The Role of Contingent Claims Analysis in Corporate Finance

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Contingent claims analysis (CCA) is a technique for determining the price of a security whose payoffs depend upon the prices of one or more other securities. The origins of CCA are found in the theory of option pricing. Although formal approaches to the evaluation of call and put options can be traced back to at least the turn of the century, the major breakthrough came a little over a decade ago in a paper by Fisher Black and Myron Scholes (1973). The Black and Scholes analysis contains a qualitative insight which may prove to be of even greater academic and practical significance than their famous quantitative formula: Corporate liabilities, in general, can be viewed as combinations of simple option contracts. This insight provides a unified framework in which to view the structure of corporate liabilities and implies that option pricing models can be used to price corporate...
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INTRODUCTION

Contingent claims analysis (CCA) is a technique for determining the price of a security whose payoffs depend upon the prices of one or more other securities. It can, for example, be used to determine the value of a convertible bond in terms of the price of the underlying stock into which the bond can be converted. It can also be used to estimate the value of the flexibility associated with a multipurpose production facility. As suggested by these brief examples, the technique is wide-ranging in that it can be applied fruitfully to a number of tactical and strategic corporate financial decision problems.

The origins of CCA are found in the theory of option pricing. Although formal approaches to the evaluation of call and put options can be traced back to at least the turn of the century, the major breakthrough came a little over a decade ago in a paper by Fisher Black and Myron Scholes (1973). Perhaps no other result in academic finance conceived entirely in theory has had so immediate and significant an impact on financial market practice. The ease of use of the Black-Scholes option pricing model and its subsequent empirical validation explain the practical success of this theoretical result. The speed at which it was adopted was, however, surely affected by the coincidence that 1973 also marked the beginning of organized stock options trading on the Chicago Board Options Exchange. The success of the CBOE and the subsequent expansion in markets to include options on fixed-income securities, currencies, and stock and bond indices have added to the practical importance of their model.

While these markets represent an increasingly larger component of the financial markets, options are, nevertheless, relatively specialized financial securities. However, the Black and Scholes (1973) analysis contains a qualitative insight which may prove to be of even greater academic and practical significance than their famous quantitative formula: Corporate liabilities, in general, can be viewed as...
combinations of simple option contracts. This insight provides a unified framework in which to view the structure of corporate liabilities and implies that option pricing models can be used to price corporate securities. Such generalized option pricing models are the quantitative foundation for contingent claims analysis.

In this paper, we present an overview of CCA and its application to a variety of corporate financial problems. The focus is on providing a functional understanding of the technique. The emphasis is, therefore, on formulating the various problems in a CCA framework and on the methods for solving the derived equations. The balance of this introduction, together with the following section, develops the basic concepts and presents in greater detail the historical development of CCA. The balance of the paper is devoted to applications of CCA broken down roughly along temporal lines: First, a section on the Black and Scholes option pricing model represents the "past" in the sense of applications which are firmly established in financial practice. The next section is the "present" in terms of those applications which are state-of-the-art in financial practice. The final section discusses those applications which are still in the development stage within academic research but which hold forth the promise of becoming a part of financial practice in the future. As the reader will discover, the boundaries among these three temporal categories are both permeable and flexible. Given the enormous range of the subject, we cannot begin to cover all the uses of CCA here and those we do cover can be done in limited depth only. We therefore call special attention to the survey article by Smith (1976) and the forthcoming book by Cox and Rubenstein as concentrated sources for further exploration of the subject.

We begin our study with a brief review of basic option analysis. There are two basic types of options: the call option, which gives the owner the right to buy a specified asset at a specified price on or before a specified date and the put option, which gives its owner the right to sell a specified asset at a specified price on or before a specified date.1 Recently, with IBM stock trading at $118 per share on the New York Stock Exchange (NYSE), the CBOE was trading both calls and puts written on 100 shares of IBM stock with an exercise price of $120 per share and a maturity of eight months. The calls were trading for $1150 and the puts for $750. These options have value because they are "rights," not obligations, to transact in the IBM stock. The owner of the IBM call options will exercise his right to purchase IBM stock in eight months only if the price then exceeds $120. The IBM put options owner will exercise his right to sell IBM stock in eight months only if the price is then below $120. The value of the options is contingent upon the value of IBM stock, i.e., the options are contingent claims.

To understand how the concept of contingent claims is related to corporate liabilities, imagine a firm's "economic" balance sheet. The left side of the balance sheet represents the economic value of the firm, whereas the right side lists the economic value of all of the firm's liabilities. For example, consider the simple firm which has only two classes of liabilities: equity and a zero-coupon bond which matures in one year with a promised principal of $10 million. One year from now, if the value of the firm exceeds $10 million, the firm will retire the debt and the equity will be worth the difference between the value of the firm and $10 million. However, if the value of the firm is less than $10 million, the firm will default. In such an event, the equity will be worth zero and the debt will be worth the value of the firm. Thus, the value of both the equity and debt is contingent upon the value of the firm; i.e., equity and debt are also contingent claims.

Black and Scholes (1973) demonstrate that corporate liabilities can be viewed as combinations of simple options contracts. The next section of this chapter demonstrates the generality of this insight and the important fact that this correspondence is not dependent on any particular option pricing model. Black and Scholes are able to derive their option pricing model because of the additional important observation that it is possible to replicate options using the underlying stock and the risk-free asset. A later section describes the replication argument and the Black and Scholes option pricing model. By combining the results of these two sections, it is possible to quantify the characterization of corporate liabilities as combinations of simple option contracts. The remaining sections describe CCA as a generalization of the Black and Scholes insights with a discussion of the pricing of several simple corporate securities, the potential role of CCA in capital budgeting decisions, and the characterization of a project's strategic value as a series of "operating options." Finally, in the appendix we present a detailed application of CCA to the analysis of large-scale investment projects that demonstrates the potential of this technique in solving complex capital budgeting and financing problems.

CORPORATE LIABILITIES AS OPTIONS:
THE BASIC CONCEPTS

To see the correspondence between corporate liabilities and options, it is first necessary to understand the most fundamental options:
calls and puts. An American call option whose price we denote by \( C(S,T,X) \) gives its owner the right to purchase one share of stock, with current price \( S \), at an exercise price, \( X \), on or before an expiration date which is \( T \) time periods from now. The call option owner will exercise his right to buy only if it is to his advantage. Figure 1 depicts the value of the call option as it depends on the stock price on the expiration date, when \( T = 0 \). Should the stock price on the expiration date be less than the exercise price, the call option owner will not exercise his right to purchase the stock and the option will expire worthless, i.e., \( C = 0 \). If, however, the stock price is greater than the exercise price, the call option will be worth \( S - X \), the difference between the stock price and the exercise price. Thus, at the expiration date, the value of the call option is

\[
C(S,0,X) = \max(S - X, 0)
\]  

(1)

Expression (1) says that the value of the call option at expiration, \( T = 0 \), is the maximum of \( S - X \) and 0. Expression (1) is also true for European call options, \( c(S,0,X) = \max(S - X, 0) \), since at expiration an American and European option are identical. Furthermore, Merton (1973) demonstrates that American and European call options written on a nondividend-paying stock have the same value. \( C(S,T,X) = c(S,T,X) \), i.e., the right to exercise prior to expiration has zero value for calls.²

An American put option, \( P(S,T,X) \), gives its owner the right to sell one share of stock \( S \) at an exercise price, \( X \), on or before its expiration date \( T \) periods from now. Again, the put option owner will exercise his right to sell only if it is to his advantage. Figure 2 depicts the value of the put option on its expiration date. If the stock price on the expiration date is greater than the exercise price, the put option owner will not exercise his right to sell the stock and the put option will expire worthless, \( P = 0 \). However, should the stock price be less than the exercise price, the put option owner will exercise his right to sell the stock and the put option will be worth \( X - S \), the difference between the exercise price and the stock price. Thus, at the expiration date the value of the put option is

\[
P(S,0,X) = \max(X - S, 0)
\]  

(2)

Expression (2) says that the value of the put option at expiration, \( T = 0 \), is the maximum of \( X - S \) and 0. Figure 2 makes it clear why puts are often characterized as an insurance contract on the stock because they pay off when "things go badly," i.e., when stock price is low. Expression (2) is also true for European puts, \( p(S,0,X) = \max(X - S, 0) \), because American and European options have identical values at expiration. Unlike call options, however, Merton (1973) demonstrates that American puts are generally more valuable than European puts, \( P(S,T,X) \geq p(S,T,X) \), because it sometimes pays to exercise the put before expiration, i.e., the right to exercise prior to expiration has value for puts.³

An important relationship between European call and put prices can be derived from Figures 1 and 2. Consider an investment posi-
tion, \( I_n \), which has purchased a European call and sold a European put on the same stock, with the same exercise price and expiration date. Therefore,

\[
I_t = c(S, T, X) - p(S, T, X)
\]  \hspace{1cm} (3)

The value of this investment position at expiration of the options is depicted in Figure 3. The value of the investment position on the expiration date is \( S - X \), the difference between the stock price and the exercise price. The investment can have negative value if the stock price is below the exercise price because the call will expire worthless and the put will be exercised against its seller. However, there is another investment position, \( I_2 \), involving no options which can replicate the payoff depicted in Figure 3. Consider buying one share of stock, \( S \), and borrowing on a discount basis \( X \) dollars for \( T \) time periods at rate \( r \), i.e., the proceeds from the loan will be \( Xe^{-rt} \), allowing for continuous discounting. Therefore

\[
I_2 = S - Xe^{-rt}
\]  \hspace{1cm} (4)

In \( T \) periods the value of this position will be \( I_2 = S - X \), since the position owns one share of stock and owes \( X \) dollars. But if these two positions have precisely the same value at \( T = 0 \), then it must be true that the initial net investment necessary to establish the positions will be the same.

\[
c(S, T, X) - p(S, T, X) = S - Xe^{-rt}
\]  \hspace{1cm} (5)

Expression (5) is well known by professional traders as “put-call parity.” The expression simply says that prices in the call, put, stock, and lending markets must be such that expression (5) is always true. If this were not the case, traders would simply buy the lower-priced alternative and sell the higher-priced alternative and earn an immediate riskless return on zero net investment.

With these fundamental options properties as background, the correspondence between options and corporate liabilities can now be established. Consider Figure 4, the economic balance sheet of a simple firm which has only two liabilities, equity, \( E \), and a single issue of zero-coupon debt, \( D \), where the equity receives no dividends and the firm will issue no new securities while the debt is outstanding. The left side of the balance sheet represents the economic value of the firm. The right side lists the economic value of all the liabilities of the firm.

Figures 5 and 6 depict the value of equity and risky debt as they depend on the value of the firm on the maturity date of the debt. If, on the debt’s maturity date, the value of the firm is greater than the promised principal, \( V > B \), then the debt will be paid off, \( D = B \), and the equity will be worth \( V - B \). However, if the value of the firm is less than the promised principal, \( V < B \), then the equity will be worthless, \( E = 0 \), since it is preferable to surrender the firm to the debtholders, \( D = V \), then repay the debt. Thus, on the maturity date of the debt, the value of equity can be represented as

\[
E(V, 0, B) = \max(V - B, 0)
\]  \hspace{1cm} (6)

Expression (6) says that the value of equity on the debt’s maturity date, \( T = 0 \), is the maximum of the difference between the value of the firm and the promised principal payment, \( (V - B) \), and zero. The value of the risky debt, \( D \), on its maturity date can be represented as

\[
D(V, 0, B) = \min(V, B)
\]  \hspace{1cm} (7)

Expression (7) says that the value of the risky debt on its maturity date, \( T = 0 \), is the minimum of \( V \) and \( B \). Both equity and risky debt
are contingent claim securities whose value is contingent on the
value of the firm.

It follows immediately from the comparison of expressions (1) and
(6) or by inspection of Figures 1 and 5 that equity in the presence of
zero-coupon risky debt is directly analogous to a European call op-
tion written on the firm value, \( V \), with an exercise price, \( B \), equal to

the debt's promised principal, and an expiration date equal to the
maturity date of the debt.

\[
E(V,T,B) = c(V,T,B)
\]  

(8)

In other words, equity can be viewed as a call option with the right
to buy the firm for \( B \) dollars \( T \) time periods from now.

Now, return to the put-call parity result, expression (5), for op-
tions demonstrated earlier. In the characterization of corporate liab-
ilities as options, the value of the firm, \( V \), is the underlying asset on
which the options are written; the debt's promised principal, \( B \), is the
exercise price; and the debt's maturity date is the option's expiration
date. With this correspondence in mind, expression (5) can be re-
arranged and restated as

\[
V = c(V,T,B) + Be^{-rT} - p(V,T,B)
\]  

(9)

But, since the value of the firm is the sum of the value of the equity
and the value of the debt,

\[
V = E + D
\]  

(10)

and since the value of the equity is given by expression (8), then it
follows that

\[
D(V,T,B) = Be^{-rT} - p(V,T,B)
\]  

(11)

The value of risky debt is equal to the price of a risk-free bond with
the same terms minus the price of a put written on the value of the
firm. Expression (11) has an intuitive interpretation. It is commonly
understood that risky debt plus a loan guarantee has the same value
as risk-free debt. The loan guarantee is like insurance, i.e., it will pay
any shortfall in the value of the firm necessary to fully repay the
debt. Figure 7 depicts the value of a loan guarantee, \( G(V,T,B) \), on the
maturity date, \( T = 0 \), of the risky debt. If on the maturity date of the
debt the value of the firm is greater than the debt's promised prin-
cipal, i.e., \( V > B \), the guarantee will pay nothing since the firm is
sufficiently valuable to retire the debt. However, if the value of the
firm is less than the promised principal, \( V < B \), the guarantor must pay
the difference between the promised principal and the value of the
firm, \( B - V \), in order that the debt be fully repaid. Thus, on the
maturity date of the debt, the value of the loan guarantee can be repre-

represented as

\[
G(V,0,B) = \text{Max}(B - V, 0)
\]  

(12)

Now compare either expressions (2) and (12) or Figures 2 and 7. It
is evident that a loan guarantee is analogous to a European put option
written on the value of the firm, i.e., \( G(V,T,Z) = p(V,T,Z) \). And, therefore, expression (11) is simply the statement that risky debt plus a loan guarantee is equal to a risk-free bond.

To demonstrate that the characterization of corporate liabilities as options goes much deeper than the simple corporate securities studied so far, assume that the debt receives coupon payments, \( \bar{c} \). Then equity can be thought of as analogous to a European call option on a dividend-paying stock where \( \bar{c} \) is the "dividend." Now assume that the coupon bond is callable under a schedule of prices \( K(T) \) where \( K(0) = B \). The equity is now analogous to an American call option on a dividend-paying stock where the exercise price changes according to the schedule \( K(T) \). Furthermore, the value of call provision can be characterized as the difference between the value of an American and European call option where the exercise price changes according to \( K(T) \). The value of call protection against redemption for the first \( T_1 < T \) time periods can be viewed as the difference between the values of two American call options on a dividend-paying stock where the first call can be exercised at any time according to \( K(T) \) and the second call can be exercised only in the last \( T - T_1 \) time periods. As is evident from these examples, the correspondence between corporate liabilities and options extends to a wide variety of securities and covenants.

As shown, equity, zero-coupon debt, and loan guarantees can be represented as combinations of simple option contracts. The correspondence is, moreover, sufficiently robust that it is possible to characterize many of the complex securities and covenants encountered in practice by similar analogies to basic options. Note that this correspondence is not dependent upon any particular option pricing model but instead is a fundamental relationship which must hold independently of how options and corporate securities are assumed to be priced. Therefore, given any option pricing model with all its direct implications for pricing stock options, that same model has corresponding direct implications for the pricing of corporate liabilities. Black and Scholes have developed a particularly attractive option pricing model which provides the means for quantifying this qualitative characterization of corporate liabilities as options. The next section describes the Black and Scholes option pricing model and the following section make explicit the implications of the model for the pricing of corporate liabilities.

**BLACK AND SCHOLES OPTION PRICING MODEL**

Historically, option pricing models have fallen into two categories: (1) ad hoc models and (2) equilibrium models. Ad hoc models generally rely only upon empirical observation or curve fitting and, therefore, need not reflect any of the price restrictions imposed by economic equilibrium. Equilibrium models deduce option prices as the result of maximizing behavior on the part of market participants. This latter approach to option pricing dates back to the work of Bachelier (1900). Although the economics and mathematics of Bachelier's work are flawed, his research pointed the way for a number of attempts to describe an equilibrium theory of option pricing, including Sprekle (1964), Boness (1964), and Samuelson (1965). All of these models essentially equate the value of an option to the discounted expected payoff to the option. The expected payoff to the option clearly depends on the assumed probability distribution of future stock prices. In addition, the proper rate to discount the expected payoff to the present must also be specified. Thus, to complete these models it is necessary to make specific and typically quite restrictive assumptions about individual risk preferences and/or the pricing structure in market equilibrium. These assumptions limit the generality (and practicality) of these early results.

Black and Scholes (1973) derive an equilibrium model of option pricing that avoids restrictive assumptions on individual risk references and market equilibrium price formation. This is made possible by their crucial insight that it is possible to replicate the payoff
to options by following a prescribed investment strategy involving the underlying stock and the risk-free asset. Under the following assumptions:

A.1. There are no transaction costs or differential taxes.
A.2. Borrowing and lending, at the same rate of interest, are unrestricted.
A.3. The short-term risk-free rate of interest, $r$, is known and constant through time.
A.4. Short sales, with full use of proceeds, are unrestricted.
A.5. Trading takes place continuously in time.
A.6. The movement of the stock price can be described by a diffusion-type process

$$dS = \alpha S dt + \sigma S dz$$

where $\alpha$ is the instantaneous expected rate of return on the stock per unit time, $\sigma$ is the assumed constant instantaneous standard deviation of the return on the stock per unit time, $dz$ is a standard Gauss-Weiner process, and it is assumed the stock pays no dividends.

Black and Scholes demonstrate that it is possible to construct a portfolio involving positions in the stock and the risk-free asset where the return to the portfolio over a short time interval exactly replicates the return to the option. In addition, Black and Scholes showed precisely how the composition of the portfolio must continually change in response to movements in the stock price and the passage of time such that the replication of the return to the option is maintained.

It is important to realize the implications of the fact that it is possible to replicate the return to options. The replication rules can be viewed as blueprints for a production technology which permits one to build synthetic options. As with any production technology, if the input markets are competitive and there is free entry into the industry, the price of the product must simply be the cost of production, i.e., there can be no excess profits. The fact that synthetic options can be constructed from existing securities does not imply that options contracts are redundant securities with no economic purpose. In the absence of options markets, individuals or institutions could achieve the desired pattern of returns only by attempting to create the options themselves using the Black-Scholes replication rules. The cost of this replication would necessarily exceed the price at which the options could be purchased if an option market existed because, in a competitive market, the price will equal the cost to the least-cost producers. Thus, all but the lowest-cost producers of options gain an economic benefit from availability of options through an organized market. Moreover, all that is necessary for the Black-Scholes pricing result to obtain is that there exist enough potential producers of options who can (to a reasonable approximation) trade continuously without significant costs or restrictions. Such a condition appears to be met by most large security-trading firms. Hence, the price at which an option trades should be well approximated by the Black-Scholes replication cost and it should not be sensitive to the specific economic reasons underlying the demand for option contracts by individuals or institutions.

The Black-Scholes option price is determined by the capital costs required to produce an identical payoff structure to the option by employing a dynamic portfolio strategy using other available securities. A general derivation of this blueprint for replicating the patterns of returns from options is given in Merton (1977). The continuous application of this replication argument results in a partial differential equation which must be satisfied by the price of the option. For example, the call option, $C(S,T,X)$ must satisfy

$$\frac{1}{2} \sigma^2 S^2 C_{SS} + rSC_T - C_T - rC = 0$$

(13)

where subscripts denote partial derivatives. This equation can be thought of as the mathematical prescription, or "recipe," for the portfolio rules governing the replication of the option. To complete the specification of the call option pricing problem, boundary and terminal conditions must be appended to the partial differential equation.

$$C(0,T) = 0$$

(13a)

$$C(S,T)/S \rightarrow 1 \text{ as } S \rightarrow \infty$$

(13b)

$$C(S,0) = \text{Max}(S - X, 0)$$

(13c)

Condition (13a) says that, should the stock become worthless, the call option becomes worthless. Condition (13b) says that, as the stock price becomes very large, the value of the call approaches the value of the stock. Condition (13c) is simply expression (1) which describes the value of the call option on the expiration date.

Black and Scholes solve equation (13), with appended conditions (13a), (13b), and (13c), for the value of a call option

$$C(S,T,X) = SN(d_1) - Xe^{-rT}N(d_2)$$

(14)

where

$$d_1 = \frac{\log(S/X) + (r + \frac{1}{2} \sigma^2)T}{\sigma T^{1/2}}$$

$$d_2 = d_1 - \sigma T^{1/2}$$
\[ d_2 = \frac{\log(S/X) + (r - \frac{1}{2}\sigma^2)T}{\sigma \sqrt{T}} \]

and \( N(\cdot) \) is the standard cumulative normal distribution function.5

While the economic and mathematical arguments necessary to derive the option pricing result expression (14) are involved, the model has several attractive features which explain its widespread use by financial practitioners. One key feature is what the model does—or, more importantly, does not—require as data or inputs. The necessary inputs include the current stock price, \( S \), the exercise price, \( X \), the time to expiration, \( T \), and the risk-free rate of interest, \( r \), all of which are observable. The fifth input is the variance rate, \( \sigma^2 \), of the return to the stock which is relatively easy to estimate from historical data. The model does not depend on the expected rate of return, \( \alpha \), on the stock. This is an important and attractive feature of the Black-Scholes model because expected rates of return are not observable and, unlike variance rates, are not easily estimated from historical data. The model requires no assumptions about investor risk preferences. This characteristic of the model can be seen to follow from our production analogy where the price of the options is determined solely by the technology or production cost structure (i.e., the supply side) and is, therefore, not affected by investor preferences (i.e., the demand side).

The Black-Scholes formula can be used to demonstrate the qualitative impact on option prices of changes in the various inputs.

1. The higher the stock price, \( S \), the higher the call price, i.e., \( S \uparrow \Rightarrow C \uparrow \).
2. The later the expiration date, \( T \), the higher the call price, i.e., \( T \uparrow \Rightarrow C \uparrow \).
3. The higher the exercise price, \( E \), the lower the call price, i.e., \( E \uparrow \Rightarrow C \downarrow \).
4. The higher the variance, \( \sigma^2 \), of stock return, the higher the call price, i.e., \( \sigma^2 \uparrow \Rightarrow C \uparrow \).
5. The higher the risk-free rate of interest, \( r \), the higher the call price, i.e., \( r \uparrow \Rightarrow C \uparrow \).

Property 1 follows from the fact that, if the stock price increases, the expected payoff to the call increases and thus the call price goes up. Property 2 is induced by two phenomena. First, if the expiration date increases, the present value of the expenditure of the exercise price goes down. Second, with a longer-lived call option the probability of the stock reaching very high levels increases. It is also true that the probability of the stock reaching very low levels increases, but the effect of downward moves on the option price are bounded, unlike upward moves, by zero, i.e., the option has limited liability. Property 3 says, if all things are the same, a call option with an exercise price of \( X_1 \) is worth less than a call option with an exercise price of \( X_2 \) where \( X_1 > X_2 \). Property 4 says that the more risky a stock the more valuable is a call option written on it. The more volatile a stock is, the higher the probability that the future stock price will be high and, therefore, the higher the probability that the payoff to the call option will be high. It is also true that increased volatility increases the probability that the future stock price will be low but, again, losses from downward moves are limited by zero. Property 5 says that, as interest rates go up, the present value today of the expenditure of the exercise price goes down.

Whether designed by practitioner or researcher, every pricing model makes abstractions from complex reality. The art of model building is to choose those abstractions which make the model tractable while capturing the essence of the real-world environment in which it is to be applied. As is evident from the discussion surrounding formula (14), the Black-Scholes option pricing model is certainly tractable. Because formula (14) follows directly from assumptions A.1–A.6, the effectiveness of the model can be judged in part by evaluating the reasonableness of those assumptions.

Assumptions A.1–A.5 are essentially institutional assumptions which imply that there are enough traders who can trade with about the same frequency as price changes at virtually zero marginal cost and without restrictions. As we have noted, these conditions appear to be met by the least-cost producers of options. A.6 is an assumption about the dynamics path followed by the price of the underlying security underlying the option. The posited diffusion process implies that the time path of the stock price is continuous, which rules out the possibility that the stock price can "gap" or jump. It is, of course, a well-known empirical fact that the prices of individual stocks do (albeit, infrequently) exhibit large changes as in the case of tender offers, and the Black-Scholes formula, expression (14), does not capture that fact. Cox and Ross (1976), Merton (1976), and Jones (1983), however, have developed models along the lines of Black-Scholes which do allow for the possibility that stock prices can change in a discontinuous fashion.

The assumption of a constant variance rate in A.6 implies that the distribution of unanticipated stock price changes is log normal. The log normal distribution is the prototype distribution used in finance to model stock returns. Unlike, for example, the normal distribution, the log normal distribution (as depicted in Figure 8) captures the
important limited liability property of securities by assigning a zero probability to negative stock prices. It is not, however, consistent with the established empirical fact that the variance rate of stock returns changes over time. Black (1975) finds that changes in this rate tend to be negatively correlated with changes in stock prices. As with jumps, this property of stock returns is not reflected in the Black-Scholes formula (expression (14)). Cox (1975) has derived a modified Black-Scholes formula for a class of stock price processes which exhibit a changing variance rate that is consistent with Black's empirical findings. Considering the importance of stock price volatility in the determination of option pricing, it is perhaps not surprising that both academics and practitioners are making a substantial research effort to understand the variance-rate process for stocks.

These analyses which take into account the jump component of stock returns and the stochastic variance rate, although important, are evolutionary—not revolutionary—steps in the further development of option pricing theory. As such, they exemplify the remarkable robustness of the Black-Scholes methodology. If, indeed, the practical significance of their model is measured only by its empirical accuracy in predicting option prices, then formula (14) does rather well even without correcting for the deviations between the posited and real-world stock price dynamics.

Black and Scholes (1972) test their model on price data gathered on options traded in the over-the-counter market from 1966 to 1969. Taking transaction costs into account, they demonstrate a close correspondence between model and market prices. Galai (1977) replicates and extends the Black-Scholes test using data from the first seven months of trading on the CBOE. His results essentially reaffirm the findings of Black and Scholes (1972).

The same arguments used by Black and Scholes to solve for the value of call options can be used to solve for the value of put options. Combining their model with the results derived earlier, the qualitative relationship between corporate liabilities and options can also be quantified. Merton (1974, 1977) formally sets forth the application of these same arguments to the problem of pricing corporate liabilities.

CONTINGENT CLAIM ANALYSIS AND THE PRICING OF CORPORATE LIABILITIES

The traditional approach to the pricing of corporate liabilities is exemplified by the organizational structure of a typical, vintage corporate finance textbook: a chapter on the pricing of equity, a chapter on long-term debt, a chapter on preferred stock, a chapter on warrants and convertible securities, etc. Each chapter employs a different valuation technique and rarely, if ever, are any attempts made to integrate the various components of the firm's capital structure as even a check on the internal consistency of these diverse valuation methodologies. In contrast, the contemporary CCA approach to the pricing of corporate liabilities begins with the firm's total capital structure and uses a single evaluation technique to simultaneously price each of the individual components of that structure. Thus, the CCA methodology takes into account the interactive effects of each of the securities on the prices of all the others and ensures a consistent evaluation procedure for the entire capital structure. In short, the pricing of corporate liabilities becomes a single, long chapter.

The development of the CCA approach began when it was first recognized that the payoff structures of risky pure-discount debt and corporate-levered equity are identical to the structure of simple call and put option strategies. These basic corporate liabilities can, therefore, be priced using the call and put option formulas derived by Black and Scholes. With this insight, further demonstrations followed rather rapidly showing that the same valuation procedure can be used to price multiple issues of coupon bonds as well as convertible securities and warrants. The procedure can, moreover, accommodate further refinements such as call provisions, sinking-fund requirements, and other covenants frequently required in the indentures of corporate securities.
As with the original Black-Scholes option pricing model, the CCA evaluation procedure does not require a past price history of similar-type corporate securities in order to price a particular security. Thus, unlike a statistical or regression model approach to valuation, the CCA model can price new types of corporate securities which have not been issued previously. This important characteristic of the approach greatly widens its range of application. One clear example is the private placement of debt with special terms where there is no observed market price. However, even in public offerings it is not uncommon to issue liabilities which are specifically linked to the asset characteristics of the firm. For example, Sunshine Mining Corporation issued a bond in the late 1970s which permits its holders to choose between receiving a specified amount of cash or a specified quantity of silver. Other bonds have been issued which are linked to oil prices and still others permit a choice of currencies.

The CCA technique can also be used to evaluate loan guarantees. Such guarantees are important in the oil pipeline industry where pipeline companies are typically financed almost entirely by debt which is guaranteed by the parent oil company. Federal deposit insurance and government guarantees of loans to private businesses can also be evaluated. With these examples as background, we now proceed with a more detailed discussion of the development of CCA.

Merton (1977) shows that the return to any corporate liability can be replicated using an investment strategy similar to the one employed by Black and Scholes. By adding the following assumption to the first five assumptions, A.1-A.5, of Black and Scholes.

\[(A.6)\] The movement of the firm value, \(V\), through time can be described by a diffusion-type equation

\[dV = (\alpha V - \bar{P})dt + \sigma Vdz\]

where \(\alpha\) is the instantaneous expected rate of return to the firm per unit time, \(\bar{P}\) is the known net total payout by the firm per unit time, and \(\sigma^2\) is the variance of the return on the firm per unit time.

Merton (1977) derives a dynamic portfolio strategy which involves mixing positions in the firm with the risk-free asset to produce a pattern of returns that exactly replicates the return to any given corporate liability of the firm. The replicating portfolio must be continually adjusted in response to changes in the value of the firm and the passage of time. The continuous application of this replication argument results in a fundamental partial differential equation which must be satisfied by the prices of all of the firm's liabilities.

For example, equity \(E(V, T, B)\) must satisfy the fundamental partial differential equation

\[\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 E}{\partial V^2} + (rV - \bar{P})\frac{\partial E}{\partial V} - rE - \bar{P} = 0\]  

where \(\bar{P}\) is the payout per unit time from the firm to the equity. To solve for the value of equity, it is necessary to append boundary and terminal conditions to equation (15).

\[E(0, T) = 0\]  

\[E(V, T)/V \to 1 \text{ as } V \to \infty\]  

\[E(V, 0) = \text{Max}(V - B, 0)\]

Condition (15a) says that when the firm is worthless the equity is worthless. Condition (15b) says that, as the value of the firm becomes very high, the value of the equity approaches the value of the firm. Condition (15c) is simply expression (6), the value of equity on the debt's maturity date.

For the case where the firm pays no dividends, \(\bar{P} = 0\), and the debt is in the form of a zero-coupon bond, \(\bar{P} = 0\), it is evident from inspection that the equity valuation problem, (15), (15a), (15b), (15c) is the same as the Black and Scholes call option problem (13), (13a), (13b), (13c). But this is precisely the general correspondence between equity and call options derived earlier. Thus, the value of equity is formally given by formula (14) for the value of a call option, where \(S\) is replaced by \(V\), \(X\) is replaced by \(B\), \(T\) is understood to be the maturity of the debt, and \(\sigma^2\) is understood to be the variance of the return to the firm.

By the same replication argument, Merton (1974) shows that the value of the risky debt must satisfy the same fundamental partial differential equation,

\[\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 D}{\partial V^2} + rVD - rD - \bar{P} = 0\]  

where \(\bar{P} = 0\) because the firm in this example is assumed to make no payouts and \(\bar{P} = 0\) because the debt is a zero-coupon bond. To solve equation (16) for the value of the debt, it is necessary to specify boundary and terminal conditions.

\[D(0, T) = 0\]  

\[D(V, T) \to B e^{-rT} \text{ as } V \to \infty\]  

\[D(V, 0) = \text{Min}(V, B)\]

Condition (16a) says that, when the firm is worthless, the debt is worthless. Condition (16b) says that, as the firm value becomes very large, the value of the risky debt approaches that of a riskless bond,
Be^{-\lambda t}, with the same terms as the risky debt. Condition (16c) is a restatement of expression (7), the value of the debt on its maturity date. Merton (1974) presents the solution to the debt valuation problem, equations (16), (16a), (16b), (16c) as:

$$D(V,T,B) = Be^{-\lambda t}N(h_1) + VN(h_2)$$  (17)

where:

$$h_1 = \frac{\log(V/B) + (r - \frac{1}{2} \sigma^2)T}{\sigma \sqrt{T}}$$
$$h_2 = \frac{\log(B/V) + (r - \frac{1}{2} \sigma^2)T}{\sigma \sqrt{T}}$$

The response of the risky debt valuation, equation (17), to changes in the inputs gives some insight into the reasonableness of the CCA characterization of risky debt.

1. The higher the value of the firm, the higher the value of the risky debt, i.e., \(V \uparrow \rightarrow D \uparrow\).
2. The later the maturity of the debt, the lower the value of the risky debt, i.e., \(T \uparrow \rightarrow D \downarrow\).
3. The higher the promised principal, the higher the value of the risky debt, i.e., \(B \uparrow \rightarrow D \uparrow\).
4. The higher the volatility of the firm value, the lower the value of the risky debt, i.e., \(\sigma^2 \uparrow \rightarrow D \downarrow\).
5. The higher the risk-free rate, the lower the value of the risky debt, i.e., \(r \uparrow \rightarrow D \downarrow\).

These derived properties are what one might expect from a reasonable model of risky debt valuation. As the value of the firm goes up, the debt becomes less risky and the value of the debt increases. As the maturity of the debt is lengthened, the present value of the receipt of the promised principal decreases; thus, the value of the debt falls. If all other things are the same, increasing the promised principal increases the debt value. If the risk of the firm increases, the value of the debt decreases because the expected losses to the debtholders from default increases. If riskless interest rates rise, the value of the debt falls.

Although the evaluation of levered equity and risky debt for the simple firm considered above is instructive, the more practical interest in CCA evolves from its ability to handle many of the complexities encountered with more realistic securities and capital structures. For example, consider a firm financed by equity, which receives dividends, and a callable coupon bond. The CCA character-

ization of the callable coupon bond, \(F(V,T,B)\), as described in Merton (1974) can be written as:

$$\frac{\sigma^2 \lambda^2}{2} V^2 F_{V^2} + (r V - \bar{d}) F_V - F_T - r F + \bar{d} = 0$$  (18)

\(F(0,T) = 0\)  (18a)
\(F(V(T),T) = K(T)\)  (18b)
\(F(V,0) = \min(V,B)\)  (18c)

where \(\bar{p} = \bar{c} + d, \bar{d} = \bar{c}, \bar{d}\) is the coupon on the debt, and \(d\) is the dollar dividend to the equity. Condition (18b) says that there is a schedule of firm values, \(V(T)\), at or above which it is optimal for the firm to call the debt at the call price, \(K(T)\), where \(K(0) = B\). The solution of this problem will provide both the value of the debt and the identification of \(V(T)\).

Another interesting application of CCA to complex securities is the evaluation of callable convertible debt, \(H(V,T,A)\). The CCA formulation of this problem is due to Ingersoll (1976) and Brennan and Schwartz (1977a). The fundamental partial differential equation is the same as equations (18), (18a), (18b), and (18c), except that conditions (18b) and (18c) are replaced by:

$$H(V(T),T) = \gamma V(T)$$  (18b')
$$H(V,0') = \min(V, \max(B, V))$$  (18c')

where \(\gamma\) is the fraction of the equity which would be held by owners of the convertible debt if all of the bonds were converted. The solution of equations (18), (18a), (18b'), and (18c') will not only give the value of the convertible bond but also the firm schedule, \(V(T)\), at or above which it is optimal to call the convertibles and force conversion. Ingersoll (1976) and Brennan and Schwartz (1977a) demonstrate that when equity receives no dividends, i.e., \(d = 0\), CCA implies:

$$\bar{V}(T) = K(T)/\gamma$$  (19)

That is, a convertible bond should be called the moment the bond's common stock value equals the call price. Ingersoll (1977) tests the implication of CCA for the optimal call policy against observed behavior. He finds that call policies of convertible-issuing corporations systematically differ from the optimal policy suggested by CCA. Even by taking the call notice and underwriting costs into account, Ingersoll (1977) is still unable to reconcile observed behavior with the model's prescription. He concludes that convertibles should sell for a premium over the model prices.

Other examples of CCA research include the work of Galai and Masulis (1976) on the effects of mergers, acquisitions, scale expansions, and spin-offs on the relative values of levered equity and risky
debt: Black and Cox (1976) on the evaluation of specific bond indentures such as safety covenants, subordination agreements, and restrictions on the financing of payouts; and Jones, Mason, and Rosenfeld (1983) on the theory and empirical testing of the implications of CCA for the valuation of capital structures comprised of multiple callable coupon bonds with sinking funds.

Most of these more complex, and interesting, applications of CCA result in partial differential equations which cannot be solved for simple formulas. However, it is possible to approximate the solutions to these problems through numerical analysis on a computer. Techniques for performing the numerical analysis of CCA type problems are described in Parkinson (1977), Brennan and Schwartz (1978a), and Mason (1978). These numerical analysis techniques have been applied to many and diverse problems, including the evaluation of callable convertible debt (Brennan and Schwartz (1982) and the pricing of loan guarantees (Jones and Mason (1980)).

The CCA model captures quantitatively what practitioners have long known to be the major determinants of value for corporate liabilities: (1) business risk, (2) financial risk, (3) level of interest rates, and (4) covenants. The notion of business risk is captured directly by \( \sigma^2 \), the variance of the rate of return to the firm. Clearly the value, and riskiness, of a firm’s liabilities is in part driven by the riskiness of its assets. CCA captures financial risk through knowledge of the value of the firm, \( V \), and the amount and timing of mandatory payouts. Two firms may have the same value and business risk, but the more levered firm’s debt will be worth less. The model is given a direct indication of the level of interest rates through the specification of \( r \), the riskless rate of interest. A security’s covenants are reflected in the boundary conditions and the payout, \( \beta \), as demonstrated in the callable coupon bond problem.

CCA, in its more general application, also provides many of the same advantages of the original Black-Scholes option pricing model in that many of the inputs required in the evaluation formula are directly observable. Mandatory payouts, maturities, and covenants are easily determined from indenture provisions. The risk-free rate, \( r \), is also observable. The variance of the return to the firm, \( \sigma^2 \), can be estimated from a time series of firm values. Again, as with the Black-Scholes model, the CCA model does not depend on \( \alpha \), the expected rate of return on the firm, nor are any assumptions concerning individual risk preferences or market equilibrium necessary.

A major obstacle in applying CCA to practical corporate financial problems is the observability of \( V \), the firm value. If the firm value can be observed in the form of all its liabilities trading, then the application goes through as described above. If, however, all of a firm’s liabilities are not traded, then certain adjustments to the procedure are necessary. For example, if all of a firm’s liabilities do not trade but there exists another traded asset which is assumed to move closely together with the unobserved firm value, the variance, \( \sigma^2 \), can be estimated from this traded asset. Then all that is necessary is that the firm has one traded claim. Since this traded claim must obey its own CCA partial differential equation, the unknown firm value can be inferred as that which gives a predicted price consistent with the observed price for the traded claim. With this knowledge of the implied firm value, the value of any other claim can then be determined. If the firm has no traded claims, but there is another traded asset which moves with the unobserved firm value, the variance can be estimated from this traded asset and a classical capital budgeting evaluation is necessary to determine the firm value. If the firm value is unobservable and there are no traded assets which move closely with the firm value, but the firm has at least two traded liabilities, then a firm value and variance rate which are consistent with observed prices can be inferred from the model.

As is evident from our examples, much research has been done with CCA in the area of pricing corporate liabilities. There is, however, still much development work to do before its full practical potential can be realized. To date, applications in the financial community have been focused on the pricing of convertible and other equity-type securities and on pricing the call option component of essentially default-free debt. Empirical testing of the model’s accuracy in pricing complex capital structures is just beginning. As practical experience accumulates and the test results come in, there will undoubtedly be changes made in the detailed formulation of the pricing equations. As has been the experience with the Black-Scholes model in its application to option pricing, we expect, however, that these changes will be evolutionary and that the basic structure of the contingent claims analysis presented here will remain essentially intact.

With these developments, CCA will become an increasingly more useful tool for analyzing corporate liability strategy and planning. The capability to simulate a virtually unlimited range of financial packages presents the opportunity to determine the firm’s most efficient capital structure. The cost in terms of reduced flexibility caused by the terms and covenants of financial instruments can be explicitly evaluated. The trade-offs between tax shields, financial distress, and the corporate need for additional funds under various contingencies can also be analyzed. Indeed, as we look to the future,
perhaps the most important application of CCA will turn out to be in the strategy and planning activities surrounding the combined capital budgeting and financing decision problem. With this in mind, we explore briefly the potential of this application in our next and concluding section.

THE ROLE OF CCA IN CAPITAL BUDGETING DECISIONS

For more than a generation, finance academics have taught the net present value method as the correct procedure to use in making capital budgeting decisions. If recent survey statistics are accurate, this view is also widely shared in practice where apparently the majority of corporations now use some version of discounted cash flow analysis to evaluate their projects. Nevertheless, these capital budgeting techniques have their critics, some of the sharpest criticisms coming from those in business policy and strategy. As a recent example, Hayes and Garvin (1982) assert that discounted cash flow methods cause a systematic undervaluation of projects because, among other reasons, the strategic value of projects is ignored. As an alternate and more direct remedy of this claimed undervaluation bias, Hayes and Garvin suggest that major investment decisions be made simply on the basis of judgment and strategic considerations, without subjecting them to the "distortions" of quantitative methods.

One does not have to embrace the suggested solutions of Hayes and Garvin to accept as a valid concern that current capital budgeting practices fail to properly account for all the important sources of value associated with a specific project. Such failures, when they occur, reflect the shortcomings of the particular evaluation technique used and not the quantitative approach itself. In this final section of the paper, we explore the potential of CCA to correct some of these shortcomings and conclude that it is especially well suited to the task of evaluating what strategists call the "flexibility" of a project.

Although not a precisely defined technical term, the flexibility of a project seems to us to be nothing more (or less) than a description of the options made available to management as part of the project. Baldwin, Mason, and Ruback (1983) call such options "operating options." As an example, the management of an electric utility faces a choice between building a power plant that burns only oil and one that can burn either oil or coal. Although the latter costs more to build, it also provides greater flexibility because management has the option to select which fuel to use and can switch back and forth, depending upon energy market conditions. In making its choice, management must, therefore, weigh the value of this operating option against its cost.

Operating facilities (such as oil refineries and chemical plants) which can use different mixes of inputs to produce the same output or the same inputs to produce various arrays of outputs, are general examples of this type of flexibility. That such options have value has, of course, long been recognized. As a practical matter, however, these option values are rarely incorporated into the capital budgeting process except, perhaps, in a qualitative fashion. With its proven record in valuing financial options, CCA shows great promise for providing the quantitative methods necessary to include explicitly the value of such options as part of the project evaluation procedure. Research along these lines is already underway, as is evident from the papers by Baldwin, Mason, and Ruback (1983), Brennan and Schwartz (1983a; 1983b), and Kester (1984). To provide a sense of the range of applications for this research, we present a brief catalog of examples of various operating options which could be evaluated by CCA.

Although a formula for the value of the option described in our electric utility example does not as yet appear in the CCA literature, there are published analyses which could be used to solve this problem. Three such papers are Margrave (1978) on the value of an option to exchange one risky asset for another; Stultz (1982) on pricing securities whose payoff is the maximum (or minimum) value of two assets; and Baldwin, and Ruback (1982) on the option value implicit in short-lived assets when prices are variable and uncertain.

Traditional capital budgeting procedures typically assume that a project will operate in each year of its anticipated lifetime. However, especially for projects involving production facilities, it may not be optimal to operate a plant in a given year because project revenue is not expected to cover variable cost. Explicit recognition of this type of management flexibility is particularly important when choosing among alternative production technologies with different ratios of variable-to-fixed costs. McDonald and Siegel (1981) use CCA to evaluate this option (not) to operate.

Another closely related type of flexibility is the option to expand or contract the scale of the project. Changes in the total output of the project can be achieved by changing the output rate per unit time or by changing the total length of time of the production run. Management may choose, for example, to build production capacity in excess of the expected level of output so that it can produce at a higher rate if the product is more successful than was originally anticipated.
Management can also choose to build a facility whose physical life exceeds the expected duration of its use, and thereby provide the firm with the option to produce more output over the life of the project by extending the production period. By choosing a plant with high maintenance costs relative to original construction costs, management gains the flexibility to reduce the life of the plant and contract the scale of the project by reducing expenditures for maintenance.

In addition to the option to temporarily shut down the project, management also has the option to terminate it. The value of this option can be substantial for large capital-intensive projects like nuclear power plants which have long construction periods. It is also important in the evaluation of projects involving new products where their acceptance in the market is uncertain. Myers and Majd (1983) provide a quantitative analysis of this option to abandon.

Just as the option to abandon can be an important source of flexibility in a project, so the option to choose when to initiate a project can be valuable. For example, the purchaser of an off-shore oil lease can choose when, if at all, during the lease period to develop the property. An analysis by Paddock, Siegel, and Smith (1982) suggests that this option can represent a significant part of the value of such leases. Their analysis implies, for example, that if the U.S. government were to require immediate development as a condition for granting such leases, then the prices paid for the leases would be considerably less than under current conditions, including, in some cases, no purchases at all.

Much the same analysis would apply to the evaluation of exploration activities. If natural resource companies were somehow committed to produce all resources discovered, then they would never explore in areas where the estimated development and extraction costs exceed the expected future price at which the resource could be sold. However, because they can choose when to initiate such development, it may pay to explore in high production cost areas in order to gain the option to produce if the price of the resource at some later date is higher than was expected. The value of the option associated with exploration has been formally analyzed by Tourinho (1979) using the Black-Scholes option pricing model.

As discussed in Kester (1984), an important strategic issue is the sequencing of investments in projects, and this, too, can be analyzed in an options-evaluation framework. Examples would be projects involving the production of basic consumer products like soap and light bulbs. In the successful marketing of such products, a brand name plays an important role, not only because of consumer recognition, but also because a brand name product is more likely to obtain "shelf space" from distributors such as supermarkets. For a firm evaluating projects to produce a number of consumer products, it may be advantageous to implement these projects sequentially rather than simultaneously. By developing a single product first, the firm can resolve at least some of the uncertainty surrounding its ability to establish a brand name and can determine the likelihood of obtaining the necessary shelf space for subsequent products. This resolved, management then has the option to proceed or not with the development of these other products. If, instead, these projects were undertaken in parallel, management would already have spent the resources and the value of the option not to spend them is lost.

Unlike the previous examples of intraproject options, the sequencing of projects involves the creation of options on one or more projects as the direct result of undertaking another project. Because the standard capital budgeting procedure is to evaluate a project on a "stand-alone" basis, the value of such interproject options can easily be missed. While neglecting such linkages may cause small errors in the evaluation of some projects, it can cause a significant undervaluation for others. Such a polar case would be research and development projects whose only source of value is the options they create to undertake other projects. More generally, interproject options are created whenever management makes an investment that places the firm in a position to use a new technology or to enter a different industry.

As discussed in Myers (1977), option analysis is important to the proper evaluation of a firm's "growth opportunities." As is well known in financial analysis, the value of a firm can exceed the market value of its projects currently in place because the firm may have the opportunity to earn a return in excess of the competitive rate on some of its future projects. The standard methodology for evaluating such projects is to discount back to the current time their net present values as of the anticipated implementation dates. This methodology implicitly assumes that the firm is committed to undertake the projects although, in fact, management need not make such a commitment before the implementation date. The standard method, therefore, neglects the value of the option not to go forward if conditions change before the implementation date.

Along with the options associated with growth opportunities, there are also "protective" or "strategic insurance" options which involve investments made by management to protect the value of current or planned future operations of the firm. An example with considerable public policy interest is the development of synthetic
fueals as energy alternatives to oil. Consider the evaluation of a synthetic fuel alternative which, if developed, would provide an energy equivalent reserve of 10 billion barrels of oil. The project requires an immediate expenditure of $20 billion for development and the cost of production is $40 per equivalent barrel of oil. Suppose that the best available long-term forecast predicts that the price of oil will remain constant in real terms at $27 per barrel.

A standard capital budgeting analysis, which takes the expected future revenues (i.e., $27 per barrel) minus the expected future costs (i.e., $40 per barrel) and discounts it back to the present, would lead to the conclusion that the project should not be undertaken even if there were no development costs. Such a procedure assumes, however, that having once developed the project, the owner will produce the synthetic fuel independently of the price received for it and, therefore, the procedure neglects the option component of the project. By spending the money for development, the project owner acquires the option to produce energy at $40 per barrel, which also means that he has the option not to produce if the price is below $40. In a world of certainty, where future oil prices always turn out to follow their forecasted path, this option to produce would never be exercised and would, therefore, have no value. As we all know, however, the future course of energy costs is far from fully predictable. As long as there is some probability that oil prices will exceed $40 per barrel, the option has a positive value. As was previously shown for financial options, the value of this operating option and, hence, the value of the project, is an increasing function of the amount of uncertainty surrounding future energy prices. CCA can be used to evaluate this option and therefore help provide an answer to the strategic question, “Is $20 billion too much to pay to ensure that future energy costs will not exceed $40 per equivalent barrel of oil?”

We describe our hypothetical project as an “insurance” option because its manifest purpose is to protect the existing capital stock and consumption patterns of the economy from unanticipated and disruptively high energy prices. It also shares the characteristic common to most insurance arrangements that the insured is better off in those events where there is no need to collect on the policy. That is, a net energy-consuming economy is more likely to be better off if energy prices remain below $40 per barrel, in which case the option to produce is not exercised.

Throughout this sampler of operating options examples, we have repeatedly noted the failure of traditional capital budgeting techniques to properly take the value of these options into account. Although ignoring any single operating option may not introduce an important error in a project’s evaluation, the cumulative error of ignoring all the operating options embedded in that project can cause a significant underestimate of its value. This is not, however, to say that standard techniques systematically and significantly undervalue all types of projects. Many classes of projects provide few, if any, operating options and, for others, the cumulative value of all such options will be small. Indeed, by neglecting the option components, these techniques may overestimate the values of some projects by failing to recognize the losses in flexibility to the firm caused by their implementation.

While rejecting the universal condemnation of current capital budgeting procedures expressed by some corporate strategists, we tend to agree with the more selective criticisms expressed by others in that community. As discussed in Myers (forthcoming), the focus of these criticisms is on the evaluation of long-horizon and broadly defined projects whose future profitability can only be imprecisely estimated. It is under just such conditions that taking account of the associated operating options is most important. As shown in our previous analyses, the value of an option is an increasing function of both the duration of the option and the amount of uncertainty surrounding the future value of the underlying asset. Hence, for projects of this sort, an evaluation technique which neglects these options can produce significant undervaluations. Moreover, as Myers (1977) has demonstrated, the choice of capital structure for the firm can also significantly affect the value of such projects. Although current capital budgeting procedures typically do make some provision for the tax deductibility of interest paid by the firm, they do not take into account the flexibility of its capital structure. Like operating flexibility, financial flexibility can be measured by the value of the financial options made available to the firm by its choice of capital structure. The interactive effects between financial and operating flexibility can be quite strong for major long-term investment projects involving considerable uncertainty. CCA would, therefore, appear to be a particularly useful tool to the corporate strategist because it provides an integrated analysis of both the operating and financial options associated with the combined investment and financing decision.

In this light, it is perhaps not surprising that the early capital budgeting applications of CCA concentrated on the evaluation of natural resource development and energy production types of projects where the scale of operations and financings is large and where the construction-development period and the production life are both long. As discussed in these analyses, the methods of fi-
financing these projects are important to their evaluation and the ones chosen are often complex. Nowhere is this more apparent than in project financings. We have resisted the temptation to accompany each of the specific applications of CCA with a technical development of the corresponding model. To have done otherwise would surely have defeated its manifest purpose as an expository overview. A survey on the application of CCA would, nevertheless, be incomplete without at least one such detailed demonstration. Therefore, in the appendix we develop a CCA model to analyze a generic large-scale project financing. Projects need not, of course, be of great complexity and size in order to justify the use of CCA. A project financing, however, provides a rich setting for illustrating the wide variety of corporate finance issues to which CCA can be applied. To provide a real-world background for the analysis in the appendix, we briefly describe here a few examples of large project financings. These examples also serve to underscore the substantive importance of this type of financing.

The construction of the Trans Alaska Pipeline System (TAPS) required the expenditure of $10 billion, which at completion made it the single most expensive construction project ever undertaken. Owned by a consortium of major oil companies, TAPS was financed entirely by debt guaranteed by these companies. The proposed mining of the tar-sand deposits in the Athabasca region of Alberta, involved not only U.S. and Canadian private interest but also the federal and provincial governments of Canada. In this proposal, Gulf Oil's Ailsands Project and Exxon's Cold Lake Project were each expected to cost between $10 and $15 billion and were to be financed by a complicated package of equity, debt, government-guaranteed debt, and tax concessions. The United States Synthetic Fuels Corporation (SFC) was established in 1980 to assist projects involving the development of commercially-viable alternative fuel technologies. SFC is empowered to provide loan guarantees, purchase commitments, price supports, direct loans, and joint venture participations as the means of extending subsidies to such energy projects. Financing packages such as these have principially been used in "hard asset" type projects and so the hypothetical project analyzed in the appendix is of this sort.

As discussed in the earlier sections, the fundamental evaluation equations of CCA are derived from arbitrage arguments involving portfolio strategies using traded securities. One might reasonably question, therefore, the validity of such equations for evaluating capital budgeting projects which are not traded. All capital budgeting procedures have as a common objective the estimation of the price that an asset or project would have if it were traded. Thus, for example, a standard discount cash flow analysis uses as a discount rate the equilibrium expected return required on a traded security in the same risk class as the nontraded project. Because the absence of arbitrage is a necessary condition for equilibrium prices, the no-arbitrage price of an option on a traded security must be the equilibrium price of an option on a corresponding nontraded project. If the undertaking of the project being evaluated would, however, significantly change the macroinvestmet opportunity set available to capital market investors, then using either the equilibrium discount rate (absence this investment) or the option model (absence this investment) will lead to an error in the project's estimated value. Such "uniqueness" of a project is likely, as an empirical matter, to be rare. Moreover, the resulting error from using the option model in such rare cases would be no different from the one arising from the standard procedure.

There are, of course, other types of quantitative models for analyzing complex capital budgeting decisions. These typically fall into two classes: (1) Monte Carlo techniques and (2) hierarchical decision trees. Monte Carlo techniques, or simulation, as proposed by Hertz (1964), suffer from the problem that the output, i.e., a probability distribution, is not easily translated into a decision. The application of decision trees to capital budgeting, as described in Magee (1964), is hampered by ambiguity in the selection of a discount rate. The CCA model has the advantage over these alternative approaches that no matter how detailed and complex the interactions, the resulting evaluations are "consistent." That is, the derived values of all the component pieces are mutually consistent with an equilibrium price structure. Thus, CCA is particularly attractive for planning models because it can be used to simulate a variety of choices with the knowledge that any unusual characteristics which surface are likely to be "true" unanticipated consequences of actions and not simply the result of inconsistencies in the model's component equations.

In summary, it is unlikely that managers will ever rely entirely upon quantitative models in making major investment decisions. We do believe, however, that CCA will become as important and commonplace a tool for capital budgeting decisions in the future as it is for financial market decisions in the present.

APPENDIX: A CCA STUDY OF A PROJECT FINANCING

In this hypothetical example, a corporation has been invited to join a consortium of companies which has the opportunity to develop a
natural resource base, e.g., coal, oil, minerals, etc. The project is to be financed by equity, senior debt, and subordinated debt guaranteed by the consortium members. In addition, the host government will provide a price guarantee for output over some early portion of the project's life. Therefore the project has three distinct phases: (1) construction, (2) operations with price guarantees, and (3) operations without price guarantees. Assume that the construction phase lasts until time $T_c$ and the project produces no cash flows during construction. Total necessary investment in the project is assumed to cover the cost and repayment of financing during construction.

The project will produce according to a known production schedule and all its production is assumed to be sold at prevailing "spot" market prices which are assumed to fluctuate. As a means of partially reducing this price risk, the host government agrees to guarantee the price of output until time $T_p$. The productive life of the project is assumed to extend to at least time $T_d$, the maturity date of the longest-term bond. Therefore

$$T_c < T_p < T_d$$

After construction, the project generates cash flow, which is proportional to its current value at various discrete points in time and all debt service must be funded from this cash flow. If current cash flow is insufficient to cover mandatory payments, senior debt has first lien on the existing cash flow. If the cash flow is sufficient to cover senior debt payments, then the junior debt guarantee will make up the current cash flow shortfall to the junior debt and the project will continue. If the cash flow is insufficient to cover the senior debt payments then equityholders have the option of either contributing more cash to make up the shortfall in payments to the senior debt or abandoning the project. If equity chooses to make up the shortfall, then the guarantor will provide any currently due payment to the junior debt and the project will continue. If equity chooses not to make up the shortfall, then all debt is due immediately. Should the cash flow prove sufficient to cover all debt payments, the equity may declare a dividend equal to the surplus. The corporation's problem is to determine, given all the terms and conditions, whether or not membership in the consortium is an attractive investment opportunity. In this formulation of the problem, the value of equity, $E^*$, will be the corporation's share of total equity and the value of the project, $V^*$, will be taken to be the corporation's share of total project value as dictated by its interest in the consortium. As will be shown, it will prove to be illuminating if the analysis keeps separate the value of the loan guarantee from the value of equity even though the same entities provide both. As is often the case in these types of problems, the "trick" to solving this problem is to recognize that the solution can be derived in a recursive manner by working backward in time.

Thus, working backward, the "first phase" of interest is the operations phase without price guarantees. Table 1 sets forth the partial differential equations which must be satisfied by equity, $E^*$, the senior debt, $D^*$, and the loan guarantee, $G^*$, in this regime. The guaranteed junior debt does not appear since its value is equal to that of a risk-free bond with the same terms. It is assumed that the junior debt's guarantors, the consortium members, are large enough that their guarantees are deemed to be risk-free. The price guarantee also does not appear because in this later phase it no longer exists.

<table>
<thead>
<tr>
<th>TABLE 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V = V^2 E^* E^* + \sum_{i=1}^{m} \delta(T - T_i) \gamma(T_i) V \gamma(T_i) V - \gamma(T_i) V - DP(T_i)$ = 0</td>
</tr>
<tr>
<td>$E^*(T,V,T)</td>
</tr>
<tr>
<td>$E^*(V,T)</td>
</tr>
</tbody>
</table>

$\frac{\partial^2 V}{\partial T^2} + \frac{\partial V}{\partial T} + \delta(T - T_i) \gamma(T_i) V \gamma(T_i) V - \gamma(T_i) V - DP(T_i) = 0$

$E^*(T,V,T)| = \text{Max}(E^*(T - DP(T) - BP(T),0))$

$E^*(V,T)| = \text{Max}(\text{Max}(V - DP(T_d)),0)$

$\frac{\partial^2 V}{\partial T^2} + \frac{\partial V}{\partial T} + \delta(T - T_i) \gamma(T_i) V \gamma(T_i) V - \gamma(T_i) V - DP(T_i) = 0$

$D^*(T,V,T)| = \text{Min}(D^*(T),DPS(T) + BPS(T))$

$D^*(V,T)| = \text{Min}(V,DPS(Td))$

$\frac{\partial^2 V}{\partial T^2} + \frac{\partial V}{\partial T} + \delta(T - T_i) \gamma(T_i) V \gamma(T_i) V - \gamma(T_i) V - DP(T_i) = 0$

$G^*(T,V,T)| = \text{Max}(\text{Min}(G^*(T),BPS(T) + BP(T),0))$

$G^*(V,T)| = \text{Max}(\text{Min}(G^*(T),DPS(Td) - V),0)$
It is assumed that the variance rate of the project's return, \( \sigma^2 \), can be specified as a function of project value and time. It is also assumed that the risk-free rate of interest, \( r \), is a deterministic function of time and that the cash flow proportionality factor, \( \gamma \), can be specified as a function of project value and time. There are \( mm \) dates during production on which cash flow is realized from the project and the last \( (mm - m) \) of these occurs during this last phase of production. The Dirac Delta function, \( \delta(x) = 0 \) for all \( x \) except \( \delta(0) = 1 \), is intended to capture the discrete nature of the cash flow. The debt payment currently due the senior debt is \( DPS(T) \), the debt payment due the junior debt is \( DJ(T) \), and \( DP(T) \) is the sum of \( DPS(T) \) and \( DJ(T) \). The senior, junior, and total bond principal outstanding at any time, \( T \), are given by \( BP(T) \), \( BP(T) \), and \( BP(T) \), respectively. Finally, \( RBS(T) \) is the value of a risk-free bond with the same terms as the senior debt.

Although these equations are complicated, they all have the same structure, and this format is precisely the one described previously. For example, recall the fundamental partial differential equation, equation (15), which Merton (1974) demonstrates must be satisfied by all corporate liabilities. As illustrated by equation (15), the total payout from the project, \( \bar{P} \), appears in the second term of the partial differential equation. Now examine each of the three equations in Table 1. In this case, the total payout from the project is the proportionality factor, \( \gamma(T) \), multiplied times the project value and this same payout appears in the second term of each equation. Returning to the fundamental partial differential equation (equation (15)), the last term in the equation, \( \bar{P} \), is the pay-out-pay-in term describing the distribution to or contribution from the security. In each of the equations in Table 1, the last term describes that security's unique pay-out/pay-in situation. For example, in the equation for equity, equation (20), the last term determines, as a function of project value and time, whether equity will receive a dividend or be required to make up a shortfall in the payment to the senior debt. Similarly, the last term in the equation for the guarantee, equation (22), determines, as a function of project value and time, whether the guarantor must make up any shortfall in a payment to the junior debt. Returning once again to the fundamental partial differential equation, equation (15), note the pattern of the three boundary conditions, \( (15a) \), \( (15b) \), and \( (15c) \). The first boundary condition gives the value that the security tends toward as project values become low. The second boundary condition gives the value that the security tends toward as project values become high. The last boundary condition gives the security's value at the end of the period of interest. This same pattern of boundary conditions is exhibited in Table 1. In the equation for equity, equation (20), boundary condition \( (20a) \) says that, as project values become low, there will come a point where the value of equity is less than the payment it is required to make to the senior debt if the project is to continue. At this point the equity will abandon the project. The schedule of project values at which this will happen is given by \( \bar{V}(T) \). If \( \bar{V}(T) = \bar{V}(T) \), the value of equity is the maximum of the difference between the project value and the total payments due debt and zero. Of course, the decision to abandon by equity holders also affects the values of other securities. Note that \( \bar{V}(T) \) appears in the first boundary condition of each of the other two security evaluation problems in Table 1. That is, the schedule \( \bar{V}(T) \), determined in the solution to equation (20), becomes input data for the solution of equations (21) and (22). This interaction illustrates one of the many ways that the solutions to a set of CCA equations can be interrelated in a cross-sectional sense. The analysis of the next earlier production phase will demonstrate the backward recursive nature of the solution of the overall problem which causes the equations to be interrelated in an intertemporal sense.

Table 2 gives the equations which must hold in the production phase characterized by the presence of the price guarantee, \( S' \). It is assumed here that the price guarantee takes the form of a schedule of guaranteed cash flows, \( CF(T) \), which at all times during this phase equals or exceeds the schedule of payments due the senior debt. Note again that all of the equations in Table 2 have the same overall format as the fundamental partial differential equation (equation (15)). The presence of the price guarantee clearly affects the value of the securities at low project values: \( PVE(T) \) is the present value of \( \text{Max} (CF(T) - DP(T), 0) \); \( PV(D(T) \) is the present value of \( DPS(T) \) evaluated out to \( T_p \); \( PVG(T) \) is the present value of \( \text{Max}(DP(T) - CF(T), 0) \) evaluated out to \( T_p \); plus the present value of \( BP(T) \), and \( PVS(T) \) is the present value of \( CF(T) \). These terms reflect the condition that no matter how low the project value becomes, the price guarantee continues to provide cash flow sufficient to ensure that equity holders will not abandon the project. Finally, take special note of the third boundary condition in each of the first three equations in Table 2. Here the recursive nature of the problem is made clear since, as an example, the solution to equation (20) is needed as a boundary condition for the solution of equation (23). This is caused by the requirement that the two solutions must match up at the time, \( T_p \), when they meet.

Finally, Table 3 sets forth the equations for the construction phase, \( T < T_p \). The schedule of total construction funds needed is
TABLE 2

\[
\begin{align*}
V_{kl}^2 E^2 + r \sum_{j=1}^{m} \delta(T_j)(T_j) V E + E' Y + r E + \\
+ \sum_{j=1}^{m} \delta(T_j) \max(\gamma_j(T_j), CF(T_j)) - DP(T_j, 0) = 0 \\
E'(0, T) = PVE(T) \\
E'(V, T) V \rightarrow 1 \text{ as } V \rightarrow \infty \\
E'(W, T) = E'(W, T) 
\end{align*}
\]

(23)

(23a)

(23b)

(23c)

\[
\begin{align*}
V_{kl} E^2 D_V + r \sum_{j=1}^{m} \delta(T_j)(T_j) V D + D' Y + r D' \\
+ \sum_{j=1}^{m} \delta(T_j) \max(\gamma_j(T_j), CF(T_j)) - DP(T_j, 0) = 0 \\
D'(0, T) = PVCD(T) \\
D'(V, T) V \rightarrow \infty \text{ as } V \rightarrow \infty \\
D'(W, T) = D'(W, T) 
\end{align*}
\]

(24)

(24a)

(24b)

(24c)

\[
\begin{align*}
V_{kl} E^2 G^2 + r \sum_{j=1}^{m} \delta(T_j)(T_j) V G + G' Y + r G' \\
+ \sum_{j=1}^{m} \delta(T_j) \max(\gamma_j(T_j), CF(T_j)) - \gamma_j(T_j) V, 0) = 0 \\
G'(0, T) = PVG(T) \\
G'(V, T) V \rightarrow 0 \text{ as } V \rightarrow \infty \\
G'(W, T) = G'(W, T) 
\end{align*}
\]

(25)

(25a)

(25b)

(25c)

\[
\begin{align*}
V_{kl} E^2 S^2 + r \sum_{j=1}^{m} \delta(T_j)(T_j) V S + S' Y + r S' \\
+ \sum_{j=1}^{m} \delta(T_j) \max(\gamma_j(T_j), CF(T_j)) - \gamma_j(T_j) V, 0) = 0 \\
S'(0, T) = PVG(T) \\
S'(V, T) V \rightarrow 0 \text{ as } V \rightarrow \infty \\
S'(W, T) = 0 
\end{align*}
\]

(26)

(26a)

(26b)

(26c)

portion of the total guarantee let at time \(T_i\). Therefore, the value of the guarantee appears as a payout term for the equity. An alternative way to interpret this condition is to think of the equityholders as having purchased the guarantees from a third-party guarantor at a price of \(G^4\). Notice that the total payout term for the project is equal to zero in each of the equations since it is assumed that the project makes no net cash distributions during construction. Of course the project value can drop to a sufficiently low level during construction that the equityholders will optimally abandon the project. The sched-
ule of firm values at which this takes place during construction is given by \( \overline{V}(T) \). If the equityholders choose to abandon, it is assumed that the project is sold, all debt is immediately due, and the offer of a price guarantee is withdrawn. The value of the project to another investor, \( \overline{W} \), during construction, \( \overline{T} \leq T \), is given by equation (27), where \( \overline{V}(T) \) is the schedule of project values at which the new owner would abandon. In this analysis, it is assumed that ownership of the project is transferred to the host government.

Having developed the three sets of equations, Tables 1–3, it is now possible to describe the solution procedure for determining (1) investment value, (2) traded securities prices, (3) nontraded securities values, (4) the effects of bond covenants, and (5) the value of operating options. The investment value to the corporation is the value of the equity, \( E \), or the solution to equation (28). Note, however, that in order to solve equation (28) it is first necessary to solve equations (20), (22), (23), (25), (27), and (30). Thus, it is not only necessary to solve recursively for the value of equity, but also for the value of the guarantee. In addition, the value of the abandoned project during construction, \( \overline{W}(\overline{T}) \), must be determined to complete the boundary conditions for equation (28). While the solution of this problem may appear formidable, there are computer programs which will perform the necessary numerical computations. Indeed, the authors have applied CCA to problems of this complexity in practice.

It is perhaps appropriate to summarize what data are needed in addition to the numerical analysis computer programs to actually carry out the solution to equation (28). The necessary data are (1) interest-rate data, (2) variance-rate data, (3) covenant descriptions, and (4) project value. Some estimate of the time path of interest rates over the life of the project is needed in order to specify \( r(T) \). One possibility for the latter is to infer a series of forward rates from the term structure of interest rates which is observed at the time of analysis. To estimate the variance of project returns, \( \sigma^2 \), it is helpful if a traded asset can be found which has volatility characteristics similar to those of the nontraded project. Alternatively, if the source of volatility in the value of the project can be attributed primarily to a single commodity, then the volatility of that commodity's price could serve as an estimate for the project's volatility. A description of the covenants (for example, coupon rates or sinking funds) can be found readily in the indentures. These data are needed to properly specify the boundary conditions and the pay-out/pay-in terms in each problem. The cash flow proportionality factor, \( \gamma(V, T) \), must also be estimated. It is assumed in this example that \( \gamma(V, T) \) is a specified function which describes the changes in the size of the natural resource base and the production schedule in response to the stochastic passage of time and changes in the level of the project value. Finally, the value of the project, \( V \), which is measured gross of investment must be estimated. It may perhaps appear circular that it is necessary to estimate the gross value of the project in order to estimate the investment value of the project. However, the pertinent decision problem for the corporation is not simply the valuation of the project but the valuation of the terms and conditions surrounding its opportunity to "buy into" the project. To make the point in a less complex example, knowing the price of IBM stock is not sufficient to deduce the value of a call option on IBM stock. The value of the call option will also depend on its terms and conditions, e.g., exercise price and maturity. The project value can be estimated as the value today of a fully operational project available at time \( T \). This project value can be determined by discounting expected cash flows by a risk-adjusted discount rate using standard capital budgeting procedures.

To illustrate how a change in the terms and conditions surrounding the opportunity affect the analysis, consider the case where the loan guarantees on the junior debt are provided by the host government instead of by members of the consortium. The valuation of equity proceeds as before except that the \( G^t \) term is now deleted from the pay-in/pay-out term of equation (28). This illustrates one of the reasons why it is convenient to separate the valuation of the guarantee from the valuation of the equity. In any negotiations between the consortium and the host government, the value of the loan guarantee and the price guarantee would be of significant interest to both in arriving at the form of subsidy to be selected. The value of the loan guarantee, \( G \), is the solution to equation (29), which in turn depends on the solution to equations (20), (22), (23), (25), (27), and (30). The value of the price guarantee, \( S \), is the solution to equation (31), which depends on the solutions to equations (20), (23), (26), (27), and (28). Note that both of these problems exhibit the same recursive and interdependent nature as the equity valuation problem. The ability to price these nontraded contracts would not only permit both the consortium and the host government to negotiate in a more enlightened manner, but also to determine the impact of the value of these financial incentives caused by changes in their terms (e.g., the amount of debt guaranteed, level of the price guarantee, maturity of the guaranteed debt, and length of the price guarantee) which might
arise during the negotiations. Thus, the system of equations can be solved for many different sets of terms and the impact on the value of the equity, loan guarantee, or price guarantee can be determined.

Another set of issues of interest to the project equityholders is the set of terms at which the nonguaranteed senior debt could be issued. Since it is the convention to issue debt at par, it is possible to test directly whether a specific combination of maturity, coupon, and sinking fund will result in equation (25) predicting a par price for the senior debt. To do so, it is necessary to first solve equations (20), (21), (22), (23), (24), (25), (27), (28), and (30) before solving equation (29). The trade-offs among different maturities, coupons, and sinking funds can be determined by simulating various combinations of these components. It is also possible to estimate the value of seniority status for nonguaranteed debt, by adjusting the pay-out/pay-in terms and boundary conditions to reflect the condition that the un-guaranteed debt and the guaranteed debt are of the same rank if seniority rights are eliminated.

The model can also be used to determine the value of the project’s operating options. As an example, suppose that the host government requires that the consortium post a surety bond in the amount of the present value of equity’s contribution to construction costs in return for the host government providing loan guarantees. The requirement takes away the owner’s operating option to abandon the project during construction and thus ensures the completion of the project. The impact of this condition on the value of equity can be readily computed as the difference between the solution to equation (28) as originally posed and the solution to equation (28) with the new boundary condition

\[ E(0, T) = -PV(t) \]  

(28’)

where PV(t) is the present value of the mandatory contribution of construction funds by the equity. Condition (28’ a) is simply a formal statement of the owner’s obligation to contribute the necessary construction funds, independently of the project’s economic values.

As another example of an operating option, consider an offer by the host government to make available to the consortium enough of the natural resource that the scale of the project could be doubled at time \( T_d \) at a cost of \( I_d \) to each consortium member. The value of this option is the difference between the solution to equation (28), where condition (20a) has been replaced by

\[ E''(V, T_d) = \text{Max}(\text{Max}(V - I_d, 0) + V - DP(T_d)) \]  

(20’c)

and the solution to equation (28) as posed.

The other classes of operating options discussed in our examples in the text can be evaluated in a similar manner using CCA. In our hypothetical project, for example, the option to initiate would apply to the consortium having the choice to begin construction at any time during a specified period. By adjusting the boundary conditions to reflect this option, CCA can estimate the value of this flexibility to delay construction.

Notes

1. Options can be of either American or European type. An American-type option allows exercise on or before the expiration date, whereas a European-type option allows exercise only on the expiration date.


4. Expression (5) is true only for European options. Merton pp. 158–59.

5. Recall from the second section that \( C(S, T, X) = c(S, T, X) \) and therefore expression (14) must hold for a European call option also.

6. Alternatively, given the Black and Scholes value of a European call option, (equation (14)), the value of a European put option follows immediately from equation (5), the put-call parity result.

7. It is important to note that \( P \) is the known net flows out of the firm. Examples of outflows would be dividends, coupon payments, sinking-fund payments, and principal repayments. Examples of inflows would be the future issuance of new securities, e.g., equity or debt, where the timing, terms, and proceeds are known for certain.

8. Instead of formally solving equations (16), (16a), (16b), (16c) for the value of risky debt, the same result could be achieved in two other ways. First, since the value of the firm is the sum of the value of the equity and debt, \( V = E + D \), the value of equity obtained from equations (15), (15a), (15b), and (15c) could simply be subtracted from \( V \). Second, given expression (11), the value of a put or loan guarantee on the firm could be computed and subtracted from the value of a risk-free bond, \( Be^{-rt} \).

REFERENCES


The Value of a Sinking Fund Provision under Interest-Rate Risk

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Most corporate bonds have sinking-fund provisions. When the bonds are subjected to interest-rate risks, and when the bond market is competitive, the provisions offer a valuable delivery option to the issuer. This paper studies the behavior of this delivery option, and the impact of the option value on the sinking-fund bond pricing.

The paper shows that issuing a sinking-fund bond is equivalent to issuing a nonsinking-fund bond and holding European interest-rate call options. Alternatively, an issuer of a sinking-fund bond may be viewed as an issuer of a nonsinking-fund serial bond, holding interest-rate put options. Further, the paper shows that the sinking-fund provision always increases the bond yield (relative to the nonsinking-fund bond). This increment should depend on the slope of the yield curve, without assuming a pure expectation theory. As long as trading is permitted, we should find the delivery option value to be low when the yield curve is upward-sloping and the option value to be high when the yield curve is downward sloping. Finally, the paper presents some empirical evidence showing that the bond market is competitive for Aa-industrial bonds at issuance and, therefore, put options associated with sinking-fund provisions are valuable.