Match Your Own Price?

Self-Matching as a Retailer’s Multichannel Pricing Strategy

Pavel Kireyev, Vineet Kumar and Elie Ofek*

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Abstract

Multichannel retailing has led to the emergence of a new form of price-matching policy. A “self-matching policy” allows a multichannel retailer to offer the lowest of its online and in-store prices to consumers with appropriate evidence of the pricing discrepancy. In contrast to competitive price-matching, a retailer matching its own price is likely to result in consumers obtaining the lower price, making it seem like an unprofitable strategy. However, we observe a variety of self-matching policies across several industries, with some retailers offering self-matching for all products while others choosing not to. Using a game-theoretic model of pricing strategy, we examine whether firms are compelled by competitive pressure to offer self-matching even though it might result in a prisoners’ dilemma. We uncover distinct and novel mechanisms that underpin the effectiveness of self-matching, and find it to be profitable across a variety of competitive scenarios. We examine multiple competitive landscapes, including as a monopolist, a mixed duopoly of a multichannel retailer competing with an e-tailer as well as two competing multichannel retailers. We find that the effectiveness of self-matching depends on the decision-making stage of consumers and the heterogeneity of their preference for channels. Self-matching strategies can also be profitably used as stores face more consumers using smartphones to discover online prices. Our findings provide insights and recommendations to managers on how and when to derive profit from self-matching pricing strategies.

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1 Introduction

Most major retailers have adopted a multichannel business model - they offer products both in-store and online. These channels attract different segments of consumers and allow retailers to customize their offerings to support different buying behaviors as well as preferences consumer have for online or store channels. Pricing has attracted significant interest in the retail trade press as a strategic tool for multi-channel retailers, and is becoming critical to retailer profitability. According to Forrester Research (Mulpuru [2012]), “…Given that the majority of retail sales are expected to be influenced by the Web to some degree, it becomes imperative for eBusiness professionals in retail to adopt cross-channel best practices, particularly related to pricing”. Retail TouchPoints (Fiorletta [2013]) notes that “Amplified price transparency due to the instant availability of information via the web and mobile devices has encouraged retailers to rethink their omnichannel pricing strategies.”

With a self-matching policy, the retailer commits to charging consumers the lowest of its online and in-store prices for the same product when consumers produce appropriate evidence of a lower price in the other channel[^1]. The most common form of this policy is an in-store to online self-match, which allows consumers to pay the lower online price while making the purchase in-store[^2].

There are several trends in retailing that make the understanding of this strategy much more important in the future. First, most retailers are becoming multi-channel, with even some of the online-only retailers establishing a physical presence. Second, the nature of the competitive landscape is evolving, from less differentiated retail to manufacturers are establishing their own retail stores, across a variety of product categories, ranging from Apple’s iconic stores to L’Oreal’s Kiehls brand with exclusive products. Retailers are also moving towards establishing strong private label brands across a range of categories and price points [Mattioli 2011, Bustillo and Lawton 2009]. Third, multi-brand retailers are building exclusive product lines to avoid price wars with their competitors. For instance, around 56% of the products sold by health products retailer GNC are exclusive or GNC branded, and electronics retailers Brookstone and Best Buy are increasing their investment in private label brand [Mattioli 2011, Bustillo and Lawton 2009]. Finally, the

[^1]: A webpage printout, or a mobile screenshot of the webpage usually suffices as appropriate evidence.
[^2]: Very few retailers offer self-matching policies in the other direction, i.e. allowing web customers to match store prices. This may likely be the result of lower online prices in equilibrium. The model we develop allows for online to in-store self-matching, but finds that the policy is not relevant as retailers have an incentive to not price higher online.
The presence of mobile-enabled shoppers is strongly impacting the pricing strategies chosen by retailers [Bustillo and Zimmerman, 2010]. These trends in retailing lead us to focus on retailers competing in a horizontally differentiated product market, which makes self-matching a primary strategic focus.

To the best of our knowledge, no prior studies have examined the implications of self-matching in multichannel retail settings. While such a policy might well be advantageous to consumers, allowing them the flexibility to shop at their preferred channel and price, from a retailer’s perspective it has the potential to have a significantly negative impact. It limits a retailer’s ability to price differently across channels that serve different types of consumers with different needs, and encourages consumers to indulge in comparing channels to obtain the lowest price, an effect we term *channel arbitrage*. Thus, at first glance, the self-matching policy may appear unprofitable, as even consumers who are loyal to a retailer shop strategically across the retailer’s online and store channels to obtain the lowest price.

We examine how *self-matching pricing* policies, which are emerging as a commonly deployed instrument for cross-channel pricing strategy, can be used as a strategic tool by the retailer to manage prices across online and store channels. Our objective is to investigate why a multichannel retailer might want to use a self-matching pricing strategy, and examine its strategic implications in a competitive setting. Our research aims at uncovering the strategic mechanisms which underlie the effectiveness of self-matching strategies, and serves to explain the significant variation observed in the adoption of self-matching policies by multichannel retailers. We explore several competitive settings, including the baseline case of a monopolist multichannel retailer, as well as duopoly competition with an e-tailer as well as another multichannel retailer. Given the rise in smartphone-enabled multichannel shoppers, we also consider the implications of increased price transparency as facilitated by mobile devices on retailer self-matching decisions. We specifically focus on the following questions:

(i) How do self-matching policies emerge in competitive markets?

(ii) Can a self-matching policy be profitable, or is it a prisoners’ dilemma where firms are compelled by competition to offer it?

(iii) How does profitability of the strategy depend on the type of competition?
(iv) How do smartphone-enabled multichannel shoppers affect retailer self-matching and pricing decisions?

Table 1 provides a survey of the in-store to online self-matching policies of retailers, which we obtained from the websites of retailers. It is interesting to observe that certain categories, such as apparel, sporting goods, department stores and discount retailers exhibit heterogeneity in self-matching adoption, with some retailers choosing to self-match and others choosing not to. In other product categories, such as home improvement, electronics and office supplies, retailers tend to uniformly offer self-matching policies. Self-matching policies may draw retailers into a prisoner’s dilemma game, whereby offering to self-match is a necessary evil required to remain competitive. However, we observe multiple markets with heterogeneity in self-matching policy adoption. It becomes important to understand why self-matching adoption varies significantly across different settings.

Whereas the rationale for competitive price-matching is well known (e.g. reducing price competition, Salop [1986]), an equivalent understanding of self-matching has not been developed, despite its increasing importance due to the growing number of markets with multichannel retailers. We address this by developing a model of self-price matching under competition in a multichannel retail setting with horizontally differentiated products. We first consider the case of a monopoly with a single multichannel retailer, then a mixed duopoly in which a multichannel retailer competes with an e-tailer, and finally a duopoly with two competing multichannel retailers.

We capture three different dimensions of consumer heterogeneity in the model. First, we feature consumers who differ in the stage of their decision making process (DMP). Decided shoppers have researched the product choices and are well aware of the product they want to purchase before they decide on which retailer/channel combination to purchase from. Undecided shoppers only recognize the need to purchase from a product category and require a visit to a physical store to shop around and find the specific version or model of the product that best fits their needs. Second, consumers differ in their preference for channels. Channel-agnostic consumers do not have a preference and can freely purchase from either channel, whereas Store consumers only purchase in-store because of high disutility associated with the online channel (including aspects like payment method, waiting time, or risk of product damage). Finally, we model consumer preferences for a retailer’s product,
<table>
<thead>
<tr>
<th>Category</th>
<th>Companies</th>
<th>Self-Match</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sporting Goods</td>
<td>Footlocker</td>
<td>Maybe</td>
<td>Up to the store &amp; who is in charge that day; online policy is “no” however</td>
</tr>
<tr>
<td></td>
<td>Sports Authority</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dick’s</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Pet Supplies</td>
<td>Petco</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pet Smart</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Electronics</td>
<td>Best Buy</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fry’s</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RadioShack</td>
<td>✓</td>
<td>Not during Thanksgiving Holidays</td>
</tr>
<tr>
<td>Home Improvement</td>
<td>Home Depot</td>
<td>✓</td>
<td>30 days</td>
</tr>
<tr>
<td></td>
<td>Lowe’s</td>
<td>✓</td>
<td>30 days</td>
</tr>
<tr>
<td>Apparel</td>
<td>Gap</td>
<td>×</td>
<td>Exception: If the item is carried in store but they ran out of a size, that price will be honored</td>
</tr>
<tr>
<td></td>
<td>J Crew</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Old Navy</td>
<td>×</td>
<td>Exception: If the item is carried in store but they ran out of a size, that price will be honored</td>
</tr>
<tr>
<td></td>
<td>Urban Outfitters</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Discount Retailer</td>
<td>Target</td>
<td>✓</td>
<td>7 days</td>
</tr>
<tr>
<td></td>
<td>Wal-Mart</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Department Store</td>
<td>J.C. Penney</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>(Low-Mid market)</td>
<td>Kohl’s</td>
<td>×</td>
<td>14 days</td>
</tr>
<tr>
<td></td>
<td>Macy’s</td>
<td>✓</td>
<td>14 days</td>
</tr>
<tr>
<td></td>
<td>Sears</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Department Store</td>
<td>Nordstrom</td>
<td>✓</td>
<td>14 days</td>
</tr>
<tr>
<td>(Upscale)</td>
<td>Neiman Marcus</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bloomingdale’s</td>
<td>×</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: In-store to online self-matching policies
with each multichannel retailer matching the horizontal or taste preferences of some consumer segments more closely than others.

We find three different but related underlying mechanisms driving the profitability and emergence of self-matching in equilibrium – the tradeoff between these effects, to the best of our knowledge, are novel and have not been identified before. First, the presence of a self-matching policy directly implies that consumers who prefer to shop in store have access to online prices at the same retailer, and can potentially pay the lower of the two, leading to a channel arbitrage effect that may potentially lower the profits of a self-matching retailer.

Second, decided consumers who have researched the product offering before visiting the store have access to the online price, whereas undecided consumers do not have access to this information and must decide whether to purchase at the store price or not. By introducing self-matching, retailers can fully segment the undecided from the decided store shoppers, and price-discriminate undecided shoppers – we term this the decision-stage segmentation effect, and we identify how it helps the retailer extract more surplus from the undecided consumer.

Third, an asymmetric policy configuration in which one firm offers a self-matching policy while the other does not can reduce the intense competition for channel-agnostic shoppers, as the self-price-matching firm has an incentive to charge a higher online price, allowing its competitor to increase prices as well. We term this the online competition dampening effect of self-matching, and find that its positive effects contribute to making self-matching a profitable strategy for the retailer that adopts the policy.

We find that self-price matching is not necessarily a prisoners’ dilemma type situation, and can actually increase firm profits when the two effects of competition dampening and decision stage segmentation dominate the negative impact of channel arbitrage. Finally, we examine how the presence of smartphone-enabled “mobile” consumers who can retrieve price information when present in store affects profitability and the adoption of price-matching in equilibrium. We find that an increase in mobile consumers can result in higher online prices and retailer profits when combined with a self-matching policy compared to the case when self-matching is not available as a strategic option, an effect that is non-intuitive and becomes apparent only upon examining the

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3We assume in the baseline model that consumers cannot invoke a self-matching policy with their mobile device, and consider the presence of mobile consumers in an extension.
mechanisms underlying self-price matching.

Our model yields a number of results which are empirically testable:

- Retailers offering to self-match will have a larger online-offline price discrepancy relative to those that do not self-match.

- Asymmetric self-matching configurations should be more pronounced in markets for relatively low-valued items, with a substantial fraction of undecided consumers and store-only consumers, with the offline price being higher than the online price in such cases.

- As the penetration of mobile consumers increases, online prices set by retailers offering to self-match are expected to grow closer to their in-store prices, i.e. we should observe more coordination across channels.

We develop an understanding of the strategic consequences of self-matching policies, provide normative insight into when and why self-matching policies may be profitable, and explore the implications of mobile consumers in multichannel retail. Our work builds upon two streams of literature that have so far not had a strong connection, with the first studying issues in multichannel retail strategy and the second focused on competitive price-matching strategies, with an implicit focus on a single channel.

Prior research in multichannel retail strategy has assumed that retailers either set the same price, or set prices differently across channels, without explicitly examining the incentives to adopt a self-matching policy. Liu et al. [2006] show that in markets where price consistency is important, bricks-and-mortar stores can refrain from opening an online arm to deter the entry of an e-tailer. Zhang [2009] studies the interaction of price advertising decisions with the retailer’s multichannel expansion strategy. Ofek et al. [2011] study the incentives retailers face in offering in-store sales assistance and operating on online channel. They find that retailers may over-invest in sales assistance when they also operate an online arm.

It is surprising to note that the literature on multichannel retailing has not focused on heterogeneity in the consumer’s decision making process, which plays an important role in channel choice. Our paper differs in that we allow for consumers to differ in their need for in-store product examination (decided versus undecided consumers) to identify the product that fits them best.
Furthermore, we do not take the requirement of price-consistency as exogenously imposed upon the retailer. Instead, we focus on the strategic incentives of the retailer to offer self-matching policies.

A well-developed literature has studied competitive price-matching decisions. Salop [1986] argued that firms price-matching each other would set higher prices than otherwise, as they no longer had an incentive to engage in price competition, effectively implying a form of tacit collusion [Zhang, 1995]. Other research has shown that differentiation in the presence of uninformed consumers can negate competition-reducing effects and that firms with lower price may offer refunds [Jain and Srivastava, 2000]. Price-matching has also been found to generate a competition enhancing effect which results from increased consumer search [Chen et al., 2001]. Since self-matching policies are a phenomenon only relevant for multi-channel retailers, the price-matching literature has generally focused on the incentives for retailers to match the prices of their competitors in a single channel, typically bricks-and-mortar contexts.

Other research in price-matching explore the impact of hassle costs [Hviid and Shaffer, 1999], the role of price-matching as a signaling mechanism [Moorthy and Winter, 2006, Moorthy and Zhang, 2006], the interaction of price-matching and assortment decisions [Coughlan and Shaffer, 2009], and the impact of product availability [Nalca et al., 2010]. Broadly, the research on competitive single-channel price-matching points to the possibility that matching guarantees may either dampen competition, increase search and enhance competition, serve as a signaling device for service and costs.

To capture these crucial factors, we examine the decision to self-match in a horizontally differentiated product market with competing retailers. In order to develop a tractable model we have had to make several modeling assumptions and choices. We relax several assumptions in Section 4, modeling the impact of smartphone-equipped consumers who have access to online price information, allowing store-only consumers the flexibility to purchase online, as well as allowing consumers to shop across stores. We further discuss these assumptions and the implications in Section 5.

2 Model

We develop a horizontally differentiated products duopoly model of multichannel retail competition. The market consists of two firms - 1 and 2, and a consumer population of size \((1 + \beta)\).
2.1 Consumers

We model consumers as being heterogeneous along multiple dimensions to appropriately capture important features of multichannel shopping environments.

**Retailer Brand Preferences** : Consumers differ in their preferences for the product offered by each one of the retailers, e.g. a consumer might be a loyal Macy’s visitor but not frequent a competitor like JC Penney’s, because of differences in the products or type of service offered by the two retailers. This is captured by allowing consumers to be distributed uniformly across a unit segment, \( x \sim U[0, 1] \). A consumer at preference location \( x \) incurs a cost \( tx \) when purchasing from retailer 1 and a cost \( t(1 - x) \) when purchasing from retailer 2. Note that the parameter \( t \) does not involve transportation costs, rather they represent horizontal retailer-consumer “misfit” costs which do not differ across channels. These misfit costs could be thought of as product preferences over a set of products of similar value, e.g. the collection of blue jeans at Macy’s compared with those at Kohl’s.

**Channel Preferences** : A segment sized \( \eta \) of channel-agnostic consumers does not have an inherent preference for either channel, and purchase from whichever channel has a lower price. In other words, they are indifferent between the online channel and the store, and hence are not concerned with waiting times, payment methods and risks associated with online purchase. The remainder \((1 - \eta)\) are store-only consumers who find the online channel insufficient. Store-only consumers purchase only in store, although they might research products online and obtain online price information if they have decided on which product to purchase. We therefore separate out the information gathering process from the purchase process. Impatient consumers may be more inclined to purchase in-store, as they would get the product sooner. In apparel and fashion, and for products that appear fragile, consumers may be concerned with the possibility of receiving an imperfect item, and hence may be more inclined to complete the purchase in-store. In the base model, store-only consumers do not visit multiple retailers, an assumption we examine in further detail in an extension (§4.4).

**Decision Stage** : Consumers differ in their decision-making process (DMP) stage for the product and channel. DMP stage heterogeneity reflects the notion that consumers may undertake...
a shopping trip without a clear idea of the exact product they wish to purchase beforehand [Mohammed, 2013]. The search process often occurs both online and in-store, with consumers narrowing the consideration set and finally choosing a product at a retailer. Decided shoppers are aware of the product they wish to purchase before making a store visit decision and can costlessly search for product information from home. Subsequently, decided shoppers decide on which retailer and which channel to purchase from. They comprise a unit-sized segment. Undecided shoppers recognize the need to purchase from a product category, but are not aware of the precise product that would fit them best. Hence, they must first visit a retailer’s store to discover an appropriate product fit. Undecided shoppers visit the store closest to their preference location $x$ and discover their preferred product at that store. Subsequently, they may purchase the product in-store, return home to purchase online, or initiate a second shopping trip, now fully aware of the precise product they wish to purchase at both retailers. Undecided shoppers comprise a segment of size $\beta$ in the population. In several product categories like apparel, fashion and sporting goods, retailers are likely to face demand from many undecided consumers, as the available styles and sizes of products are important factors in the consumer decision process.

We normalize the travel cost for the first shopping trip to be zero. To capture the reality of physical and economic travel costs, subsequent trips incur a cost that is high enough in the baseline model to ensure no consumer visits multiple stores or the same store multiple times. The cost of visiting a retailer’s website to shop online is zero as long as the consumer is at home. In the base mode, consumers cannot use mobile devices to access online prices while in-store, but we later examine this possibility in §4 by including a segment of consumers with mobile internet access.

Table 2 depicts the different consumer segments we include in the model. We denote the four segments of consumers as US, UC, DS and DC depending on their decision stage and their channel

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4Undecided consumers do not form expectations of pricing conditional on observed asymmetries in matching policies. This assumption is realistic, as few consumers would cite the existence of a matching policy as their reason for choosing a particular store to browse at. It is much more likely that they choose the store based on their retailer preferences. Furthermore, undecided consumers would be unaware of the product they wish to purchase, limiting their ability to form expectations over its price under different self-matching configurations.

5After consumers discover the exact product they wish to purchase in-store, they form responsive rational expectations of online pricing. Furthermore, after having discovered their ideal product, they are able to uncover the associated product at the competitor’s store.

6The exact bound on travel costs is given in the appendix.
preferences, and the number of these consumers in the population is indicated in the corresponding cell of the table.

2.2 Retailers

We model two multichannel retailers as situated at the endpoints of the unit consumer interval. Both retailers carry a similar assortment and offer horizontally differentiated products of value \( v \) in both channels.

Consumers consider purchasing one of the available products. We model a two-stage game in which the retailers must first decide on self-matching price policies, and then on prices. In the analysis that follows, we find that retailers never set lower prices in-store than online. Hence, the only relevant matching policy is the offline-online self-match.

We denote by \( SM_i = 0 \) the decision of a retailer \( i \) not to self-match and by \( SM_i = 1 \) the decision to self-match, so that the game includes four possible subgames - \((0, 0), (1, 1), (1, 0) \) and \((0, 1) \) representing \((SM_1, SM_2)\). In each subgame, retailer \( j \in \{1, 2\} \) sets price \( p_{jk} \) in channel \( k \in \{on, s\} \), where \( on \) stands for the online or internet channel, and \( s \) stands for the store channel. When the retailer offers to self-match, consumers who retrieve the match will pay the lowest of the two prices set by the retailer from the matching channel. To tractably focus on price-match as well as pricing decisions, we normalize all costs to zero. Table 3 summarizes the notation used throughout.

The sequence of events in the model is depicted in Figure 1. Firms first simultaneously decide upon a price self-matching strategy, and then determine their price levels. Consumers, depending on their type (decided or undecided and store or channel-agnostic) make product and channel decisions, and determine which option to choose (including the no-purchase option). Consumers also ask for a price-match if a firm has chosen to self-match their own cross-channel prices. Finally,
Table 3: Summary of Notation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1^s$</td>
<td>In-store price for retailer 1</td>
</tr>
<tr>
<td>$p_1^{on}$</td>
<td>Online price for retailer 1</td>
</tr>
<tr>
<td>$p_2^s$</td>
<td>In-store price for retailer 2</td>
</tr>
<tr>
<td>$p_2^{on}$</td>
<td>Online price for retailer 2</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Total profit</td>
</tr>
<tr>
<td>$v$</td>
<td>Consumer valuation of the product</td>
</tr>
<tr>
<td>$t$</td>
<td>Retailer differentiation</td>
</tr>
</tbody>
</table>

Consumer Heterogeneity

1. Size of decided segment of consumers
2. Size of undecided segment of consumers
3. Fraction of channel-agnostic consumers
4. Fraction of store-only consumers
5. $t \cdot x$ for $x \in [0, 1]$ Degree of retailer preference

Figure 1: Sequence of Events in Model

firm profits are realized.

2.3 Consumer Decision Making and Utility

Table 4 summarizes the implications on the consumers’ choice set, their information set, and the availability of proof for self-matching (i.e. evidence of a lower online price while in-store) that varies
Table 4: Consumer Choice and Information Sets

<table>
<thead>
<tr>
<th>Consumer Type</th>
<th>Choice Set Information Set</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In-Store</td>
</tr>
<tr>
<td>DC</td>
<td>$u_1^s$</td>
</tr>
<tr>
<td>DS</td>
<td>$u_1^s$</td>
</tr>
<tr>
<td>UC ($x \leq 1/2$)</td>
<td>$u_1^{ss}$</td>
</tr>
<tr>
<td>US ($x \leq 1/2$)</td>
<td>$u_1^{ss}$</td>
</tr>
<tr>
<td>UC ($x &gt; 1/2$)</td>
<td>$\times$</td>
</tr>
<tr>
<td>US ($x &gt; 1/2$)</td>
<td>$\times$</td>
</tr>
</tbody>
</table>

We obtain consumer utilities for their choice of retailer and channel, beginning with the case when neither firm self-matches as the baseline case, and then examine the other price matching strategies in turn. Consumers in all cases have the option not to make a purchase, and obtain a zero utility for the no purchase option.

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Under $(SM_1, SM_2) = (0, 0)$, the decided shopper may either purchase online or in-store from either retailer. The utilities for decided shoppers are given as follows:

$$u_1^{om} = v - p_1^{om} - tx, \quad u_1^s = v - p_1^s - tx$$
$$u_2^{om} = v - p_2^{om} - t(1 - x), \quad u_2^s = v - p_2^s - t(1 - x)$$

where $v$ is the value of the product, $p_1^{om}$ and $p_2^{om}$ are the online prices set by firms 1 and 2 respectively, $t$ measures the extent of horizontal differentiation between firms, and $x$ is the shopper’s distance (in preference space) to the product offered by firm 1. Store-only decided shoppers do not consider the option of purchasing online.

Undecided shoppers who first visit retailer 1 to browse for products face utilities similar to decided shoppers above, except in the offline channel, they are restricted the store they choose to browse at. This captures the idea that it is more difficult to comparison shop across stores than in the online channel. Undecided channel-agnostic consumers can either purchase in the store they first visited to browse, or return online and purchase from either retailer. Undecided store-only consumers make a purchase decision at the store they first visit, which is a better match to their...
preference than the competing retailer’s store.

**Effect of Price Self-Matching**

We next examine how the price self-matching strategies of the firms impact consumer utilities. Decided consumers who shop at the store channel of a retailer with self-matching, and can request a *price match for a specific product* to obtain the *lowest of the online and store prices*. Thus, when both retailers offer a self-match pricing policy, i.e. under \((SM_1, SM_2) = (1, 1)\), decided consumers can either purchase online and pay the online price, or purchase in-store and pay the lowest of the online and in-store prices of the retailer they purchase from. Decided consumers obtain the following utilities from their retailer and channel choices:

\[
egin{align*}
    u_{on}^1 &= v - p_{on}^1 - tx, \\
    u_{on}^2 &= v - p_{on}^2 - t(1 - x), \\
    u_1^s &= v - \min(p^s_1, p^m_1) - tx \\
    u_2^s &= v - \min(p^s_2, p^m_2) - t(1 - x).
\end{align*}
\]

These expressions are similar to the utilities derived for decided consumers under \((0, 0)\), except that decided consumers may now pay the lowest of the online and in-store prices at a particular retailer if they purchase offline, as captured in the different representation of \(u_1^s\) and \(u_2^s\).

Undecided shoppers face the same utilities under \((1, 1)\) as under \((0, 0)\) as they cannot redeem matching policies when they visit a retailer’s store without making a second trip or invoking a match using a mobile device\(^7\).

Utilities in the asymmetric subgames \((1, 0)\) and \((0, 1)\), where only one retailer offers price self-matching are defined correspondingly. Table 4 summarizes the relevant utilities, where

\[
egin{align*}
    u_1^{ss} &= v - p^s_1 - tx, \\
    u_1^s &= \begin{cases} 
        u_1^{ss} & \text{if } SM_1 = 0, \\
        v - \min(p^s_1, p^m_1) - tx & \text{if } SM_1 = 1,
    \end{cases} \\
    u_1^{om} &= v - p^m_1 - tx,
\end{align*}
\]

The remaining utilities for decisions involving firm 2 - \(u_2^{ss}, u_2^s\) and \(u_2^{om}\) - are defined similarly.

\(^7\)That is, although the consumers have responsive rational expectations, when in-store, they lack the evidence required to claim a pricing policy, as beforehand it was impossible for them to search for the product online.
3 Analysis

Throughout the following analysis we restrict our attention to interior solutions for expository clarity and tractability, although most results would naturally extend to corner solutions by continuity.\footnote{Formally, we require \( v \) and \( \beta \) sufficiently small. The exact bounds are given in the appendix. We also consider the implications of large \( v \) in an extension. For the purpose of clarity, we further restrict \( \beta \) to not be too large in the following analysis. All proofs in the appendix consider the full range of \( \beta \).}

3.1 Benchmark Monopoly

Consider a benchmark scenario, in which a multichannel monopolist retailer, located at the center of the 0−1 Hotelling line, chooses a self-matching policy and sets prices. We examine the conditions under which the firm chooses to self-match.

**Proposition 1.** A monopolist will never find it profitable to self-match prices across channels.

The monopolist will price to extract the highest possible surplus from each consumer segment. Under \( SM = 0 \), channel-agnostic consumer will purchase online at a price \( p_{1}^{on} = v - t/2 \). Store-only consumers will purchase in-store at a price \( p_{1}^{s} = v - t/2 \). The monopolist will then earn profits of:

\[
\Pi_{1}^{SM=0} = \eta(v - t/2) + (1 - \eta)(v - t/2) + \beta(\eta(v - t/2) + (1 - \eta)(v - t/2))
\]

\[
= (v - t/2)(1 + \beta).
\]

The retailer chooses to self-match, it will earn profits of:

\[
\Pi_{1}^{SM=1} = (v - t/2)(1 + \beta),
\]

which is no greater than \( \Pi_{1}^{SM=0} \).

We note that these profits are the same as a monopolist serving a consumer segment of size \((1 + \beta)\) in a single channel. The retailer will gain the same profit from store-only consumers as well as from channel-agnostic consumers, and will therefore have no incentive to offer a self-matching policy.\footnote{More specifically, the retailer is indifferent between offering and not offering a self-matching policy, but for any non-zero cost of administering the policy, it will choose not to offer.}
3.2 Duopoly

To examine the effect of duopoly competition, we model two firms locating at the endpoints of a Hotelling segment of unit length. We first consider the case of a multichannel retailer competing with an online-only e-tailer. This setting provides intuition for many of the results that hold when two multichannel retailers compete across both store and online channels.

Channel-agnostic consumers will purchase from the channel offering the lowest price, whereas store-only consumers will purchase only from a store.

We focus on pure strategy solutions and use the subgame perfect equilibrium solution concept to identify the equilibria of this two-stage self-matching policy game. Consumers purchase from the retailer/channel combination for which they obtain the highest utility. Firms anticipate consumer decisions, and set prices accordingly. Finally, self-matching policy equilibria emerge as a consequence of the firms’ anticipation of pricing outcomes under different policy arrangements. Retailers set prices to satisfy the demand they face, taking into account consumer heterogeneity and competitor response. Consumers have a retailer taste preference utility of $t \cdot x$ for Firm 1, where $x$ is their preference location on the Hotelling line and $t \cdot (1 - x)$ for firm 2. We define the function $\Phi_1$ as the proportion of consumer demand obtained by firm 1, when competing in a duopoly with firm 2, where both firms are differentiated horizontally. Specifically, $\Phi_1(p_1, p_2; t) := \frac{1}{2} + \frac{(p_2 - p_1)}{2t}$.

Mixed Duopoly: Multichannel retailer and e-tailer

In several markets, multichannel retailers find themselves competing primarily with online-only retailers, or e-tailers. The issue is whether self-matching might prove to be an effective strategy for a multichannel retailer in such a competitive landscape. We consider the case of asymmetric competition, where the focal multichannel firm 1 is competing with an online-only firm 2 (e-tailer). As before, firm 1 is located at $x = 0$ while firm 2 is located at $x = 1$ on the Hotelling linear city. Firm 1 has the option of offering a self-matching policy in stage 1. Subsequently, both firms set

\[^{10}\text{In Section 4.2 we allow store-only consumers to purchase online if the cross-channel price difference is sufficiently large.}\]
prices and compete for consumer demand. Under (0, 0) the firms earn profits:

\[
\Pi_{1,0}^0 = \eta (1 + \beta) \Phi_1(p_{11}^m, p_{21}^m)p_{11}^m + (1 - \eta)(1 + \beta)p_1^s,
\]

Decided & Undecided Channel Agnostic

\[
\Pi_{2,0}^0 = \eta (1 + \beta)(1 - \Phi_1(p_{11}^m, p_{21}^m))p_{21}^m.
\]

Decided & Undecided Channel Agnostic

Firm 1 will serve as a monopoly for store-only consumers, but will compete with firm 2 for channel-agnostic consumers. In this case, undecided channel-agnostic consumers located to the right of \(x = 1/2\) will purchase from the etailer, but only after “free-riding” by browsing and learning about the product category at firm 1’s store. The firms will set competitive prices online, \(p_{11}^m = p_{21}^m = t\), and firm 1 will set monopoly price in-store \(p_1^s = v - t\).

Under the (1, 0) subgame, firms earn profits

\[
\Pi_{1,0}^1 = \eta (1 + \beta) \Phi_1(p_{11}^m, p_{21}^m)p_{11}^m + (1 - \eta)(1 + \beta)(1 - \Phi_1(p_{11}^m, p_{21}^m))p_{21}^m + (1 - \eta)(1 + \beta)p_1^s.
\]

Decided & Undecided Channel Agnostic

\[
\Pi_{2,0}^1 = \eta (1 + \beta)(1 - \Phi_1(p_{11}^m, p_{21}^m))p_{21}^m.
\]

Decided Store

Undecided Store

Store-only consumers now pay different prices depending on whether or not they are decided. In particular, decided store-only consumers redeem the matching policy, and pay the online price, whereas undecided store-only consumers fail to redeem the policy (because of their lack of evidence), and pay the in-store price. The retailer faces a channel arbitrage effect when it allows the consumer to obtain a price match resulting in the lower price being effectively charged in both channels to consumers who request the match. We might intuitively expect self-matching to be an unprofitable strategy in such a setting, since store consumers shopping at the multichannel retailer will obtain a lower price through a price-match, and in any case these consumers would not have defected to the e-tailers because of their preference for the store channel. Thus, it would seem like a losing strategy from the viewpoint of the multichannel retailer. However, while this intuition is correct, we find it to be incomplete in characterizing the equilibrium strategies. Focusing on the self-matching strategy chosen by the multichannel firm, we find the following result.

**Proposition 2.** In a mixed duopoly featuring a multichannel retailer (firm 1) offering a store and an online channel and an e-tailer offering only an online channel (firm 2):

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(i) The multichannel retailer only offers a self-matching policy for low product valuation or a large number of undecided consumers. Otherwise, the retailer will not offer to self-match.

(ii) Prices in the online channel are higher for both retailers when the multichannel retailer chooses self-matching.

The multichannel retailer earns greater profits when self-matching for low $v$ because of two effects - channel arbitrage and online competition dampening. Consider what happens when $v$ is large. Under $(0, 0)$, the multichannel retailer is able to charge all store-only consumers the monopoly price, which increases in $v$, as they have no competitive option, whereas under $(1, 0)$, decided store-only consumers will redeem the self-matching policy and claim the lower online price, i.e. the channel arbitrage effect. For large $v$ this effect is especially detrimental to a self-matching retailer. On the other hand, we observe that the prices in the online channel are higher for both retailers when the multichannel retailer self-matches.

To see why, consider the pricing incentives in the online channel without self-matching: a lower online price by the multi-channel retailer increases its share of channel-agnostic consumers under both equilibria, and the resulting pricing equilibrium under $(0, 0)$ is competitive in the online channel, with both retailers pricing to reflect the strength of retailer preferences, i.e. $p_1^{on} = p_2^{on} = t$. However, in the self-matching equilibrium, reducing online price has an additional negative effect on the multi-channel retailer induced by channel arbitrage. To mitigate the effects of channel arbitrage on decided store-only consumers, the multichannel retailer has an incentive to charge a higher online price, as this is the price decided store-only consumers will redeem by price-matching. Since the online price levels of both retailers are strategic complements, the e-tailer will also increases its price level in response. We refer to this effect of self matching as the online competition dampening effect. Note that channel arbitrage enables online competition dampening.

The emergence of self-matching in equilibrium depends on the trade-off in payoffs from channel arbitrage and online competition dampening. For high $v$, the negative impact of channel arbitrage exceeds the positive impact of online competition dampening. Hence, self-matching emerges only for low values of $v$.

Specifically, the firms will set prices $p_1^{on} = t + \frac{4t(1-\eta)}{3\eta(1+\beta)}$, $p_2^{on} = t + \frac{2t(1-\eta)}{3\eta(1+\beta)}$, $p_1^s = v - t$ for $v > 2t + \frac{4t(1-\eta)}{3\eta(1+\beta)}$, and $p_1^{on} = v - t$, $p_2^{on} = v/2$, $p_1^s = v - t$ for $v \leq 2t + \frac{4t(1-\eta)}{3\eta(1+\beta)}$. 

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Examining the impact on the multichannel retailer and e-tailer from the former adopting a price matching policy, we find the following:

**Corollary 1.** In mixed duopoly competition, comparing the self-matching policy to a no-matching policy adopted by the multi-channel retailer, we find the following:

(i) The e-tailer always makes higher profits when the multichannel retailer uses a self-matching policy.

(ii) For the multichannel retailer, the difference in profits between a self-matching and no-matching policy decreases as the value of the product increases, the degree of differentiation decreases, and the number of undecided consumers increases.

We observe that adopting a self-matching policy has positive effects on the competing retailer due to reduced competition in the online channel, which allows the e-tailer to increase prices. However, the e-tailer’s price level is still lower than the multi-channel retailer because it does not internalize the benefit of raising its price through the store channel, like the multichannel retailer is able to do. We might expect that this effect might be reinforced if the e-tailer were to add a store channel, which we examine in §3.2 below.

We examine this mixed duopoly scenario of multichannel competition for two reasons. First, this type of asymmetric competition is becoming more important for multichannel retailers, e.g. several retailers find that Amazon and potentially other online retailers are their primary competitors. Second, it is useful to obtain the intuition for the multichannel duopoly case, since similar effects apply there. However, there are additional mechanisms that affect the profitability of price self-matching in the case of a duopoly with multichannel retailers, and we proceed to examine that case next.

**Multichannel Duopoly: Competition between and across channels**

We now turn to the case of two competing multichannel retailers. We first study each one of the possible self-matching policy subgames individually and conclude with the result highlighting the main conditions under which self-matching policies emerge in equilibrium (Proposition 3). This is followed by an examination of the profitability of self-matching policies.
No Self-Matching - (0,0): In the (0,0) subgame, both decided and undecided shoppers pay the same price if they purchase from the same channel. The profit functions of firms 1 and 2 can be written as

\[
\Pi_{1,0}^0 = \eta (1 + \beta) \Phi_1(p_{1,0}^o, p_{2,0}^o) p_{1,0}^o + (1 - \eta) \left( \Phi_1(p_1^s, p_2^s) + \frac{\beta}{2} p_1^s \right),
\]

\[
\Pi_{2,0}^0 = \eta (1 + \beta) (1 - \Phi_1(p_{1,0}^o, p_{2,0}^o)) p_{1,0}^o + (1 - \eta) \left( \Phi_1(p_1^s, p_2^s) + \frac{\beta}{2} \right) p_2^s.
\]

Undecided store-only shoppers purchase from the store they first visit to learn about products. As a result, each retailer effectively has a monopoly segment of undecided consumers of size 1/2 that purchases the product in-store, represented by the last term in the profit expression above.

Solving for the equilibrium in the pricing subgame, we find that the prices are \( p_{1,0}^o = p_{2,0}^o = t \) and \( p_1^s = p_2^s = (1 + \beta) \) for \( v/t > 3/2 + \beta \), and \( p_1^s = p_2^s = v - t/2 \) for \( v/t \leq 3/2 + \beta \). The firms charge a higher price in-store than online to extract surplus from undecided store-only shoppers. We find that prices in the less-differentiated online channel are driven down to \( t \), denoting the strength of consumer preference for specific retailers. However, in the store channel, the firms are able to charge a higher price and consequently extract more surplus from consumers due to the presence of undecided and store-only consumers. If there were no undecided consumers, i.e. as \( \beta \to 0 \), the store prices would become equal to \( t \) as well.

Symmetric Self-Matching - (1,1): When both firms offer self-matching policies, decided store-only consumers retrieve the online price in-store. Recall that undecided store-only consumers cannot invoke the price-match as they do not have a chosen product until they visit the store. The self-matching policy thus becomes a device for price-discrimination between decided and undecided
in-store buyers. To see this, consider retailer profits:

\[ \Pi_{1,1}^1 = (1 + \eta \beta) \Phi_1(p_{11}^m, p_{21}^m) p_{12}^m + (1 - \eta) \frac{\beta}{2} p_1^s, \]

Channel-Agnostic & Decided Store

Undecided Store

\[ \Pi_{2,1}^2 = (1 + \eta \beta)(1 - \Phi_1(p_{11}^m, p_{21}^m)) p_{22}^m + (1 - \eta) \frac{\beta}{2} p_2^s. \]

The entire unit segment of decided shoppers and the \((\eta \beta)\) segment of undecided channel-agnostic shoppers purchase the product for its online price, either by buying online, or by redeeming a matching guarantee in-store (channel arbitrage). Undecided store-only buyers must pay the in-store price. In equilibrium, the firms set prices \(p_{11}^m = p_{21}^m = t\) online and \(p_{12}^s = p_{22}^s = v - t/2\) in-store. That is, the firms charge decided shoppers and undecided channel-agnostic shoppers the competitive price \(t\), while charging undecided store-only shoppers the monopoly price \(v - t/2\).

The price-discrimination strategy applies to the store-only market and works for two reasons. First, firms compete more heavily for decided than they do for undecided consumers, as the latter can only purchase from the store they choose to visit first. As a result, each firm would prefer to charge undecided store-only shoppers a higher price than decided store-only shoppers. The firm cannot achieve such segmentation and price discrimination with a no-matching policy, as both consumer types would purchase in the same channel, the store, paying the in-store price.

However, with a self-matching policy, the firm can charge different prices to store-only consumers based on their DMP stage by setting a high in-store price paid by undecided consumers and a lower online price for decided consumers who can redeem the self-match. Thus the firm can effectively segment and price discriminate and based on the decision state of consumers, with the undecided consumers paying a higher price than the decided segment. We refer to this effect as the \(\text{decision-stage segmentation effect}\) of self-matching.

**Asymmetric Self-Matching - (1,0):** We explore the subgame in which retailer 1 offers a self-matching policy and retailer 2 does not. By symmetry, similar results follow in the \((0, 1)\) subgame. Observe that a decided store-only shopper who visits retailer 1’s store can purchase from the store and have the option to pay the lower of the online and store price, i.e. \(p_{11}^m = \min(p_{11}^m, p_{12}^s)\).
However, if the consumer visits retailer 2’s store, she faces a price of $p^s_2$. Consumers will thus weight their preference for the retailer brand (denoted by their location $x$) as well as the above prices in deciding which store to visit. These choices of consumers then lead to the following profit functions for retailers:

\[
\Pi^{1,0}_1 = \eta(1 + \beta) \Phi_1(p^{on}_1, p^{on}_2) p^{on}_1 + (1 - \eta) \Phi_1(p^{on}_1, p^s_2) p^{on}_1 + (1 - \eta) \frac{\beta}{2} p^s_1,
\]

\[
\Pi^{1,0}_2 = \eta(1 + \beta)(1 - \Phi_1(p^{on}_1, p^{on}_2)) p^{on}_2 + (1 - \eta) \left((1 - \Phi_1(p^{on}_1, p^s_2)) + \frac{\beta}{2}\right) p^s_2.
\]

If the online price $p^{on}_1 < p^s_2$, it induces consumers who are closer in preference location to firm 2, i.e. those with $x > \frac{1}{2}$ to visit store 1 and obtain the lower online price using a self-match. Thus, the fraction of store consumers who now shop at retailer 1’s store channel is given by $\Phi_1(p^{on}_1, p^s_2)$, and firm 1 obtains a potentially larger share of the store-only segment of consumers.

Again, we begin with the equilibrium of the pricing subgame, and find that the price levels chosen by the firms are higher than the competitive price of $t$. We find that firms set online prices at $p^{on}_1 = t \left(\frac{2}{3} + \frac{1+\beta}{3(1+\eta\beta)}\right)$ and $p^{on}_2 = t \left(\frac{5}{6} + \frac{1+\beta}{6(1+\eta\beta)}\right)$ and in-store prices $p^s_1 = v - t/2$ and $p^s_2 = p^s_2 + \beta t/2$ for $v/t > t \left(\frac{4}{3} + \frac{1+\beta}{6(1+\eta\beta)} + \frac{\beta}{2}\right)$. Otherwise, $p^{on}_1 = t + \frac{(1-\eta)(2v-3t)}{4-\eta(1-3\beta)}$, $p^{on}_2 = t + \frac{(1-\eta)(2v-3t)}{2(4-\eta(1-3\beta))}$ and $p^s_1 = p^s_2 = v - t/2$.

Note that equilibrium online prices in the asymmetric self-matching case are greater than those set in the other subgames. This results from the impact of the online competition dampening effect introduced in §3.2. Firm 1 suffers from the channel arbitrage effect, as decided store-only shoppers now claim its lower online price when they purchase in-store. Although firm 1 steals in-store demand from firm 2 by enabling a lower price in its store, the net effect of channel arbitrage for firm 1 is profit reducing. To mitigate channel arbitrage, the firm has an incentive to charge a higher online price. Since prices across firms in the online channel are strategic complements, this effect in turn leads to higher online price set by the competitor. As seen in §3.2, channel arbitrage enables online competition dampening in the asymmetric self-matching equilibrium. Note that the decision-state segmentation effect is also present under $(1,0)$, as the self-matching firm 1 is able to charge undecided store-only consumers the monopoly in-store price.
Asymmetry in self-matching policies drives down firm 2’s in-store price and profits. Firm 1’s channel arbitrage problem affects firm 2, as firm 2 must set its in-store price to compete with the online price set by firm 1 in the market for decided store-only consumers, and can no longer leverage the reduced competition for these decided store-only consumers induced by the undecided store-only segment of consumers.

We next examine whether the prices across channels are more coordinated in the case of a self-match, and find this not to be the case in the following result.

**Corollary 2.** *The price discrepancy across channels grows with \( v \) for a firm offering to self-match.*

Higher value products lead to a greater price disparity across channels in the presence of self-matching, since it allows the firm to price discriminate undecided store-only consumers more effectively and extract more surplus from them.

Note that the results detailed above in this section are the equilibria in the pricing sub-game, which is conditional on the self-matching policy chosen earlier (see Figure 1 for the timeline), and are therefore partial equilibrium results. We now examine the full equilibrium results of the game beginning with the price-matching strategy choices.

### 3.3 Self-Matching Policy Equilibria

For a policy configuration to emerge in equilibrium, it must be the case that no firm would do better by unilaterally deviating to offer a different policy. Hence, \((1, 1)\) is an equilibrium if and only if firm 2 earns lower profits in the asymmetric subgame than it does when both firms self-match. \((0, 0)\) is an equilibrium if and only if firm 1 earns lower profits in the asymmetric subgame than it does when no firm offers a self-matching policy. An asymmetric equilibrium exists in all other cases.

**Proposition 3.** In a duopoly featuring two multichannel retailers, both offering store and online channels, we find that self matching policies are determined by the following focal regions based on product value and the number of undecided consumers:

**Region A:** *Firms will offer asymmetric self-matching policies.*

**Region B:** *There is a symmetric equilibrium where neither firm will self-match.*
**Region C:** There is a symmetric equilibrium with both firms offering self-matching policies.

Region A is characterized by low values of $v$ and high values of $\beta$ whereas region C is characterized by the inverse, and region B has intermediate values of both $v$ and $\beta$.

We find that no single price matching strategy always dominates, and that there are three main regions where there are different different price matching policies are equilibrium strategies, and that all three settings that we have previous examined (no matching, asymmetric matching and symmetric matching) can thus result in equilibrium, depending on consumer preferences and retailer differentiation.

To understand the intuition behind the emergence of different equilibria, we begin by detailing firms’ best responses to competitor decisions. We focus in turn on how the effects we have discussed above, i.e. channel arbitrage, online competition dampening, and decision stage segmentation vary based on the product value $v$ and the number of undecided shoppers, $\beta$. Figure 2 shows how firm best response functions evolve and translate into equilibria as $v$ increases from left to right.

The first arrow captures firm 1’s best response when firm 2 does not offer a matching policy. When $v$ is low, the product valuation of undecided consumers is low, and firm 1 earns very little from segmenting consumers by their decision stage and enabling price discrimination. The negative channel arbitrage effect is also low, as the highest price firm 1 can charge in-store is bounded by the low product value. However, the firm does benefit from online competition dampening if it chooses self-matching, and this positive effect outweighs the arbitrage, leading firm 1 chooses to match at low $v$.

As $v$ grows, firm 1 is able to charge higher in-store prices to capture more surplus from the undecided store segment. Online competition dampening ceases to grow in $v$ as firms reach a point when they must compete for channel agnostic consumers. The negative impact of arbitrage outweighs the positive impact of competition dampening, and the firm no longer prefers to self-match. At a high $v$, channel arbitrage ceases to increase in $v$ as firms reach a point where they must compete for store-only consumers, but discrimination continues to grow as under a self-matching policy, undecided store-only shoppers pay monopoly price, which is increasing in $v$. The positive impact of discrimination outweighs the negative impact of arbitrage. Firm 1 once again prefers to self-match.
The second arrow in figure 2 captures firm 1’s best response when firm 2 offers to self-match. At low \( v \), firm 1 cannot gain by matching as it would lead to arbitrage and nullify the dampening effect. Hence, firm 1 prefers to not match. As \( v \) grows, so does the positive impact of decision stage segmentation followed by price discrimination. At high enough \( v \), the positive impact of discrimination will outweigh the negative impact of arbitrage and dampening. The firm will then prefer to match.

The third arrow in figure 2 shows the equilibria that emerge as a consequence of firm strategic behavior. At low \( v \), the firms play an counter-coordination game. Specifically, only one retailer can gain by self-matching, and the other cannot increase its payoffs by also self-matching, as it would lead to channel arbitrage and also nullify the online competition dampening effect. These effects would decrease profits from both decided store-only and channel-agnostic consumers. As \( v \) grows, the negative impact of channel arbitrage outweighs the positive impact of decision state segmentation, and neither firm will offer to self-match. Eventually, \( v \) will be sufficiently large so that channel arbitrage outweighs the benefits of online competition dampening, but decision state segmentation is also strong. At this stage, we enter a region where multiple equilibria exist, and the self-matching game becomes a coordination game. Either both of the retailers offer to self-match, or neither one does.

For very high \( v \), the decision state segmentation effect will dominate. Firms charge a very high price to price-discriminate undecided store shoppers with the self-match, leading to an incentive for both firms to self-match. In equilibrium, both firms will offer self-matching policies. As we discuss in the following section, firms may be playing in a setting resembling the prisoner’s dilemma game, since each firm acts on its individual incentive to self-match even though both firms may be worse off in a symmetric self-matching equilibrium than a no-matching equilibrium. In such a case, the availability of self-matching as a strategy serves to reduce industry profits.

Figure 3 captures the impact of changes in \( \beta \) on firm best responses and equilibrium outcomes. As the top arrow shows, increases in \( \beta \) make firm 1 more likely to offer a self-matching policy for low \( v \). As \( \beta \) increases, the size of the undecided segment increases, leading to an increase in the dampening effect on undecided channel agnostic consumers. The arbitrage effect does not change by as much for low \( v \), increasing the range of \( v \) for which firm 1 will match. For large \( v \), increases in \( \beta \) lead to an increase in the arbitrage effect, as the competitive in-store price charged
by firms in a no-matching equilibrium grows in $\beta$ (recall that both decided and undecided in-store consumers pay this price under no-matching). The arbitrage effect grows faster than dampening, and discrimination takes effect at larger $v$ as firms can set monopoly prices in-store for larger $v$ than if $\beta$ were lower. Hence, the negative effect of arbitrage overtakes the other two effects, and firm 1 has less of an incentive to self-match at higher $v$.

The bottom arrow in figure 2 illustrates the impact of increasing $\beta$ on firm 2’s best response strategy. As $\beta$ increases, for intermediate $v$, the impact of an increase in price discrimination outweighs the negative effect of arbitrage. This is because firm 2 loses significant demand on decided store-only shoppers if it does not match, and hence the negative effect of arbitrage if it does match is mitigated. Increases in $\beta$ motivate firm 2 to match at lower $v$. In summary, increasing $\beta$ increases the asymmetric equilibrium region for low $v$, shrinks the region where both firms do not match, and shrinks the region where both firms self-match as a unique equilibrium at high $v$.

Retailers may find different consumer segments more profitable for different values of $v$. At low $v$, when the dampening effect dominates, retailers adopt an asymmetric equilibrium and benefit
most from channel-agnostic consumers. At intermediate values of $v$, prices charged to decided store shoppers grow in the symmetric no-matching equilibrium, and retailers benefit most from decided store-only shoppers. At high values of $v$, the decision state segmentation effect has the maximum impact, and both retailers self-match and benefit most from the higher prices paid by undecided store-only shoppers.

### 3.4 The Profitability of Self-Matching

We began the paper attempting to understand if self-matching ends up as a prisoners’ dilemma, where competing multichannel retailers are forced to adopt the policy as a competitive effect, which then ends up reducing profits for both. We begin by exploring the profitability of firms under different equilibrium configurations.

**Proposition 4.** Examining the profit implications of the self-matching mechanism, we find the following:

1. In an asymmetric equilibrium, only the firm offering to self-match earns greater profits compared to the case when neither firm offered a matching policy.

2. In symmetric self-matching equilibrium, both firms can earn higher profits with self-matching strategies when product valuation is high, compared to the setting where such an option were
not available. However with lower values of product valuation, self-matching can result in lower profits for both firms.

We find that firms can earn greater profit in both types of equilibria, asymmetric and symmetric self-matching. Recall, from proposition 3, that at lower product values, we are more likely to be in region A, where asymmetric self-matching strategies are chosen in equilibrium. Whereas, at high values, we are in region C, leading to symmetric strategies. We find that the availability of self-matching as a strategy can enhance profits for at least one firm, and potentially for both competing firms at low and high product value. At intermediate product value, we find that self-matching can occur exclusively due to competitive reactions, and can result in a prisoners’ dilemma like situation where both firms decide to adopt self-matching, but both would have been better off (making higher profits) had self-matching not been available as a strategic pricing tool.

Note that whenever there are multiple equilibria present, our focus has been on the equilibrium that is Pareto-optimal, and we find that in in our setting, we always have a unique equilibrium that satisfies this criterion (except for labeling the firms differently in the case of an asymmetric equilibrium). Figure 4 presents a graphic of the Pareto-optimal equilibria that emerge under different values of \(v\) and \(\beta\), assuming that in a scenario with multiple equilibria firms would coordinate on the one that yields greater profits to both.
4 Extensions

The base model analyzed in §2 focused on developing an understanding of the mechanisms underlying the effectiveness of self-matching. Here, we aim to examine additional variables as well as relax some of these assumptions previously made, with a view to increasing the range of applicability of the findings from above. We consider four extensions, beginning with the presence of mobile consumers, evaluating how strategic decisions evolve when store-only consumers also have the online option, examining what happens when retailers offer the popular “buy now, pick up in store” choice, and when store-only consumers can shop at multiple stores before making a purchase.

4.1 Impact of Mobile Consumers

In the main model we assumed that undecided consumers cannot invoke a self-matching policy because they are in-store at the time of decision with no internet access and hence no evidence of a lower online price. Suppose that a fraction \( \mu \) of consumers has access to the internet in-store. We refer to these consumers as “mobile” to reflect the notion that consumers with internet-enabled smartphone devices can easily obtain online price information while in-store. Undecided store-only mobile consumers will invoke a self-matching policy if the online price offered by a firm is lower than its in-store price.

The existence of mobile consumers thus impacts the retailers’ incentives to self-match. On the one hand, self-matching retailers are less able to price-discriminate undecided in-store consumers if a fraction of them no longer retrieves the high in-store price. On the other hand, retailers still maintain monopoly power over the undecided in-store consumers who invoke the policy and retrieve the online price, which gives rise to an incentive to increase online prices. Consider the profits retailers earn if they both offer to self-match:

\[
\Pi_{1,1}^1 = (1 + \eta \beta) \Phi_1(p_{1,1}^m, p_{2,1}^m)p_{1,1}^m + (1 - \eta) \frac{\beta}{2} ((1 - \mu)p_{1}^s + \mu p_{1}^m),
\]

\[
\Pi_{1,1}^2 = (1 + \eta \beta)(1 - \Phi_1(p_{1,1}^m, p_{2,1}^m))p_{2,1}^m + (1 - \eta) \frac{\beta}{2} ((1 - \mu)p_{2}^s + \mu p_{2}^m).
\]

The retailers will set \( p_{1}^m = p_{2}^m = t \left( (1 - \mu) + \mu \frac{1 + \beta}{1 + \beta} \right) \) and \( p_{1}^s = p_{2}^s = v - t/2 \). Note that the
online prices are increasing in \( \mu \). Decided consumers obtain product and price information in advance of their shopping, and undecided channel-agnostic consumers can obtain product match information through the online channel after their first store visit. Observe that having access to price information with a mobile device is only relevant for undecided store-only consumers.

We model mobile consumers as visiting a store, and having access to both retailers’ online prices while in-store. However, for store-only mobile consumers, choosing the competing retailer would involve making a trip to the other store, which would involve high travel costs, and in our baseline framework, these costs are high enough to discourage store-to-store comparison shopping.

Undecided store-only consumers with mobile thus are like decided consumers in the sense that they have access to online prices of both retailers. However, they are different because they do not have the option of deciding which store to visit (now that they are at a store). So, having a higher proportion of mobile consumers is not equivalent to having more decided consumers, and has a more nuanced effect. We detail how mobile consumers impact retailers’ incentives to self-match in Proposition 5 below.

**Proposition 5.** When two multichannel retailers compete for consumers, some of whom use a mobile device in store to obtain price information in the online channel, we find the following:

1. As the fraction of mobile consumers increases, retailers become less likely to offer self-matching policies for high product values, but more likely for low product values.
2. Retailer profit can increase in fraction of mobile consumers.

We find that the presence of mobile consumers can give rise to equilibria with symmetric self-matching policies. Figure 5 presents a cross-section of Figure 4 at fixed \( \beta \). The conditions for symmetric self-matching policies to emerge in equilibrium for high values become more stringent as \( \mu \) grows. This is because the existence of mobile consumers erodes the decision stage segmentation effect of self-matching, which reduces the firms’ incentives to self-match. On the other hand, mobile consumers enhance the online competition dampening effect, which encourages retailers to self-match. A symmetric self-matching equilibrium emerges for low \( v \) as firm 2 finds it beneficial to deviate from the (1, 0) scenario by offering a self-matching policy. By doing so, firm 2 no longer needs to set a low in-store price and lose out on profit in the market for decided store-only
consumers. It can leverage the existence of undecided mobile consumers to further dampen online competition.

Thus, overall, we observe that the existence of mobile consumers need not decrease the profitability of a self-matching retailer (see Appendix for details). On the contrary, mobile consumers can motivate retailers to charge higher online prices, increasing the profitability of self-matching policies. Therefore, we find that given current technology trends, retailers would find it worthwhile to more carefully examine whether self-matching is an appropriate strategy option.

4.2 Store-Only Consumers Purchasing Online

Throughout the duopoly analysis we assumed that store consumers have high channel-misfit costs of shopping online, which is why they never shop via the online channel. Recall that the channel-agnostic consumers faced no costs for shopping online or in store. We now relax this assumption so that the store-only consumers face a channel-misfit cost of $s$ while shopping through the online channel, so store-only consumers may consider purchasing online if the online price is sufficiently low and the advantage of the low price offsets the channel misfit costs. We restrict attention to the case when the these misfit costs are low compared to the product value, so that there is a

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11 We only require a high enough cost so that consumers in equilibrium would not consider buying online. Note that a misfit cost of $v$ would effectively ensure this outcome.
non-trivial tradeoff.\footnote{We still require that the cost of visiting multiple brick-and-mortar stores is sufficiently large so that consumers do not make multiple trips.}

\textbf{Proposition 6.} Symmetric self-matching is the unique equilibrium when the product value is high relative to the costs faced by store consumers face when shopping in the online channel.

In this case, the channels become more effective substitutes for store-only consumers. This result points to the robustness of proposition\footnote{3} as in both cases, high $v$ imply symmetric self-matching policies in equilibrium. The only difference is that the firms will charge $p^{**} = p^{on*} + s$ instead of $p^{**} = v - t/2$, and hence $p^{on*} > t$, as the firms would have incentive to increase online prices, as online prices now determine how high the firms can price offline.

\subsection*{4.3 Buy Online, Pick-Up In-Store}

Many retailers offer policies that allow consumers to place an online order, and pick-up the product in store, which combines the convenience of online shopping with some advantages of the retail store environment, and offers the ability obtain the product immediately. These policies are closely connected to self-matching policies, if the product is in-stock at the store when the consumer buys it online. As before, decided store-only consumers will be able to claim the online price by purchasing the product online for in-store pick-up, while undecided store-only buyers cannot use this option since they would want to make a decision on products after they are able to evaluate a set of choices in the store. We find that the results from the main analysis carry over in this case.

\subsection*{4.4 Travel Costs}

In the main model we assumed that consumers visit at most one store in a single shopping trip and that they make at most one such trip to reflect the reality of high travel costs in the offline channel. If the value of the product is sufficiently large (relative to the cost of making an additional trip), we would expect consumers to make multiple visits and search all channels for price information before making a purchase decision. As a result, firms will compete with equal intensity for online and in-store buyers. Hence, we find that firms will set the same price $t$ across both channels in equilibrium, regardless of whether or not they offer to self-match. If there is at least a small cost associated with the implementation of a self-matching policy, neither retailer will offer to self-match.
5 Discussion, Limitations and Conclusion

Self-matching policies, where multichannel retailers match their own price across channels (typically online and store) have become commonly used, but not universally by retailers across a wide variety of markets, including sporting goods, apparel, electronics, office supplies and home furnishings. Beginning with the observation that such self-matching is likely to lead consumers to obtain the lower of the online and store prices, or channel arbitrage, we examine whether self-matching leads to a destructive prisoners’ dilemma like situation with lower profits for both firms, or whether there are mechanisms by which self-matching policies may be profitably used in competitive settings.

We develop a framework with multichannel retailers competing against both e-tailers (mixed duopoly) as well as other multichannel retailers (multichannel duopoly). Firms in the model choose a price self-matching policy in the first stage, and then set price levels in the second stage. Products of the retailers are horizontally differentiated, where consumers have a better match with one of the two retailers. Consumers are heterogeneous along two additional dimensions, decision-stage and channel preference, leading to three aspects of heterogeneity modeled in our framework. For decision-stage heterogeneity, we have decided consumers, who are aware of which product they wish to purchase and can examine all the channel and retailer options for price prior to their shopping trip. Undecided consumers, on the other hand, have a general idea of the product category, but have not narrowed down a product, and require a visit to a store for that purpose. For channel preference, we model consumers who are channel-agnostic and examine both channels as well as store-only shoppers who have strong preferences for a bricks-and-mortar store.

We uncover two different mechanisms by which self-matching can enhance firm profits, and in several cases, these effects can overcome the adverse effects of channel arbitrage. First, we find that self-matching can lead to online competition dampening, an effect that arises because a self-matching firm has less of an incentive to reduce online prices to the competitive level in an asymmetric equilibrium, in order to minimize the damage from channel arbitrage. Second, we find that self-matching allows a multi-channel retailer to practice decision-stage segmentation, i.e. to separate out the decided and undecided consumers. When the retailer is able to charge decided store consumers the lower online price by allowing a price-match, it can price even higher in-store and extract surplus from the undecided store consumer. Self-matching is profitable when the positive
effects of decision-stage segmentation and online competition dampening overcome the negative effects of channel arbitrage. We find that the profitability of self-matching strongly depends on the value of the product, the nature of the competition as well as the degree of consumer heterogeneity.

We also examine the case when consumers can use mobile devices in store to obtain instant pricing information from the online channel, and find that self-matching can surprisingly increase retailer profitability under appropriate conditions. Given the increasing prevalence of mobile enabled shoppers, this finding has the potential to to be an important strategy decision for retailers.

We believe this is the first research rigorously examining the idea of self-matching, but the present paper has several limitations, and research would benefit from extending the work along multiple dimensions. First, for reasons of tractability, we do not model competitive price matching policies. Such policies have been extensively explored in the literature and it would be interesting to examine whether they complement or substitute for self-matching policies in a homogenous product market. Second, it would be useful to consider a richer model of consumer decision making, where consumers could visit a retailer’s store, then decide whether to visit a second based on expectations of price as well as the benefits they may obtain. Such an effort would connect with the search literature, and it would be interesting to examine whether self-matching leads to more search and larger consideration sets or less. Finally, although we expect that the mechanisms we have detailed here to apply to the case where there are ex-ante differences among retailers (based on costs, or loyalty etc.), there might be additional insight obtained in modeling this more general case.

An interesting implication for consumer advocates also stems from this research. Consumers increasingly demand consistency across channels, which has led retailers to strive for an omni-channel paradigm [Dahlhoff and Kireyev 2012]. Omni-channel posits that consumers must be offered a consistent experience across the many different channels they may use to interact with the retailer. Self-matching policies, by design, offer retailers the flexibility of setting different prices across channels, while promising consumers a consistent experience, in line with the philosophy of omni-channel. Although self-matching policies appear to offer consistency across channels, our model predicts that price discrepancy may actually grow, decreasing the welfare of undecided shoppers. Therefore, advocating price consistency may actually decrease welfare for certain types of consumers.

Broadly, our findings suggest that although self-matching policies may at first appear to an
unprofitable but necessary evil, they can have more subtle and positive competitive implications. They are not like a prisoners’ dilemma and can be profitably used as a strategic choice. In sum, multichannel retailers should carefully tailor their price self-matching decisions, taking into account their market, product and consumers characteristics and well as competitive reactions.

References


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Mathematical Appendix

Proof of Proposition 1

To ensure an interior solution, assume \( v > t \). Then, monopoly prices are given by \( p_1^{on} = v - t/2 \) and \( p_1^s = v - t/2 \). As the online price is no less than the in-store price, if the monopolist offers an offline-to-online match it will set the same prices and the policy will not be relevant. Assuming the firm serves all segments, profits are given by:

\[
\Pi_1^{SM=0} = \eta p_1^{on} + (1 - \eta)p_1^s + \beta(\eta p_1^{on} + (1 - \eta)p_1^s),
\]
\[
\Pi_1^{SM=1} = \min(p_1^{on}, p_1^s) + \beta(\eta \min(p_1^{on}, p_1^s) + (1 - \eta)p_1^s),
\]

From the profit expressions it is clear that the monopolist cannot earn larger profits under \( SM = 1 \). Any pricing level the monopolist adopts under \( SM = 1 \) can be achieved under \( SM = 0 \). This is because the \( SM = 1 \) strategy is restrictive in that all consumers retrieve the same price, either by purchasing from the same channel, or by redeeming the self-match. Hence, the monopolist will always prefer \( SM = 0 \), which weakly dominates \( SM = 1 \).

Proof of Proposition 2

First, we note that the derivatives of the proportion function \( \Phi_1 \) to be \( \frac{\partial \Phi_1(p_1, p_2)}{\partial p_1} = -\frac{1}{2t} \) and \( \frac{\partial \Phi_1(p_1, p_2)}{\partial p_2} = \frac{1}{2t} \).

Suppose that a multichannel retailer competed with an online-only e-tailer. Assume \( v > 2t \) to ensure that all markets are fully covered. Assume \( v < 4t \) and \( \beta > \frac{1}{3}(\frac{2}{\eta} - 5) \) to ensure that the multichannel firm has positive online sales. Under \((0, 0)\) the firms earn profits

\[
\Pi_1^{0,0} = \eta(1 + \beta)\Phi_1(p_1^{on}, p_2^{on})p_1^{on} + (1 - \eta)(1 + \beta)p_1^s,
\]
\[
\Pi_2^{0,0} = \eta(1 + \beta)(1 - \Phi_1(p_1^{on}, p_2^{on}))p_2^{on}.
\]
Taking the FOCs with respect to the prices, we solve:

\[
\frac{\partial \Pi_1^{0,0}}{\partial p_1^{on}} = \eta(1 + \beta) \left[ \Phi_1(p_1^{on}, p_2^{on}) + p_1^{on} \frac{\partial \Phi_1(p_1^{on}, p_2^{on})}{\partial p_1^{on}} \right] = 0
\]

\[
\frac{\partial \Pi_1^{0,0}}{\partial p_1^s} = (1 - \eta)(1 + \beta) > 0, \text{ implying a corner solution.}
\]

We obtain the corresponding FOCs for firm 2 and solve for the equilibrium corresponding to the best responses of both firms. All channel-agnostic consumers will purchase online, whereas store-only consumers will buy from the multichannel firm’s store. The firms will set competitive prices online, \(p_1^{on} = p_2^{on} = t\), and firm 1 will set monopoly price in-store \(p_s^1 = (v - t)\).

Under the \((1, 0)\) subgame of competition between a self-matching multichannel retailer with an e-tailer, firms earn profits

\[
\Pi_1^{1,0} = \eta(1 + \beta) \Phi_1(p_1^{on}, p_2^{on}) p_1^{on} + (1 - \eta) (p_1^{on} + \beta p_s^1),
\]

\[
\Pi_2^{1,0} = \eta(1 + \beta) (1 - \Phi_1(p_1^{on}, p_2^{on})) p_2^{on}.
\]

As under \((0, 0)\), channel-agnostic consumers will purchase online and store-only consumers will purchase from the multichannel firm’s store. Decided store-only consumers redeem the matching policy, and pay the online price, whereas undecided store-only consumers fail to redeem the policy and pay the in-store price. The firms will set prices \(p_1^{on} = t + \frac{4t(1-\eta)}{3\eta(1+\beta)}, p_2^{on} = t + \frac{2t(1-\eta)}{3\eta(1+\beta)}, p_s^1 = v - t\) for \(v > 2t + \frac{4t(1-\eta)}{3\eta(1+\beta)}\), and \(p_1^{on} = v - t, p_2^{on} = v/2, p_s^1 = v - t\) for \(v \leq 2t + \frac{4t(1-\eta)}{3\eta(1+\beta)}\).

Suppose that \(\beta > \frac{4-7\eta}{3\eta}\). Then, \(\Pi_1^{1,0} > \Pi_0^{1,0}\) if \(v < z_1 = t \left(\frac{7}{3} + \frac{8(1-\eta)}{9\eta(1+\beta)}\right)\). Otherwise, if \(\beta \leq \frac{4-7\eta}{3\eta}\), then \(\Pi_1^{1,0} > \Pi_0^{1,0}\) if \(v < z_2 = 3t\). Hence, there exists a \(z_0 = \min\{z_1, z_2\}\), such that for \(v < z_0\), the multichannel retailer will prefer to self-match.

**Proof of Corollary**

A comparison of the e-tailer’s profits, \(\Pi_2^{1,0} - \Pi_2^{0,0}\) reveals that it earns greater profits when the multichannel retailer offers a self-matching policy. This is because the e-tailer sets a higher price, and earns a greater fraction of demand.
We find that for $\Delta \Pi_1 = \Pi_1^{1,0} - \Pi_1^{0,0}$,

$$\frac{\partial \Delta \Pi_1}{\partial v} < 0, \quad \frac{\partial \Delta \Pi_1}{\partial t} > 0, \quad \text{and} \quad \frac{\partial \Delta \Pi_1}{\partial \beta} < 0$$

which suggests that the difference in the multichannel retailer’s profits between self-matching and no-matching is decreasing in $v$ and $\beta$, but increasing in $t$.

**Proof of Corollary 2**

The firm offering to self-match will set monopoly price $v - t/2$ in-store to leverage the demand from its captive segment of undecided store-only consumers. The online price it sets is increasing at a slower rate than the monopoly price in the $(1,0)$ subgame for low $v$, while in both $(1,1)$ for all $v$ and in $(1,0)$ for high $v$ the online price is constant in $v$. Hence, the price discrepancy across channels grows with $v$.

*Below, we refer to $m$ as the cost of undertaking a second shopping trip for undecided store-only shoppers. We derive bounds on $m$ that ensure the consumer behavior specified in our assumptions.*

**Proof of Proposition 3**

First, we consider each subgame separately. Then, we compare the profits from each subgame to derive the bounds for the equilibrium results. The following constraints need to be imposed:

- $v > 3t/2$ ensures that all markets are fully covered,
- $\beta < 5/3$ ensures that no firm sets such a high online price to earn zero demand from decided consumers in the $(1,0)$ subgame,
- $v < t \left( \frac{9\beta^2 + 24\beta + 25}{36\beta} + \frac{(1+\beta)^2}{36\beta(1+\beta\eta)} + \frac{3\beta^2 + 8\beta + 5}{18\beta(1+\beta\eta)} \right)$ ensures that no firm wants to price exclusively for its captive segment of undecided store-only shoppers and forego all demand for decided store-only shoppers,
- $m > v - \frac{3t}{2}$ ensures that no undecided store-only shoppers switch stores after their first visit, and that no consumer returns home and visits the store a second time to redeem a self-matching policy.
No Matching - (0,0)

Channel-agnostic consumers will purchase online. Store-only consumers will buy in-store. All consumers will pay the price set in the channel they buy from. The firms will earn profits

\[ \Pi_{1,0} = \eta(1 + \beta)\Phi_1(p_{1,0}^o, p_{2,0}^o)p_{1,0}^o + (1 - \eta)\left(\Phi_1(p_1^s, p_2^s) + \frac{\beta}{2}\right)p_1^s, \]

\[ \Pi_{2,0} = \eta(1 + \beta)(1 - \Phi_1(p_{1,0}^o, p_{2,0}^o))p_{2,0}^o + (1 - \eta)\left((1 - \Phi_1(p_1^s, p_2^s)) + \frac{\beta}{2}\right)p_2^s, \]

Again, we solve for the first order conditions:

\[ \frac{\partial\Pi_{1,0}^j}{\partial p^s_j} = 0, \quad \text{and} \quad \frac{\partial\Pi_{2,0}^j}{\partial p_{1,0}^o} = 0 \]

for \( j \in \{1, 2\} \), and check for corner solutions. We find the equilibrium prices to be \( p_{1,0}^m = p_{2,0}^m = t \) and \( p_1^s = p_2^s = t(1 + \beta) \) for \( v/t > 3/2 + \beta \), and \( p_1^s = p_2^s = v - t/2 \) for \( v/t \leq 3/2 + \beta \). The in-store price is larger than the online price as firms have an incentive to price higher for their captive segment of store-only consumers.

Asymmetric Self-Matching - (1,0)

Channel-agnostic consumers will purchase online, and pay the online price. Decided store-only consumer will buy in-store, but will redeem the online price (as it will be lower) if they buy from the self-matching firm. They will pay the in-store price if they buy from the non-self-matching firm. Undecided store-only consumers will buy from the store they first visit and pay the in-store price. The firms will earn profits

\[ \Pi_{1,1} = \eta(1 + \beta)\Phi_1(p_{1,1}^o, p_{2,1}^o)p_{1,1}^o + (1 - \eta)\left(\Phi_1(p_1^s, p_2^s) + \frac{\beta}{2}\right)p_1^s, \]

\[ \Pi_{2,1} = \eta(1 + \beta)(1 - \Phi_1(p_{1,1}^o, p_{2,1}^o))p_{2,1}^o + (1 - \eta)\left(1 - \Phi_1(p_1^s, p_2^s) + \frac{\beta}{2}\right)p_2^s. \]
In equilibrium, the firms set online prices $p_{on}^1 = t \left( \frac{2}{3} + \frac{1+\beta}{3(1+\eta \beta)} \right)$ and $p_{on}^2 = t \left( \frac{5}{6} + \frac{1+\beta}{6(1+\eta \beta)} \right)$ and in-store prices $p_{s}^1 = v - t/2$ and $p_{s}^2 = p_{on}^2 + \beta t/2$ for $v/t > t \left( \frac{4}{3} + \frac{1+\beta}{6(1+\eta \beta)} + \frac{\beta}{2} \right)$. Otherwise, $p_{on}^1 = t + \frac{(1-\eta)(2v-3t)}{4-\eta(1-3\beta)}$, $p_{on}^2 = t + \frac{(1-\eta)(2v-3t)}{2(4-\eta(1-3\beta))}$ and $p_{s}^1 = p_{s}^2 = v - t/2$.

**Symmetric Self-Matching - (1,1)**

Channel-agnostic consumers will purchase online, and pay the online price. Decided store-only consumers will buy in-store, but will redeem the online price of the store they purchase from. Undecided store-only consumers will buy in the store they first visit and will pay the in-store price. The firms will earn profits

$$\Pi_{1,1}^1 = (1 + \eta \beta) \Phi_1(p_{on}^1, p_{on}^2) p_{on}^1 + (1 - \eta) \frac{\beta}{2} p_{s}^1,$$

$$\Pi_{2,1}^1 = (1 + \eta \beta)(1 - \Phi_1(p_{on}^1, p_{on}^2)) p_{on}^2 + (1 - \eta) \frac{\beta}{2} p_{s}^2.$$

and set prices $p_{on}^1 = p_{on}^2 = t$ online and $p_{s}^1 = p_{s}^2 = v - t/2$ in-store.

**Equilibrium Analysis**

A self-matching configuration is an SPNE if no firm has the incentive to unilaterally deviate. Equivalently, for (0, 0) to be an SPNE, firm 1 must not have the incentive to deviate to (1, 0). For (1, 1) to be an SPNE, firm 2 must not have the incentive to deviate to (1, 0). For (1, 0) or (0, 1) to be an SPNE the self-matching firm must not prefer (0, 0) and the non-self-matching firm must not prefer (1, 1). By comparing the profits at the equilibrium prices defined above, we can construct equilibrium regions.

Let $\beta' = \sqrt{25 \eta^2 + 448 \eta + 256 - 108/11} + \frac{5}{22}$. The results in proposition 3 focus on the region where $\beta < \beta'$ for clarity of exposition. In this proof, we provide an extended analysis, including the region where $\beta \geq \beta'$. Define

- $z_1 = \min \left\{ \frac{10 \beta + 27}{18} + \frac{\beta(1+\beta)}{9(1+\beta \eta)}, \frac{9 \beta + 59}{36} + \frac{7(1+\beta)}{36(1+\beta \eta)}, \frac{13}{6} + \frac{8(1-\eta)}{3(8+\eta(1+9\beta))} \right\}$,
- $z_2 = \max \left\{ \frac{9 \beta + 59}{36} + \frac{7(1+\beta)}{36(1+\beta \eta)}, \frac{10 \beta + 27}{18} + \frac{\beta(1+\beta)}{9(1+\beta \eta)} \right\}$,
Figure 6: Pareto equilibrium regions for fixed $\eta$.

- $z_3 = \frac{35}{18} + \beta - \frac{1+\beta}{9(1+\beta)\eta}$.

Then, for $v/t < z_1$, $(1, 0)$ and $(0, 1)$ are SPNE. For $z_1 < v/t < z_2$, $(0, 0)$ is the unique equilibrium for $\beta < \beta'$ and $(1, 1)$ is the unique equilibrium for $\beta > \beta'$. For $z_2 < v/t < z_3$, both $(0, 0)$ and $(1, 1)$ are SPNE. For $v/t > z_3$, $(1, 1)$ is the unique SPNE.

Note that for sufficiently large $\beta$, firms prefer to offer symmetric self-matching policies at intermediate $v$. This is because, as illustrated in figure 3 in the paper, as $\beta$ grows, firm 1 has an incentive to match for lower $v$ given that firm 2 also matches. As the critical threshold of $v$ becomes lower, it may cross the threshold at which the other firm no longer prefers to match, yielding an equilibrium where both firms match for intermediate $v$. Figure 6 shows the equilibrium regions for the full range of $\beta$ values.

To summarize, for low $\beta$, as $v$ increases, there will first be an asymmetric solution, then $(0, 0)$, then both $(0, 0)$ and $(1, 1)$, and then uniquely $(1, 1)$. For high $\beta$, as $v$ increases, there will first be an asymmetric solution, then $(1, 1)$, then both $(0, 0)$ and $(1, 1)$, and then uniquely $(1, 1)$.

**Proof of Corollary 4**

A comparison of profits in the $(1, 0)$ subgame reveals that $\Pi_{1,0}^1 > \Pi_{1,0}^2$ everywhere. A comparison of profits earned by firm 1 in the $(1, 1)$ subgame and in the $(0, 0)$ subgame reveals that $\Pi_{1,1}^1 > \Pi_{0,0}^1$ if $v > \frac{5t}{2} + \beta t = z_4$, which is strictly greater than $z_3$. 

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Proof of Proposition 5

The proof of proposition 5 proceeds just as that the previous proposition, except with an extra parameter $\mu$ representing fraction of “mobile” consumers, or consumers who can costlessly search for online information while in-store. The $\mu$ segment will be relevant for undecided store-only shoppers, as these will only be able to claim a self-matching policy if they have access to the internet in-store. The non-mobile undecided store-only shoppers will not be able to claim a self-matching policy, and will have to pay the in-store price. We require the following restrictions:

- $v > \frac{3t}{2} + \frac{\beta \mu (1-\eta)}{1+\beta \eta}$ ensures that all markets are fully covered,
- $\beta < \frac{5-2\mu}{3}$ ensures that no firm sets such a high online price to earn zero demand from decided shoppers in the (1, 0) subgame,
- $v < t \left( \frac{1}{\beta} + \frac{4\mu + 16\eta + 3 \beta \eta - 4 \mu \eta + 2}{12 \eta} - \frac{(1+2\mu)^2(1-\eta)^2}{36 \eta (1+\beta \eta)^2} + \frac{(1+2\mu)(1-\eta)(2 \mu + 11 \eta - 2 \mu \eta - 5)}{36 \eta (1+\beta \eta)} \right)$ ensures that no firm wants to price exclusively for its captive segment of undecided store-only shoppers and forego all demand for decided store-only shoppers,
- $m > v - \frac{3t}{2} + \mu t - \frac{\mu t (1+\beta)}{1+\beta \eta}$ ensures that no undecided store-only shoppers switch stores after their first visit, and that no consumer returns home and visits the store a second time to redeem a self-matching policy.

No Firms Self-Match - (0,0)

The equilibrium prices under (0, 0) emerge just as in proposition 3, as mobile consumers behave just as the rest of the consumers.

One Firm Self-Matches - (1,0)

Undecided store-only consumers who are mobile will redeem the self-matching policy if they first visit the store that offers the policy. Profits are
\[ \Pi_{1,0}^1 = \eta(1 + \beta)\Phi_1(p_{1n}^o, p_{2n}^o)p_{1n}^o + (1 - \eta)\left(\Phi_1(p_{1n}^o, p_{2n}^s)p_{1n}^o + \frac{\beta}{2}(1 - \mu)p_1^s + \mu p_{1n}^o\right), \]

\[ \Pi_{2,0}^1 = \eta(1 + \beta)(1 - \Phi_1(p_{1n}^o, p_{2n}^o))p_{2n}^o + (1 - \eta)\left(1 - \Phi_1(p_{1n}^o, p_{2n}^s)\right) + \frac{\beta}{2} p_{2n}^s. \]

In equilibrium, the firms set online prices \[ p_{1n}^o = t\left(\frac{(1+2\mu)(1+\beta)}{3(1+\beta\eta)} + \frac{2(1-\mu)}{3}\right) \]

and in-store prices \[ p_{2n}^o = p_{2n}^s + \beta t/2 \] for \( v/t > t \left(\frac{(1+2\mu)(1+\beta)}{6(1+\beta\eta)} + \frac{8-2\mu+3\beta}{6}\right). \] Otherwise, \[ p_{1n}^o = \frac{t+2v+2nt-2v+2\beta\mu t+3\beta nt-2\beta \mu t}{4-\eta+3\beta \eta}, \]

\[ p_{2n}^o = \frac{5t+2v-nt-2v+2\beta \mu t+6\beta nt-\beta \mu t}{4-\eta+3\beta \eta} \] and \( p_{1n}^s = p_{2n}^s = v - t/2. \)

**Both Firms Self-Match - (1,1)**

The firms will earn profits

\[ \Pi_{1,1}^1 = (1 + \eta \beta)\Phi_1(p_{1n}^o, p_{2n}^o)p_{1n}^o + (1 - \eta)\left(\frac{\beta}{2}(1 - \mu)p_1^s + \mu p_{1n}^o\right), \]

\[ \Pi_{2,1}^1 = (1 + \eta \beta)(1 - \Phi_1(p_{1n}^o, p_{2n}^o))p_{2n}^o + (1 - \eta)\left(\frac{\beta}{2}(1 - \mu)p_2^s + \mu p_{2n}^o\right). \]

and set prices \( p_{1n}^o = p_{2n}^o = t(1 - \mu) + \frac{\mu t(1+\beta)}{1+3\beta \eta} \) online and \( p_{1n}^s = p_{2n}^s = v - t/2 \) in-store.

**Equilibrium Analysis**

For tractability, set \( \eta = 0.2 \) and \( \beta = 0.5. \) Let

\[ y_0 = \frac{2385\mu - 41(529\mu^2 + 1408\mu + 1936)^{1/2} + 7810}{4004}, \]

\[ y_1 = \frac{33\mu + 331}{198} + \frac{4}{11(2+\mu)}, \]

\[ y_2 = \frac{64\mu + 395}{198} + \frac{3}{88(1-\mu)}, \]

\[ y_3 = \frac{32\mu + 427}{198} + \frac{3}{22(1-\mu)}. \]

Then, for \( v/t < y_0, \) \( (1, 1) \) is the unique SPNE. For \( y_0 < v/t < y_1, (1, 0) \) and \( (0, 1) \) are SPNE. For \( y_1 < v/t < y_2, \) \( (0, 0) \) is the unique equilibrium. For \( y_2 < v/t < y_3, \) both \( (0, 0) \) and \( (1, 1) \) are SPNE.
For $v/t > y_3$, $(1, 1)$ is the unique SPNE.

To prove the associated proposition, note that $y_0, y_1, y_2$ and $y_3$ are all increasing in $\mu$, so that holding constant $v/t$, an increase in mobile consumers shrinks the equilibrium region that admits self-matching policies. Also, note that there is now a new region for $v/t < y_0$ that allows for symmetric self-matching which grows at a slower rate than the existing regions shrink.

**Example of increasing profits with mobile consumers**

In the $(1, 1)$ equilibrium for large $v$, $\eta = 0.2$ and $\beta = 0.5$, the retailers’ profits are increasing in $\mu$ if $v/t < 5/2 + 8\mu/11$, which is possible if $\mu < 0.83$. Furthermore, the retailers’ profits are larger than when $\mu = 0$ if $v/t < 5/2 + 4\mu/11$, which is possible if $\mu < 0.72$. This shows that retailers in symmetric self-matching equilibria need not fear the existence of mobile consumers.

**Proof of Proposition 6**

We require that $v > s + t/2 + t(1 + \beta)/(1 + \beta\eta)$ for the analysis of the equilibria below.

**No Firms Self-Match - $(1,1)$**

In this subgame the two firms will price $p_1^o = p_2^o = t$ and $p_1^s = p_2^s = t(1 + \beta)$ for $s > \beta t$, and $p_1^o = p_2^o = \frac{t(1 + \beta) - s(1 - \eta)}{1 + \beta\eta}$ and $p_1^s = p_2^s = p_1^o + s$ for $s < \beta t$.

**One Firm Self-Matches - $(1,1)$**

In the asymmetric subgame, the firms will price $p_1^o = \frac{t(1 + \beta)}{1 + \beta\eta}$, $p_2^o = \frac{t}{2} + \frac{t(1 + \beta)}{2(1 + \beta\eta)}$ online, and $p_1^s = p_2^s + s$, $p_2^s = \frac{t(1 + \beta)(2 + \beta\eta)}{2(1 + \beta\eta)}$ in-store for $s > \beta t/2$. For $s < \beta t/2$, firm 2 will set $p_2^o = \frac{t(1 + \beta) - s(1 - \eta)}{1 + \beta\eta}$ and $p_2^s = \frac{t + s}{1 + \beta\eta}$.

**Both Firms Self-Match - $(1,1)$**

Under symmetric self-matching, the firms will set $p_1^o = p_2^o = \frac{t(1 + \beta)}{1 + \beta\eta}$ online and $p_1^s = p_2^s = p_1^o + s$ in-store.

**Equilibrium Analysis**

A comparison of profits yields that $(1, 1)$ is the unique equilibrium solution.