Utility and Risk Preferences

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ABSTRACT. Utility theory has been a primary method used for the analysis of decision making under uncertainty for the last 60 years. We describe the rationale and uses of the approach in the case of decisions involving money. Our focus is the question of which particular utility function should be used by a decision maker.

Introduction

Playing games of chance for money is an activity that has diverted people for at least 6 millennia, so it is reasonable to suppose that an interest in how one should think about maximizing wealth under uncertainty has existed at least as long.

The starting point for all students of decision analysis is Expected Monetary Value (EMV) (Raiffa 1968). If there are \( i \) possible courses of action under consideration, and \( n \) possible outcomes \( x_1, \ldots, x_n \), and if \( p_{ij} \) is the probability that action \( i \) leads to monetary outcome \( x_j \), then EMV suggests that one select the action \( i \) that maximizes \( \sum_j p_{ij}x_j \). The justification for this criterion – and it’s a good one – is that any other criterion, if used repeatedly, will assuredly (almost certainly) leave you less well off at the end. Though few people get to repeat the same gamble over and over again, EMV is still relevant in our lives because we often make decisions involving financial uncertainty, and using EMV to make them will almost certainly be the right criterion for choice.

There are two very notable exceptions when EMV should not be used. The first is when payoffs are multiplicative rather than additive. In decision analysis, as opposed to other fields such as finance, it is usual to think of uncertain payoffs as incremental to existing wealth, \( w + \xi \), as opposed to multiplicative, \( w\xi \). The latter type we will refer to as “investments.” Using EMV to select among investments would be a big mistake. The investment offering a 50–50 chance of either 1.50 (a 50 percent return) or 0.60 (a 40 percent loss) has a positive EMV, but if it is accepted repeatedly it will almost certainly lead you to ruin (because \( 1.50 \times 0.60 < 1 \)). Only investments for which \( E\log\xi > 0 \) have the property that repetitions will lead inexorably to profit. The second notable exception – and our primary focus in this article – is that EMV is not appropriate “when risk is an issue.” The caveat refers to situations where repetition is not a realistic option for overcoming an early bad outcome. The argument that a 50–50 bet between +$50,000 and –$20,000 will
make you rich if repeated often enough is not relevant if a loss at the first iteration makes you financially unable or unwilling to continue.

The problem is not necessarily that as you become poorer you become “more risk averse,” because even a decision maker with constant risk aversion would not use EMV. Rather it is because gains and losses are not equivalent. Gaining $100,000 is a delight, losing $100,000 is painful. For most people, the downside pain is greater than the upside delight. No scholar, to my knowledge, has managed to argue that there is a precise relationship between the value of gains and monetarily equal losses. It is generally accepted that the relationship varies between individuals, varies for one individual as his or her wealth changes and even, though this is less well studied, as his or her “attitude to life” changes.

In this chapter our focus will be on advising the “rational actor”; a decision maker not buffeted by psychological distractions such as regret (Bell 1982; Loomes and Sugden 1982), disappointment (Bell, 1983; Loomes and Sugden 1986), or anxiety (Wu 1999). Although these can be real concerns to real people, if they are genuine concerns that a decision maker wishes to seriously avoid they can be accommodated by multiattribute theory (Keeney and Raiffa 1976).

**Stochastic Dominance**

If \( \tilde{x} \) and \( \tilde{y} \) are resolved by some common uncertainty such that with probability \( p_i \) the outcomes are \( x_i \) and \( y_i \), respectively, and if \( x_i \geq y_i \) then surely no rational person would prefer \( \tilde{y} \) over \( \tilde{x} \). Clearly \( \tilde{x} \) dominates \( \tilde{y} \). One alternative is said to stochastically dominate another if for all sums of money, \( a, P(\tilde{x} \geq a) \geq P(\tilde{y} \geq a) \). But now suppose that the two are resolved independently, though the individual gambles have the same probabilities and outcomes as before. Even though \( x_i \geq y_i \) for all \( i \) is still true, it could happen that the actual outcome of \( \tilde{y} \) is superior to that of \( \tilde{x} \). A person who suffers regret might think that it was not so obvious that \( \tilde{x} \) still dominates \( \tilde{y} \). A rational actor should argue that if \( \tilde{x} \) is selected, the actual outcome of \( \tilde{y} \) is irrelevant and can be ignored.

**Strength of Preference**

It has been appreciated at least since the eighteenth century (Bernoulli 1738; Luce and Raiffa 1957) that if EMV fails because losing an amount of money is more painful than gaining it is good, then the analytic solution is to create a scale that reflects the real value of money to a person. People seem quite comfortable making judgments of the form “the increment from having $10,000 to having $20,000 is about as attractive as the increment from having $20,000 to $35,000.” The problem for scholars has been to validate these statements by connecting them to action. One solution is to use some yardstick like “work expended.” If you claim to be indifferent between working 6 weeks for $10,000 and working 10 weeks for $20,000, and also between 6 weeks for $20,000 and 10 weeks for $35,000,
I might conclude that the increments $10,000 \rightarrow 20,000$ and $20,000 \rightarrow 35,000$ had equal strength of preference.

The problem with the yardstick approach is that money and hours worked might interact; your distaste for working might increase more rapidly when rich because of the more attractive uses you have for leisure time. Nor does it help with decisions under uncertainty unless decisions involving uncertain numbers of hours worked can be solved via expectation.

Utility Theory

It was not until 1947 when John von Neumann and Oskar Morgenstern published the second edition of their book *Theory of Games and Economic Behavior* that a satisfying axiomatization of risky choice was offered to justify the use of value functions – which in this context we refer to as utility functions. Consider two axioms:

(i) Independence from Irrelevant Alternatives (IIA): If $A$ is the best alternative from among some set of alternatives, then $A$ is the best alternative from any subset of those alternatives (if the subset includes $A$).

(ii) Independence from Irrelevant States (IIS): If a particular outcome has the same probability of occurring for all alternatives in a set, then the best alternative in that set is independent of the level of the particular outcome.

IIA implies transitivity ($A > B$ and $B > C$ implies $A > C$). For if $A$ is the best from the set of alternatives $\{A, B, C\}$ then IIA implies $A > B$ and $B > C$. If $B > C$ then transitivity holds ($A > B, B > C$ and $A > C$) or if $C > B$ then transitivity holds ($A > C, C > B$ and $A > B$). Either way IIA $\Rightarrow$ Transitivity. If we further assume that the decision maker is able to select the best alternative for any set of alternatives (comparability) then IIA implies that there exists a rank order (simple order) of all alternatives (Krantz et al. 1971; Fishburn 1970).

Let us restrict ourselves for the moment to gambles having $n$ equally likely payoffs. Let $(x_1, \ldots, x_n)$ be the payoffs of such a gamble where the ordering of the $x$s is irrelevant because they are all equally likely. Let $\phi$ be the simple order implied by IIA and comparability. In this context the interpretation of IIS is that tradeoffs among, say, the payoffs $x_1, \ldots, x_j$ are independent of the values of the remaining payoffs $x_{j+1}, \ldots, x_n$. This is true for any subset of the attributes. By a well-known result for ordinal value functions (Debreu 1960; Gorman 1968a, 1968b) $\phi$ is additive; that is, for some functions $\Psi$, $u_1, \ldots, u_n$ we have

$$\Psi(\phi(x_1, \ldots, x_n)) = \sum_{i=1}^{n} u_i(x_i).$$

By symmetry of the $x$s (the ordering doesn’t matter because they are equally likely) we have $u_i = u_j = u$ for all $i$ and $j$. Hence, gambles may be ranked according to the quantity $\sum u(x_i)$, where $u$ is the utility function.
Because this argument applies for any $n$, the quantity $\sum_{i=1}^{n} p_i u(x_i)$ may also be used to compare gambles where $p_i$ are rational numbers. *Continuity* makes this criterion valid for all values of $p$.

**Uses of Utility Theory**

There are three kinds of applications for the theory. The first, most obviously, is to enhance the ability of a decision maker to act coherently. If there are many alternatives and many possible outcomes the cognitive load may be greatly simplified by first estimating an appropriate utility function, estimating the relevant probabilities and then applying the expected utility formula. Relative to EMV the only extra step is to estimate the utility function. If multiple decisions are to be made, the utility function need be estimated only once.

A second use of utility is as a means of delegating decisions to an agent. Just as I would not expect someone to acquire artwork on my behalf without a thorough discussion of my tastes, a financial agent should have some sense of my taste for gambles. The utility function is the information the agent needs.

Finally, and this is where the theory has found the most application, utility functions are useful for investigating general rules of behavior (e.g., Grayson 1960).

**Assessment of the Utility Function**

Nothing in the theory requires that different people have different utility functions. But people do react quite differently to the prospect of risk. Suppose you are given a lottery ticket that gives you a 50–50 chance of winning zero or $100,000. Would you rather accept the risk or trade the opportunity for a guaranteed $40,000? Some people will answer this one way, some people the other way. The answer, of course, tells us how $u(40,000)$ relates to $1/2 u(0) + 1/2 u(100,000)$.

By asking the decision maker to answer a carefully selected set of similar stylized questions, it is possible to estimate a person’s utility function. Farquhar (1984) gives a review of estimating procedures. The practical problem in assessment is that even if a decision maker agrees that IIA and IIS make sense, he or she may nevertheless give answers that are inconsistent with those axioms. If the answers do conflict then resolution can be obtained only by “fitting” the utility function as closely as possible to the stated preferences, or by confronting the decision maker with his inconsistencies and asking him to reconcile them.

A third way around this dilemma is to find ways to restrict the “degrees of freedom” the decision maker has in selecting a utility function. For example, if we discover that the decision maker’s preferences, although perfectly consistent with the theory, nevertheless suggest that $u(20,000) > u(30,000)$ we would feel justified in thinking something was wrong.

One frequent “error” in judgment is that people are very unreliable in their judgment about small gambles (Schlaifer 1962). A person might say that they would exchange a 50–50 gamble between zero and $10 for $4, and say they would
exchange a 50–50 gamble between zero and $10,000 for $3,500. No plausible utility function can capture that kind of risk attitude “in the small” and “in the large.” Schlaifer warned that it was inadvisable to rely on any assessments that were not in terms of sums of money of the order of the respondent’s assets.

**Risk Aversion**

It is to be expected that rational actors will prefer more money to less (u is increasing), and that risk is bad (u is concave). How does one measure risk aversion? If one’s wealth is w then a 50–50 gamble between zero and $100 is really a gamble between w and w + 100. If c(\hat{x}, w) is the certainty equivalent of \hat{x} when you have wealth w, that is, u(w + c(\hat{x}, w)) = Eu(\hat{x}) = w + \hat{x}, then risk aversion is equivalent to the statement that \( w + c(\hat{x}, w) < w + E(\hat{x}) \equiv w + \hat{x} \).

Pratt (1964) and Arrow (1971) discovered that a useful measure of (local) risk aversion (for small gambles) is \(-u''/u'\) because approximately,

\[
\hat{x} = c(w, \hat{x}) + \frac{1}{2} E(\hat{x} - \hat{x})^2 \left[ -u''(w)/u'(w) \right].
\]

In this expression the risk premium of \( \hat{x} \) at w, the difference between the EMV, \( \hat{x} \), and the certainty equivalent, c(\hat{x}, w), is proportional to the variance of the payoffs, E(\hat{x} - \hat{x})^2. The proportion depends on the individual (through \( u \)) and on the particular wealth level (through w).

**Constant Risk Aversion**

Although the degree of risk aversion might vary with wealth, a first approximation (generalizing EMV) might assume that risk aversion is independent of wealth. This has many practical advantages. First it means that in order to make a decision we do not need to know the decision maker’s wealth. Many people do not know their own wealth with any precision, and for an agent, the lack of knowledge may be worse. It also means that for repetitive situations (recall the EMV discussion) the same choice among gambles will always have the same answer. Pfanzagl (1959) was an early proponent that constant risk aversion made sense as a prescription (how people should behave) as much as it was a description (how people really behave). He showed that a person with constant risk aversion had to have a utility function that was either linear (zero risk aversion as a special case) or exponential; \( u(w) = -e^{-cw} \) where the constant \( c \) varied with the decision maker. Note that \(-u''(w)/u'(w) = c\) for the exponential.

If a decision maker believes that constant risk aversion is a good match for her preferences, then in principle, a single assessment is sufficient to nail down the value of \( c \).

**Relative Risk Aversion**

The von Neumann–Morgenstern axiomatization of decreasing marginal value [money gets incrementally less valuable as wealth increases] through gambles,
although elegant, is somewhat dissatisfying. The intuitive idea of strength of
preference has nothing (or seemingly nothing) to do with uncertainty. Are these
two concepts indeed one and the same?

If \( u(w) \) is the strength of preference function for money, then we may write
\( u(w) = u^*(v(w)) \). Sarin (1982) gives axioms that lead to the conclusion that
\( u(w) = v(w) \). Dyer and Sarin (1982) defined \textit{relative risk aversion} as the degree of risk
aversion of \( u \) relative to that implicit in \( v \). Bell and Raiffa (1988) argued that
relative risk aversion should be constant. Their notion is that any residual risk
aversion, over and above that expressible by a varying value function must be an
"intrinsic" distaste for risk. If a 50–50 gamble between, say, a 10 value-point gain
and a 7 value-point loss is just acceptable, then that should be true no matter what
the underlying decision problem (whatever wealth level if the uncertainty is about
money, or even uncertainty about health issues, so long as the gains and losses
have the same value implications). This hypothesis has been supported by Smidts
(1997).

Whether constant or not, it is of great interest to understand what factors
influence the level of relative risk aversion for a rational actor.

\textbf{Decreasing Risk Aversion}

It is clear that \( u \) should be increasing and exhibit risk aversion (\( u \) concave, or
\( c(\tilde{x}, w) < \tilde{x} \)) for all uncertain gambles and wealth levels \( w \). It also seems reason-
able to suppose that risk aversion should decrease as a person gets wealthier, that
is, \( c(\tilde{x}, w^*) < c(\tilde{x}, w) \) for \( w > w^* \).

If a person finds \( \tilde{x} \) just equivalent to a sure event \( c \) at wealth level \( w^* \), then at
wealth level \( w > w^* \) a decreasingly risk averse person should be more willing to
take a risk, so that now \( \tilde{x} \) becomes preferable to \( c \). This is equivalent to requiring
that the Pratt–Arrow measure \(-u''(w)/u'(w)\) is decreasing in \( w \), or that \( u''(w) <
\ u'(w)u''(w) \). Because \( u'(w) \) is positive, and \( u''(w) \) is positive, this means that \( u''(w) \)
has to be positive. A positive third derivative is not a guarantee of decreasing risk
aversion however.

How far might risk aversion decrease? Can we assume for example that
\( \lim_{w \to \infty} -u''(w)/u'(w) = 0 \)? I think the answer must be yes. For one thing, the
relevance of EMV depends on most people being approximately risk neutral for
small gambles, or similarly for any given wealth level there comes a point when
the stakes are small enough that a person becomes risk neutral, or nearly so. For
any gamble there must be some wealth level, no matter if very large, that makes
the person with that wealth level regard the gamble as essentially riskless.

\textbf{Contextual Uncertainty}

So far we have examined risk averse behavior assuming that \( w \) is known. Anyone
owning a car or home, or with job insecurity, knows that wealth is always uncertain.
The myriad of financial uncertainties that face us all may be termed \textit{contextual
uncertainty}. Surely as contextual uncertainties grow we should become more risk
averse, a condition described by Gollier and Pratt (1996) as risk vulnerability. Pratt and Zeckhauser (1987) proposed that if \( \tilde{w}, \tilde{x}, \) and \( \tilde{y} \) are probabilistically independent, and if \( \tilde{w} + \tilde{x} < \tilde{w} \) and \( \tilde{w} + \tilde{y} < \tilde{w} \) then also \( \tilde{w} + \tilde{x} + \tilde{y} < \tilde{w} + \tilde{y} \). In the special case where \( \tilde{x} = \tilde{y} \), so that one is just an independent repetition of the other, the condition implies that repetition can’t turn a bad gamble into a good one. Pratt and Zeckhauser term their condition proper risk aversion. It is hard to see why a rational actor would not want to satisfy this condition. Pratt and Zeckhauser show that any utility function that is a mixture of exponentials \( u(w) = \int_0^\infty g(s)e^{-sw}ds \) for any probability distribution \( g(s) \) is proper. This is an extraordinarily broad set of functions that includes, for example, the power and logarithmic functions.

Both risk vulnerability and proper risk aversion are based on the notion that adding uncertainty increases risk (Eeckhoudt et al. 1996). Another way to increase risk is by multiplying the size of an uncertainty. If \( k > 1 \) then \( k\tilde{x} \) will always be riskier than \( \tilde{x} \) for a risk averse person, that is, if \( Eu(w + \tilde{x}) < u(w) \) then \( Eu(w + k\tilde{x}) < Eu(w + \tilde{x}) \). Let us say that if \( \tilde{z}_1 \) and \( \tilde{z}_2 \) are independent but related risks in that \( \tilde{z}_2 = k\tilde{z}_1 \) for \( k > 1 \), then \( \tilde{z}_2 \) is a larger risk than \( \tilde{z}_1 \), and \( \tilde{z}_1 \) is a smaller risk than \( \tilde{z}_2 \).

Suppose there are two contextual uncertainties affecting wealth that complicate a decision maker’s choice between two further gambles \( \tilde{x} \) and \( \tilde{y} \). The contextual uncertainties are due to be resolved after the choice between \( \tilde{x} \) and \( \tilde{y} \) is made. It cannot harm and might be of definite benefit if either or both of the contextual uncertainties were to be resolved before the choice is made.

But what if only one of the contextual uncertainties can be resolved early? If one is larger than the other (in the sense described above) surely it would make sense to prefer resolving the larger rather than the smaller contextual uncertainty. Bell (1995b) showed that the only increasing, risk averse and decreasingly risk averse utility function to satisfy this condition is the linear plus exponential form, \( u(w) = w - be^{-cw} \) where \( b \) and \( c \) are positive constants.

This family of functions satisfies all of the properties that I would regard as requirements for a rational actor: in addition to being increasing, risk averse and decreasingly risk averse, it satisfies risk vulnerability, is proper, and in the limit is risk neutral.

**Measuring Riskiness**

When can it be said that one gamble is riskier than another? There are many studies of how people measure riskiness (e.g., Weber 1988), but for our rational actor any measure of riskiness should be compatible with utility theory. If EMV truly is an appropriate criterion “except when risk is an issue” it should be possible to express \( Eu(w + \tilde{x}) \) as some equivalent expression involving \( w, \tilde{x} \) and some measure of riskiness \( R(\tilde{x}) \), say \( f(\tilde{x}, R(\tilde{x}), w) \). We know that for small gambles the certainty equivalent of \( \tilde{x} \) is approximately \( \tilde{x} - \frac{1}{2}\sigma^2(-u'(w)/u'(w)) \), so that \( R(\tilde{w}) \) is approximately equivalent to the variance. Certainly the variance of a distribution, or its relation, the standard deviation, is a widely understood measure
of dispersion. Though variance is compatible with risk aversion “in the small,” it is rarely so “in the large” (Fishburn 1977). The quadratic utility function \( u(w) = aw^2 + bw + c \) has \( EU(w + \bar{x}) = aw^2 + bw + c + (b + 2aw)\bar{x} + a\bar{x}^2 + a\bar{x}^2 \), which is of the form \( f(\bar{x}, \sigma^2, w) \), but this is unsatisfactory not least because the quadratic is never decreasingly risk averse. The exponential may be written as \( EU(w + \bar{x}) = -\exp(-cw) \exp(-c\bar{x})E \exp(-c(\bar{x} - \bar{x})) \), which is in the right format, if we take \( E \exp(-c(\bar{x} - \bar{x})) \) as a measure of riskiness, but the exponential is not decreasingly risk averse.

Suppose \( \bar{x} \) is preferred to \( \bar{y} \) at wealth level \( w_1 \). Suppose that at a higher wealth level, \( w_2 \), the preference is now \( \bar{y} \) over \( \bar{x} \). Could it be that at an even higher wealth level, \( w_3 \), the decision maker might revert to preferring \( \bar{x} \) to \( \bar{y} \)? Most people think not. They reason that the switch in preference as wealth went from \( w_1 \) to \( w_2 \) implies that \( \bar{y} \) must be riskier than \( \bar{x} \); any further increase in wealth makes you even less risk averse, and this means \( \bar{y} \) remains preferred to \( \bar{x} \). But this presupposes that each decision maker has a (perhaps implicit) ranking of gambles according to riskiness and that this ranking is independent of wealth. As Bell (1988b, 1995a) shows, only a handful of utility functions satisfy this condition, and of those, only the linear plus exponential function is compatible with the other desiderata.

If EMV is the criterion of choice when wealth is high we might expect that any switches in preference between gambles should be in favor of the gamble with the higher EMV. That is, if \( \bar{x} > \bar{y} \) at \( w_1 \) but \( \bar{y} > \bar{x} \) at \( w_2 > w_1 \), then \( \bar{y} > \bar{x} \). This, too, is true for an increasing, risk averse and decreasingly risk averse utility function only if it is linear plus exponential.

In this case \( EU(w + \bar{x}) = a(w + \bar{x}) - \exp(-c(w + \bar{x}))E \exp(-c(\bar{x} - \bar{x})) \), which is of the form \( f(\bar{x}, R((\bar{x}), w) \text{ if we set } R(\bar{x}) = E \exp(-c(\bar{x} - \bar{x})) \). Thus, this is the only measure of riskiness that is compatible with utility theory (Weber and Milliman 1997).

**Utility for Consumption**

A closely related concept of utility is for consumption. So far, we have thought of \( w \) as an endowment to be used in a beneficial but unspecified manner for the decision maker over time. But now let us be a little more concrete and assume that the quality of life that results from spending \( x \) dollars \( i \) years from now is \( u_i(x) \). Making lots of simplifying assumptions we can connect the two kinds of utility functions by means of the maximization problem:

\[
    u(w) = \max_{x_1,\ldots,x_n} \sum_{i=1}^{n-1} u_i(x_i) + u_n\left(w - \sum x_i\right).
\]

To simplify still further we can reduce the consumption problem down to the relevant question of how much to spend this year:

\[
    u(w) = u_1(x) + u_2(w - x),
\]

where \( x \) is current consumption and \( w - x \) is “savings.” An interesting question, studied by Kimball (1990, 1993) is how current consumption is influenced by
Utility and Risk Preferences

uncertainty in savings. Because of the potential confusion between utility as we have been discussing it, and utility for consumption, I prefer to present Kimball’s analysis in terms of hedging, the intuition being the same.

Hedging

Suppose you face a 50–50 gamble between zero and \( \tilde{x} \). If a coin lands heads, you get \( \tilde{x} \), if it lands tails, you get nothing. Suppose you are permitted to hedge by modifying the gamble to \( \tilde{x} - h \) if heads and \( +h \) if tails (\( h \) could be a positive or negative amount). Kimball shows that for small risks the optimal \( h \) is approximately \( 1/4\sigma^2 u''(w)/u'(w) \). If \( \tilde{x} = 0 \) we would expect \( h \) to be negative. If \( u''(w) \) is negative then \( u''' \) needs to be positive. Kimball calls the quantity \( -u''(w)/u'(w) \) the degree of prudence. It is interesting to consider how prudence should vary with wealth.

One would certainly suppose that the more risk averse a person is the greater desire there will be to hedge. Similarly, we would expect \( h \) to increase as the riskiness of \( \tilde{x} \) increases. In the particular case of linear plus exponential we have the hedging problem:

\[
\max_h \frac{1}{2} (w - h - be^{c(w-h)}) + \frac{1}{2} (w + \tilde{x} + h - be^{-c(w+\tilde{x}+h)}) R(\tilde{x}),
\]

where \( R(\tilde{x}) = E e^{-c(\tilde{x}-\tilde{x})} \).

The optimal \( h \) is found by solving

\[
1 + bce^{-c(w-h)} = 1 + bce^{-c(w+\tilde{x}+h)} R(\tilde{x}),
\]

which implies

\[
w - h = w + \tilde{x} + h - \frac{1}{c} \log R(\tilde{x}), \quad \text{so} \quad h = \frac{\tilde{x}}{2} + \frac{1}{2c} \log R(\tilde{x})
\]

The optimal hedge is to transfer half the EMV plus a “prudence premium” of \( \frac{1}{2c} \log R(\tilde{x}) \).

Note that although the premium does increase with the riskiness of \( \tilde{x} \) it does not decline with \( w \). Should it decline with \( w \)? Certainly we would expect the decision maker to feel the urgency to hedge declining as she gets wealthy, but should the size of the optimal hedge change? Not necessarily. Because the hedge is EMV neutral, and costless, its only purpose is to reduce risk. The unique aspect of the linear plus exponential is that the relative riskiness of gambles is independent of wealth; so the optimal hedge is also independent of wealth, in this case.

Derived Utility

If one day your utility function is \( u(w) \), and the next day you are given an unresolved contextual uncertainty of \( \tilde{z} \), then your de facto derived utility function is \( Eu(w + \tilde{z}) \). That is true because you should pick \( \tilde{x} \) over \( \tilde{y} \) only if \( Eu(w + \tilde{x} + \tilde{z}) \geq Eu(w + \tilde{z} + \tilde{y}) \). As it happens the linear plus exponential family is consistent with contextual uncertainty, though the “\( b' \)” parameter changes from \( b \) to \( bE e^{-c\tilde{z}} \). But suppose the decision maker is to decide between \( \tilde{x} \) and \( \tilde{y} \) while simultaneously
owning an *unresolved* decision between \( \tilde{z}_1 \) and \( \tilde{z}_2 \). In this case the *derived utility function* may not even be concave (Bell 1988a). Added to these complexities, a rational actor should anticipate future, uncertain, additions to wealth.

Is there any realistic hope of identifying a suitable utility function? The answer lies with the original concept of EMV. EMV is commonly used as an approximate guide to action, even though it has limitations. Similarly, utility theory should be thought of as a guide to action in cases where “risk is an issue.” In this spirit there is a danger in making the theory overly complicated. For most applications an approximate utility function, perhaps even the linear plus exponential function, will be close enough.

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