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A Model of Contract Guarantees for Credit-Sensitive, Opaque Financial Intermediaries*

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1. Introduction

As discussed in Merton (1993, Sections 5 and 6) and here in the section to follow, the effective delivery of many financial services depends critically on the credit-worthiness of the provider financial institution. Such service activities are said to be 'credit-sensitive'. The intermediary's credit standing can cause significant externality-like effects on the various business activities of the intermediary, even when there are no interconnections among them. For example, the announcement by a U.S. investment bank that it is even thinking of entering into a new merchant-banking activity of extending bridge financing and other interim risk-taking positioning for restructuring firms can materially and negatively affect its over-the-counter derivatives-products business for corporate customers because those customers may perceive the risk of the merchant-banking involvement as jeopardizing the bank's ability to fulfill its obligations on its long-dated contractual agreements. Thus the potential merchant-banking business affects the derivative-products business although there is no overlap of personnel, customer base, location, or employee skill sets between them.

The shared credit standing of the institution's individual businesses can therefore cause a significant failure of the principle of 'value-additivity', which complicates decentralization of the capital budgeting and financial decisions, The issue of monitoring credit quality are made more complex because those intermediaries such as banks and insurance companies that are principals to customer contractual agreements tend to be 'opaque' institutions, as defined in Ross (1989) and Merton

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(1993). This paper addresses this complexity with a model of incentive contracts as a substitute for direct monitoring of the institution.

2. Default Risk on Customer Contracts

2.1. A Central Managerial Issue for Intermediaries

Credit risk or more precisely the prospect of *contract default* by any firm is, of course, a concern to all transactors with that firm, whatever its business. However, unlike most firms, the efficiency of the central business activities of many financial intermediaries depends critically on their customer liabilities being default-free. As discussed in Merton (1989, 1992), controlling contract default is inexorably connected with the central management of risk within the financial firm.

The focus of analysis here is on opaque intermediaries which serve their principal function by issuing liabilities of a certain type to customers, and manage their assets to facilitate this principal function. A straightforward example is a property and casualty insurance company that issues more or less customized insurance contracts to its policyholders, and invests almost exclusively in securities traded in the capital markets. A more subtle but major example is an organized derivative-security exchange. Some classify such exchanges as financial markets, not intermediaries. However, unlike a typical stock or bond exchange, the financial futures and options exchanges serve the fundamental intermediation function of guaranteeing the performance of contracts traded on their exchanges. Buyers and sellers of those contracts have the clearing corporation of the exchange – not each other – as their respective counter-party. Thus, the exchange issues liabilities to both classes of its customers. It would vastly reduce the efficiency of the derivative-security markets if their customers had to 'diversify' against contract-default risk by spreading their otherwise homogeneous transactions across a large number of different exchanges. It is therefore absolutely essential that the clearing corporation of such exchanges have the very highest credit-standing with its customers.

To see why credit-worthiness is a much larger issue for intermediaries than for firms in general, it is helpful to draw a formal distinction between the 'customers' and the 'investors' of the firm. Calling attention to the distinction between customers and investors of nonfinancial firms is rarely necessary, because it is generally obvious. Few would confuse the customer who buys a car from an automobile firm with the shareholder, lender, or other investor who buys its securities. Similarly, no one would confuse a customer who changes money at a bank or takes out a loan from it with an investor who owns shares in the bank. But, the customers of many types of intermediaries receive a promise of services in the future in return for payments to the firms now. Financial services of this type such as insurance and retirement annuities usually involve payments to the customer of specified amounts of money, contingent on events and the passage of time. Those promised future services are liabilities of the firm, both economically and in the accounting sense. Since investors in the firm also hold its liabilities, the distinctions between customers and investors is not always so clear for such intermediaries.

The distinction between customers and investors can however be made. *Customers* who hold the intermediary's liabilities are identified by *their strict preference to have the payoffs on their contracts as insensitive as possible to the fortunes of the intermediary itself.* For example, the function served by a life insurance policy is to provide its beneficiaries with a specified cash payment in the event of the insured party's death. That function is less efficiently performed if the contract calls instead for the death benefit to be paid in the joint event that the insured party dies *and* the insurance company is solvent. Even if the insurance company offers an actuarially fair reduction in the price of the insurance to reflect the risk of insolvency, a risk-averse customer would prefer the policy with the least default risk. Indeed, on introspection, I doubt that many real-world customers would consciously agree to accept non-trivial default risk on a $200,000 life insurance policy in return for a large reduction in the annual premium, say from $400 to $300. Such results obtain *even* in theoretical models in which the customer has all of the relevant information necessary to assess the default risk of the insurer. In most real-world cases, the customers do not have the relevant information, and this fact *a fortiori* makes the potential welfare loss from customer-contract default even greater.

Theoretical counterarguments against this distinction assert that the customer may be able to eliminate the effect of this default risk either by trading in the securities of the life insurance company ('hedging') or by entering into a large number of tiny insurance contracts with many different companies ('diversification'). Such a case can perhaps be made for frictionless, complete-market economies. But, the *very* economic role of the intermediary is to service those entities (its customers) who *cannot* trade efficiently and who *cannot* enter contracts costlessly. A major rationale for the existence of intermediaries is to reduce the costs which households and firms would otherwise incur to manage risks directly by transacting in the financial markets.

By contrast, *investors* in the liabilities issued by an intermediary (e.g., stocks or bonds) *expect* their returns to be affected by its profits and losses. Indeed, their function is to allow the intermediary to better serve its customers by shifting the burden of the risk-bearing and resource commitment from customers to investors. The investors of course expect to be compensated for this service by an appropriate expected return. The resulting increase in efficiency of customer contracts from this shift in risk-bearing makes customers better off. Note that although the functional roles of 'customers' and 'investors' are distinct, the same individual or firm can be both a customer of *and* an investor in a particular intermediary. Thus, I can both buy an insurance policy from a particular insurance company and also hold its shares as part of my investment portfolio. The growing role for over-the-counter or custom contractual arrangements such as long-maturity currency and interest rate swaps has made some of the most profitable intermediary activities quite credit-sensitive.

The distinction between an investor-held and a customer-held liability claim is not unique to financial intermediaries. For example, a customer who buys a warranty on a new car from an automobile manufacturer wants the repairs paid for in the event that the car is defective. In fact, the customer's contract pays for repairs in the joint event that the car is defective *and* the automobile manufacturer is financially solvent. If given a choice, customers would prefer not to accept additional default risk in return for an actuarially fair reduction in the cost of the warranty. Much the same point can be made about the implicit contract with customers to ensure that spare parts are available in the future for repairs. Although it can become quite significant for a financially distressed firm, default risk is probably a secondary consideration for most customers of an automobile manufacturer. In contrast, because of the substantial size and long duration of many financial contracts such as annuities and life insurance, default risk is a first-order issue for customers of financial intermediaries. Thus, the success of a financial intermediary depends not only on charging adequate prices to cover its production costs, but also on providing adequate assurances to its customers that promised payments will be made.

In sum, customers in general are likely to know less about the firm's business prospects than its investors. However, the larger cost of customers instead of investors bearing default risk of the firm is not simply a consequence of customers being less-well-informed than investors. The 'wedge' of additional cost between customers and investors is primarily the result of customers 'internalizing' risks of the firm that investors can eliminate by diversification. That is, *the efficiency of customer contracts is diminished if they are exposed to default.* The term 'credit-sensitive' is used to describe the businesses of an intermediary that are significantly affected by changes in customer perception of the credit standing of the intermediary. Business activities that require customers to hold contractual liabilities of the intermediary are credit-sensitive.

There are essentially three ways for an intermediary with credit-sensitive activities to provide assurances against default risk to the customers who hold its liabilities: 1) By *hedging*: the firm holds assets which have payouts that 'match' those promised on its contractual liabilities and it chooses a 'transparent' structure so that customers can easily verify that such a matching policy is being followed. 2) By *insuring*: the firm acquires guarantees of its customer liabilities from an AAA-credit-rated private-sector or government third party. The providing of such guarantees is a large financial-intermediation business, which is itself quite credit-sensitive. 3) By *capital cushions*: the firm raises additional capital beyond that required for the funding of the physical investments and working capital needed to run the intermediary. Included in this category is the common practice of collateralizing contract performance as for example with repurchase agreements, futures contracts, and margin loans. The distinction between the collateral approach and hedging is that the collateral assets are not chosen to match the promised obligations on the contract. Assurance capital typically takes the form of equity although debt that is subordinated to customer contractual claims can sometimes be used.

There are economic deadweight costs to the intermediary from any of these three ways used to provide assurance of performance on customer contracts. Hedging is typically effective only when combined with considerable transparency. Ongoing disclosure required for transparency in a fluid structure can be expensive in direct costs and can provide the firm's competitors with proprietary information. A rigid asset-liability structure (e.g., unit trust) as an alternative imposes costs from lost flexibility. External monitoring of disclosures is costly to the firm's customers. Deadweight losses from using equity capital cushions can be high because of its tax disadvantage and the agency costs between managers/insider equityholders and external equityholders. Furthermore, customers often cannot be assured at low cost that the amount of capital cushion is adequate to protect their claims against the firm. Agency and monitoring costs between the firm and the external guarantor of its customer obligations are the main deadweight costs for the insurance approach.

This paper focuses on the insuring method for ensuring performance on customer contracts. It develops a formal model of optimal contracting decisions for an opaque intermediary with credit-sensitive businesses in an environment of agency costs. Optimal arrangements are derived which minimize deadweight costs of the firm-guarantor-customer system.

The model assumes that lack of transparency and transaction costs effectively prohibit customers from hedging or diversifying the default risk of the contracts purchased from the intermediary. Hence, contract-default risk depends on total (not just systematic) risk of the intermediary. As in the standard risky-debt and loan-guarantee models (e.g., Merton, 1992), the guarantee premium depends on total risk also. The agency costs for guaranteeing contract performance are shown to be an increasing function of the total risk of the intermediary.

The formal model derived here provides a theoretical foundation for the agency-cost structure posited in the risk-capital model in Merton and Perold (1993). This same agency-cost structure is assumed in Merton (1993) that develops a model for capital-budgeting and acquisition/divestiture decisions by credit-sensitive opaque intermediaries. As shown there, the potentially large and direct effect of an intermediary's credit standing on its operating cash flows can create significant synergistic-like effects across individual businesses within an intermediary, even when there are none of the normal 'physical' or 'relationship' synergies present. The credit-sensitivities of individual businesses within an intermediary can thus cause a significant failure of the 'value-additivity' principle, even without any synergistic interactions among the operations of those businesses. Efficient decentralization of the capital budgeting and financial decisions is therefore likely to be more difficult to implement for intermediaries with credit-sensitive business activities than for typical multi-business, non-financial firms.

To illustrate those effects, Merton (1993) presents a hypothetical example of an opaque intermediary with three separate one-year business operations. In addition to the failure of value additivity, the example shows that decentralized capital-cost or agency-cost allocations based on the marginal cost of each business will leave an aggregate unallocated cost for the intermediary. It is also shown that passive diversification strategies within an opaque intermediary will not increase its market value and may decrease it. However, hedging by the intermediary may increase value.

3. A Formal Model for Opaque Financial Firms

There arc three entities involved; a financial-intermediary firm, its customers, and an external guarantor that guarantees contract performance by the intermediary for its customer contracts. Initially (at $t = 0$), the assets and liabilities of the intermediary as well as the class of businesses it will pursue are known. Although the intermediary is restricted to remain in the same business classes, it is permitted to either expand or contract its assets and liabilities.

The 'opaqueness' of the intermediary is captured as follows: The market values of the assets and customer liabilities of the firm are continuously observable by the managers of the firm. For $t > 0$, these values can only be observed by 'outsiders' (i.e., the customers or guarantor or external equityholders) if they conduct an audit of the firm which causes a value-verification cost. As in the Townsend (1979) model, these audit costs capture the explicit deadweight agency costs to the collective. That is, if the managers of the firm and the guarantor were one in the same, then there would be no need to audit. As will be described, the model implicitly also allows for the possibility of agency costs between managers and equityholders. However, managers' and equityholders' interests are otherwise taken to be coincident.

Define the *surplus* of the financial intermediary, denoted at time t by $S(t)$, to be the difference between the market value of the intermediary's assets and the *default-free* value of its customers' liabilities. It is assumed that the customers will only do business with the firm if their contracts have no risk of default. If there were no costs and the intermediary were fully transparent, the customers could ensure the default-free nature of their contracts by simply requiring immediate liquidation of the firm if the surplus falls to zero (or some other predetermined positive value). With the posited costs and the opaqueness of the firm, an external guarantor is introduced that insures customers against losses from any shortfall (i.e., $S(t) < 0$).

The Guarantor: The guarantor guarantees performance on customer-contract liabilities for the indefinite future. In return, it receives an initial fee and it imposes covenant restrictions that the firm must have an initial surplus S_0 and that its equityholders receive no payouts or other distributions of value unless the surplus is shown to exceed S_0 or until the firm is liquidated. The guarantor has the right at any time to audit the firm, but it must pay the value-verification cost. If such an audit finds $S(t) \leq 0$, then the guarantor can instantly seize the assets. Otherwise, the firm continues operations. If the firm confirms by audit that its surplus exceeds S_0, then the 'excess' assets can be reverted to its equityholders. The process for liquidating the firm begins with an audit confirming the value of the assets and liabilities of the firm, followed by the payment (or collateralization) of all customer liabilities in full at their default-free value. If, after satisfaction of such liabilities *and* the payment of verification costs, there is a positive surplus value, then this surplus is distributed to the equityholders. If the net surplus is negative, then the guarantor seizes the assets and makes up the shortfall to customers. Management has the right to liquidate the firm at any time. Once liquidation takes place and customer obligations are satisfied, the guarantor's obligations are terminated.

The guarantor tries to structure a contract with the firm to minimize costs. To minimize its costs, the guarantor would prefer not to audit unless the firm is insolvent. Hence, it chooses a contract with the firm structured to induce management to liquidate the firm when insolvency occurs. Although in principle, induced liquidation at any non-positive $\underline{S}$ would work as well, choosing a $\underline{S} < 0$ would leave the firm open to gaming exposure by the guarantor who could make 'surprise' audits. If such an audit occurs when $\underline{S} < S \leq 0$, then the equityholders lose. Hence, $\underline{S} = 0$ is the contract-equilibrium liquidation level. As an incentive, the guarantor promises a payment to the firm's equityholders in the circumstance that the surplus is non-positive. The magnitude of the payment is a decreasing function of the size of the shortfall, $(-S(t))$. In particular a 'linear' structure is selected with the payment to the firm given by $\max[0, a + bS(t)]$ for $S(t) \leq 0$ where a and b are positive constants selected by the guarantor.

Firm: The intermediary sells its products to customers and purchases the liability guarantee at $t = 0$. Subject to the requirement of an initial surplus, $S(0) = S_0$, it extracts its profits from initial customer contracts at $t = 0$ and distributes them to the equityholders. The firm is permitted to issue additional customer liabilities (after $t = 0$), but it must retain all fees in the firm until 'reversion' of excess assets or full liquidation. It must also stay within the same lines of business. 'Passive' assets are held in the firm to secure customer liabilities. It is posited that from the perspective of the equityholders, the 'excess' of assets over liabilities, the surplus earns less than the full return these net assets would earn if they were held externally to the firm (e.g., in a 'transparent' unit trust). Examples of reasons for this below-market return are management expenses, regulatory reserve requirements, tax disadvantages of holding passive assets within the firm and possibly Jensen-Grossman-Hart free-cash flow (or asset) agency costs between managers and shareholders. Other than this 'haircut' on asset return for agency costs, the assumed coincident interests of managers and shareholders are to maximize the value of the firm. This constrained value-maximizing behavior, together with the below-market returns to surplus (assets) in the firm, will lead to an optimal strategy to either revert excess assets or fully liquidate the firm if the surplus becomes sufficiently large.

3.1. Core Environmental Assumptions

Surplus Dynamics: The surplus, assets minus default-free value of customer liabilities, follows an arithmetic Brownian motion given by

$$\mathrm{d}S = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}z, \tag{1}$$

where μ and σ are constants known to all and $\mathrm{d}z$ is a Wiener process. If assets earned a 'full' fair-rate-of-return, then (on a risk-adjusted basis) there would be a growth term, $rSdt$, in (1) where r is the riskless interest rate (a constant in this model). For analytical simplicity, it is assumed that these earnings are exactly dissipated by the various expense, tax, reserve, and agency costs described in **Firm.** Note that $S(t)$ is not a speculative price and therefore, the possibility that $S(t) < 0$ does not violate limited liability or other no-arbitrage conditions. μ and σ are in units of \$ and therefore, their magnitudes reflect the scale of operations.

Equilibrium Pricing: Since $S(t)$ is assumed not to be observable outside the firm, there is no other publicly-traded asset which has returns perfectly correlated with $\mathrm{d}S$. If such an asset existed with price $Y(t)$ and dynamics denoted by $\mathrm{d}Y = \alpha_Y Y\,\mathrm{d}t + \sigma_Y Y\,\mathrm{d}z$, then equilibrium asset pricing is assumed to satisfy

$$\alpha_Y = r + \Pi\sigma_Y, \tag{2}$$

where Π is the risk-premium for exposure to '$\mathrm{d}z$-risk'. E.g., if the return on the market portfolio were given by $\alpha_M\,\mathrm{d}t + \sigma_M\,\mathrm{d}z_M$ and the Capital Asset Pricing Model holds, then $\Pi = \rho[(\alpha_M - r)/\sigma_M]$ where $\mathrm{d}z\,\mathrm{d}z_M = \rho\,\mathrm{d}t$. Π is assumed to be constant.

Verification Costs: The cost of auditing and verifying the value of the firm's surplus at time t is given by

$$C_1 + C_2|S(t)|, \tag{3}$$

where C_1 and C_2 are non-negative constants. Parametrically, C_1 is probably an increasing function of σ, reflecting both the scale and uncertainty (and hence, difficulty to value) of the operation.

4. Analysis of the Model

We simplify the analysis by initially assuming that full liquidation is the only way to distribute 'excess' assets to the equityholders. We then show how to modify the analysis to accommodate partial liquidation in the form of surplus-asset reversion.

Valuing the Firm: Let $F(S)$ denote the value of the firm at time t for $S(t) = S$. By Itô's Lemma and (1),

$$E_t[\mathrm{d}F/F] = \left\{\left[\frac{1}{2}\sigma^2 F''(S) + \mu F'(S)\right]/F\right\}\,\mathrm{d}t,$$

and the (*signed*) standard deviation of its return is $F'(S)\sigma/F(S)$, where primes on F denote derivatives. From equilibrium-pricing (2), it follows that F satisfies:

$$0 = \frac{1}{2}\sigma^2 F''(S) + \gamma F'(S) - rF(s), \tag{4}$$

where $\gamma = \mu - \Pi\sigma$. The boundary conditions for (4) reflect the liquidation rules:

$$F(\underline{S}) = \max[0, a + b\underline{S}] \tag{5a}$$

$$F(\overline{S}) = \overline{S} - C_1 - C_2\overline{S} = (1 - C_2)\overline{S} - C_1, \tag{5b}$$

where $\underline{S}(\leq 0)$ and $\overline{S}(\geq C_1/(1 - C_2)$ are chosen to maximize F. This maximization leads to the 'high-contact' conditions

$$\begin{aligned} F'(\underline{S}) &= b \quad \text{for} \quad \underline{S} > -a/b; \\ &= 0 \text{ otherwise} \end{aligned} \tag{6a}$$

$$F'(\overline{S}) = (1 - C_2). \tag{6b}$$

The general solution to (4) is

$$F(S) = A_1 e^{\lambda_1 S} + A_2 e^{-\lambda_2 S}, \tag{7}$$

where

$$\lambda_1 = [(\gamma^2 + 2r\sigma^2)^{1/2} - \gamma]/\sigma^2 > 0$$

$$\lambda_2 = [(\gamma^2 + 2r\sigma^2)^{1/2} + \gamma]/\sigma^2 = \lambda_1 + 2\gamma/\sigma^2 > 0,$$

and A_1 and A_2 are constants to be determined from (5) and (6).

Let $F^*(S)$ denote the value of the firm *if* $\underline{S} = -\infty$, i.e., the value of the firm if management chose *never* to reveal that the firm is insolvent. Form (5), (6), and (7), it follows that

$$F^*(S) = \frac{(1 - C_2)}{\lambda_1} e^{\lambda_1 [S - S^*]}, \tag{8}$$

where the level of surplus at which voluntary liquidation occurs is $\overline{S} = S^*$ with

$$S^* \equiv 1/\lambda_1 + C_1/(1 - C_2). \tag{8a}$$

Management will choose $(\underline{S}, \overline{S})$ so as to maximize the value of the firm. As discussed, the guarantor must choose an incentive-compatible contract with the firm to induce the management to liquidate the firm if the surplus reaches zero. Since $\underline{S} = -\infty$ is a feasible choice by management, the guarantor must select

a and b so that $F(S)$ with $\underline{S} = 0$ exceeds $F^*(S)$ for $0 \leq S \leq S^*$. From (5a), $F(0) = a$ if $\underline{S} = 0$. Therefore, for $F(0) \geq F^*(0)$, it follows from (8) that the choice for a must satisfy

$$a \geq \frac{(1 - C_2)}{\lambda_1} e^{-\lambda_1 S^*}. \tag{9}$$

Assuming that (a, b) are chosen to ensure that $\underline{S} = 0$, we have from (5), (6), and (7) that $(A_1, A_2, \overline{S})$ satisfy

$$a = A_1 + A_2, \tag{10a}$$

$$(1 - C_2)\overline{S} - C_1 = A_1 e^{\lambda_1 \overline{S}} + A_2 e^{-\lambda_2 \overline{S}}, \tag{10b}$$

$$b = \lambda_1 A_1 - \lambda_2 A_2, \tag{10c}$$

$$(1 - C_2) = \lambda_1 A_1 e^{\lambda_1 \overline{S}} - \lambda_2 A_2 e^{-\lambda_2 \overline{S}}. \tag{10d}$$

It follows from (10b) and (10d) that

$$A_1 = e^{-\lambda_1 \overline{S}}[(1 - C_2)(\lambda_2 \overline{S} + 1) - \lambda_2 C_1]/(\lambda_1 + \lambda_2), \tag{11a}$$

$$A_2 = e^{\lambda_2 \overline{S}}[(1 - C_2)(\lambda_1 \overline{S} - 1) - \lambda_1 C_1]/(\lambda_1 + \lambda_2). \tag{11b}$$

If b is adjusted to keep $\underline{S} = 0$, then from (8a), (10a), (11a), and (11b), we can express the parameteric tradeoff between a and $\overline{S}$ as

$$a(\overline{S}) = \frac{(1 - C_2)e^{\lambda_2 \overline{S}}}{(\lambda_1 + \lambda_2)}(\lambda_1(\overline{S} - S^*) + e^{-(\lambda_1 + \lambda_2)\overline{S}}[\lambda_2(\overline{S} - S^*) + (1 + \lambda_2/\lambda_1)]). \tag{12}$$

From (12), we have that

$$\frac{da}{d\overline{S}} = \frac{(1 - C_2)\lambda_1}{(\lambda_1 + \lambda_2)}[\lambda_2(\overline{S} - S^*) + 1](e^{\lambda_2 \overline{S}} - e^{-\lambda_1 \overline{S}}). \tag{13}$$

Armed with (13), we can now determine the equilibrium-contract terms by analyzing the value of the guarantee and the optimal behavior of the guarantor.

Valuing the Guarantee: Let $G(S)$ denote the value of the guarantee at time t if $S(t) = S$. From the same reasoning that led to (4), G will satisfy

$$0 = 1/2\sigma^2 G''(S) + \gamma G'(S) - rG, \tag{14}$$

subject to

$$G(\underline{S}) = a + C_1 + C_2|\underline{S}| - b|\underline{S}| + |\underline{S}|, \tag{15a}$$

$$G(\overline{S}) = 0. \tag{15b}$$

The solution (14)–(15) with $\underline{S} = 0$ is

$$G(S; a, \overline{S}) = (a + C_1)[e^{\lambda_1 \overline{S} - \lambda_2 S} - e^{\lambda_1 S - \lambda_2 \overline{S}}]/[e^{\lambda_1 \overline{S}} - e^{-\lambda_2 \overline{S}}]. \tag{16}$$

As discussed, the guarantor picks a strategy and the parameters a and b in the firm's compensation schedule to minimize the cost to the guarantor. The minimum of audit costs occurs with a strategy that leads to no guarantor-paid-for-audits when $S > 0$ and a single audit at the first time that $S = 0$. This is achieved by picking a and b so that the optimal behavior of the firm is to voluntarily liquidate at $S = 0$ (i.e., to pick $\underline{S} = 0$). From (16), for all $\overline{S}$, $(\partial G/\partial a) > 0$ for fixed $\overline{S}$ and $(\partial G/\partial \overline{S}) > 0$ for fixed a. Hence, minimizing G implies choosing both the smallest feasible a and $\overline{S}$, subject to (9) and (12). From (13), $\mathrm{d}a/\mathrm{d}\overline{S} > 0$ for $\overline{S} \geq S^*$. Therefore, in that region, the minimum a corresponds to the minimum $\overline{S}$. It follows from (9) that the equilibrium a and $\overline{S}$ are given by

$$a = (1 - C_2)e^{-\lambda_1 S^*}/\lambda_1, \tag{17a}$$

$$\overline{S} = S^*, \tag{17b}$$

and from (10a), (10c), and (11), the equilibrium b is

$$b = (1 - C_2)e^{-\lambda_1 S^*}. \tag{17c}$$

Hence, in equilibrium, the value of the firm is given by

$$F(S) = F^*(S). \tag{18}$$

Note: By inspection of F^* in (8), $F(S)$ is distributed log-normal (within boundaries of $0 \leq S \leq S^*$) because $S(t)$ is normally distributed. That is, the *endogenously* determined value of the firm is lognormally distributed, even though the surplus of the firm's asset value is normally distributed. Thus, we have here an example of log-normally distributed stock returns endogenously derived instead of exogeneously specified.

5. Efficient Contracting to Minimize Deadweight Agency Costs

The deadweight agency costs are the verification costs paid when the firm is liquidated. The value of those costs can be determined by the same method used to find F and G. Let $H(S)$ denote the value of a security which receives C_1 if the firm is liquidated at $S = 0$ and $C_1 + C_2 S^*$ if the firm is liquidated at $S = S^*$. In effect, H is the amount one would pay to the auditors for a prepayment of audit services. As with (4) and (14), H satisfies

$$0 = 1/2\sigma^2 H''(S) + \gamma H'(S) - rH, \tag{19}$$

subject to

$$H(0) = C_1, \tag{20a}$$

$$H(S^*) = C_1 + C_2 S^*. \tag{20b}$$

The solution to (19)–(20) is

$$H(S) = \frac{[C_1(1 - e^{-\lambda_2 S^*}) + C_2 S^*]e^{\lambda_1 S} + [C_1(e^{\lambda_1 S^*} - 1) - C_2 S^*]e^{-\lambda_2 S}}{e^{\lambda_1 S^*} - e^{-\lambda_2 S^*}}. \tag{21}$$

From (21) and $0 \leq C_2 < 1$, there exists a $S^+, 0 < S^+ < S^*$, such that $H'(S^+) = 0$ and $H(S^+)$ is a minimum of H. It follows that the optimal (deadweight-cost-minimizing) initial-surplus requirement for the contract is

$$S_0 = S^+. \tag{22}$$

This condition is the initial (and maintenance) surplus specified in the optimal contract between the firm and the guarantor.

6. Alternative Versions and Extensions of the Model

An alternative model to (1) for the surplus dynamics, posits that the *ratio* of assets-to-default-free value of customer liabilities, $X(t)$, follows a geometric Brownian motion with an asset-return 'drag': $\mathrm{d}X = (\alpha - \eta)X\,\mathrm{d}t + \sigma X\,\mathrm{d}z$ with $X(t) = 1$ as the point of insolvency. The advantage of the arithmetic Brownian motion (1) is that the combination of businesses within a firm, $S(t) = \Sigma_1^n S_j(t)$, will just change parameters:

$$\begin{aligned} \mathrm{d}S &= \Sigma_1^n \,\mathrm{d}S_j(t) \\ &= (\Sigma_1^n \mu_j)\,\mathrm{d}t + \Sigma_1^n \sigma_j \,\mathrm{d}z_j \\ &= \mu\,\mathrm{d}t + \sigma\,\mathrm{d}z. \end{aligned} \tag{23}$$

As noted at the outset an alternative to the voluntary and full liquidation at $S = \overline{S}$ is that the firm can distribute just the 'excess' surplus over the initial required surplus S_0 and the guarantee remains in force. This partial-liquidation alternative would change condition (5b) to

$$F(\overline{S}) = F(S_0) + \overline{S} - S_0 - C_1 - C_2\overline{S}, \tag{5b$'$}$$

and boundary condition (15) to

$$G(\overline{S}) = G(S_0). \tag{15$'$}$$

An easy generalization of the model is to permit different audit-cost parameters, C_1 and C_2 depending on whether the liquidation is from insolvency or excess surplus. Also in (23), the dynamics of business activity and profitability of business j, represented parametrically by (μ_j, σ_j), could depend on the perceived credit rating of the firm.

References

Barnea, A., Haugen, R. A., and Senbet, L. W. (1985) *Agency Problems and Financial Contracting*, Prentice-Hall, Englewood Cliffs, N.J.

Diamond, D. W. and Verrecchia, R. E. (1982) Optimal managerial contracts and equilibrium security prices, *Journal of Finance* **37**, 275–287.

Grossman, S. J. and Hart, O. D. (1982) Corporate financial structure and managerial incentives, in J. J. McCall (ed.), *The Economics of Information and Uncertainty*, University of Chicago Press, Chicago.

Jensen, M. C. (1986) Agency costs of free cash flow, corporate finance, and takeovers, *American Economics Review* **76**, 323–329.

Merton, R. C. (1989) On the application of the continuous-time theory of finance to financial intermediation and insurance, *The Geneva Papers on Risk and Insurance* **14**, 225–262.

Merton, R. C. (1992) *Continuous-Time Finance*, Revised Edition, Basil Blackwell, Oxford.

Merton, R. C. (1993) Operation and regulation in financial intermediation: A functional perspective, in P. Englund (ed.), *Operational and Regulation of Financial Markets*, The Economic Council, Stockholm.

Merton, R. C. and Bodie, Z. (1992) On the managemet of financial guarantees, *Financial Management* (Winter), 87–109.

Merton, R. C. and Perold, A. F. (1993) Theory of risk capital in financial firms, *Journal of Applied Corporate Finance* **5**, 16–32.

Ross, S. A. (1989) Institutional markets, financial marketing and financial innovation, *Journal of Finance* **44**, 541–556.

Townsend, R. M. (1979) Optimal contracts and competitive markets with costly state verification, *Journal of Economic Theory* **21**, 265–293.