Internet Appendix for:

Social Risk, Fiscal Risk, and the Portfolio of Government Programs

Samuel G. Hanson David S. Scharfstein Adi Sunderam
Harvard University

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Contents

A Programs that impact the tax base

In this section, we show how the analysis should be modified if the program under consideration adds to the tax base. Suppose that the program adds to the tax base so that tax revenue is given by

$$T_t = \ell_t^* \tau_t \left(Y_t + W_t(q)\right) = \left(1 - \eta\tau_t\right)\tau_t \left(Y_t + W_t(q)\right).$$

Thus, household consumption is given by

$$\begin{aligned}
C_t\left(\ell_t^*\right) &= \left(Y_t + W_t(q)\right)\left(\ell_t^*\left(1-\tau_t\right) - \frac{\left(\ell_t^* - 1 + \eta\right)^2 - \eta^2}{2\eta}\right) + (\text{Net trade govt bonds})_t \\
&= \left(Y_t + W_t(q)\right)\left(\ell_t^*\left(1-\tau_t\right) - \frac{\left(\ell_t^* - 1 + \eta\right)^2 - \eta^2}{2\eta}\right) + T_t - G_t - X_t(q) \\
&= \left(Y_t + W_t(q)\right)\left(\ell_t^* - \frac{\left(\ell_t^* - 1 + \eta\right)^2 - \eta^2}{2\eta}\right) - G_t - X_t(q) \quad [\text{since } T_t = \ell_t^*\tau_t\left(Y_t + W_t(q)\right)] \\
&= \left(Y_t + W_t(q)\right)\left(\left(1-\eta\tau_t\right) - \frac{\left(1 - \eta\tau_t - 1 + \eta\right)^2 - \eta^2}{2\eta}\right) - G_t - X_t(q) \quad [\text{since } \ell_t^* = 1 - \eta\tau_t] \\
&= \left(Y_t + W_t(q)\right)\left(1 - \frac{1}{2}\eta\tau_t^2\right) - G_t - X_t(q) \quad [\text{simplifying}]
\end{aligned}$$

The optimal tax rate is

$$\tau_t = \frac{1 - \sqrt{1 - 4\eta\frac{T_t}{Y_t + W_t(q)}}}{2\eta}.$$

Thus, we have

$$\frac{\partial\tau_t}{\partial T_t} = \frac{1}{1 - 2\eta\tau_t}\frac{1}{Y_t + W_t(q)},$$

so

$$\frac{\partial C_t}{\partial\tau_t} \times \frac{\partial\tau_t}{\partial T_t} = -\frac{\eta\tau_t}{1 - 2\eta\tau_t} = -h'\left(\tau_t\right).$$

And we have

$$\frac{\partial\tau_t}{\partial W_t(q)} = -\frac{\left(1 - \eta\tau_t\right)\tau_t}{\left(1 - 2\eta\tau_t\right)\left(Y_t + W_t(q)\right)},$$

so

$$\frac{\partial C_t}{\partial\tau_t} \times \frac{\partial\tau_t}{\partial W_t(q)} = \left(1 - \eta\tau_t\right)\tau_t \times \frac{\eta\tau_t}{\left(1 - 2\eta\tau_t\right)} = \left(1 - \eta\tau_t\right)\tau_t \times h'\left(\tau_t\right).$$

The first-order condition for D_0 is the same in the baseline case where the program has no effect on the tax base. Specifically, we have

$$0 = u'\left(C_0\right)\frac{\partial C_0}{\partial\tau_0}\frac{\partial\tau_0}{\partial T_0}\frac{\partial T_0}{\partial D_0} + \beta E\left[u'\left(C_1\right)\frac{\partial C_1}{\partial\tau_1}\frac{\partial\tau_1}{\partial T_1}\frac{\partial T_1}{\partial D_1}\right].$$

Using the fact that $\frac{\partial C_t}{\partial \tau_t}\frac{\partial \tau_t}{\partial T_t} = -h'(\tau_t)$, $\frac{\partial T_0}{\partial D_0} = -1$, and $\frac{\partial T_1}{\partial D_0} = \left(R_f + D_0 \frac{\partial R_f}{\partial D_0}\right)$, this implies that

$$u'(C_0)\, h'(\tau_0) = \left(R_f + D_0 \frac{\partial R_f}{\partial D_0}\right) \beta E\left[u'(C_1)\, h'(\tau_1)\right]. \tag{1}$$

However, the fact that the program impacts the tax base will impact the first-order condition for q. As in the baseline model, the first-order condition for q still takes the form

$$0 = u'(C_0)\left(\frac{\partial C_0}{\partial q} + \frac{\partial C_0}{\partial \tau_0}\frac{\partial \tau_0}{\partial q}\right) + \beta E\left[u'(C_1)\left(\frac{\partial C_1}{\partial q} + \frac{\partial C_1}{\partial \tau_1}\frac{\partial \tau_1}{\partial q}\right)\right].$$

But we now have

$$\begin{aligned}
\frac{\partial C_0}{\partial \tau_0}\frac{\partial \tau_0}{\partial q} &= \frac{\partial C_0}{\partial \tau_0}\frac{\partial \tau_0}{\partial W_0}\frac{\partial W_0}{\partial q} + \frac{\partial C_0}{\partial \tau_0}\frac{\partial \tau_0}{\partial T_0}\frac{\partial T_0}{\partial q} \\
&= (1-\eta\tau_0)\,\tau_0 h'(\tau_0)\, W_0'(q) - h'(\tau_0)\, X_0'(q)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial C_1}{\partial \tau_1}\frac{\partial \tau_1}{\partial q} &= \frac{\partial C_1}{\partial \tau_1}\frac{\partial \tau_1}{\partial W_1}\frac{\partial W_1}{\partial q} + \frac{\partial C_1}{\partial \tau_1}\frac{\partial \tau_1}{\partial T_1}\frac{\partial T_1}{\partial q} \\
&= (1-\eta\tau_1)\,\tau_1 h'(\tau_1)\, W_1'(q) - h'(\tau_1)\left(X_1'(q) + D_0\frac{\partial R_f}{\partial q}\right)
\end{aligned}$$

This is because the program impacts both government expenditures and the tax base. As a result the program has two independent effects on tax rates and, thus, the distortionary costs of taxation.

Thus, can write the effect of changing q on household consumption at time 0 as:

$$\begin{aligned}
\frac{\partial C_0}{\partial q} + \frac{\partial C_0}{\partial \tau_0}\frac{\partial \tau_0}{\partial q} &= W_0'(q)\,(1 - \frac{\eta}{2}\tau_0^2) - X_0'(q) + (1-\eta\tau_0)\,\tau_0 h'(\tau_0)\, W_0'(q) - h'(\tau_0)\, X_0'(q) \\
&= W_0'(q)\,(1 + \frac{\tau_0}{2}h'(\tau_0)) - (1 + h'(\tau_0))\, X_0'(q)
\end{aligned}$$

which follows from

$$\begin{aligned}
\left(1 - \frac{\eta}{2}\tau^2\right) + (1-\eta\tau)\,\tau h'(\tau) &= \left(1 - \frac{\eta}{2}\tau^2\right) + (1-\eta\tau)\,\tau\frac{\eta\tau}{1-2\eta\tau} \\
&= 1 + \frac{1}{2}\tau\frac{\eta\tau}{1-2\tau\eta} = 1 + \frac{1}{2}\tau h'(\tau).
\end{aligned}$$

Similarly, the effect on consumption at time 1 is

$$\begin{aligned}
\frac{\partial C_1}{\partial q} + \frac{\partial C_1}{\partial \tau_1}\frac{\partial \tau_1}{\partial q} &= W_1'(q)\,(1 - \frac{\eta}{2}\tau_1^2) - X_1'(q) + (1-\eta\tau_1)\,\tau_1 h'(\tau_1)\, W_1'(q) - h'(\tau_1)\left(X_1'(q) + D_0\frac{\partial R_f}{\partial q}\right) \\
&= W_1'(q)\,(1 + \frac{\tau_1}{2}h'(\tau_1)) - (1 + h'(\tau_1))\, X_1'(q) - h'(\tau_1)\, D_0\frac{\partial R_f}{\partial q}
\end{aligned}$$

Thus, the optimal scale of the program satisfies

$$\begin{aligned} 0 &= u'(C_0)\Big(W_0'(q)(1+\frac{\tau_0}{2}h'(\tau_0)) - (1+h'(\tau_0))X_0'(q)\Big) \\ &\quad +\beta E\left[u'(C_1)\left(W_1'(q)(1+\frac{\tau_1}{2}h'(\tau_1)) - (1+h'(\tau_1))X_1'(q) - h'(\tau_1)D_0\frac{\partial R_f}{\partial q}\right)\right]. \end{aligned} \tag{2}$$

This expression can be compared to the expression in the baseline model which is

$$\begin{aligned} 0 &= u'(C_0)(W_0'(q) - (1+h'(\tau_0))X_0'(q)) \\ &\quad +\beta E\left[u'(C_1)\left(W_1'(q) - (1+h'(\tau_1))X_1'(q)X_1'(q) - h'(\tau_1)D_0\frac{\partial R_f}{\partial q}\right)\right]. \end{aligned} \tag{3}$$

Comparing the two expression, we see that there is an additional benefit of the form $W_t'(q)\frac{\tau_t}{2}h'(\tau_t))$ associated with programs that expand the tax base.

When programs can add to the tax base, the need to manage fiscal risk can reinforce the government's desire to manage social risk, partially alleviating the normal tension between fiscal and social risk management. Specifically, programs that add to the tax base are valuable because they help keep tax rates and the associated deadweight losses low. And, programs are especially valuable if they tend to lift the tax base in time 1 states when it would otherwise decline.

B Optimal debt management problem

Our model embeds the core intuitions present in the state-contingent debt management problem studied in Bohn (1990), Aiyagari, Marcet, Sargent, and Seppala (2002), and Bhandari, Evans, Golosov, and Sargent (2016). In this problem, the government chooses riskless debt and its issuance q of a risky security with time 0 price $P = -X_0$ and state-contingent time 1 payoffs X_1 to minimize the cost of distortionary taxes. This problem can be studied in our framework by setting $W_t = X_t$, so that the net benefits of the project are zero. In this case, the choice of q only impacts household consumption insofar as it impacts tax revenues. Specifically, the first-order condition for q says that the government wants to hedge background fiscal risk by issuing risky securities that have low returns in states where other government spending (G_1) is unexpectedly high, so that the tax rate τ_1 is unexpectedly high.

Letting P and q denote the price of the risky security and the quantity of the risky security that the government issues, the government's budget constraints are

$$\begin{aligned} T_0 + qP + D_0 &= G_0 + \overline{D} \\ T_1 &= G_1 + qX_1 + R_f D_0. \end{aligned}$$

Letting z denote the representative household's holding of the risky asset, the household budget constraints are

$$\begin{aligned} C_0 &= Y_0(1-\frac{\eta}{2}\tau_0^2) + \overline{D} - B_0 - zP - T_0 \\ C_1 &= Y_1(1-\frac{\eta}{2}\tau_1^2) + zX_1 + R_f B_0 - T_1. \end{aligned}$$

Atomistic households choose their holdings of the riskless bond (B_0) and of the risky security (z) taking prices and taxes as given. Thus, the Euler equation for B_0 is

$$u'(C_0) = R_f \beta E[u'(C_1)]$$

and Euler equation for z is

$$u'(C_0) P = \beta E[u'(C_1) X_1].$$

Imposing market clearing ($B_0 = D_0$ and $q = z$) and substituting in the government's budget constraints into the household budget constraints, we have

$$\begin{aligned} C_0 &= Y_0(1 - \frac{\eta}{2}\tau_0^2) - G_0 \\ C_1 &= Y_1(1 - \frac{\eta}{2}\tau_1^2) - G_1. \end{aligned}$$

Thus, the government's problem is

$$\max_{q, D_0} \left\{ u\left(Y_0(1 - \frac{\eta}{2}\tau_0^2) - G_0\right) + \beta E\left[u\left(Y_1(1 - \frac{\eta}{2}\tau_1^2) - G_1\right)\right]\right\},$$

and where R_f is implicitly defined by

$$u'\left(Y_0(1 - \frac{\eta}{2}\tau_0^2) - G_0\right) = R_f \beta E\left[u'\left(Y_1(1 - \frac{\eta}{2}\tau_1^2) - G_1\right)\right]$$

and P is implicitly defined by

$$u'\left(Y_0(1 - \frac{\eta}{2}\tau_0^2) - G_0\right) P = R_f \beta E\left[u'\left(Y_1(1 - \frac{\eta}{2}\tau_1^2) - G_1\right) X_1\right].$$

Finally, τ_0 and τ_1 are defined by

$$\begin{aligned} Y_0 \tau_0 (1 - \eta \tau_0) &= \overline{D} - D_0 + G_0 - Pq \\ Y_1 \tau_1 (1 - \eta \tau_1) &= R_f D_0 + G_1 + X_1 q, \end{aligned}$$

The first order condition for D_0 is

$$\left(1 + q\frac{\partial P}{\partial D_0}\right) h'(\tau_0) u'(C_0) = \beta E\left[u'(C_1) h'(\tau_1)\left(R_f + D_0 \frac{\partial R_f}{\partial D_0}\right)\right].$$

where, for a small increase in D_0, $(1 + q\partial P/\partial D_0)$ is the marginal decline in tax revenue at 0 and $R_f + D_0 \partial R_f / \partial D_0$ is marginal increase in tax revenue at 1. The first order condition for q is

$$\left(P + q\frac{\partial P}{\partial q}\right) h'(\tau_0) u'(C_0) = \beta E\left[u'(C_1) h'(\tau_1)\left(X_1 + D_0 \frac{\partial R_f}{\partial q}\right)\right]$$

where, for a small increase in q, $(P + q\partial P/\partial q)$ is the marginal decline in tax revenue at 0 and $X_1 + D_0 \partial R_f / \partial q$ is marginal increase in tax revenue at 1.

Using the definition of the stochastic discount factor, this implies that at an optimum we have

$$1 = E\left[\overbrace{M_1 \frac{h'(\tau_1)}{h'(\tau_0)}}^{\text{Fiscal SDF}} \times \overbrace{\frac{R_f + D_0 \frac{\partial R_f}{\partial D_0}}{1 + q \frac{\partial P}{\partial D_0}}}^{\text{Marginal fiscal return on riskless bonds}}\right]$$

and

$$1 = E\left[\overbrace{M_1 \frac{h'(\tau_1)}{h'(\tau_0)}}^{\text{Fiscal SDF}} \times \overbrace{\frac{X_1 + D_0 \frac{\partial R_f}{\partial q}}{P + q \frac{\partial P}{\partial q}}}^{\text{Marginal fiscal return on risky security}}\right].$$

Thus, both optimality conditions can be written in the standard Euler equation form $1 = E[SDF \times RET]$. Here the relevant stochastic discount factor—the fiscal SDF—is the product of the private SDF, M_1, and the ratio of marginal tax distortions at time 1 and 0, $h'(T_1)/h'(T_0)$. Similarly, the relevant return here is not the private return. Instead, it is a fiscal return that accounts for the general equilibrium effects that arise when $u''(\cdot) < 0$. Specifically, we have

$$\begin{aligned}
\frac{\partial R_f}{\partial q} &= \frac{R_f \beta E[u''(C_1) h'(\tau_1) X_1] + u''(C_0) h'(\tau_0) P}{\beta E[u'(C_1) - R_f D_0 u''(C_1) h'(\tau_1)]} \leq 0 \\
\frac{\partial R_f}{\partial D_0} &= \frac{R_f^2 \beta E[h'(\tau_1) u''(C_1)] + u''(C_1) h'(\tau_0)}{\beta E[u'(C_1) - R_f D_0 u''(C_1) h'(\tau_1)]} \leq 0
\end{aligned}$$

where the former assumes that $X_1 \geq 0$. We also have

$$\begin{aligned}
\frac{\partial P}{\partial q} &= -\frac{u''(C_0) h'(\tau_0) P^2 + \beta E[u''(C_1) h'(\tau_1) X_1^2]}{u'(C_0) + u''(C_0) h'(T_0) qP} \\
\frac{\partial P}{\partial D_0} &= -\frac{u''(C_0) h'(\tau_0) P + \beta R_f E[u''(C_1) h'(\tau_1) X_1]}{u'(C_0) + u''(C_0) h'(\tau_0) qP}
\end{aligned}$$

which are ambiguous.

There is a parallel here to the literature on intermediary-based asset pricing (He and Krishnamurthy [2013], Brunnermeier and Sanikov [2014]). In those models, households indirectly hold financial assets via intermediaries. And, because there are frictions between the household sector and the financial sector, the stochastic discount factor that prices financial assets equals households' stochastic discount factors with an adjustment that captures these intermediation frictions. A similar result obtains in our model because tax distortions mean that the government perceives a financing wedge term between itself and the households it represents.

C Approximate model solution with no borrowing

In this section, we explore the approximate solution to our model in the case where the government cannot borrow at $t = 0$. In this case, the optimal value of q satisfies a quadratic equation that can be solved in closed form. However, much like in the case where $\gamma > 0$ and $\eta = 0$ many of the comparative statics are ambiguous due to competing income and substitution effects.

Suppose that $\overline{D}=0$ and that the government cannot borrow at $t=0$ so that $D_0=0$. Then the optimal level of q satisfies

$$\begin{aligned} 0 &= \left(1-\gamma(\tilde{C}_0-\overline{C})\right)\left(W_0-X_0-\left(\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{Y}}\left(\frac{T_0}{\overline{T}}-\frac{Y_0}{\overline{Y}}\right)\right)X_0\right) \\ &\quad +\beta E\left[\left(1-\gamma(\tilde{C}_1-\overline{C})\right)\left(W_1-X_1-\left(\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{Y}}\left(\frac{T_1}{\overline{T}}-\frac{Y_1}{\overline{Y}}\right)\right)X_1\right)\right] \end{aligned}$$

where

$$\begin{aligned} T_t &= G_t+qX_t \\ \tilde{C}_t &= Y_t-\left(\bar{Y}\frac{\eta}{2}\bar{\tau}^2+\overline{h}'\left(T_t-\overline{T}\right)-\hat{\eta}_Y\left(Y_t-\overline{Y}\right)\right)+\left(W_t-X_t\right)q-G_t. \end{aligned}$$

Thus, we obtain

$$\begin{aligned} 0 &= \left(1-\gamma\left(Y_0-\overline{C}-\left(\bar{Y}\frac{\eta}{2}\bar{\tau}^2+\overline{h}'\left(G_0+qX_0-\overline{T}\right)-\hat{\eta}_Y\left(Y_0-\overline{Y}\right)\right)+q\left(W_0-X_0\right)-G_0\right)\right) \\ &\quad \times\left(\left(W_0-X_0\right)-\left(\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{Y}}\left(\frac{G_0+qX_0}{\overline{T}}-\frac{Y_0}{\overline{Y}}\right)\right)X_0\right) \\ &\quad +\beta E\left[\begin{array}{c} \left(1-\gamma\left(Y_1-\overline{C}-\left(\bar{Y}\frac{\eta}{2}\bar{\tau}^2+\overline{h}'\left(G_1+qX_1-\overline{T}\right)-\hat{\eta}_Y\left(Y_1-\overline{Y}\right)\right)+q\left(W_1-X_1\right)-G_1\right)\right) \\ \times\left(\left(W_1-X_1\right)-\left(\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{Y}}\left(\frac{G_1+qX_1}{\overline{T}}-\frac{Y_1}{\overline{Y}}\right)\right)X_1\right) \end{array}\right] \\ &= q^2\times\overbrace{\left[\begin{array}{c} \gamma\left(W_0-X_0\left(1+\overline{h}'\right)\right)\times\left(\bar{\eta}\frac{\overline{T}}{\overline{Y}}\frac{X_0^2}{\overline{T}}\right) \\ +\beta E\left[\gamma\left(W_1-X_1\left(1+\overline{h}'\right)\right)\times\left(\bar{\eta}\frac{\overline{T}}{\overline{Y}}\frac{X_1^2}{\overline{T}}\right)\right] \end{array}\right]}^{A} \\ &\quad +q\times\overbrace{\left[\left(\begin{array}{c} -\left(1-\gamma\left(Y_0-\overline{C}-\bar{Y}\frac{\eta}{2}\bar{\tau}^2-\overline{h}'\left(G_0-\overline{T}\right)-G_0+\hat{\eta}_Y\left(Y_0-\overline{Y}\right)\right)\right)\times\left(\bar{\eta}\frac{\overline{T}}{\overline{Y}}\frac{X_0^2}{\overline{T}}\right) \\ -\gamma\left(W_0-X_0\left(1+\overline{h}'\right)\right)\times\left(W_0-X_0\left(1+\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{Y}}\left(\frac{G_0}{\overline{T}}-\frac{Y_0}{\overline{Y}}\right)\right)\right) \\ -\beta E\left[\left(1-\gamma\left(Y_1-\overline{C}-\bar{Y}\frac{\eta}{2}\bar{\tau}^2-\overline{h}'\left(G_1-\overline{T}\right)-G_1+\hat{\eta}_Y\left(Y_1-\overline{Y}\right)\right)\right)\times\left(\bar{\eta}\frac{\overline{T}}{\overline{Y}}\frac{X_1^2}{\overline{T}}\right)\right] \\ -\beta E\left[\gamma\left(W_1-X_1\left(1+\overline{h}'\right)\right)\times\left(W_1-X_1\left(1+\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{Y}}\left(\frac{G_1}{\overline{T}}-\frac{Y_1}{\overline{Y}}\right)\right)\right)\right] \end{array}\right)\right]}^{B} \\ &\quad +\overbrace{\left[\begin{array}{c} \left(1-\gamma\left(Y_0-\overline{C}-\bar{Y}\frac{\eta}{2}\bar{\tau}^2-\overline{h}'\left(G_0-\overline{T}\right)-G_0+\hat{\eta}_Y\left(Y_0-\overline{Y}\right)\right)\right) \\ \times\left(W_0-X_0\left(1+\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{Y}}\left(\frac{G_0}{\overline{T}}-\frac{Y_0}{\overline{Y}}\right)\right)\right) \\ +\beta E\left[\begin{array}{c} \left(1-\gamma\left(Y_1-\overline{C}-\bar{Y}\frac{\eta}{2}\bar{\tau}^2-\overline{h}'\left(G_1-\overline{T}\right)-G_1+\hat{\eta}_Y\left(Y_1-\overline{Y}\right)\right)\right) \\ \times\left(W_1-X_1\left(1+\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{Y}}\left(\frac{G_1}{\overline{T}}-\frac{Y_1}{\overline{Y}}\right)\right)\right) \end{array}\right] \end{array}\right]}^{C} \end{aligned}$$

In other words, the optimal value of q^* satisfies the following quadratic equation in q :

$$0=A\times q^2+B\times q+C.$$

The solutions to this first-order condition are:

$$q^+ = \frac{-B+\sqrt{B^2-4AC}}{2A} \text{ and } q^- = \frac{-B-\sqrt{B^2-4AC}}{2A}.$$

The second order condition for an optimum is $2Aq^* + B < 0$. Thus, assuming that the discriminant $B^2 - 4AC > 0$, the optimum us

$$q^* = q^- = \frac{-B-\sqrt{B^2-4AC}}{2A}.$$

Comparative statics on q^* for any exogenous parameter θ, then follow from

$$\frac{\partial q^*}{\partial \theta} = -\frac{\left[A'(\theta) q^{*2} + B'(\theta) q^* + C'(\theta)\right]}{\left[2A(\theta) q^* + B(\theta)\right]} \propto \left[A'(\theta) q^{*2} + B'(\theta) q^* + C'(\theta)\right].$$

As in the case where $\gamma > 0$ and $\eta = 0$, many of these comparative statics will ambiguous due to competing income and substitution effects.

Special case where $\gamma > 0$ and $\eta = 0$ Whether or not the government can borrow, the solution is:

$$\begin{aligned} q^* &= \frac{1-\gamma\left(E\left[Y_1-G_1\right]-\overline{C}\right)}{\gamma} \frac{(W_0-X_0)+\beta E\left[W_1-X_1\right]}{(W_0-X_0)^2+\beta\left(E\left[W_1-X_1\right]\right)^2+\beta\left(Var\left[W_1-X_1\right]\right)} \\ &- \frac{\left((Y_0-G_0)-E\left[Y_1-G_1\right]\right)(W_0-X_0)}{(W_0-X_0)^2+\beta\left(E\left[W_1-X_1\right]\right)^2+\beta\left(Var\left[W_1-X_1\right]\right)} \\ &- \frac{\beta Cov\left[Y_1-G_1, W_1-X_1\right]}{(W_0-X_0)^2+\beta\left(E\left[W_1-X_1\right]\right)^2+\beta\left(Var\left[W_1-X_1\right]\right)}. \end{aligned}$$

In this Ricardian case, we obtain the same solution whether or not the firm can borrow. This is because the way that expenditures are financed in this case is irrelevant for welfare.

Special case where $\gamma = 0$ and $\eta > 0$ When government cannot borrow at $t = 0$ to smooth taxes over time, the solution is:

$$\begin{aligned} q^* &= \frac{(W_0-X_0)+\beta E\left[W_1-X_1\right]}{\left(\bar{\eta}/\overline{Y}\right)\left[(X_0^2)+\beta\left(E\left[X_1\right]\right)^2+\beta Var\left[X_1\right]\right]} \\ &- \frac{\left(\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{Y}}\left(\frac{G_0}{\overline{T}}-\frac{Y_0}{\overline{Y}}\right)\right)X_0+\left(\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{Y}}E\left[\frac{G_1}{\overline{T}}-\frac{Y_1}{\overline{Y}}\right]\right)\beta E\left[X_1\right]}{\left(\bar{\eta}/\overline{Y}\right)\left[(X_0^2)+\beta\left(E\left[X_1\right]\right)^2+\beta Var\left[X_1\right]\right]} \\ &- \frac{\beta Cov\left[X_1, \left(G_1-\frac{\overline{T}}{\overline{Y}}Y_1\right)\right]}{\left[(X_0^2)+\beta\left(E\left[X_1\right]\right)^2+\beta Var\left[X_1\right]\right]} \end{aligned}$$

By contrast, when the government can borrow, the solution is:

$$
\begin{aligned}
q^* \quad = \quad & \frac{1}{\bar{\eta}/\overline{Y}} \frac{(W_0 - X_0) + \beta E[W_1 - X_1]}{(1+\beta)^{-1}(X_0 + \beta E[X_1])^2 + \beta Var[X_1]} \\
& - \frac{\bar{h}'/\left(\bar{\eta}/\overline{Y}\right) + (1+\beta)^{-1}\left(\overline{D} + (G_0 + \beta E[G_1]) - \frac{\overline{T}}{\overline{Y}}(Y_0 + \beta E[Y_1])\right)}{(1+\beta)^{-1}(X_0 + \beta E[X_1])^2 + \beta Var[X_1]} (X_0 + \beta E[X_1]) \\
& - \frac{\beta Cov\left[X_1, \left(G_1 - \frac{\overline{T}}{\overline{Y}} Y_1\right)\right]}{(1+\beta)^{-1}(X_0 + \beta E[X_1])^2 + \beta Var[X_1]}.
\end{aligned}
$$

The differences between these two solutions stem from the fact that, when $\eta > 0$, government chooses debt in an attempt to smooth tax burden over time. And, the ability to smooth taxes and the corresponding tax distortions impacts the optimal choice of program scale.

Specifically, comparing the first and third terms of these two expression we see that—since $(1+\beta)^{-1}(X_0 + \beta E[X_1])^2 < (X_0^2) + \beta (E[X_1])^2$—the government adjusts program scale more elastically in response to changes in the discount net benefits ($(W_0 - X_0) + \beta E[W_1 - X_1]$) or fiscal risk hedging considerations ($\beta Cov\left[X_1, \left(G_1 - \frac{\overline{T}}{\overline{Y}} Y_1\right)\right]$) when the government is allowed to simulatenously choose program scale and debt issuance. This is an instance of Samuelson's (1947) Le Chatelier Principle which says that comparative statics are smaller in magnitude when an optimizing agents is not permitted to adjust certain control variables. Furthermore, comparing the second terms of these expressions, we see that the inability to borrow to smooth expected tax distortions over time naturally changes the expected fiscal costs calculusus. Specifically, since the government cannot borrow to smooth taxes, it will adjust program scale in an attempt to smooth expected tax distortions.

Obviously, household welfare must be lower when we require government to set $D_0 = 0$ as we do here. And, one would generically expect that imposing the constraint that $D_0 = 0$ would lead the government to choose a smaller level of q.