THE VALUE OF PRE-DECISION SIDE BETS FOR UTILITY MAXIMIZERS*

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If an alternative is attractive only at certain wealth levels of a decision maker, typically above some critical level, then there will be occasions when it is better to enter into an independent side bet prior to making a decision, than to accept or reject the alternative outright. (DECISION ANALYSIS; OPTIONS; RISK AVERSION)

Consider a decision maker with a logarithmic utility function for wealth, assets of $10,000, and an open offer to pay $5,000 for equally likely payoffs of either $15,000 or $0. Since \( \log 10,000 = \frac{1}{2} \log 20,000 + \frac{1}{2} \log 5,000 \), this decision maker is just indifferent between taking the deal and not. Suppose that, before he decides, he agrees with a friend on a side bet in which both stake $1,000 on the toss of a coin. A contingent strategy, in which our decision maker accepts the original deal if and only if he wins the side bet, proves to have higher expected utility than simply accepting or rejecting the deal outright:

\[
\frac{1}{2} [\frac{1}{2} \log 21,000 + \frac{1}{2} \log 6,000] + \frac{1}{2} \log 9,000 > \log 10,000.
\]

That our decision maker was initially indifferent about taking the deal is not an essential part of the story. This particular side bet enhances the expected utility of the decision maker for any initial asset level in the range $9,497 to $10,348. Therefore even when a deal is clearly attractive or clearly unattractive it may pay to see if a side bet could not make the opportunity even more valuable.

The Option Inherent in a Decision

The side bet strategy works in the neighborhood of any wealth level at which an alternative switches from being unattractive to being attractive. Thus the only utility maximizers for whom a side bet strategy can never be optimal are those with linear or exponential utility functions. Figure 1 illustrates the situation in the interval around the indifference point. Since the decision maker will accept a deal only if its certainty equivalent is positive, the value of this opportunity is zero below the indifference point and positive above. Instead of being satisfied with a zero value at the indifference wealth level, the decision maker can take advantage of the convexity of the value curve by randomizing his wealth level.

The “hockey-stick” shape of this value curve will be familiar to those who have studied financial options (e.g., Bookstaber 1981). It is important to understand that the contingent strategy arising from the side bet is of quite a different character than the usual option.

An option (as the term is typically used) derives its value from the owner’s ability to wait for resolution of some or all of the uncertainty concerning an alternative’s value before deciding whether to accept or reject it. Thus the mathematics of valuing options is identical to that of valuing information.

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In our example, however, value is created without any resolution of uncertainty concerning the underlying gamble. We artificially create uncertainty in the decision maker’s wealth in order to take advantage of the changing (usually decreasing) level of risk aversion (Pratt 1964) of the decision maker.

Such a device may have been overlooked because creating uncertainty seems, on the face of it, to be counterproductive for a risk averse person. Other authors (e.g., Kwang 1965) have found reasons for risk averse people to accept nonpositive expected value gambles but their explanations usually involve some nonlinearity in the underlying economics of the deal, such as higher returns for larger investments. In our example the deal never changes, only its value to the decision maker.

A very close analogy to our example comes from dynamic or multiperiod decision making where it has long been recognized that a risk averse person may act in a risk-prone manner in period 1, if other opportunities, also offered in period 1, need not be decided upon until period 2. The presence of these period 2 opportunities distorts the decision maker’s incentives in period 1. Our side bet strategy depends on exactly the same mathematics (see next section); the contribution of this note is to recognize that all decisions may be regarded as “period 2 decisions” and that appropriate “period 1 decisions” (side bets) may be constructed at will by the decision maker.

Prescriptive Implications

There are a number of potential objections to the idea of using side bets to improve the value of close calls. One could argue that side bets will be difficult to set up. After all this “friend” is probably risk averse too. But my example still works even with the disadvantageous odds normally available in casinos. Another objection would be that the time needed to set up the side bets isn’t normally available. Yet only Wall Street traders and respondents to utility assessment questionnaires need to respond immediately to offers.
There's not much evidence though of people randomizing their assets before undertaking major decisions and this will usually be wise. For one thing the gains tend to be small depending as they do on the third derivative of the utility function. Also they occur at all only when a deal is all or nothing: if it is possible to take any fraction of a deal the need for side bets disappears.

Even so there are a number of real-life situations where asset randomization seems intuitively sensible. For example, an entrepreneur with an idea she believes will work, but without financial backers who agree, would be justified trying to raise the necessary capital in Atlantic City. A graduate student, unsure whether he can afford the expense of a Ph.D., who is considering getting a job, might do well to gamble his remaining assets at the roulette wheel. If he succeeds he can pursue the Ph.D. without the distraction of money worries, if he fails he will be forced to get a job—as he likely would have done in the absence of the side bet.

From both mathematical and intuitive viewpoints the side bet deserves serious consideration as a solution technique of the decision analyst.

**Mathematical Details**

Suppose the decision maker has a utility function for wealth $u(x)$ and the offer of an investment opportunity with uncertain net return $\tilde{y}$. This investment will be acceptable at wealth level $x$ only if $v(x) = Eu(x + \tilde{y}) \geq u(x)$. Assume that there is a critical wealth level $x^*$ which separates two wealth intervals in which $\tilde{y}$ is and is not acceptable. (For a discussion of when there can be at most one such critical point see Bell 1987.) In the time period between the offer of $\tilde{y}$ and the subsequent decision, the decision maker's interim utility function is $\max (u(x), v(x))$. If $u$ and $v$ cross at $x^*$ then $u'(x^*) \neq v'(x^*)$ so that $\max (u(x), v(x))$ is locally convex at $x^*$ (see Figure 2). As Raiffa 1968 points out, a decision maker with any portion of her utility function convex should accept appropriate side bets to "concaveify" the function.

The asset levels $x_0$ and $x_1$ defined implicitly by the equations

$$u'(x_0) = v'(x_1) = (v(x_1) - v(x_0))/(x_1 - x_0)$$

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**Figure 2.** Utility as a Function of Wealth after an Offer But before a Decision.
and pictorially by the tangent common to both $u$ and $v$, are the desired final wealth levels for an ideal side bet. For an arbitrary wealth level $x$ in $(x_0, x_1)$ it is better to take a side bet offering a probability $(x - x_0)/(x_1 - x_0)$ of winning $x_1 - x$ and a probability $(x_1 - x)/(x_1 - x_0)$ of losing $x - x_0$ than to accept or reject $y$ immediately. This is because the expected utility of the contingent strategy is

$$[(x_1 - x)u(x_0) + (x - x_0)v(x_1)]/(x_1 - x_0) \quad \text{or} \quad u(x_0) + (x - x_0)(v(x_1) - u(x_0))/(x_1 - x_0)$$

which exceeds both $v(x)$ and $u(x)$.

In the example, $x_0$ and $x_1$ prove to be $9,029 and $10,780 respectively. The optimal side bet, a probability 0.554 at $780 and 0.446 at $971 leads to an expected utility of log 10,053, a gain of $53 over the status quo and $2 over that achieved by our ad hoc $1000 side bet.

References