Technical Note on the Economics of Incentive Alignment

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Introduction

In this technical note, we describe the newsvendor problem and characterize the solution for it. We will then use the newsvendor problem and its solution to illustrate incentive-alignment problems and solutions in supply chains. The newsvendor problem is a simple inventory model that has been well studied in operations research. See Production and Operations Analysis by Steven Nahmias, McGraw-Hill/Irwin, 2001 for a more detailed treatment of the newsvendor and other inventory models.

The Newsvendor Problem

Consider a vendor of newspapers. He has to decide how many newspapers to stock. He can purchase newspapers from the newspaper publisher at a wholesale price of \( w \) per copy, who, in turn, prints the newspapers at a cost \( c \) per copy with \( c < w \). He then retails the newspapers at a retail price of \( r \) with \( r > w \). Newspapers that cannot be sold because demand was less than the number of copies stocked have to be salvaged at \( s < c < w \) per copy. Demand for newspapers, \( D \), is uncertain and is assumed to be distributed with density function \( f(\cdot) \) and distribution function \( F(\cdot) \). If the demand, \( D \), exceeds the quantity of newspapers stocked, \( q \), then the newsvendor has an opportunity cost of understocking, \( c_u = r - w \), per copy of excess inventory. On the other hand, if demand is below the quantity stocked, the newsvendor incurs an overstocking cost of \( c_o = w - s \) per copy of newspaper that is overstocked. The newsvendor’s problem can then be written as

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We can solve for the optimal quantities from the first-order conditions because the second order conditions $s f(q) - r f(q) < 0$ imply that the newsvendor’s objective function is globally concave. From the first-order conditions, we get

$$\int_0^q sf(x)dx + \int_q^\infty rf(x)dx - wq = 0$$

and

$$\frac{r - w}{r - s} = F(q)$$

Equation 1 can be rewritten as

$$\frac{c_u}{c_u + c_o} = F(q)$$

This equation characterizes the optimal inventory policy for any arbitrary demand distribution.

**Explanation for the Sidebar on “The Economics of Incentive Alignment”**

The afore-mentioned sidebar uses a uniform demand function for simplicity and derives the following table (Table 1).

<table>
<thead>
<tr>
<th>Costs and Profits</th>
<th>Traditional Contract</th>
<th>Revenue-Sharing Contract</th>
<th>Markdown-Money Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail Price</td>
<td>$1.00</td>
<td>$1.00</td>
<td>$1.00</td>
</tr>
<tr>
<td>Printing Cost</td>
<td>$0.45</td>
<td>$0.45</td>
<td>$0.45</td>
</tr>
<tr>
<td>Wholesale Price</td>
<td>$0.80</td>
<td>$0.45</td>
<td>$0.80</td>
</tr>
<tr>
<td>Vendor’s Share of Revenue</td>
<td>100%</td>
<td>65%</td>
<td>100%</td>
</tr>
<tr>
<td>Compensation Vendor Receives for Unsold Copies</td>
<td></td>
<td></td>
<td>$0.60</td>
</tr>
<tr>
<td>Vendor’s Understocking Cost</td>
<td>$0.20</td>
<td>$0.20</td>
<td>$0.20</td>
</tr>
</tbody>
</table>

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Using the equations above, we can verify the “Traditional Contract” column in Table 1 quite easily. The retail price is $1.00, the wholesale price $.80, salvage value is $0, and the publisher’s printing cost is $.45. The vendor’s overstocking cost is thus $.80 and the vendor’s understocking cost is $.20. Since demand is assumed to be uniformly distributed between 100 and 200 copies, the vendor will stock such that $F(q) = \frac{2}{2 + 8}$. That is, the vendor will stock 120 copies. Since the wholesale price is $.80 and the printing cost is $.45, the publisher makes a margin of $.35 per issue or $42.00 ($0.35*120 copies) in profit. To calculate the vendor’s profits, we substitute parameter values into equation (1) and get
\[\int_{0}^{q}(rx+(q-x)s)f(x)dx + \int_{q}^{\infty}rqf(x)dx - wq =
\]$1.00*100 + \int_{100}^{120} (\frac{1}{100})*120 \frac{1}{100} dx + \int_{120}^{200} \frac{1}{100}$dx - .80*120 = $22.00

Applications of the Newsvendor Problem to Supply Chain Incentives

The optimal inventory for the vendor is characterized in equation 1. However, the understocking and overstocking costs for the supply chain are very different from those of the vendor. The supply chain’s understocking cost is $r-c$ and overstocking cost is $c-s$. Since $w > c$, $r-c > r-w$. That is, the supply chain’s understocking cost exceeds the vendor’s understocking cost. Note, however, that the sum of the overstocking and understocking costs for the supply chain is equal to the sum of the overstocking and understocking costs for the vendor. It follows that the ratio of the understocking to the sum of the understocking and overstocking costs of the vendor is strictly less than the corresponding ratio for the supply chain. Thus, the vendor will choose to stock strictly less quantity than is optimal for the entire supply chain.

Revenue Sharing

One method for aligning incentives is revenue sharing. Let the vendor’s share of revenues be $k$ per newspaper. Hence, the publisher’s share of the revenues is $1-k$. Let the wholesale price under revenue sharing be $w_r$. Then, the vendor will
stock $q = F^{-1}\left(\frac{kr-w_r}{kr-s}\right)$ following the logic in equation (2). Substituting parameter values from the “Revenue-Sharing Contract” column of Table 1, we get $\frac{kr-w_r}{kr-s} = \frac{.65 \times $1.00 - $0.45}{.65 \times $1.00} = \frac{4}{13}$. Thus the vendor will stock $q = F^{-1}\left(\frac{4}{13}\right) = 131$ copies. The vendor’s profit is given by the following equation, which is a slightly modified version of equation (1) where the modification reflects the revenue sharing arrangement.

$$\int_{0}^{q} (rkx+(q-x)s)f(x)dx + \int_{q}^{\infty} rkqf(x)dx - w_rq$$  \hspace{1cm} (3)

Substituting parameter values into equation (3) from Table 1, we get $$.65 \times $1.00 \times 100 + \int_{100}^{131} (1.00 \times .65x) \frac{1}{100} dx + \int_{131}^{\infty} 1.00 \times .65 \times 131 \frac{1}{100} dx - .45 \times 131 = $23.08. The publisher’s profits are given by $\int_{0}^{q} (r(1-k)x)f(x)dx + \int_{q}^{\infty} r(1-k)qf(x)dx + w_rq - cq$$.

Again, substituting parameter values from Table 1, we get the publisher’s expected daily profit to be $$.35 \times $1.00 \times 100 + \int_{100}^{131} (1.00(1-.65)x) \frac{1}{100} dx + \int_{q}^{\infty} 1.00(1-.65) \times 131 \frac{1}{100} dx + .45 \times 131 - .45 \times 131 = $44.17. The supply chain profits of $67.25 is the sum of the vendor’s and publisher’s expected daily profits.

Markdown Money

Another way for the publisher to boost the retail quantity is to increase the subsidy for unsold inventory also known as markdown money. By paying the vendor markdown money, the publisher reduces the vendor’s overstocking cost and makes the vendor willing to stock more inventory. Of course, paying the vendor markdown money is costly for the publisher. But it may be worth it as long as the increase in vendor demand compensates for the increased cost due to markdown money. The vendor’s stocking quantity when he receives markdown money of $m$ per unsold newspaper is $q = F^{-1}\left(\frac{r-w}{r-s-m}\right)$ which is clearly greater than $F^{-1}\left(\frac{r-w}{r-s}\right)$. Now, we are ready to derive the numbers in the “Markdown Money” column in Table 1. $$\frac{r-w}{r-s-m} = \frac{$1.00 - $0.80}{$1.00 - $0.00 - $0.60} = \frac{1}{2}$$.

When the demand for newspapers is uniformly distributed between 100 and 200 copies, the vendor will stock $q = F^{-1}\left(\frac{r-w}{r-s-m}\right) = F^{-1}\left(\frac{1}{2}\right) = 150$ copies. The vendor’s profits are given by
Equation (4) is a modification of equation (1) to reflect markdown money paid by the publisher to the vendor. Substituting parameters into equation (4), we get

\[ \int_0^q (rx + (q - x)(s + m)) f(x) dx + \int_q^\infty r q f(x) dx - w q \]

The publisher’s expected profit is a function of markdown money as well and is given by the equation below.

\[ w q - \int_0^q (q - x)(m)) f(x) dx - c q \]

Substituting for parameter values, we get

\[ w q - \int_0^q (q - x)(m)) f(x) dx - c q = .80 * 150 - \int_0^{150} (150 - x)(.60)) \frac{1}{100} dx - .45 * 150 = .45 \]

The supply chain profits is the sum of the vendor and publisher profits.