Journal of Banking and Finance 1 (1977) 3–11.

AN ANALYTIC DERIVATION OF THE COST OF DEPOSIT INSURANCE AND LOAN GUARANTEES

An application of modern option pricing theory

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It is not uncommon in the arrangement of a loan to include as part of the financial package a guarantee of the loan by a third party. Examples are guarantees by a parent company of loans made to its subsidiaries or government guarantees of loans made to private corporations. Also included would be guarantees of bank deposits by the Federal Deposit Insurance Corporation. As with other forms of insurance, the issuing of a guarantee imposes a liability or cost on the guarantor. In this paper, a formula is derived to evaluate this cost. The method used is to demonstrate an isomorphic correspondence between loan guarantees and common stock put options, and then to use the well developed theory of option pricing to derive the formula.

1. Introduction

The essential functions of a bank are to lend money to firms and individuals and to serve as a riskless repository for the short-term funds of firms and individuals. The bank charges interest on the loans and pays interest or provides noncash services to depositors for the use of their funds.

The traditional advantages to depositors of using a bank rather than making direct market purchases of fixed-income securities are economies of scale, smaller transactions costs, liquidity, and convenience. However, these are important advantages only if deposits can be treated as riskless. Otherwise, to determine which bank to use, the depositor must assume the role of a security analyst and analyze the balance sheets of the bank, its management, and overall market conditions to determine the risks. Even if such analyses are performed, it would be prudent for the depositor to diversify his holdings across many banks. Moreover, these analyses and holdings would have to be revised as conditions changed.

Hence, for the small depositor particularly, there are large information and surveillance costs to be saved if the institutional structure of the bank were such that the safety of the deposits was assured without requiring these analyses. While this could be accomplished by requiring that banks invest only in short-term

*Aid from the National Science Foundation is gratefully acknowledged.

government securities, such a requirement would not allow banks to perform their other function, which is to lend money to firms and individuals. Moreover, such a restriction would require surveillance to ensure compliance. While such surveillance could be carried out in a centralized fashion (e.g. by a government regulatory agency), the financial burden from noncompliance would still be borne by the depositor.

A sensible alternative choice would be to have third-party guarantees where the capability and willingness of that party to meet its obligations are beyond question. For the scale of the banking system, this almost certainly means that the third-party would be the government or one of its agencies. While these guarantees could take the form of guarantees on the loans made by banks, a less expensive and more efficient alternative form is to guarantee the deposits. Indeed, one observes widespread use of such deposit guarantees although their institutional forms differ. In the United States, there is the separately-funded Federal Deposit Insurance Corporation (FDIC) for commercial banks and the Federal Savings and Loan Insurance Corporation for savings and loan associations. In other countries the government may own the banks or there may simply be a widespread belief that the government will act to, in effect, guarantee deposits. Indeed, it is probably fair to say that although the FDIC is separately funded, there is a further belief that the U.S. government would take the necessary actions to protect depositors in the event of a major default by banks that bankrupted the FDIC. To the extent such implicit guarantees are politically binding, they impose a cost on the guarantor which is essentially the same as for explicit guarantees.

In this paper, a systematic theory for determining these costs is presented. As will be shown, techniques for solving this problem can be found in the seemingly unrelated area of Finance called Option Pricing theory. Indeed, the properties of deposit insurance viewed as a security are isomorphic to those of a put option.[1] This is fortunate because recent research has led to a major 'breakthrough' in the development of an option pricing theory.

In a seminal paper, Black and Scholes (1973) developed an explicit formula for pricing call options[2] on common stocks. While there had been a number of earlier efforts dating back at least to 1900, the Black–Scholes analysis is a major advancement for two reasons. First, the important assumptions required to derive the formula are substantially weaker than in the earlier efforts. Second, the inputs required by the formula are either directly observable variables or ones

[1]The essential terms of the contract are defined later in the paper. For further discussion of put options, see Black and Scholes (1973), Merton (1973a, b), Brennan and Schwatrz (1977), and Parkinson (1977).

[2]A call option gives its owner the right to buy a specified number of shares of a given stock at a specified price per share on or before a specified date. For an alternative derivation of the Black–Scholes model, see Merton (1973a). Smith (1976) provides an excellent survey article of the research on option pricing theory.

that can be readily estimated. Hence, the formula can be empirically tested, and if proved satisfactory, it can then be used as a practical tool.

Such tests have been made in at least two studies with generally favorable outcomes.[3] Moreover, the Black–Scholes formula is probably the analytical tool most widely used by investors and market makers in the rapidly growing organized option exchange markets.

While call options are a very specialized type of financial instrument, it is straightforward to see that the Black–Scholes techniques can be applied to the pricing of corporate liabilities in general. Hence their original analysis has led to a unified theory for the pricing of virtually any financial claim on the firm.[4] Further, because the formulas derived do not require a history of market prices for the type of security being evaluated, it shows great promise as an appraisal tool for evaluating nontradeable securities, such as insurance contracts, which is the substantive subject of this paper.[5]

2. A model for pricing deposit insurance

As discussed in the introduction, the development of the deposit-insurance pricing model has as its foundation the isomorphic relationship between deposit insurance and common stock put options. Hence, before developing the model, I make a brief digression to summarize the relevant findings in option pricing theory.

The essential terms of a 'European'[6] put option on a common stock are that its owner has the right to sell a specified number of shares of a given stock at a specified price per share – the 'exercise price' – on a specified date – the 'expiration date'. A put option purchase is different from the sale of a futures contract because the put owner has a choice whether or not to 'exercise his option' to sell at the specified price. Indeed, if this option is not exercised on the expiration date, the contract expires and is worthless. Hence, if on the expiration date the stock price per share, S, is higher than the exercise price per share, E, the put owner would clearly not exercise his right to sell the stock at the exercise price when he could sell it on the open market at a higher price. In this case, the owner would allow the put to expire worthless. However, if on the expiration date the stock price was lower than the exercise price, then the put owner would exercise his right, and the value of the put option would be the difference between the exercise price and the stock price, $(E-S)$, times the number of shares specified in the put contract.

[3]See Black and Scholes (1972) and Galai (1975).

[4]See Merton (1974) and Smith (1976).

[5]Brennan and Schwartz (1976) have used a similar model to evaluate the cost of certain insurance company guarantees of equity-based life insurance plans.

[6]The term 'European' is applied to options that can only be exercised on the expiration date. An 'American' type option can be exercised on or before the expiration date.

Thus, the value of a put on one share of stock at the expiration date can be written as

$$P(0)=\mathrm{Max}\,[0, E-S], \tag{1}$$

where $P(T)$ is the price of a put with length of time T to go before expiration.

Since the value of the put at expiration depends on the stock price, its value prior to expiration will depend on the probability distribution for the range of stock prices on the expiration date.

Using the standard 'frictionless' market assumptions and the additional assumption that the stochastic process generating the stock's returns can be described by a diffusion process with a constant variance per unit time,[7] Black and Scholes impose the condition of 'no arbitrage opportunities' to derive a formula for the value of a put option. The formula can be written as

$$P(T)=E\mathrm{e}^{-rT}\Phi(y_2)-S\Phi(y_1), \tag{2}$$

where:

$$y_1 \equiv \left\{\log(E/S)-\left(r+\frac{\sigma^2}{2}\right)T\right\}\Big/\sigma\sqrt{T},$$

$$y_2 \equiv y_1+\sigma\sqrt{T},$$

$\Phi(\cdot)$ is the cumulative normal density function, S is the current price per share for the stock, r is the market rate of interest per unit time on riskless securities, and σ^2 is the variance rate per unit time for the (logarithmic) rate of the return on the stock. While eq. (2) may appear formidable, it only requires as inputs the interest rate, the exercise price, the current stock price, the length of time until expiration, and the variance rate. Of these, only the variance rate on the stock is not directly observable, and it can be reasonably estimated. Note, more importantly, that neither the expected return on the stock nor investors' preferences are required as inputs. Hence the formula is robust with respect to both these nonobservable variables. This completes the digression.

With this as background, consider the following simple model of a firm that borrows money by issuing a single homogeneous debt issue. The terms of the debt are that the firm promises to pay a total of B dollars on a specified date (the 'maturity' date) and in the event that the promised payment is not made, the firm defaults to the bondholders all the assets of the firm. There are no interim or

[7]The model can be derived in various modified forms to allow for relaxation of most of the 'frictionless' market assumptions. Also, the constant variance rate assumption is not necessary. See Smith (1976) for a discussion of these modifications. For expositional convenience, the original Black–Scholes assumptions are retained throughout the paper.

coupon payments required on the debt, and so the debt is a term discount issue.

On the maturity date, if the value of the firm's assets, V, is larger than the promised payment on the bond issue, B, then it is in the interests of the equity holders for the management to make the payment (by selling assets if necessary). Hence, the value of the debt issue at this point will be B, and the value of equity will be $V-B$. However, if on the maturity date the value of the firm's assets is less than the promised payment, then the management will be unable to make the payment even by selling assets. Hence, the firm is defaulted to the bondholders and the value of the debt issue will be V. The value of the equity will be zero.

In an abbreviated form, at the maturity date, the value of the debt can be written as $\mathrm{Min}\,[V, B]$ and the value of the equity as $\mathrm{Max}\,[0, V-B]$. As long as there is a positive probability that the value of the assets on the maturity date can be less than the promised payment, then there is a positive probability of bankruptcy, and the debt is risky. In another paper [Merton (1974)], I have analyzed the evaluation of such risky debt along Black–Scholes lines.

Consider the impact of a third-party guarantee of the payment to the bondholders where there is no uncertainty about the obligations of the guarantee being met. The terms of the guarantee are that in the event the management does not make the promised payment to the bondholders, the guarantor will meet these payments. However, if such an event occurs, the firm will default its assets to the guarantor. In effect, the guarantor has ensured that the value of the firm's assets on the maturity date will be at least B dollars. Like a traditional insurance policy, the guarantee has value to the insured and imposes a cost on the insurer. Hence, the firm would normally be expected to pay for the guarantee an amount at least equal to its actuarial cost.

To determine this cost, I begin by reexamining the payoffs to the various claims on the maturity date. If the value of the firm's assets exceed the promised payment, then, as without the guarantee, the bondholders receive B and the equityholders receive $V-B$. However, now if the value of the assets is less than the promised payment, then the bondholders receive B, the equityholders receive nothing, and the third-party guarantor has a net payout or loss of $(B-V)$, the discrepancy between the promise payment and the value of assets.

In an abbreviated form, at the maturity date, the value of equity is the same with or without the guarantee, $\mathrm{Max}\,[0, V-B]$; the value of the debt is always B and therefore riskless; and the value of the guarantor's 'claim' is $\mathrm{Min}\,[0, V-B]$ which is nonpositive. In effect, the result of the guarantee is to create an additional cash inflow to the firm of $-\mathrm{Min}\,[0, V-B]$ dollars. But, $-\mathrm{Min}\,[0, V-B]$ can be rewritten as $\mathrm{Max}\,[0, B-V]$. Hence, if $G(T)$ is the value to the firm of the guarantee when the length of time until the maturity date of the bond is T, then

$$G(0) = \mathrm{Max}\,[0, B-V]. \tag{3}$$

By comparing eq. (3) with eq. (1), we see that the payoff structure of the loan guarantee is identical to that of a put option, where in (3) the promised payment, B, corresponds to the exercise price, E, and the value of the firm's assets, V, corresponds to the common stock's price, S. Essentially, by guaranteeing the debt issue, the guarantor has issued a put option on the assets of the firm which gives management the right to sell those assets for B dollars on the maturity date of the debt.

Hence, using the identical arguments used by Black and Scholes to derive the value of a put option written in eq. (2), we can derive a formula for the value of the guarantee, and it can be written as

$$G(T)=B\mathrm{e}^{-rT}\Phi(x_2)-V\Phi(x_1), \tag{4}$$

where:

$$x_1 \equiv \left\{\log(B/V)-\left(r+\frac{\sigma^2}{2}\right)T\right\}\Big/\sigma\sqrt{T},$$

$$x_2 \equiv x_1+\sigma\sqrt{T},$$

V is the current value of the assets of the firm, and σ^2 is now the variance rate per unit time for the logarithmic changes in the value of the assets.

Eq. (4) can be used to evaluate the cost to the guarantor of guaranteeing a discount debt issue with a face value of B dollars and maturity T.

Let $B\exp[-R(T)T]$ be the market value of the debt when there is no guarantee, where $R(T)$ is the promised yield. Clearly, the market value of the debt with a guarantee is $B\exp[-rT]$, and therefore,

$$G(T)+B\exp[-R(T)T]=B\exp[-rT],$$

or

$$\frac{G(T)}{B\mathrm{e}^{-rT}}=1-\mathrm{e}^{-[R(T)-r]T}. \tag{4'}$$

Eq. (4′) gives the cost of the loan guarantee as a fraction of the amount of money raised. I have [Merton (1974, p. 457)] computed values of $[R(T)-r]$ for a variety of maturity dates, variance rates, and firm values. Inspection of these values will demonstrate that the cost of loan guarantees can be substantial.

Now, suppose that the firm is a bank and the debt issue corresponds to deposits. Because most deposits are of the demand type, the model assumption of a term-debt issue is not strictly applicable. However, if one reinterprets the length of time until maturity as the length of time until the next audit of the bank's assets, then from the point of view of the guarantor, the model's structure is reasonable even for demand deposits. Therefore, from the point of view of the guarantor, deposits can be treated as if they were term and interest bearing.

For deposit insurance where both principal and interest are guaranteed, the insured deposits will be riskless and their current value can be written as

$$D = Be^{-rT}. \tag{5}$$

If g is the cost of the guarantee per dollar of insured deposits, i.e. $G(T)/D$, then from (4) the formula for g can be written as a function of two variables:

$$g(d,\tau) = \Phi(h_2) - \frac{1}{d}\Phi(h_1), \tag{6}$$

where:

$$h_1 \equiv \left\{\log(d) - \frac{\tau}{2}\right\} \Big/ \sqrt{\tau},$$

$$h_2 \equiv h_1 + \sqrt{\tau},$$

$d \equiv D/V$ is the current deposit-to-asset value ratio, and $\tau \equiv \sigma^2 T$ is the variance of the logarithmic change in the value of the assets during the term of the deposits. Hence, as long as the deposit-to-asset value ratio and the volatility of the underlying assets remain fixed, the cost of deposit insurance per dollar of deposits is constant. As one would expect, the change in the cost with respect to an increase in the deposit-to-asset value ratio is positive, and is given by

$$\frac{\partial g}{\partial d} = \Phi(h_1)/d^2.$$

The change in the cost with respect to an increase in τ is also positive, and is given by

$$\frac{\partial g}{\partial \tau} = \Phi'(h_1)/(2d\sqrt{\tau}),$$

where the prime denotes the first derivatives. Hence, an increase in either the variance rate of the assets or the length of time that the insurance is in force will increase the cost per dollar of deposits.

From eq. (6), we have that changes in the market rate of interest will have no effect on the cost of deposit insurance unless such changes affect the deposit-to-asset value ratio. To develop some sense of the magnitudes implied by eq. (6), table 1 displays the cost per dollar of deposits for different values of d and τ. It is

Table 1
Cost of deposit insurance per dollar of insured deposits.

Cost of deposit insurance ($)	Deposit-to-asset value ratio	τ
0.00055	0.85	0.00600
0.00040	0.85	0.00550
0.00028	0.85	0.00500
0.00018	0.85	0.00450
0.00011	0.85	0.00400
0.00326	0.90	0.00600
0.00274	0.90	0.00550
0.00223	0.90	0.00500
0.00176	0.90	0.00450
0.00132	0.90	0.00400
0.00093	0.90	0.00350
0.00060	0.90	0.00300
0.00015	0.90	0.00200
0.01209	0.95	0.00600
0.01102	0.95	0.00550
0.00992	0.95	0.00500
0.00880	0.95	0.00450
0.00765	0.95	0.00400
0.00647	0.95	0.00350
0.00528	0.95	0.00300
0.00287	0.95	0.00200
0.00172	0.95	0.00150
0.00072	0.95	0.00100
0.00033	0.95	0.00075
0.03089	1.00	0.00600
0.02958	1.00	0.00550
0.02820	1.00	0.00500
0.02676	1.00	0.00450
0.02523	1.00	0.00400
0.02360	1.00	0.00350
0.02185	1.00	0.00300
0.01784	1.00	0.00200
0.01545	1.00	0.00150
0.01262	1.00	0.00100
0.01093	1.00	0.00075
0.00892	1.00	0.00050
0.00631	1.00	0.00025
0.00564	1.00	0.00020
0.00489	1.00	0.00015
0.00399	1.00	0.00010
0.00282	1.00	0.00005
0.00126	1.00	0.00001

typical for banks to have relatively low volatility assets and high deposit-to-asset ratios. The reader is reminded that V is something like the 'fair' or 'market' value of the assets and not the book value. To put the various τ values in table 1 is perspective, the $\tau = 0.003$ would correspond to a one-year term where the volatility of the assets is similar to those historically observed from holding long-term U.S. government bonds. Lower values of τ correspond to either less volatile assets or a shorter term. The tables include values where the cost exceeded \$0.0001 per dollar of deposits. The $d=1$ case is included as the theoretically limiting case where asset value equals deposits.

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