Therefore, since we are keeping the contract constant at $(p^*, t^*)$, the buyer’s expected profit does not change after the change in the court’s decision rule.
Consider now the seller expected utility associated with the contract \((p^*, t^*)\) and the court’s policy \(E\). Using (6), Lemma 1 and recalling that \(\theta\) is uniformly distributed on \([0, 1/2]\), without loss of generality we can write this as

\[
\int_{E} \theta \left[ qH V(p^* + t^* - c_N - g(\theta)) + (1 - qH) V(p^* + t^* - c_N + f(\theta)) \right] 2 \, d\theta + \\
+ \frac{3}{4} V(p^* + t^* - c_N) + \left( \frac{1}{4} - \bar{\theta}_E \right) V(\hat{c}R + \Delta + t^*)
\]

\[(A.5)\]

Notice next that, using (A.4) above we have that the second and third term of the seller’s expected utility do not change if we change the court’s decision rule from \(E\) to \(E'\). Therefore, we focus on the first term of the seller’s expected utility and we let

\[
V(\theta) = qH V(p^* + t^* - c_N - g(\theta)) + (1 - qH) V(p^* + t^* - c_N + f(\theta))
\]

\[(A.6)\]

Now define the following three sets of values of \(\theta\)

\[
S_1 = \{ E \cap E' \}
\]

\[
S_2 = \{ E \cap ([0, 1/2] \setminus E') \}
\]

\[
S_3 = \{ ([0, 1/2] \setminus E) \cap E' \}
\]

\[(A.7)\]

We can then write the first term of (A.5) as:

\[
\int_{E} \theta \, V(\theta) \, d\theta = \int_{S_1} \theta \, V(\theta) \, d\theta + \int_{S_2} \theta \, V(\theta) \, d\theta.
\]

\[(A.8)\]

If we switch the court’s decision rule from \(E\) to \(E'\) the corresponding term becomes

\[
\int_{E'} \theta \, V(\theta) \, d\theta = \int_{S_1} \theta \, V(\theta) \, d\theta + \int_{S_3} \theta \, V(\theta) \, d\theta.
\]

\[(A.9)\]

Obviously, the quantity in (A.9) is strictly larger than the quantity in (A.8) if and only if

\[
\int_{S_3} \theta \, V(\theta) \, d\theta > \int_{S_2} \theta \, V(\theta) \, d\theta.
\]

\[(A.10)\]

Notice now that the function \(V(\theta)\) is monotonically increasing in \(\theta\). Indeed, using (2), \(g'(\theta) < 0\) and \(V''(\cdot) < 0\) we obtain:

\[
\frac{dV(\theta)}{d\theta} = -qH \ g'(\theta) \ [V'(p^* + t^* - c_N - g(\theta)) - V'(p^* + t^* - c_N + f(\theta))] > 0.
\]

\[(A.11)\]
We can now change the variable in the integrals in (A.10) using \( y = \theta^2 \) and denote
\[
\begin{align*}
Y_2 &= \{ y \mid y = \theta^2, \theta \in S_1 \} \\
Y_3 &= \{ y \mid y = \theta^2, \theta \in S_2 \}
\end{align*}
\]
Therefore the inequality in (A.10) is satisfied if and only if
\[
\int_{Y_3} \mathcal{V}(\sqrt{y}) \, dy > \int_{Y_2} \mathcal{V}(\sqrt{y}) \, dy.
\]
Notice next that using (A.4) we get that
\[
\int_{Y_3} dy = \int_{Y_2} dy
\]
Finally, recall that using (A.7) and (A.12) we know that for every \( y_2 \in Y_2 \) and every \( y_3 \in Y_3 \) we must have that \( y_3 \geq y_2 \). Therefore, using the fact that \( \mathcal{V}(\sqrt{y}) \) is an increasing function of \( y \) and (A.14), we can now deduce that (A.13) must hold, and therefore that (A.10) must hold as well.

Therefore, for a given contract \((p^*, t^*)\), the seller’s expected utility increases if the court changes her decision rule from \( \mathcal{E} \) to \( \mathcal{E}' \). Since we have already shown that this change leaves the expected profit of the seller unchanged, this is enough to prove our claim. ■

Proof of Proposition 2: Using Lemmas 2 and 1, we know that an optimal decision rule for the court exists, then, up to a set of \( \theta \)s of Lebesgue-measure zero it is a solution to the following problem.

\[
\begin{align*}
\max_{\theta^*} & \quad [1 - (\theta^*)^2] \left[ \hat{e} R + \Delta + c_N - p^* \right] - t^* - \psi(\hat{e}) \\
\text{s.t.} \quad & \int_{\theta^*}^{1/2} \theta \left[ q_H V(p^* + t^* - c_H) + (1 - q_H) V(p^* + t^* - c_L) \right] \, d\theta + \frac{3}{4} V(p^* + t^* - c_N) + (\theta^*)^2 V(\hat{e} R + \Delta + t^*) \geq V(\Delta) \\
& \psi'(\hat{e}) = [1 - (\theta^*)^2] \, R
\end{align*}
\]
Since a solution to problem (A.15) exists unique by standard arguments, this observation is sufficient to prove our existence and uniqueness claim.

Therefore, there only remains to show that the solution to problem (A.15) has \( \theta^* \in (0, 1/2) \). Let \( \mathcal{L}(\theta^*, \lambda, \mu) \) denote the Lagrangean associated with problem (A.15). The derivative of \( \mathcal{L}(\theta^*, \lambda, \mu) \) with respect to \( \theta^* \) can be written as
\[
\frac{\partial \mathcal{L}(\theta^*, \lambda, \mu)}{\partial \theta^*} = 2 \theta^* (p^* - c_N - \hat{e} R - \Delta) + 2 \theta^* \frac{V(\hat{e} R + \Delta + t^*) - \mathcal{V}(\theta^*)}{V'(\hat{e} R + \Delta + t^*)} - \frac{2 (\theta^*)^3 R^2}{\psi''(\hat{e})}
\]
where $V(\theta^*)$ is defined as in (A.6) above.

Consider now the value of the expression in (A.16) when $\theta^* = 1/2$. Recall that when $\theta^* = 1/2$ no risk is associated with the unforeseen contingencies under the contract $(p^*, t^*)$. This is because $g(1/2) = f(1/2) = 0$. Using the same method as in the proof of Lemma 1 we then immediately have that for $\theta^* = 1/2$ it must be the case that $p^* - c_N = \hat{e}R + \Delta$. Therefore we obtain that

$$\frac{\partial \mathcal{L}(1/2, \lambda, \mu)}{\partial \theta^*} = -\frac{R^2}{4 \psi'('e)} < 0$$

which directly implies that in the solution to problem (A.15) it is impossible that $\theta^* = 1/2$.

Consider now the limit of the expression in (A.16) as $\theta^*$ approaches zero. Using again the function $V(\cdot)$ defined in (A.6) above, we can write this limit as

$$\lim_{\theta^* \to 0} \frac{\partial \mathcal{L}(\theta^*, \lambda, \mu)}{\partial \theta^*} = \lim_{\theta^* \to 0} \frac{2 \theta^* V(\theta^*)}{V'(\hat{e} + \Delta + t^*)}$$

Since $V(\cdot)$ is concave and bounded above, using the fact that both $f(\cdot)$ and $g(\cdot)$ diverge to $+\infty$ when $\theta^*$ approaches zero we can conclude that the value of $V(\theta^*)$ diverges to $-\infty$. Using l'Hôpital’s rule we can then rewrite the limit above as

$$\lim_{\theta^* \to 0} \frac{2 \theta^* V(\theta^*)}{V'(\hat{e} + \Delta + t^*)} = \lim_{\theta^* \to 0} \frac{(\theta^*)^2 V'(\theta^*)}{V'(\hat{e} + \Delta + t^*)}$$

(A.17)

Substituting the expression for $V'(\cdot)$ that we computed in (A.11) into (A.17) yields that the sign of the limit in (A.17) is the same as the sign of

$$\lim_{\theta^* \to 0} -q_H (\theta^*)^2 g'(\theta^*) [V'(p^* + t^* - c_N - g(\theta^*)) - V'(p^* + t^* - c_N + f(\theta^*))]$$

(A.18)

Notice next that the term in square brackets in (A.18) remains positive as $\theta^*$ approaches zero. This is simply because both $f(\cdot)$ and $g(\cdot)$ diverge to $+\infty$ as $\theta^*$ approaches zero, and because $V''(\cdot) < 0$. Therefore the sign of the limit in (A.18) is the same as the sign of $\lim_{\theta^* \to 0} - (\theta^*)^2 g'(\theta^*)$. Therefore, using assumption (3) above we can now conclude that

$$\lim_{\theta^* \to 0} \frac{\partial \mathcal{L}(\theta^*, \lambda, \mu)}{\partial \theta^*} > 0$$

(A.19)

which directly implies that it is impossible that the solution to problem (A.15) has $\theta^* = 0$. ■