MATHEMATICS OF DERIVATIVE SECURITIES

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Foreword

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The two-day discussion meeting on mathematical models in finance at the Royal Society of London in November 1993 and its subsequent publication in Philosophical Transactions provided signal recognition to the rapidly advancing but still comparatively new discipline which relates mathematical finance theory and finance practice. But this strong signal was outdone by the extraordinary commitment of time, breadth of scholarship, and economic resources devoted to the Mathematical Finance Programme of six-month duration at the Isaac Newton Institute for Mathematical Sciences. That the contents of this volume represent only a small portion of the work produced testifies to the scale of the Programme's contributions. The fine introductory chapter by the two co-editors happily removes any need for me to comment here on the content of the volume. It is enough to say that the chapters focused for the most part on derivative securities are representative of the high quality and mathematical sophistication of research in the field. Instead I try my hand at locating the relation between mathematical research in finance theory and finance practice, as it has evolved and in prospect.

The essence of finance theory is the study of allocation and deployment of economic resources, both spatially and across time, in an uncertain environment. To capture the influence and interaction of time and uncertainty effectively requires sophisticated analytical tools. Indeed, mathematical models of modern finance contain some truly elegant applications of probability and optimization theory. But, of course, all that is elegant in science need not also be practical; and surely, not all that is practical in science is elegant. Here we have both. Over the past two decades, the mathematically complex models of finance theory have had a direct and important influence on finance practice. This conjoining of intrinsic intellectual interest with extrinsic application is central to research in modern finance.

The origins of much of the mathematics in modern finance can be traced to Louis Bachelier's 1900 dissertation on the theory of speculation. This work marks the twin births of both the continuous-time mathematics of stochastic processes and the continuous-time economics of derivative-security pricing. Although largely lost to financial economists for more than a half century, this same mathematics rediscovered by Kiyoshi Ito and Paul Samuelson, has been an essential tool in the development of modern mathematical finance theory.
Through the resulting theory’s influence on finance practice, this mathematics has played a fundamental role in supporting the creation of new financial products and markets around the globe. In the present and the impending future, that role is expanding to support the design of entirely new financial institutions, decision-making by senior management, and public policy on the financial system. To underscore that point, I begin with a few remarks about financial innovation of the past, this to be followed by some observations on the directions for change in the future.

New financial product and market designs, improved computer and telecommunications technology and advances in the theory of finance during the past quarter-century have led to dramatic and rapid changes in the structure of global financial markets and institutions. The scientific breakthroughs in financial modeling in this period both shaped and were shaped by the extraordinary flow of financial innovation which coincided with those changes. To put this into perspective, we have only to recall the fall of Bretton Woods leading to floating-exchange rates for currencies, the development of the national mortgage market in the United States, the first oil shock, and the creation of the first listed options exchange which accompanied publication of the famous Black-Scholes option-pricing model, all in 1973; ERISA and the subsequent development of the U.S. pension fund industry and the first money-markets fund with check writing that took place in 1974; and the fact that 25 years ago total assets in all U.S. mutual funds amounted to $48 billion. Today those funds are some 50 times larger, with one institution, Fidelity, accounting for approximately $450 billion. In this same period, average daily trading volume on the New York Stock Exchange grew from 12 million shares to more than 300 million. Even more dramatic were the changes in Europe and in Asia. The cumulative impact has significantly affected all of us—as users, producers, or overseers of the financial system.

Nowhere has this been more the case than in the development, refinement and broad-based implementation of contracting technology. Derivative securities such as futures, options, swaps and other contractual agreements, the underlying substantive subject of this mathematical volume, provide a prime example. Those innovations in financial-contracting technology have improved efficiency by expanding opportunities for risk sharing, lowering transactions costs and reducing information and agency costs. Some observers see the extraordinary growth in the use of derivatives as fad-like, but a more likely explanation is the vast saving in transactions costs derived from their use. The cost of implementing financial strategies for institutions using derivatives can be one tenth to one twentieth of the cost of executing them in the underlying cash-market securities. The significance of reducing spread costs in financing can be quite dramatic for corporations and for sovereigns: for instance, a 1 percent (i.e., 100 basis point) reduction in debt-spread cost in Italian government debt would reduce the deficit by 1.25 percent of the Gross Domestic Product of Italy.
Further improved technology, together with growing breadth and experience in the applications of derivatives, should reduce transactions costs secularly as both users and producers of derivatives move down the learning curve. Like retail depositors with automatic teller machines in banks, initial resistance by institutional clients to contractual agreements can be high, but once used, customers tend not to return to the traditional alternatives for implementing financial strategies.

A central process in the past two decades has been the remarkable rate of globalization of the financial system. Inspection of the diverse financial systems of individual nation-states would lead one to question how much effective integration across geopolitical borders could have taken place since those systems are rarely compatible in institutional forms, regulations, laws, tax structures, and business practices. Still, significant integration did take place. This was made possible in large part by derivative securities functioning as adapters. In general, the flexibility created by the widespread use of contractual agreements, other derivatives, and specialized institutional designs provides an offset to dysfunctional institutional rigidities. More specifically, derivative-security technologies provide efficient means for creating cross-border interfaces among otherwise incompatible domestic systems, without making widespread or radical changes within each system. For that reason, future development of derivative-security technology and markets within smaller and emerging-market countries could help form important gateways of access to world capital markets and global risk-sharing. Furthermore, derivatives and other contracting technologies are likely to play a significant role in the financial engineering of the major transitions required for European Monetary Union and for restructuring financial institutions in Japan.

As the preceding remarks are intended to indicate, innovation is a central force driving the financial system toward greater economic efficiency with considerable economic benefit having accrued from the changes over the past several decades. Moreover, both finance research and practitioner experience over that period has lead to vast improvements in our understanding of how to use the new financial technologies to manage risk. Despite all this, we still find today an intense uneasiness among managers, regulators, politicians, the press, and the public over these new derivative-security activities and the associated risks to financial institutions, relative to their more traditional risks such as commercial, real-estate, and less-developed-country lending and loan guarantees. Why? One conjecture about the sources of this collective anxiety holds that their implementation has required major changes in the basic institutional hierarchy and in the infrastructure to support it. As a consequence, the knowledge base required to manage and oversee financial institutions differs considerably from the traditional training and experience of many financial managers and government regulators. Changes of this sort are threatening. It is difficult to deal with change that is exogenous with respect to one's traditional knowledge base and framework and therefore comes
to seem beyond our control. Less understanding of the new environment can create a sense of greater risk even when the objective level of risk in the system is unchanged or actually reduced. That knowledge gap may widen since the current pace of financial innovation is anticipated to accelerate into the 21st Century. Moreover, greater complexity of products and the need for more rapid decision-making will likely increase the reliance on models, which implies a growing place for elements of mathematical maturity in the managerial knowledge base. Managing this knowledge gap offers considerable challenge to private institutions and government as well as considerable opportunity to schools of management and engineering and to university departments of mathematics.

The successful financial-service providers and governmental overseers in the impending future will be those that can address the dysfunctional aspects of innovation in financial technology while still fully exploiting their functional benefits. What types of research and training will be needed to manage financial institutions? With this in mind, consider only a few thoughts on the direction of product and service demands of users of the financial system.

The household sector of users in the more developed financial systems have experienced a secular trend of disaggregation in financial services. There are those who see this trend continuing with existing products such as mutual funds being transported into technologically less developed systems. Perhaps so, especially in the more immediate future. However, deep and wide-ranging disaggregation has left households with the responsibility for making important and technically complex, micro financial decisions involving risk (such as detailed asset allocation and estimates of the optimal level of life-cycle saving for retirement) that they had not had to make in the past, are not trained to make in the present, and are unlikely to execute efficiently even with attempts at education in the future. I therefore believe that the trend will shift toward more aggregated financial products and services, which are easier to understand and more tailored toward individual profiles. Those products and services will include integration of human capital considerations, hedging, and income and estate tax planning into the asset-allocation decisions and the creation of financial instruments that eliminate 'short-fall' or 'basis' risk for the households with respect to targeted financial goals such as tuition for children's higher education and desired consumption-smoothing throughout the life-cycle (e.g., preserving the household's standard of living in retirement). Paradoxically, making the products more user-friendly and simpler to understand for customers will create considerably more complexity for the producers of those products. Hence, financial-engineering creativity and the technological and transactional bases to implement that creativity, reliably and cost-effectively, are likely to become a central competitive element in the industry. The resulting complexity will require more elaborate and highly quantitative risk-management systems within financial-service firms and a parallel need for more sophisticated approaches to government over-
sight. Neither of these can be achieved without greater reliance on mathematical financial modeling, which in turn will be feasible only with continued improvements in the sophistication and accuracy of financial models.

Nonfinancial firms currently use derivative securities and other contractual agreements to hedge interest rate, currency, commodity, and even equity price risks. With improved lower-cost technology and learning-curve experience, this practice is likely to expand. Eventually, this alternative to equity capital as a cushion for risk could have a major change on corporate structures as more firms use hedging to substitute for equity capital; thereby moving from publicly traded shares to private closely-held shares. The big potential shift in the future, however, is from tactical applications of derivatives to strategic ones. For example, a hypothetical oil company with crude oil reserves and gasoline and heating oil distribution but no refining could complete the vertical integration of the firm by using contractual agreements instead of physical acquisition. Thus, by entering into contracts that call for the delivery of crude oil by the firm on one date in return for receiving a mix of refined petroleum products at a prespecified later date, the firm in effect creates a synthetic refinery. Real-world strategic examples in natural gas and electricity are described in Harvard Business School case studies, 'Enron Gas Services' (1994) and 'Tennessee Valley Authority: Option Purchase Agreements' (1996), by Peter Tufano. It is no coincidence that the early applications are in energy- and power-generation industries that need long-term planning horizons and have major fixed-cost components on a large scale with considerable uncertainty. Since energy and power generation are fundamental in every economy, this use for derivatives may become mainline applications in both developed and developing countries. Eventually, such use of derivatives will become standard tools for implementing strategic objectives.

A major requirement for the efficient broad-based application of these contracting technologies in both the household and nonfinancial-firm sectors will be to find effective organizational structures for ensuring contract performance, including global clarification and revisions of the treatment of such contractual agreements in bankruptcy. The need for assurances on contract performance is likely to stimulate further development of the financial-guarantee business for financial institutions. Such institutions will have to improve further the efficiency of collateral management as assurance for performance. As has been known for some two decades, the mathematical models for pricing and hedging derivative securities can be applied directly to the valuation and risk-exposure measurement of financial guarantees.

A consequence of all this prospective technological change will be the need for greater analytical understanding of valuation and risk management by users, producers, and regulators of derivative securities. Furthermore, improvements in efficiency from derivative products will not be effectively realized without concurrent changes in the financial 'infrastructure'—the institutional interfaces between intermediaries and financial markets, regulatory
practices, organization of trading, clearing, settlement, other back-office facilities, and management-information systems. To perform its functions as both user and overseer of the financial system, government will need to innovate and make use of derivative-security technology in the provision of risk-accounting standards, designing monetary and fiscal policies, implementing stabilization programs, and financial-system regulation.

In summary, in the distant past, mathematical models had only limited and side-stream effects on finance practice. But in the last quarter century, such models have become mainstream to practitioners in financial institutions and markets around the world. In the future, mathematical models will surely play an indispensable role in the functioning of the global financial system.

Even this brief discourse on the virtues of the application of mathematical models in finance practice would be negligently incomplete without an added word of caution about their use. At times we can lose sight of the ultimate purpose of the models when their mathematics become too interesting. The mathematics of financial models can be applied precisely, but the models are not at all precise in their application to the complex real world. Their accuracy as a useful approximation to that world varies significantly across time and place. The models should be applied in practice only tentatively, with careful assessment of their limitations in each application. This stated, the reader should not allow this cautionary note on practical limitations to get in the way of enjoying the elegance of the mathematics still to come.

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