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On the Cost of Deposit Insurance When There Are Surveillance Costs

A model for evaluating the cost of deposit insurance is derived that explicitly takes into account surveillance or auditing costs and provides for random auditing times. The method used to derive this evaluation formula exploits the isomorphic correspondence between loan guarantees and common stock put options. Because of these auditing costs, the equilibrium rate of return on deposits will be below the market interest rate even in a competitive banking industry with no transactions costs. Further, it is shown that the auditing cost component of the deposit insurance premium is, in effect, paid for by the depositors, and the put option component is paid for by the equity holders of the bank.

In an earlier paper on the cost of deposit insurance and loan guarantees (Merton 1977*a*), I demonstrated an isomorphic correspondence between loan guarantees and common stock put options. Using this correspondence and the well-developed theory of option pricing, a formula was derived to evaluate these liabilities. If the guarantor chooses to audit only at the end of a finite, specified period, then the same analysis applies to the evaluation of demand-deposit guarantees. While the reason given for the finite time between audits was the cost to the guarantor of "continuous surveillance," no explicit recognition of these costs was presented in the model.

Although based on the same structure, the model developed in this paper extends the earlier development to take into account explicitly surveillance or auditing costs and to provide for random auditing times. Under the assumption of free entry into the banking industry, the equilibrium interest rate on deposits is derived. Further, it is shown that, in effect, depositors "pay" for the surveillance costs and the equity holders of the bank "pay" for the put option component of the deposit guarantee.

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Assumptions of the Model

In this paper, a bank is defined to be an institution that holds financial assets and finances their purchase by equity and by issuing deposits that are fully insured. From the point of view of depositors, each bank is a perfect substitute for every other bank, and therefore $(R + s)$ must be the same for all banks where R is the interest rate on deposits and s is the rate paid in the form of services. The banking industry is assumed competitive in the sense that there are no barriers to entry.

It is assumed that the deposits are guaranteed by the government or one of its agencies (e.g., the FDIC). While the formal analysis does not require that the guarantor be the government, it does require that the capability and willingness of the guarantor to meet its obligations are essentially beyond question. Otherwise, a second guarantor might be required for the first one; and a third for the second; and so on. The resulting "layering" of surveillance costs would probably be inefficient. Moreover, in the absence of detailed information and analysis, depositors might well use the size of the guarantor as a selector for greater safety, and this tendency could make it difficult for the "guarantor industry" to the banking system to have a competitive structure.

To develop the model, the following additional assumptions are made:

1. Trading in securities takes place continuously in time.

2. The securities and exchange markets are "sufficiently perfect," and asset-return dynamics are such that securities are priced so as to satisfy the security market-line equation in the (continuous time version of the) capital asset pricing model.[1]

3. There exists an exchange market where some investors and institutions (including the banks and FDIC) can borrow or lend at the same rate of interest, r, which is taken to be constant through time. Therefore, to avoid arbitrage by these investors, $r \geqslant R + s$.

4. At least some investors face a transactions cost for lending in the exchange market that can be measured for the jth investor as tc_j per unit time per dollar. So, the return per unit time for lending by the jth investor is $r_j = r - tc_j$. Hence, if $R + s > r_j$, then the jth investor will lend indirectly through deposits, and if $R + s < r_j$, then the jth investor will lend directly through the exchange market.

5. Although individual deposits are of the demand type, it is assumed that the dynamics for aggregate deposits for a given bank, D, are

1. The security market-line equation states that the equilibrium expected excess return on a security is proportional to the expected excess return on the market where the proportionality factor, called "beta," is equal to the ratio of the security's return covariance with the market to the variance of the return on the market. The capital asset pricing model is thoroughly discussed in Jensen (1972). See Merton (1973, p. 878) for the continuous-time version used in this paper.

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nonstochastic and described by $dD/dt = gD$ where the percentage growth in deposits, g, is a known constant.[2] The dynamics for the value of a given bank's assets, V, can be described by a diffusion-type stochastic process with stochastic differential equation:

$$\begin{aligned} dV &= [\alpha V - (R + s)D]dt + dD + \sigma V dz \\ &= [\alpha V - (R + s - g)D]dt + \sigma V dz, \; V > 0 \\ &= 0, \; V = 0, \end{aligned}$$

where α is the instantaneous expected rate of return on the assets per unit time; σ^2 is the instantaneous variance of the return per unit time and is assumed constant; dz is a standard Gauss-Wiener process.

6. The FDIC charges the bank a one-time premium to insure all the deposits of the bank in perpetuity provided that the bank is solvent (i.e., $V > D$). For purposes of this provision, solvency of the bank is determined by audit.[3] If, at the time of an audit, the bank is found to be solvent, then the bank continues operations unaffected. If, at the time of an audit, the bank is insolvent (i.e., $V < D$), then the FDIC liquidates the bank and pays off the depositors. It uses a random-time audit procedure where the event of an audit is Poisson-distributed with characteristic parameter λ, and successive audit times are independently and identically distributed. There is a cost per audit to the FDIC which can be described by a first-degree homogeneous function $C(V, D)$. Therefore, the audit cost per dollar of deposits C/D, can be written as $c(x)$ where $x \equiv V/D$. It is assumed that $c(x)$ is a continuous and bounded function for all $x \geqslant 0$.

The Evaluation of FDIC Liabilities

When the FDIC insures a bank's deposits, it receives a premium and assumes a liability of actual and potential future cash outflows for surveillance costs and for any "shortfall" between assets and deposits in the event of the bank's liquidation. In this section, a formula for the value of these liabilities is derived using the methods pioneered by Black and Scholes in the evaluation of options and corporate liabilities.[4] For convenience, the values will be derived treating the cash outflows as positive inflows, and, therefore, the derived values

2. To keep the bank's treatment of deposits from becoming a "Ponzi game," the growth rate of aggregate deposits, g, is assumed to be less than or equal to the total return on deposits, $R + s$.

3. The one exception is when $V = 0$ and $g < R + s$. In this case, the bank will be unable to pay the promised return, $R + s$, on deposits. This "default" is assumed to cause an immediate audit by the FDIC. This exception can be formally handled in the framework of the text by having λ "jump" to infinity when $V = 0$ and $g < R + s$.

4. See Black and Scholes 1973. Smith (1976) is an excellent review article that covers many applications of this method to the pricing of financial instruments. In particular, Brennan and Schwartz (1976) have used the method to evaluate the cost of certain insurance company guarantees of equity-based life insurance plans.

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will be positive. As with standard cost function analysis, these values will be the "cost" to the FDIC.

Assume that the value of the liability can be written as a twice-differentiable function $P(V, D)$ with continuous first derivatives.[5] The return on the liability and its change in price over a short time interval will depend not only on the changes in V and D, but also on whether an audit takes place during that interval. If an audit takes place, then there is a cash flow of $C(V, D)$. In addition, if the bank is insolvent, then there is a second cash flow of $(D - V)$ and the liability of the FDIC ceases (i.e., $P = 0$). If dR_P is the rate of return on the liability, then, using Itô's Lemma,[6] we can write dR_P (conditional on $P(V, D) = P$) as

$$\begin{aligned} P\,dR_P &= L[P(V, D)]\,dt + \sigma V \frac{\partial P}{\partial V}\,dz, \text{ if no audit occurs,} \\ &= L[P(V, D)]\,dt + \sigma V \frac{\partial P}{\partial V}\,dz + C(V, D), \\ &\quad \text{if an audit occurs and } V > D \qquad (1) \\ &= L[P(V, D)]\,dt + \sigma V \frac{\partial P}{\partial V}\,dz + C(V, D) + D - V - P, \\ &\quad \text{if an audit occurs and } V < D \end{aligned}$$

where L is an operator defined by

$$L \equiv 1/2\sigma^2 V^2 \frac{\partial^2}{\partial V^2} + [\alpha V - (R + s - g)D] \frac{\partial}{\partial V} + gD \frac{\partial}{\partial D}$$

and, with probability one, no more than one audit occurs in an instant.

If the event of an audit is independent of the return on the stock market, then from (1) the only source of systematic risk in the return on P is through its dependence on V (i.e., the dz term in [1]). Therefore, the beta of the return on P, β_P, can be written in terms of the beta of the return on V, β_V, as

$$\beta_P = \frac{\sigma V \dfrac{\partial P}{\partial V}}{P} \frac{\beta_V}{\sigma}. \qquad (2)$$

From assumption 2 and (2), the equilibrium expected return per unit time on P, α_P, must satisfy

5. While this is stated as an assumption, I have proved elsewhere (1977*b*) that if such a "smooth" solution exists then it is the equilibrium valuation function; i.e., other "candidate" solutions that are either discontinuous or have discontinuous first derivatives at any interior point cannot be the equilibrium solution.

6. For references on Itô's Lemma and extensions and for a treatment of stochastic differential equations that combine "mixed" Wiener and Poisson processes, see Merton (1976, esp. p. 130, n. 8).

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$$\alpha_P - r = V \frac{\partial P}{\partial V} (\alpha - r)/P. \tag{3}$$

Because the event of audit is Poisson distributed, the probability of an audit over the next instant is λdt and the probability of no audit is $(1 - \lambda dt)$. Taking expectations in (1), we have

$$\begin{aligned} P\alpha_P &= L[P(V, D)] + \lambda C(V, D) && \text{for } V > D \\ &= L[P(V, D)] + \lambda[C(V, D) + D - V - P] && \text{for } V < D. \end{aligned} \tag{4}$$

Combining (3) and (4) and simplifying terms, we have P must satisfy the partial differential equations:

$$1/2\sigma^2 V^2 \frac{\partial^2 P}{\partial V^2} + [rV - (R+s-g)D] \frac{\partial P}{\partial V} + gD \frac{\partial P}{\partial D} - rP + \lambda C(V, D) = 0, \text{ for } V > D, \tag{5a}$$

$$1/2\sigma^2 V^2 \frac{\partial^2 P}{\partial V^2} + [rV - (R+s-g)D] \frac{\partial P}{\partial V} + gD \frac{\partial P}{\partial D} - (r+\lambda)P + \lambda[C(V, D) + D - V] = 0, \text{ for } V < D. \tag{5b}$$

Equation system (5) can be converted to ordinary differential equations by the change in variables $x \equiv V/D$ and $p \equiv P/D$ where x is the asset-to-deposit ratio and p is the FDIC liability per dollar of deposits. If $p_1 \equiv p(x)$ for $x \geqslant 1$ and $p_2 \equiv p(x)$ for $0 \leqslant x \leqslant 1$, then from (5) we have

$$1/2\sigma^2 x^2 p_1'' + [(r-g)x - (R+s-g)]p_1' - (r-g)p_1 + \lambda c(x) = 0, x \geqslant 1 \tag{6a}$$

and

$$1/2\sigma^2 x^2 p_2'' + [(r-g)x - (R+s-g)]p_2' - (r+\lambda-g)p_2 + \lambda[c(x) + 1 - x] = 0, x \leqslant 1 \tag{6b}$$

where primes denote derivatives.

The boundary conditions for (6) are as follows:

$$p_1(1) = p_2(1) \quad [\text{continuity of } p(x)], \tag{7a}$$

$$p_1'(1) = p_2'(1) \quad [\text{continuity of } p'(x)], \tag{7b}$$

$$p_2(0) = \lambda[1 + c(0)]/(r + \lambda - g), \tag{7c}$$

$$p_1(x) \text{ is bounded as } x \to \infty. \tag{7d}$$

Boundary conditions (7a) and (7b) are required by continuity, and (7c) and (7d) are proved in the Appendix.

The general homogeneous solutions to (6) will be confluent hypergeometric functions. However, in the particular case where the growth rate in aggregate deposits equals the total return on deposits

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(i.e., $g = R + s$), the homogeneous solutions are simple polynomials. In light of the substantial simplification in the solutions and because it appears to do little violence to the general substantive properties of the results, I adopt this additional assumption for the balance of the paper.

With this assumption, the complete solution to (6) can be written as

$$p_1(x) = a_1 x + a_2 x^{-\delta} + Q_1(x), \qquad x \geq 1, \tag{8a}$$

$$p_2(x) = b_1 x^k + b_2 x^\zeta + \frac{\lambda}{\mu + \lambda} - x + Q_2(x), \qquad x \leq 1, \tag{8b}$$

where the various parameters and functions are defined as follows:

$$\begin{aligned}
Q_1(x) &\equiv \frac{\delta}{\mu(1+\delta)}[x^{-\delta}\textstyle\int^x y^{\delta-1}c(y)dy - x\int^x y^{-2}c(y)dy],\\
Q_2(x) &\equiv \frac{\delta}{\mu(k-\zeta)}[x^{\zeta}\textstyle\int^x y^{-\zeta-1}c(y)dy - x^k\int^x y^{-k-1}c(y)dy],\\
k &\equiv 1/2[1 - \delta + \sqrt{(1+\delta)^2 + \gamma}] > 1,\\
\zeta &\equiv 1 - \delta - k < 0,\\
\mu &\equiv r - g = r - (R + s) > 0,\\
\gamma &\equiv 8\lambda/\sigma^2 > 0,\\
\delta &\equiv \frac{2\mu}{\sigma^2} > 0,
\end{aligned} \tag{9}$$

and a_1, a_2, b_1, and b_2 are constants to be chosen so as to satisfy the boundary conditions (7).

The solutions in (8) depend on the form of the cost function, $c(x)$. However, for a variety of functions, the integrations for the $Q_i(x)$ in (9) can be performed to yield a closed-form solution. For example, if c is a polynomial then the Q_i are also polynomials, or if c is exponential then the Q_i are incomplete gamma functions. While the interested reader can try these various forms, I will assume in further exposition the special case where the audit cost per dollar of deposits is constant (i.e., $c[x] = K$).

In that special case, the solution to (8), including boundary conditions (7), can be written as

$$p_1(x) = \frac{\lambda K}{\mu} - \frac{1}{\mu(\mu+\lambda)(\delta+k)}[\mu^2(k-1) + \lambda(\lambda K k - \mu)]x^{-\delta}, \; x \geq 1, \tag{10a}$$

$$p_2(x) = \frac{\lambda(K+1)}{(\mu+\lambda)} - x + \frac{1}{(\delta+k)}[1 + \frac{\delta}{\mu(\mu+\lambda)}(\mu^2 + \lambda^2 K)]x^k, \; x \leq 1. \tag{10b}$$

Inspection of (10) shows that, unlike the analysis in Merton (1977*a*) the FDIC liability per dollar of deposits is not a monotonically decreas-

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ing function of the asset-to-deposit ratio. The reason is that there are two sources of the FDIC liability: (1) the guarantee of deposits (i.e., the "put option part") which is a monotonically decreasing function, and (2) the surveillance or audit cost which is a monotonically increasing function. The latter increases even though the cost per audit is constant because the expected number of audits prior to a "successful" audit where the bank is found to be insolvent is an increasing function of the asset-to-deposit ratio. Clearly, for a very small asset-to-deposit ratio ($x << 1$) an increase in that ratio will cause a tiny increase in the probability of more than one audit prior to liquidation, and it will cause almost a dollar-for-dollar reduction in the liability of the FDIC to the depositors when the bank is (almost certainly) found to be insolvent. Hence, in this region, the liability would be expected to decline in response to an increase in x. However, for larger values of x, the trade-off generally shifts more in the other direction until for $x >> 1$ the increase in audit costs completely swamps the small reduction in deposit liability. Figure 1 plots some typical patterns.

The reader is reminded that the FDIC never "observes" itself as having the liability given by $p_2(x)$ because, if it audits a bank and finds it insolvent, the bank is liquidated and the FDIC liability per dollar of deposits at that point is $1 - x + K$. However, to determine the correct solution in the "observed" region ($x > 1$), it is necessary to solve for $p_2(x)$.

A comparative statics analysis of the impact on the FDIC liability, $p_1(x)$, of changes in the parameter values will generally show a lack of monotonicity for the same reason that a change in the asset-to-deposit ratio did. Hence, any such formal analysis is postponed until equilibrium conditions are derived.

The Evaluation of Bank Equity

To determine the equilibrium deposit rate, it is first necessary to determine the value of bank equity-holder's shares. The procedure used follows along the same lines used to derive the value of FDIC liabilities.

Consider a bank that has paid the necessary premium to the FDIC and currently has assets with value V and deposits in the amount of D. Assume that the value of the bank's shares can be written as a twice-differentiable function $F(V, D)$ with continuous first derivatives. As was true for the FDIC liability evaluation, the return on the shares over a short time interval will depend on whether an audit takes place during that interval as well as on changes in V and D. If an audit takes place and the bank is solvent (i.e., $V > D$), then there will be no impact on the shares' return. However, if there is an audit and the bank is insolvent, then the shares become worthless. As was implicitly as-

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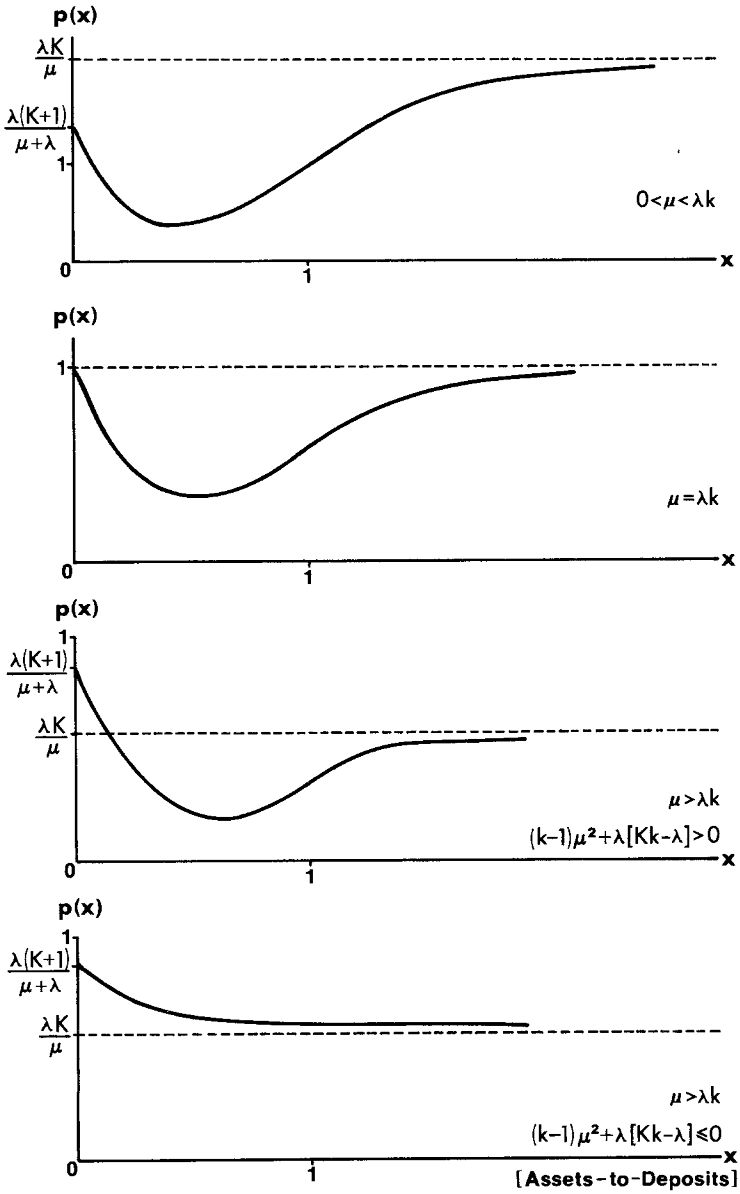

FIG. 1.—FDIC liability per dollar of deposits, $p(x)$

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sumed in the asset-dynamics description in 5, the equity holders receive no dividend payouts. Hence, using the identical procedure used to derive equations (5a) and (5b), we have that F must satisfy

$$1/2\sigma^2V^2 \frac{\partial^2 F}{\partial V^2} + [rV - (R+s-g)D] \frac{\partial F}{\partial V} + gD \frac{\partial F}{\partial D} - rF = 0, \quad \text{for } V > D, \tag{11a}$$

$$1/2\sigma^2V^2 \frac{\partial^2 F}{\partial V^2} + [rV - (R+s-g)D] \frac{\partial F}{\partial V} + gD \frac{\partial F}{\partial D} - (r+\lambda)F = 0, \quad \text{for } V < D. \tag{11b}$$

If we define $f(x) \equiv F/D$, the value of equity per dollar of deposits, then (11) can be rewritten as a system of ordinary differential equations:

$$1/2\sigma^2x^2f_1'' + \mu x f_1' - \mu f_1 = 0, \qquad \text{for } x \geqslant 1, \tag{12a}$$

$$1/2\sigma^2x^2f_2'' + \mu x f_2' - (\mu+\lambda)f_2 = 0, \qquad \text{for } x \leqslant 1, \tag{12b}$$

where $f(x) = f_1(x)$ when $x \geqslant 1$ and $f(x) = f_2(x)$ when $x \leqslant 1$ and the simplifying assumption that $g = R + s$ has been used.

The boundary conditions to be applied to (2) are

$$f_1(1) = f_2(1), \tag{13a}$$

$$f_1'(1) = f_2'(1), \tag{13b}$$

$$f_2(0) = 0, \tag{13c}$$

$$\lim_{x \to \infty} [f_1(x)/x] = 1. \tag{13d}$$

Boundary conditions (13a) and (13b) are continuity requirements; (13c) follows because with probability one the bank will be liquidated and the shares will be worthless; (13d) follows because the only liability of the bank, deposits, expressed in units of per dollar of deposits, is bounded.

The complete solution to (12) with boundary conditions (13) can be written as

$$f_1(x) = x - \frac{(k-1)}{(\delta+k)} x^{-\delta}, \; x \geqslant 1, \tag{14a}$$

$$f_2(x) = \frac{(1+\delta)}{(\delta+k)} x^{k}, \; x \leqslant 1. \tag{14b}$$

Inspection of (14) shows that the equity per dollar of deposits is a monotonically increasing function of the asset-to-deposit ratio. As is usually the case for limited-liability, levered equity, $f(x)$ is a strictly convex function for $x < 1$. However, unlike the standard case, it is strictly concave for $x > 1$.

In the usual limited-liability, levered-equity case (see Merton 1974), the equity position can be viewed as ownership of the assets levered by

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an unlimited-liability, riskless-debt issue combined with an implicit put option on the value of the assets. In the case of the bank equity, the position is the same except the rate paid on the "riskless debt part" is not r but $(R + s)$. It is the positive spread, $\mu = r - (R + s)$, that induces the concavity. However, the equity is still "levered" in the sense that the percentage change in $f(x)$ for a given percentage change in x is always greater than one.

Not only does the positive spread induce concavity but it also causes many of the usual comparative statics results, such as "an increase in the volatility of the underlying assets will increase the value of levered equity," not to obtain. The common sense of this departure is that the bank has, in addition to its tangible assets as measured by V, a valuable "intangible" asset: As long as it is solvent the bank pays less than the riskless rate on its deposits. By increasing the volatility of its assets when currently solvent, it increases the likelihood of becoming insolvent and thereby increases the likelihood of losing this intangible asset. Indeed, this effect is strong enough to obtain even when the bank is moderately insolvent. However, if it is already substantially below the solvency level, then by increasing the volatility of its assets the bank will increase the likelihood of becoming solvent by the time of the next audit.

On the Equilibrium Deposit Rate

Consider an investor or firm that is deciding whether or not to enter the banking industry. If the proposed investment is to have initial financial assets of V and initial deposits of D, then the initial equity investment required, I_e, is $I_e = V + H(V, D) - D$ where $H(V, D)$ is the one-time premium charged by the FDIC for insuring the bank's deposits. To be willing to enter the banking industry, the value of bank equity after entering must be greater than or equal to the initial equity investment required. That is, from (14a),

$$Df_1(x) - I_e = D\left[1 - \frac{(k-1)}{(\delta+k)}x^{-\delta} - h\right] \geqslant 0 \qquad (15)$$

where $h \equiv H/D$ is the premium per dollar of deposits and (14a) is the relevant formula because initially V must exceed D.

If the FDIC is operated on a "no subsidy"–"no excess profits" basis, then the premium h must be chosen to cover costs (i.e., $h = p_1[x]$). Substituting for h from (10a) into (15), we have as a necessary condition for entry,

$$\left[1 - \frac{\lambda K}{\mu}\right]\left[1 - \frac{\lambda k x^{-\delta}}{(\mu+\lambda)\,(\delta+k)}\right] \geqslant 0. \qquad (16)$$

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Moreover, for $\mu > 0$ and $x \geqslant 1$ a necessary and sufficient condition for (16) to obtain is that $\mu \geqslant \lambda K$, with the equality in (16) holding if and only if $\mu = \lambda K$.

If there are no barriers to entry into banking, then a necessary condition for equilibrium is $\mu \leqslant \lambda K$, and, for an "interior" and sustaining equilibrium, $\mu = \lambda K$. Therefore, the equilibrium deposit rate, R^* is given by

$$R^* = r - s - \lambda K. \tag{17}$$

As is the usual case for a constant-returns-to-scale industry, the scale of each bank is indeterminate, but the industry size as measured by deposits is determined from the aggregate demand for deposits by investors according to the schedule implied by 4.

The equilibrium spread between the market interest rate and the total rate of return on deposits exactly equals the expected auditing costs per deposit per unit time. So, in effect, depositors pay for the cost of surveillance, and the bank's equity holders pay for the deposit guarantee which is, in fact, an asset value guarantee.

The equilibrium premium charged by the FDIC per dollar of deposits can be written as

$$p_1^*(x) = 1 - \frac{(k^*-1)}{(\delta^*+k^*)} x^{-\delta^*} \tag{18}$$

where $k^* = 1/2[1 - \delta^* + \sqrt{(1+\delta^*)^2 + \gamma}]$ and $\delta^* = 2\lambda K/\sigma^2$. The equilibrium value of equity per dollar of deposits can be written as

$$f_1^*(x) = x - \frac{(k^*-1)}{(\delta^*+k^*)} x^{-\delta^*}. \tag{19}$$

To derive the formulas, it was assumed that once a bank selected a volatility rate for its assets it could not change it. However, comparative statics applied to (19) will show that if a bank "cheats" on the variance rate then it will do so in the direction of choosing *less volatile* assets not more volatile assets, provided the bank is solvent and there is a positive spread on the deposit rate. This finding is in sharp contrast to the usual result for levered equity as was discussed in the previous section and provides an attractive "stabilizing" side effect to this structure. However, this result will no longer obtain once the bank is sufficiently insolvent (i.e., once x is sufficiently smaller than one).

A final note on this point. If the FDIC chose to completely subsidize depositors by charging a premium that produced an equilibrium spread of zero (i.e., $\mu = 0$ and $R^* = r - s$), then the equilibrium value of equity would be

$$f_1^*(x) = x - 1 + 1/k^* \tag{20}$$

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with $\delta^* = 0$, and in this case, if the banks cheated, then they would choose more volatile assets.

Conclusion

Although the model presented in the paper is simple, it does allow for audit costs and random auditing. By explicit recognition of these costs, it leads to an equilibrium rate of return on deposits which is below the market interest rate even in a competitive banking industry with no transactions costs.

There are many directions in which the model could be extended. For example, the introduction of stochastic deposits and finite-time guarantees with intermittent payments to the FDIC would be important steps in improving the realism of the model. Further improvements would be to make the audit rate parameter, λ, state dependent based on the information learned by the FDIC at the last audit and to choose the λ function to minimize FDIC costs.

Finally, the same type of model can be applied to the "other side" of bank operations, namely, demand loans. Because banks have surveillance costs of their loans, they face, with respect to their customers, a surveillance problem similar to those of the FDIC in this paper. Hence, using a similar analysis, a positive spread between bank demand-loan rates and the exchange market rate can be derived.

Appendix

In this Appendix, the boundary conditions (7c) and (7d) used to solve for the FDIC liability per dollar of deposits, $p(x)$, are shown to be the appropriate ones.

For boundary condition (7d) to be valid, it is sufficient to show that $p(x)$ is a bounded function. To prove it is bounded, consider the worst situation: let D_0 be the level of deposits at time zero. The FDIC can guarantee its ability to pay off all deposits at any time if it invests D_0 dollars in the riskless-asset-earning interest rate r because $g < r$. To cover the audit costs, suppose it was known for certain that the bank would never be insolvent and therefore the FDIC would have to pay the audit costs forever. By assumption, the cost per audit per dollar of deposits, $c(x)$, is a bounded function. Let M be chosen such that $c(x) \leq M$ for all x. Clearly, the current value of the cost of auditing using M as the cost per audit per dollar of deposits and assuming that auditing will go on forever is an upper bound to the actual liability to the FDIC created by auditing costs. Under these conditions, the only uncertainty about the stream of audit costs is the timing of the audits. However, by assumption, the event of an audit is independent of the return on the market. Therefore, the current value of the cost of auditing using M for the cost per audit per dollar of deposits is equal to the present value of the expected auditing costs discounted at the riskless rate of interest.

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If an audit takes place at time t, then the cost of the audit will be $D(t)M = D_0e^{gt}M$. Because the event of an audit is Poisson distributed, the probability of an audit during the time interval $(t, t + dt)$ is λdt. Therefore, the expected audit costs for the time interval $(t, t + dt)$ is $\lambda D(t)Mdt = \lambda D_0e^{gt}Mdt$, and the present value of the expected audit costs is $e^{-rt}(\lambda D_0e^{gt}M)dt = \lambda D_0Me^{-\mu t}\,dt$ where $\mu = r - g > 0$. To compute the current value of all audit costs, we simply integrate over t from zero to infinity, that is,

$$\int_0^\infty \lambda D_0 M e^{-\mu t} dt = \frac{D_0\lambda M}{\mu}.$$

Therefore, the total FDIC liability at time zero, $D_0p(x)$, is less than or equal to $D_0 + D_0\lambda M/\mu$, or $p(x) \leqslant (1 + \lambda M/\mu)$. Hence, for $\mu > 0$, $p(x)$ is a bounded function.

Boundary condition (7c) gives the value of $p(x)$ for $x = 0$. From the posited dynamics for V and $g = R+s$, $V = 0$ is a natural absorbing barrier. Hence, given that $V = 0$, it is a certainty that at the time of the next audit the FDIC will have to pay the full amount of the deposits and the audit cost. However, once these are paid, there will be no further liability for the FDIC. Hence, the only source of uncertainty is the time of the audit.

Therefore, as in the previous analysis, the current value of the liability will equal the present value of the expected cost discounted at the riskless rate of interest. If the next audit takes place at time t, then the payout will be $D(t) + c(0)D(t) = [1 + c(0)]D_0e^{gt}$. Because the event of an audit is Poisson distributed, the probability that the next audit takes place in time interval $(t, t + dt)$ is given by the exponential distribution, $\lambda \exp[-\lambda t]dt$. Consequently, the current value of the FDIC liability $D_0P(0)$ is given by

$$\begin{aligned} D_0P(0) &= \int_0^\infty [1 + c(0)]D_0e^{gt}\cdot e^{-rt}\cdot \lambda e^{-\lambda t}dt \\ &= \lambda[1 + c(0)]D_0/(\mu+\lambda), \end{aligned}$$

and $p(0) = \lambda[1 + c(0)]/(\mu+\lambda)$, which is boundary condition (7c). As discussed in footnote 3, in the case where $g < R + s$ the depositors can not be paid by the bank, and therefore an audit takes place immediately. This condition can be treated formally by letting λ go to infinity when $V = 0$. Taking this limit in the above expression, gives $p(0) = 1 + c(0)$.

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