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CONTINGENT CLAIMS APPROACH TO MEASURING AND MANAGING SOVEREIGN CREDIT RISK

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This paper proposes a new approach to measure, analyze, and manage sovereign risk based on the theory and practice of modern contingent claims analysis (CCA). The paper provides a new framework for adapting the CCA model to the sovereign balance sheet in a way that can help forecast credit spreads and evaluate the impact of market risks and risks transferred from other sectors. This new framework is useful for assessing vulnerability, policy analysis, sovereign credit risk analysis, and design of sovereign risk mitigation and control strategies. Applications for investors in three areas are discussed. First, CCA provides a new framework for valuing, investing, and trading sovereign securities, including sovereign capital structure arbitrage. Second, it provides a new framework for analysis and management of sovereign wealth funds being created by many emerging market and resource rich countries. Third, the framework provides quantitative measures of sovereign risk exposures which facilitates the design of new instruments and contracts to control or transfer sovereign risk.

0 Introduction

Vulnerability of a national economy to volatility in the global markets for credit, currencies, commodities, and other assets has become a central concern of policymakers, credit analysts, and investors everywhere. This paper proposes a new comprehensive approach to measure, analyze, and manage sovereign risk based on the theory and practice of modern contingent claims analysis (CCA). In this approach, the sectors of a national economy are viewed as interconnected portfolios of assets, liabilities, and guarantees. We measure the sensitivities of the market values of these portfolios to "shocks" in underlying market risk factors, and we illustrate how to use contingent claims analysis to quantify sovereign credit risk and risks that are transferred from other sectors to the public sector.

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Contingent claims analysis is a proven approach to analyzing and managing private-sector risk. A contingent claim is any financial asset whose future payoff depends on the value of another asset. The prototypical contingent claim is an option—the right to buy or sell the underlying asset at a specified exercise price by a certain expiration date. A call is an option to buy; a put is an option to sell. CCA is a generalization of the option pricing theory pioneered by Black–Scholes (1973) and Merton (1973). Option pricing methodology has been applied to a wide variety of contingent claims. When applied to the analysis and measurement of credit risk, CCA is commonly called the "Merton Model."[1] It is based on three principles: (i) the values of liabilities are derived from assets; (ii) assets follow a stochastic process; and, (iii) liabilities have different priority (i.e., senior and junior claims). Equity can be modeled as an implicit call option and risky debt modeled as the default-free value of debt minus an implicit put option.

The Merton Model was first adapted and applied commercially by KMV (now Moody's KMV) and is now firmly established as the theoretical basis for several applied models that are widely used in the investment industry to measure and evaluate credit risk for corporate firms and financial institutions.[2] The innovation in this paper is to adapt the Merton Model and apply it at the aggregate level to the sovereign balance sheet and to sectors of the economy.[3] We substitute the term "junior claim" for equity in describing the sovereign balance sheet.

The traditional approach to analyzing a country's ability to service its foreign-currency denominated debt was developed by economists who specialize in macroeconomics and international trade. It usually involves forecasting national saving, investing, exports, imports, and funds flows. These macroeconomic flows are often supplemented with partial data on the government debt, and vulnerability is usually assessed with accounting ratios, such as debt to GDP. By contrast, our contingent claims approach focuses on the risk-adjusted economic balance sheet of the sovereign (combined government and monetary authorities) and is able to more accurately forecast the nonlinear behavior of sovereign bond prices and credit spreads.

The paper is organized as follows. In Section 1, we give a conceptual overview of our approach and explain the basic features of the quantitative model. In Section 2, we show how the model can be used to analyze and forecast the credit spread on a country's foreign-currency denominated sovereign debt. In Section 3, we demonstrate its real-world application showing how the sovereign CCA risk indicators can be calculated, with country examples, and application of the framework to a specific country—Brazil, during the 2002–2005 period. Section 4 explores actual and potential applications of this framework for investment management: (i) valuing, investing, and trading sovereign securities including Sovereign Capital Structure Arbitrage (SCSA) strategies; (ii) design and management of sovereign wealth funds; and, (iii) design and valuation of new sovereign risk transfer instruments and contracts.

1 Overview of the contingent-claims balance-sheet approach

Balance sheet risk is the key to understanding credit risk and default probabilities whether the balance sheet is of a corporation, a financial institution or a sovereign. All of the entity's assets and liabilities are measured at their current market values. Random changes in financial inflows, outflows, and fluctuations in market prices cause uncertainty in the values of the entity's assets and liabilities. The total value of all assets could decline to below the level of promised payments on the debt causing distress and/or default.

Figure 1 illustrates the key relationships, and Box 1 presents the Merton Model equations. The value of total assets in relation to the promised payments is illustrated in Figure 1(a), where the expected rate of return on assets is called the drift and denoted by the Greek letter mu (μ). The uncertainty in future asset value is represented by a probability distribution at the time horizon T. At the end of the period, the value of assets may be above the promised payments, indicating that debt service can be made, or below the promised payments, leading to default.

The probability that the asset value will be below the promised payments is the area below the promised payments in Figure 1(a). It is the "actual" default probability.[4] The asset-return probability distribution used to value contingent claims is not the "actual" one but the "risk-adjusted" or "risk-neutral" probability distribution, which substitutes the risk-free interest rate for the actual expected return in the distribution. This risk-neutral distribution is the dashed line in Figure 1(b) with expected rate of return r, the risk-free rate. Thus, the "risk-adjusted" probability of default calculated using the "risk-neutral" distribution is larger than the actual probability of default for all assets which have an actual expected return (μ) greater than the risk-free rate r (that is, a positive risk premium).[5] This is illustrated in Figure 1(b) which shows that the area below the promised payments of the "risk-neutral" distribution (dashed line) is larger than the area below the promised payments of the "actual" distribution (solid line).

The calculation of the "actual" probability of default is outside the CCA/Merton Model but it can be combined with an equilibrium model of underlying asset expected returns to produce estimates that are consistent for expected returns on all derivatives,

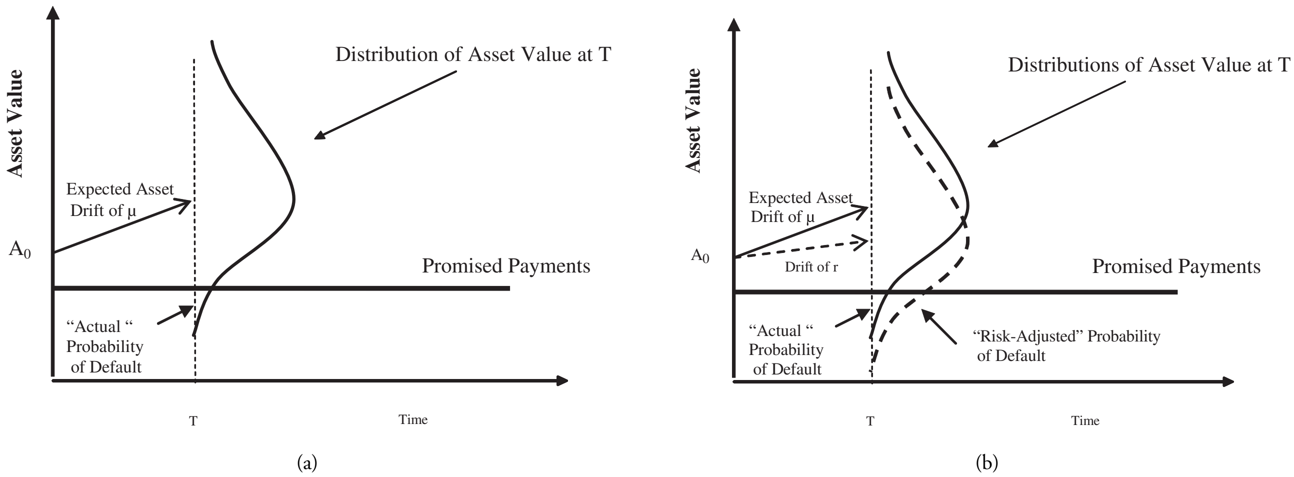

Figure 1 Probability distribution of asset value in relation to the promised payments.

The value of assets at time t is $A(t)$. The asset return process is $dA/A = \mu_A dt + \sigma_A \varepsilon \sqrt{t}$, where μ_A is the drift rate or asset return, σ_A is equal to the standard deviation of the asset return, and ε is normally distributed, with zero mean and unit variance. The probability distribution at time T is shown in (a).

Default occurs when assets fall to or below the promised payments, B_t. The probability of default is the probability that $A_t \leq B_t$ which is: $\text{Prob}(A_t \leq B_t) = \text{Prob}(A_0 \exp[(\mu_A - \sigma_A^2/2)t + \sigma_A \varepsilon \sqrt{t}] \leq B_t) = \text{Prob}(\varepsilon \leq -d_{2,\mu})$. Since ε: $N(0,1)$, the "actual" probability of default is $N(-d_{2,\mu})$, where $d_{2,\mu} = \frac{\ln(A_0/B_t) + (\mu_A - \sigma_A^2/2)t}{\sigma_A\sqrt{t}}$. $N(\bullet)$ is the cumulative standard normal distribution.

Shown in (b) is the probability distribution (dashed line) with drift of the risk-free interest rate, r.

Risk adjusted probability of default is $N(-d_2)$, where $d_2 = \frac{\ln(A_0/B_t) + (r - \sigma_A^2/2)t}{\sigma_A\sqrt{t}}$.

Box. 1

Merton Model Equations for Pricing Contingent Claims

The total market value of assets at any time, t, is equal to the market value of the claims on the assets, equity and risky debt maturing at time T:

$$\text{Assets} = \text{Equity} + \text{Risky Debt}$$
$$A(t) = J(t) + D(t)$$

Asset value is stochastic and in the future may decline below the point where debt payments on scheduled dates cannot be made. The equity can be modeled and calculated as an implicit call option on the assets, with an exercise price equal to the promised payments, B, maturing in $T - t$ periods. The risky debt is equivalent in value to default-free debt minus a guarantee against default. This guarantee can be calculated as the value of a put on the assets with an exercise price equal to B.

$$\text{Risky Debt} = \text{Default-Free Debt} - \text{Debt Guarantee}$$
$$D(t) = Be^{-r(T-t)} - P(t)$$

We omit the time subscript at $t = 0$.

The value of the equity is computed using the Black-Scholes-Merton formula for the value of a call:

$$J = AN(d_1) - Be^{-rT}N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{A}{B}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T},$$

r is the risk-free rate.
σ is the asset return volatility.
$N(d)$ is the cumulative probability of the standard normal density function below d.
The "risk-neutral" or "risk-adjusted" default probability is $N(-d_2)$.
The formula for the "delta" of the put option is $N(d_1) - 1$.
The yield to maturity on the risky debt, y, is defined by:

$$D = Be^{-yT}$$
$$y = \frac{\ln(B/D)}{T}$$

And the credit spread is $s = y - r$

Example: Assuming that: $A = \$100$,
$\sigma = 0.40$ (40%),
$B = \$ 75$
$r = 0.05$ (5%)
$T = 1$ (one year)

The value of the equity is \$32.367, the value of risky debt is \$67.633; the yield to maturity on the risky debt is 10.34%, and the credit spread 5.34%. The risk adjusted probability of default is 26%.

conditional on the expected return on the asset. The reason is that one does not have to estimate expected returns to use the CCA/Merton models for the purpose of value or risk calculations.

The fact that losses will be incurred in the event of default means that the debt is "risky," and therefore its value is less than if it were default-free. In CCA, the value of risky debt is calculated as the default-free value of debt minus an implicit put option.[6] The implicit put option is the value of a put option on the underlying assets with the strike price equal to the promised payments. Equity (the most junior claim) is modeled as an implicit call option on the assets with the strike price equal to the promised payments. By substituting into the balance sheet identity that total assets always equals total liabilities (including equity), we obtain:

$$\begin{aligned} \text{Assets} &= \text{Equity} + \text{Risky Debt} \\ &= \text{Implicit Call Option} \\ &\quad + \text{Default-Free Debt} \\ &\quad - \text{Implicit Put Option} \end{aligned}$$

The parameters of the risky debt on the CCA balance sheet can be used to obtain measures of risk exposures. Measures include probabilities of default,[7] credit spreads on debt, and the "distance-to-distress" (number of standard deviations of asset volatility the asset is away from the promised payments). Other risk measures are the sensitivity of the implicit option to the underlying asset. Option theory suggests a number of ways to measure exposure to risk. The most common one, "delta," is the change in the value of an option as the value of the underlying asset changes. Figure 2 illustrates how the value of the put option, and the value of its delta change as underlying asset values change. "Delta" is thus an appropriate measure of the risk in both risky debt and of a financial guarantee (formula for the delta of a put option is in Box 1).

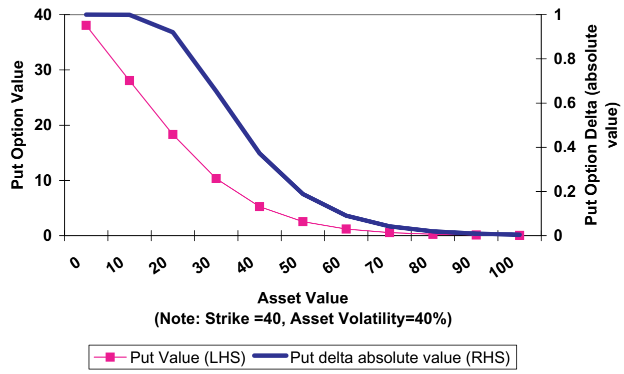

Figure 2 Implicit put option value and delta.

1.1 Accounting balance sheet of the sovereign

The accounting balance sheet of the government and monetary authorities is the starting point from which the contingent claims sovereign balance sheet will be constructed. Figure 3 is a simplified version of the sovereign accounting balance sheet.[8]

The assets are comprised of:

- *Foreign reserves*—Net international reserves of the public sector.
- *Net Fiscal Asset*—Items related to fiscal assets and liabilities are taxes, revenues, and expenditures. Expenditures can be divided into discretionary expenditures and non-discretionary expenditures

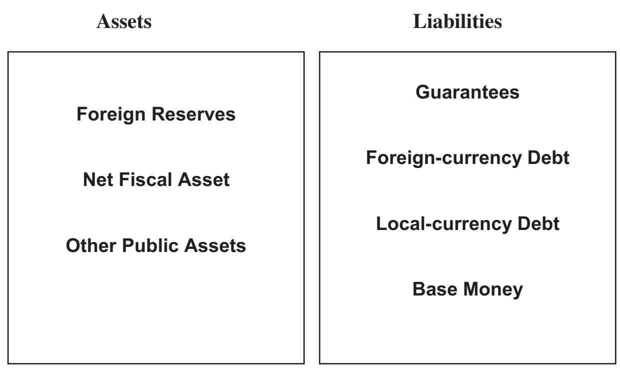

Figure 3 Stylized accounting balance sheet for the sovereign (Combined government and monetary authorities).

(e.g., defense, education, core infrastructure, welfare, etc. that will not be given up before giving up on paying the debt). Under stress situations, the government maintains the non-discretionary expenditures and cuts the discretionary expenditures. Subtracting the present value of non-discretionary expenditures from the present value of taxes and revenues gives the *net fiscal asset*,[9] similar to the present value of the primary fiscal surplus over time (the present value of fiscal surplus minus interest payments).

- *Other Public Assets*—These include equity in public enterprises, value of the public sector's monopoly on the issue of money, and other financial and non-financial assets. (Assets which do not generate value and will likely never be sold and become part of fiscal revenues (e.g., public land) might be included in a purely accounting statement, but will not be part of an economic balance sheet.)

Liabilities consist of:

- *Base money*—Base money is a liability of the monetary authorities and thus a liability on the sovereign balance sheet. Base money consists of currency in circulation, bank reserves (required bank reserves, excess reserves, vault cash).[10]
- *Local-currency debt*—Local-currency debt of the government and monetary authorities, held outside of the monetary authorities and the government.
- *Foreign-currency debt*—Sovereign debt denominated in foreign currency, usually held primarily by foreigners.
- *Guarantees*—Implicit or explicit financial guarantees to "too-important-to-fail" banks, other financial institutions or contingent pension/social obligations.

Appendix provides greater detail on the sovereign balance sheet.

1.2 Sovereign contingent claims balance sheet and valuation of sovereign risky debt

The analytical sovereign contingent claims balance sheet is the economic balance sheet of the combined government and the monetary authorities.[11] In order to apply the Merton Model to the sovereign, certain items on the balance sheet need to be rearranged. We begin by subtracting the guarantees to the too-important-to-fail entities from the asset side. Sovereign assets now consist of foreign reserves, net fiscal asset, other assets, minus guarantees to too-important-to-fail entities. Liabilities consist of foreign-currency denominated debt plus what we will call local-currency liabilities (local-currency debt and base money). Sovereign distress or default on foreign-currency debt occurs when the sovereign assets are insufficient to cover the promised payments on the foreign-currency debt. We will define a "distress barrier" as the present value of the promised payments on foreign-currency debt. While the promised payments are known with a fair degree of certainty over a time horizon, there is much more uncertainty about sovereign assets. The assets may increase or decrease for a number of reasons. A decline in sovereign assets to a level below the distress barrier will lead to serious distress and/or default. Default is thus effectively driven by three elements: value of sovereign assets, the volatility of sovereign assets and the distress barrier. For the calculations here we adopt a KMV-like measure of the distress barrier equal to short-term debt plus one-half of long-term debt plus interest payments up to time t.[12]

The value of the sovereign foreign-currency debt to the holders of such debt depends on the default-free value and also on losses if the sovereign were to default on the debt. The value of the sovereign foreign-currency debt is "risky." Any time a lender makes a loan to a sovereign, an *implicit* guarantee of that loan is created. To see how, consider the following identity, which holds in both a functional and a valuation sense:

$$\text{Risky Sovereign Debt} + \text{Implicit Guarantee} \equiv \text{Default-free Debt}$$

This equation can be re-written as

$$\text{Risky Sovereign Debt} \equiv \text{Default-free Debt} - \text{Implicit Guarantee}$$

Lending to sovereigns thus consists of two functionally distinct activities: pure default-free lending and the bearing of default risk by the lender. This implicit guarantee embedded in the risky debt is equal to the expected losses from default, which depends on the value and volatility of sovereign assets. Thus risky debt derives its value from (stochastic) sovereign assets, i.e., its value can be seen as contingent on sovereign assets.

Since some items on both the asset side and the liability side of the balance sheet are measured in different currencies, we will measure all items in a single reserve currency. In an emerging market country with a "soft" currency, the numeraire for the analysis is a "hard" reserve currency, e.g., US dollars. The distress barrier is related to the debt obligations in a "hard" currency.[13] For developed economies with a "hard" currency the numeraire will be in the "hard" currency as well. (See Appendix for more details on this issue). While the analysis is the same in units of local currency, the analysis is simpler in foreign currency terms since our goal is to measure credit risk in the foreign-currency debt (i.e., obligations in "hard" currencies). Figure 4 shows the CCA sovereign balance sheet.

2 Using implicit options to measure CCA balance sheet values and risk

The balance sheet in Figure 4 has two liabilities whose value is derived from sovereign assets

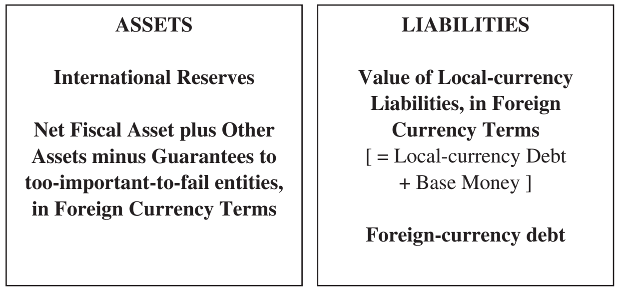

Figure 4 Contingent claims sovereign balance sheet.

and can be valued as contingent claims. The foreign-currency debt can be viewed as a "senior claim" and the local-currency liabilities as a "junior claim." Seniority of sovereign liabilities is not defined through legal status as in the corporate sector, but may be inferred from examining the behavior of government policymakers during periods of stress. In times of stress, governments often make strenuous efforts to remain current on their foreign-currency debt, efforts that effectively make such debt senior to domestic currency liabilities.[14] The payment of foreign-currency debt requires the acquisition of foreign currency, which the government has a more limited capacity to produce. In contrast, the government has much more flexibility to issue, repurchase, and restructure local-currency debt. See Appendix for more details on the sovereign CCA balance sheet.

The foreign-currency debt can be modeled as default-free value of foreign-currency debt minus an implicit put option. Sovereign local-currency liabilities in Figure 4 are modeled as an implicit call option. Sovereign local-currency liabilities have certain "equity-like features" since money and local-currency debt can be issued in large amounts even if this causes dilution/reduction in their value. In this sense, base money and local-currency debt are similar to "shares" on a sovereign balance sheet. Excessive issue of these "shares" can cause inflation and price changes similar to the case where excessive issue of corporate shares dilute existing holders' claims and reduce the price per share on a corporate balance sheet. The local-currency liabilities times the exchange rate is like the "market cap" of the sovereign in the international financial market.

2.1 Implicit options create risk interlinkages between the sovereign and other sectors

Financial guarantees can transfer risk across different sectors in the economy. The implicit guarantees governments extend to banks and other financial institutions can result in the accumulation of large unanticipated risks in the balance sheet of the public sector, especially if the government is not explicitly conscious of the magnitude of those guarantees and their potential for rapid increases in their risk profile in a weakening economic environment.

The explicit or implicit guarantee to the too-important-to-fail banks or financial institutions can be modeled with put options. Economic balance sheets can be used to demonstrate the interdependence among sectors. One sector is "long" certain implicit options; another is "short" the same implicit options. These implicit options thus create important risk inter-linkages among different sectors. For the sovereign balance sheet, we can see that there are two different types of implicit put options. The first is the implicit put option associated with the foreign-currency debt. The holders of risky debt are "short" this put option and the sovereign is "long" this put option. The second type of implicit put option is associated with the guarantee to too-important-to-fail financial institutions and other entities. The government is "short" these financial sector put option(s) and the financial sector is "long" those same option(s).

3 Contingent claims analysis estimates sovereign asset values and volatility from market prices

Given the conceptual definition of sovereign distress, how does one go about estimating it? The main challenge is deriving an accurate estimate for the *market value and volatility of sovereign assets*. While the levels and amounts of contractual debt are relatively easy to determine from balance sheet information, the same is not true when measuring the value of sovereign assets or its volatility. The market value of sovereign assets is not directly observable and must therefore be estimated. There are several ways to value an asset:

1. Determine value from observed market prices of all or part of the asset. This method can use a market price quote, direct observation, bid-ask quote or other similar direct measures.
2. Determine value by a comparable or adjusted comparable. A sophisticated version of obtaining a comparable value is the present value of a discounted expected cash flows—such as the primary surplus—with an appropriate discount rate.
3. Determine value from an implied value where the balance sheet relationships between assets and liabilities allow the observed prices of liabilities to be used to obtain the implied value of the assets.

The three methods have different advantages and disadvantages. The first method is straightforward but difficult to apply because only a few components of sovereign assets have directly observable market prices. International reserves are both observable and have a market value, yet the remaining items lack observable market prices. The second method, using comparables, is commonly used but also has short-comings. Future cash flows are difficult to project. The appropriate risk-adjusted discount rate and all of the relevant components that underlie the cash flow projections for tangible and intangible items included in the asset value estimation are difficult to determine. Furthermore, it is unclear how asset volatility should be best measured under the first two methods.

The third method, the approach adopted in this paper, circumvents the problems in the first two methods. It estimates sovereign asset value and volatility indirectly from information on observable values of the liability side of the balance sheet. This approach relies on the relationship between assets and liabilities. Since liabilities are contingent claims on assets, CCA can be used to get an "implied" estimate of the sovereign assets. The calculation of implied values is a very common technique in the finance world.[15] The collective view of many market participants is incorporated in the observable market prices of liabilities and the change in the market price of these liabilities will determine its volatility. The contingent claims approach implicitly assumes that market participants' views on prices incorporate forward-looking information about the future economic prospects of the sovereign. This assumption does not imply that the market is always right about its assessment of sovereign risk, but that it reflects the best available collective forecast of the expectations of market participants. Implementing contingent claims analysis to derive the implied sovereign asset value and volatility requires several steps and assumptions. From the observed prices and volatilities of market-traded securities, one can estimate the implied values and volatilities of the underlying assets.

3.1 *Calculating implied assets and implied asset volatility*

In the Merton Model for firms, banks and nonbank financials with traded equity, E, is an implicit call option which is a function of assets, A, volatility of assets, the distress barrier, B, risk-free rate, r, and the time horizon, t. The value of E is the formula for the call option.

$$E = f_1\left(A_{Firm}, \text{volatility of firm assets}, B, r, t\right). \tag{1}$$

There is a second equation that links equity and equity volatility to the same five parameters.[16]

$$E * \text{volatility of Equity} = f_2\left(A_{Firm}, \text{volatility of firm assets}, B, r, t\right). \tag{2}$$

The value and volatility of equity can be observed. Equations (1) and (2) are used to solve for the two unknowns, asset value and volatility.

Since the market value of sovereign assets cannot be observed directly, a similar calibration procedure can be used for the sovereign balance sheet to estimate implied assets and asset volatility. The prices in the international markets (including foreign currency market), together with information from domestic market prices, provide the market information for the value and volatility of certain liabilities on the public sector balance sheet.[17] The balance sheet in Figure 4 has liabilities structured in a way that we can observe the market value of the junior claims and the distress barrier of foreign-currency debt so as to be able to adapt the Merton Model to the sovereign. On the simplified sovereign balance sheet, the local-currency debt of the government, held outside of the monetary authorities, and base money are local-currency liabilities which are modeled as a call option on the sovereign assets.

This model combines money and sovereign local-currency debt together to get local currency liabilities (LCL). The distress barrier, $B_{\text{Sovereign}}$, is calculated as short-term foreign-currency debt plus one-half of long-term foreign-currency debt plus interest payments up to time t. A simple two claim CCA framework is used to calibrate the sovereign balance sheet by calculating implied sovereign assets, $V_{\text{Sovereign}}$, and asset volatility.

The value of local-currency liabilities in foreign currency terms $LCL_{\$}$ is

$$LCL_{\$} = f_1(V_{\text{Sovereign}}, \text{ volatility of sovereign assets}, B_{\text{Sovereign}}, r, t). \quad (3)$$

A second equation links $LCL_{\$}$ and its volatility to the same five parameters.

$$LCL_{\$} * \text{volatility of } LCL_{\$} = f_2(V_{\text{Sovereign}}, \text{ volatility of sovereign assets}, B_{\text{Sovereign}}, r, t) \quad (4)$$

Since local-currency liabilities have some similarities to "shares," the value of money and local-currency debt times the exchange rate is like the "market cap" of the sovereign. The volatility of the local-currency liabilities comes from the volatility of the exchange rate and the volatility of the quantities of money and local-currency debt (issued or repurchased).

Value of local-currency liabilities in foreign currency terms, $LCL_{\$}$, is a call option of sovereign assets in foreign currency terms, $V_{\$\text{Sov}}$, with its strike price tied to the distress barrier for foreign-currency denominated debt B_f derived from the promised payments on foreign-currency debt and interest payments up to time t.

$$LCL_{\$} = V_{\$\text{Sov}} N(d_1) - B_f e^{-r_f T} N(d_2) \quad (5)$$

The formula for the value of local-currency liabilities in foreign currency terms is

$$LCL_{\$} = M + B_{d\$,t=0} = \frac{(M_{LC} e^{r_d T} + B_d) e^{-r_f T}}{X_F} \quad (6)$$

The volatility of the local-currency liabilities is

$$\sigma_{\$LCL} = f(M, B_d, r_d, \sigma_M, \sigma_d, \sigma_{X_F}, X_F, \rho_{D_d, X_F}, \rho_{M,Dd\$}), \quad (7)$$

where the definitions of the variables in Eqs. (6) and (7) are:

M_{LC} base money in local currency terms; r_d domestic interest rate; r_f foreign interest rate; domestic currency denominated debt is B_d (derived from the promised payments on local-currency debt and interest payments up to time t); X_F forward exchange rate; σ_{X_F} volatility of forward exchange rate; σ_{D_d} volatility of domestic debt in local currency terms; ρ_{D_d,X_F} correlation of forward exchange rate and volatility of domestic debt in local currency terms; $\rho_{M,D_{d\$}}$ correlation of money (in foreign currency terms) and local-currency debt (in foreign currency terms); σ_{MLC} volatility of money (in local currency terms); σ_M volatility of money (in foreign currency terms); and $\sigma_{D_{d\$}}$ volatility of local-currency debt (in foreign currency terms).

Equations (3) and (4) can be rewritten as the two key equations relating assets and local-currency liabilities:

$$LCL_{\$} = V_{\$\text{Sov}} N(d_1) - B_f e^{-r_f T} N(d_2) \quad (8)$$

$$LCL_{\$} \sigma_{\$LCL} = V_{\$Sov} \sigma_{\$Sov} N(d_1). \quad (9)$$

Equations (8) and (9) can be used to calculate the two unknowns: sovereign asset value and sovereign asset volatility. Note that if the exchange rate is floating the volatility comes largely from the exchange rate. If the exchange rate is "managed" or "fixed" there is little or no volatility in the exchange rate but, to keep the exchange rate stable, more money and local-currency debt must be issued and bought back (via sterilization operations). There is thus higher volatility in the *quantities* of local-currency liabilities from the issue and repurchase operations as the counterpart to less volatility in the exchange rate. (An analogy: A firm that tries to fix its stock price must issue and repurchase shares with the result that the "market cap," shares times stock price, still has volatility.)

The calibrated parameters of the sovereign CCA balance sheet can be used to obtain quantitative sovereign risk measures. These include risk exposures for risky debt, such as distance-to-distress, probabilities of default, spreads on debt, the sensitivity of the implicit put option (i.e., expected losses) to the underlying asset (the delta), and other measures.[18]

A large component of the spread on sovereign foreign currency debt is the credit spread to compensate for the risk of default. The credit spread on sovereign foreign-currency debt (from spread equation in Box 1) is a function of: (i) the ratio of sovereign asset, $V_{\$\mathrm{Sov}}$ to the default barrier, B_f; (ii) the volatility of sovereign assets, $\sigma_{\$Sov}$; and (iii) horizon and risk-free interest rate. As the ratio of asset to distress barrier declines and/or $\sigma_{\$Sov}$ increases, the spread increases in a *nonlinear* way and can become sharply higher. A decline in foreign currency reserves, lower fiscal revenues, and/or a rise in the foreign debt default barrier will increase spreads.[19]

Risk managers working in modern financial institutions would find it difficult to analyze the risk exposure of their financial institutions if they relied solely on their income and cash flow statements and did not take into account (mark-to-market) balance sheets or information on their institution's derivative or option positions. Country risk analysis that relies only on a macroeconomic flow-based approach is deficient in a similar way, given that traditional analysis does not take into account the volatility of assets. Note that when the volatility of assets in the CCA balance sheet equations are set to zero, the values of the implicit put options go to zero. Something very similar to the traditional macroeconomic accounting balance sheet is the result. If we subsequently calculate the changes from one period to the next, the traditional flow-of-funds is the result, since the change in assets is equal to changes in cash/reserves and changes in the book value of debt. Thus, the flow-of-funds can be seen as a special deterministic case of the CCA balance sheet equations when volatility is set to zero and annual changes are calculated. Similarly, if the volatility of the assets of the too-important-to-fail financial institutions is set to zero, the implicit put options and contingent financial guarantees go to zero as well. The measurement of risk transmission between sectors is lost.

4 Application to countries

Using the calibration technique described above, the implied market value of sovereign assets and implied volatility of sovereign assets were calculated on a weekly frequency. The risk indicators were also calculated. Figure 5 shows the model risk indicator (risk-neutral default probability) compared to credit default swap (CDS) spreads on sovereign foreign-currency debt. Note that CDS or bond credit spreads *were not used as inputs* into the calculation of the risk indicator. As can be seen in Figure 5 the risk indicator has a high correlation with sovereign spreads.

Robustness checks suggest that the distance-to-distress, model credit spread, and risk-neutral probability of default are useful for evaluating sovereign vulnerabilities (Gapen *et al.* 2005). The evidence indicates that the book value of foreign-currency liabilities along with market information from domestic currency liabilities and the exchange rate contain important information about changes in the value of foreign-currency liabilities and credit risk premium. The nonlinearities and inclusion of volatility in the option pricing relationship used in this analysis contributes to the high degree of explanatory power and correlation with actual data.[20]

The sovereign distance-to-distress (d_2) to CDS spreads for 11 countries in 2002–2004 period

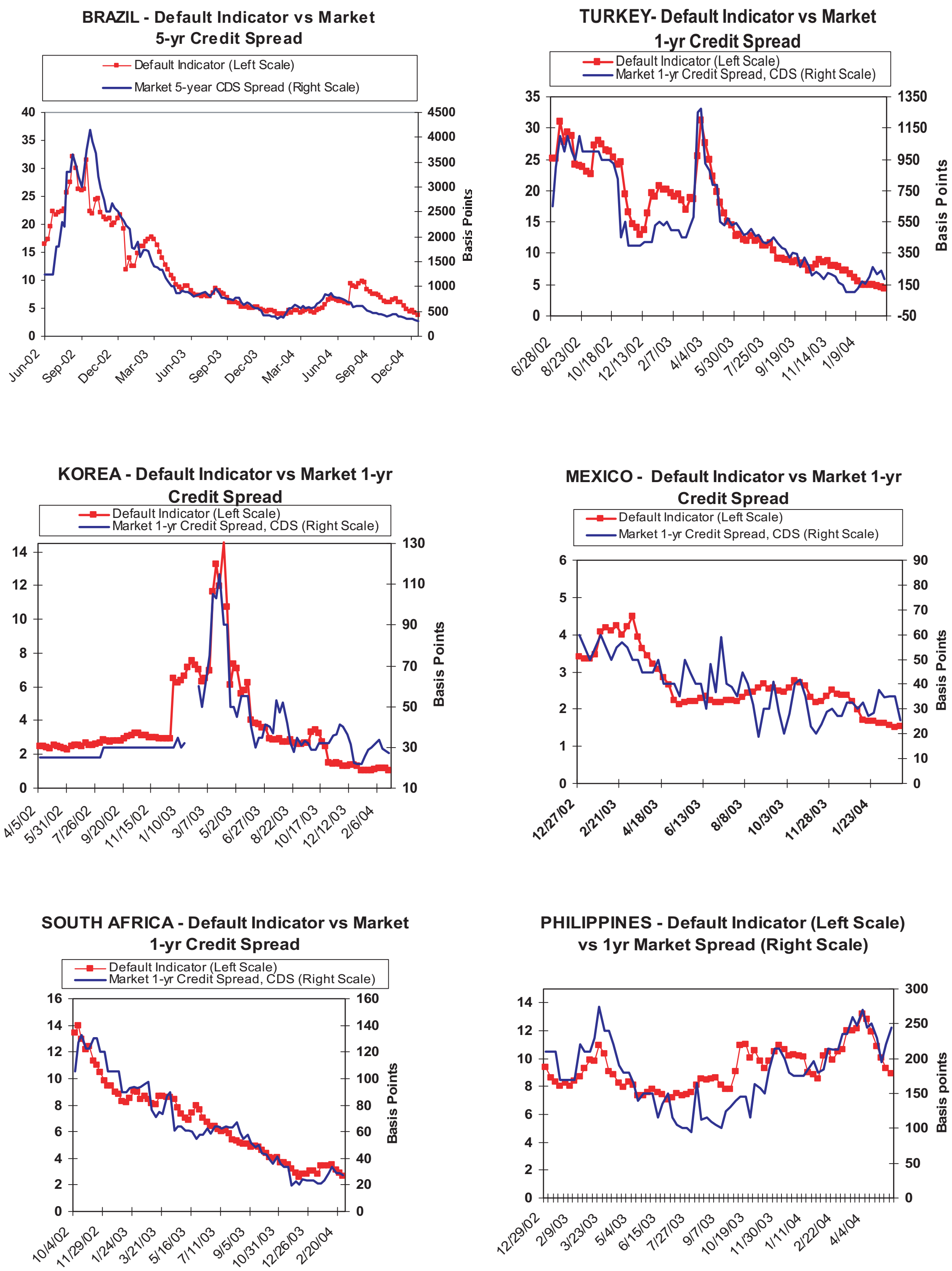

Figure 5 Risk indicators from sovereign CCA model compared to market spreads.
Source: MFRisk model and Bloomberg.

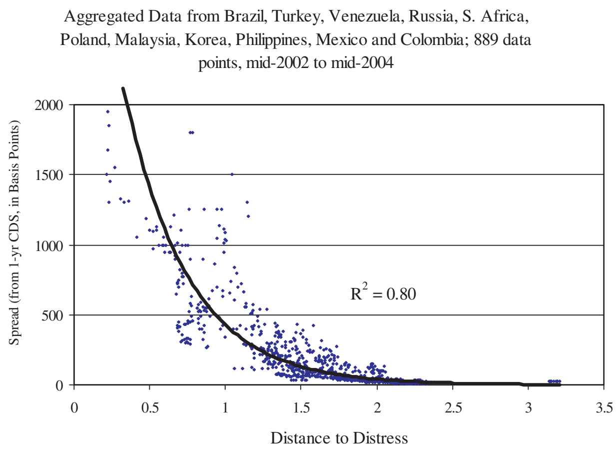

Figure 6 Sovereign spreads from CDS vs. model distance-to-distress.
Source: IMF Working Paper 155/05 using MFRisk model.

(1 year CDS spread is used) is shown in Figure 6. The correlation is high (R-squared is 0.8) and the nonlinear relationship of distance-to-distress with spreads can be seen. Correlations of debt-to-GDP ratios with spreads are very low, around 0.15 or less,[21] which is not surprising since no forward-looking market information or volatility is included in the accounting debt-to-GDP indicator.

4.1 *Application of CCA model to Brazil in volatile period 2002–2005*

The calibration of the CCA model for Brazil uses information from the forward exchange rate is used as an input into the sovereign contingent claims calibration (Eq. (6)). The volatility of the forward exchange rate in Brazil was particularly high in October 2002, 80–90 percent, whereas the volatility in calm periods was in the 10–30 percent range.[22]

The exchange rate level and its volatility, values of sovereign local-currency liabilities (base money and local-currency debt) and the foreign-currency debt distress barrier were used as inputs to estimate implied values and volatilities of the sovereign asset at various points in time. Brazil's implied sovereign assets were close to the distress barrier in 2002 and since have risen far above it. See below in Figure 7.

The relationship of foreign exchange price volatility, sovereign asset volatility, and sovereign spreads is shown in Figure 8. Brazil's implied sovereign assets, measured in billion of US $, were low in 2002 and the implied asset volatility was very high (panel A). This relationship corresponded to high volatility of the forward exchange rate and a low (depreciated) forward exchange rate (panel B). The high volatility of the sovereign asset and its low level led to much higher spreads on sovereign foreign-currency debt. The model estimated actual five-year spreads (blue diamonds in panel C) track close to actual spreads (dots in panel C) both of which are graphed versus the level of sovereign implied assets (billions of US $). Panel D is a graph of the forward exchange rate (horizontal axis) vs. the one-year domestic interest rates in Brazil. High domestic interest rates (near

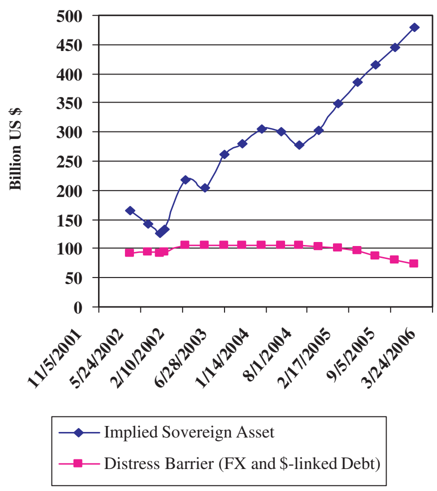

Figure 7 Implied sovereign asset value vs distress barrier (external and $-linked debt).
Source: MFRisk model.

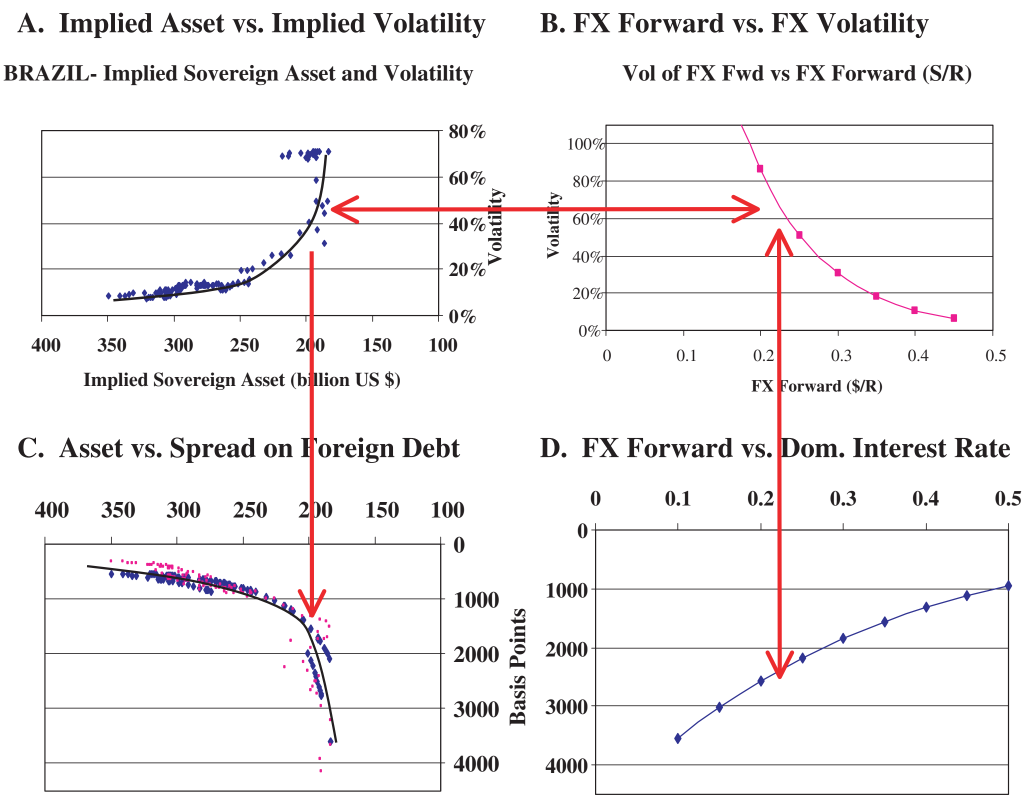

Figure 8 Nonlinear changes in value and volatility of sovereign assets, foreign exchange rates, credit spreads, and domestic interest rates in the case of Brazil 2002–2005.
Source: MFRisk model and Bloomberg.

2000–3000 basis points) correspond to a lower (more depreciated) forward exchange rate.

5 Applications of sovereign CCA from the perspective of investors[23]

Investors can apply sovereign CCA framework in numerous ways. First, it provides a relative value framework for the contingent claims on sovereign assets and can be a helpful tool in sovereign relative value trading or what can be called sovereign capital structure arbitrage. Second, CCA has important implications for the rapidly growing area of sovereign wealth funds. Emerging market governments have amassed large reserves and many governments have or are setting up sizeable sovereign wealth funds. The CCA framework developed here has important applications to this growing field of sovereign asset and wealth management. Third, once the sovereign risk exposures are calculated, new ways of transferring sovereign risk can be explored and potential new instruments and risk transfer contracting arrangements can be developed. The development and application of such instruments and contracts is known as the Alternative Risk Transfer (ART).

5.1 Capital Structure Arbitrage for firms and potential for relative value and Sovereign Capital Structure Arbitrage

Capital Structure Arbitrage (CSA) involves taking long and short positions in different instruments and asset classes in a firm's capital structure. It uses relative value techniques based on structural models for valuation across markets. Since the development of the CDS market in the late 1990s, there has been a rapid growth of CSA for corporate securities. Here we briefly describe CSA valuation and trading strategies for corporate and financial firm securities and outline a framework for SCSA.

The market value of risky debt from bonds and CDS can be compared to the "fair value" derived from a CCA model using equity market information. Trading strategies are designed to take advantage of apparent pricing discrepancies. Equity, equity options, senior debt, convertible debt, CDS, and asset swaps are among the instruments that can be used in CSA trading strategies. Summaries of market and CSA strategies are provided by Berndt and de Melo (2003), Jain (2005), CreditGrades (2002), and Toft (2003) among others. CreditGrades is a model developed by RiskMetrics Group and others that provides a framework for relative valuation. The original CreditGrades model (2002) included a diffusion of a firm's assets and a first passage time default with a stochastic default barrier. The model was modified to incorporate equity derivatives (Stamicar and Finger, 2005). Recent research has studied the relationship between the volatility skew implied by equity options and CDS spreads (Hull *et al.*, 2004). They establish a relationship between implied volatility of two equity options, leverage and asset volatility. This approach is, in fact, another way of implementing Merton's model to get spreads and risk-neutral default probabilities directly from the implied volatility of equity options.

A popular trade strategy is to trade equity against the CDS. Using a structural model calibrated with an equity and asset skew, which is most easily done with information from equity options, the "fair value" CDS spread can be obtained from the contingent claims model using equity market information. If the equity looks cheap relative to the observed CDS the strategy is to buy equity stock and buy protection in the CDS market. If equity prices go up or spread widens then the strategy earns money. Another strategy, if equity volatility is expensive relative to CDS spreads, is to write put options on equity and buy protection. If equity prices increase (and volatility declines) or spreads widen the strategy earns money.

The sovereign CCA model provides a framework for valuing sovereign foreign-currency debt, local-currency debt, foreign currency value of base money and local-currency debt, CDS on foreign-currency debt, and other claims. The benefit is that the sovereign CCA model provides "fair value" estimates of risky debt and CDS using as inputs the exchange rate, exchange rate volatility, local-currency liabilities, book value of foreign debt, and other parameters. Many different sovereign capital trading strategies are possible using a variety of instruments, including FX spot and forwards, FX options, local-currency debt, foreign-currency debt, CDS on foreign-currency debt, and inflation or indexed debt.

Using the Brazil 2002–2005 environment for context, we construct a hypothetical trade strategy. The strategies described here are motivated by the comovements in the flexible exchange rate (which affect the value of the local-currency liabilities) and the credit spread on foreign-currency debt. When the exchange rate is depreciated and very volatile the model spreads (and actual spreads) are high. One strategy is to go long in FX and sell protection. If FX appreciates the trade makes money. If spreads decline and the fees earned from selling protection decline, the strategy earns money. If both occur, money is earned on both the FX and CDS protection strategy. If there is a further depreciation and even higher spreads the strategy loses money. The example in Table 1 shows the initial state where the exchange rate is 0.23 \$/LC, sovereign distress barrier is \$100 million, the value of local-currency

Table 1 Hypothetical example: Sovereign capital structure trades—long FX, sell protection.

	Initial state	Final state	Profit or Loss
Exchange Rate $/LC	0.23	0.30	
Distress Barrier	100	100	
LCL$	112	146	
Implied Asset	202	239	
Implied Asset Vol	56%	45%	
RNDP (1-yr)	14%	3%	
Example positions			
Long FX ($/LC)	2326	3030	705
CDS: Fee for selling protection	2990	690	2300
		Total	3005

liabilities is $112 million and using the CCA model the implied sovereign assets are $202 million, the implied asset volatility is 56%, and the risk neutral default probability is 14%. The initial position is 10,000 long units of the exchange rate and CDS protection is sold on a certain notional. If the final state is one with more appreciated exchange rate and corresponding higher sovereign asset, lower asset volatility and lower default probability, then both legs of the strategy earn money. In such a state the FX position earns a profit and protection can be bought for much less, earning a profit on the original sale of protection via CDS.

Note that an alternative strategy could have been to go short FX to hedge selling CDS protection. In this case, the final state in Table 1 would have led to profits from the sale of protection and loss on the FX position (profits of 2300 minus 705 = 1595 < 3005). However, if the exchange rate depreciated, instead of appreciating, and if CDS spreads stayed the same, profit would have been made on the FX trade. If the CDS spreads widened, profit would be made on the FX trade and lost on the CDS trade. There are a myriad of possible strategies. Convergence arbitrage is one strategy if model "fair values" diverge from observed levels and the bet is that they will converge over a certain horizon period. Volatility trades are possible as well.

Preliminary work shows that the skew from FX options and its relation to sovereign CDS spreads has parallels to the relation between equity option skew and corporate CDS spreads seen in corporate capital structure analyses/models. Many of the strategies designed for corporate capital structure trades can be adapted to sovereign capital structure and relative value trades.

In a broader economic setting, the economy-wide CCA balance sheet model incorporating the financial and corporate sectors can be utilized to design relative value and other trading strategies. These can be extended to stock indexes, individual stocks of firms or banks and interest rate derivatives. There are a variety of trading strategies including international positions in other countries and in the S&P, VIX, foreign bonds, etc. The sovereign CCA framework has also recently been extended to value sovereign local-currency debt. This extension can be included in trading strategies.[24]

5.2 *Applications to sovereign asset and wealth management*

The CCA approach to measuring, analyzing, and managing sovereign risk can be applied to the analysis and management of sovereign wealth funds. Fund managers can combine CCA balance sheets with a Value-at-Risk (VaR) type approach adapted to the sovereign. Macro risk management dovetails with national wealth management. The analytical framework allows the quantification of the sovereign risk-adjusted balance sheet so that it can be viewed as a "sovereign portfolio" consisting of assets, liabilities, and contingent liabilities (whose values can be measured as implicit put options). This quantitative risk-oriented approach has two important advantages. First, it is a potentially useful new tool with which to gauge the risk reduction benefits of holding liquid foreign currency reserves versus other financial instruments for managing risk. In many countries the build-up of reserves has been much larger than is justified by short-term liquidity needs. Asian countries with booming export sectors and commodity exporters have amassed large reserve positions. Reserves in excess of the required liquid reserves can be invested in higher return but less-liquid instruments. The framework described here can be used to assess investment strategies that provide likely optimal hedging, diversification/risk reduction tailored for the specific risk characteristics of a country.[25] Excess reserves should be invested in instruments that improve the diversification of the "sovereign portfolio."

The summary of the long and short "positions" on the sovereign balance sheet are listed below.

Sovereign Asset-at-Risk (SAaR) analysis evaluates investment strategies which, along with other policies, keep the tail of the probability distribution of the sovereign asset portfolio above a threshold for a given confidence level (e.g., 5% or 10%). The sovereign asset portfolio is the reserves, fiscal and other assets including the contingent liabilities. If the sovereign debt structure of the country in question includes significant foreign-currency denominated debt, there may be additional or complementary debt targets. For example, a target for the expected loss associated with the foreign-currency debt (i.e., the credit spread associated with the implicit put option) so as to try to achieve a specific target rating of, for example, 0.5% probability of default or less for a one-year horizon. If countries with large excess reserves do not have significant amounts of foreign-currency debt, the SAaR is the more relevant target than a default probability target. If the SAaR exceeds the target threshold levels, policy makers can adjust various components of the sovereign balance sheet to lower the risk, for example:

- Use fiscal, debt and other policies that change fiscal surplus, the amount and maturity of

Sovereign's Portfolio of "Positions"

Sovereign asset (Reserves, Fiscal, and Contingent liabilities):		
Reserves FX (liquid)	$R_{FX\ \text{Liquid}}$	long
Reserves (Investments)	$R_{FX\ \text{Invested}}$	long
PV of primary surplus	$PV(T - G)$	long
Contingent Liability to banks and other too-important-to-fail entities	$-\alpha_G P_F$	short put options
Sovereign Debt and Other Liabilities:		
Risky LC Debt	$-B_{GLC}$	short default-free LC debt and
	$+P_{GLC}$	long dilution/inflation/default put option
Risky FX Debt	$-B_{GFX}$	short default free foreign debt and
	$+P_{GFX}$	long default put option
Base Money	$-M_{BM}$	short (long-term liability of MA)

outstanding government local currency and foreign-currency debt. Use financial sector and other policies such as capital controls.

- Make changes in asset allocation of sovereign wealth funds with respect to the risks, volatility and covariance of the fund vis a vis other components of the sovereign balance sheet.
- Use derivative securities to either hedge or better diversify the sovereign balance sheet as described in detail in Sec. 5.3.

These strategies imply that the optimal composition of the investments in wealth funds should take into account the risk profile of the sovereign.[26] Consider four countries with different economies and different risk profiles, such as China, Chile, Algeria, and South Africa. China's risk exposures are to higher oil and copper prices and to a slowdown in the US consumer market. Chile's risk exposures are to higher oil prices, lower copper prices and to a sudden stop in capital flows. Algeria is at risk of lower oil prices. South Africa is at risk of higher oil prices and lower prices of gold and other minerals. The sovereigns have various exposures from tax revenues, expenditures, risks of banking system crises, and to capital inflows and outflows. Should the asset allocation for the sovereign wealth portfolio for each of these countries be the same? Obviously not. Such different risk exposures argue for viewing the asset allocation policy decision in an integrated context including all the country's exposures. CCA provides a framework for assessing each economic sector's assets and liabilities, which allows policy makers to take a holistic view when formulating asset policy decisions.

5.3 Applications of the sovereign CCA framework for design of new instruments to transfer risk and the "ART" of sovereign risk management

The application of CCA to analyze risk exposures in the sectors of an economy offers a rich framework for comparing alternative ways to control and transfer risk. There are several benefits. First, CCA gives the interrelated values and risk exposure measures across sectors. Understanding these values and risk exposures can help identify particularly vulnerable situations and potential chain reactions of default. Identification allows formulation of various alternative ways to control and transfer risk. Second, the framework dovetails with risk-management strategies involving *explicit derivatives and swaps* used by the private and public sectors to control, hedge or transfer risk.[27] The field of ART includes a variety of instruments and contracts used by firms, financial institutions, and insurance companies. Many of these tools and techniques can be applied to transfer sovereign risk (directly or indirectly).

Risk can be controlled or transferred by a direct change in the financial structure (the structure of assets and liabilities), by managing guarantees (i.e., policies to limit the contingent liabilities to too-important-to-fail entities) or via risk transfer. This section will explore how the framework described in the paper could lead to some potential new ways to transfer sovereign risk and a way to value new risk transfer instruments or contracts. This is the "ART" of sovereign and macro risk management.

In general, there are three ways to transfer risk, *diversification, hedging, and insurance*. Risk concentration can be reduced by diversification to parties who have a comparative advantage in bearing various risks. If the balance sheets of corporations and financial institutions are weak when the economy is weak—as it is generally the case—then it is precisely when tax revenue is low, and the cost of debt service is high, which contributes to higher sovereign risk. This observation offers a powerful argument for diversification of the sovereign exposure to local shocks.

The financial markets, especially in emerging markets, are often "incomplete," meaning that they provide only limited possibilities to shift risk across various entities and groups. In such situations, diversification through international capital mobility is the obvious alternative. The sovereign CCA framework could be used in conjunction with diversification, hedging, or mitigation of sovereign risk in various ways:

- *Diversification and hedging related to management of foreign reserves*—A sovereign holds foreign currency reserves, in part, to cushion against potential losses of the monetary authorities or government. CCA can be used to assess the costs of increasing reserves via issue of foreign debt, local-currency debt, money or contingent capital contracts against the benefits of having a cushion to mitigate losses.
- *Contingent reserves or contingent sovereign capital*—Corporations sometimes contract for contingent equity or debt purchases triggered under pre-agreed conditions. Similarly, governments could make arrangements with external public or private sector entities for pre-agreed purchase of government local-currency debt under specific circumstances such as a sudden stop in capital flows or certain revenue losses, commodity price drops or natural disasters. The value of such contingent capital can be compared to the costs of increasing paid-in capital reserves via debt issues. This macrofinance framework could be used to calculate value-at-risk for the sovereign balance sheet which would help determine the appropriate level of foreign currency reserves and contingent reserves or contingent sovereign capital.
- *Sovereign bonds with special features*—GDP-linked bonds or bonds with specific roll-over clauses can help manage risk. Indexed bonds such as commodity-linked bonds linked to major exports such as oil or copper, or catastrophe bonds (CAT) and similar instruments.
- *Diversification*—Certain methods of diversifying, hedging, and risk mitigation provide benefits to other sectors and can have an indirect benefit for the sovereign. Asset diversification in the banking sector could indirectly benefit the sovereign. A bank that invests part of its assets in domestic government bonds enhances its exposure to local macro shocks; the value of government bonds will be low precisely when the value of the loan book is low. Therefore, in such economies, banks should hedge the exposure of their loan book by investing in non-domestic assets—such as bonds.
- *Equity swaps as a method of diversifying internationally*—An equity swap would enable a small country to diversify internationally without violating possible restrictions on investing capital abroad. Suppose that small-country pension funds who already own the domestic equity were to enter into swaps with a global pension intermediary (GPI). In the swap, the total return per dollar on the small country's stock market is exchanged annually for the total return per dollar on a market-value weighted-average of the world stock markets. The swap effectively transfers the risk of the small-country stock market to foreign investors and provides the domestic investors with the risk-return pattern of a well-diversified world portfolio. Since there are no initial payments between parties, there are no initial capital flows in or out of the country. Subsequent payments, which may be either inflows or outflows, involve only the *difference* between the *returns* on the two stock market indices, and no "principal" amount flow. Equity swaps can be used to reduce a country's risk of dependence on specific exports, for example Taiwan could reduce its dependence on electronics products. The Taiwanese government would pay returns on the world electronics portfolio in exchange for returns on another industry, say the automobile industry. Thus swaps are a way in which countries could focus on industries which they have a comparative advantage and still pursue efficient risk diversification.[28]

6 Conclusions

The high cost of international economic and financial crises highlights the need for a comprehensive framework to assess the robustness of countries' economic and financial systems. This paper proposes a new approach to measure, analyze, and manage sovereign risk based on the theory and practice of modern CCA. We illustrate how to use CCA to model and measure sovereign risk exposures and analyze policies to offset their potentially harmful effects. The paper provides a new framework for adapting the CCA model to the sovereign balance sheet in a way that can help forecast credit spreads and evaluate credit and market risks for the sovereign and risks transferred from other sectors. This new framework is useful for assessing vulnerability, policy analysis, sovereign credit risk analysis, sovereign capital structure, and design of sovereign risk mitigation and control strategies. Applications of this framework for investors in three areas are discussed. First, CCA provides a new for valuing, investing, and trading sovereign securities, including SCSA. Second, it provides a new framework for sovereign asset and wealth management which is particularly applicable to the increasingly large sovereign wealth funds being created by many emerging market and resource rich countries. Third, the framework provides quantitative measures of sovereign risk exposures which allow for potential new ways of transferring sovereign risk. New instruments and risk transfer contracting arrangements can be developed using ART tools applied to sovereign risk management.

Appendix – Public sector CCA balance sheet and its calibration using the contingent claims approach

This Appendix describes how the segregated contingent claim balance sheet of the monetary authorities and the government can be combined and how the implied sovereign assets and asset volatility can be calculated and risk indicators estimated.[29] Useful insights can be obtained when one views relationship between the assets and liabilities of the public sector[30] as separate balance sheets of the government and monetary authorities where there are cross-holdings and financial guarantees between these two public sector "partners." Under this structure, the assets of the monetary authority include foreign reserves, credit to the government and other claims.[31] The liabilities of the monetary authority partner are base money and financial guarantees to the government, including guarantees to supply foreign currency to service the sovereign foreign-currency denominated debt. The assets of the government partner include the net fiscal assets and other assets. Liabilities include credit to the monetary authority (and could include local-currency debt held by the monetary authority), local-currency debt held outside of the government and monetary authority, financial guarantees and foreign-currency denominated debt.

Figure A.1 shows the structure of this segregated balance sheet. This simplified framework is not meant to be a comprehensive catalogue of all the guarantees, the nature of which varies by country and by the detailed structure of the relationship between monetary authorities and the government. There also may be implicit financial support from the monetary authorities to the government via purchase of government local-currency debt under certain circumstances, but this is not shown here. The action of the monetary authority "partner" of buying additional government local-currency debt entails issue of additional base money. There are also "options," that the government and the monetary authorities have to "default" on the obligations to convert local currency into foreign currency. Similarly the government could "forcibly" restructure local-currency debt or dictate "mandatory" purchases of government bonds by certain public

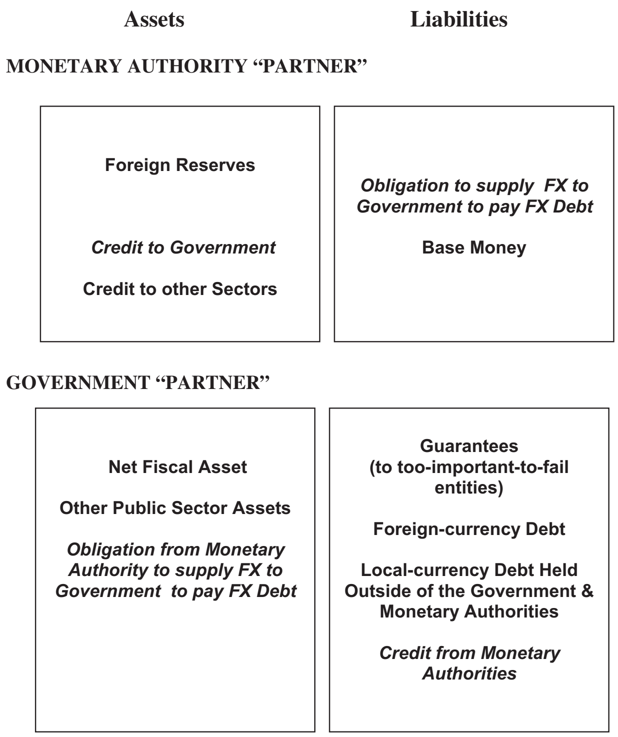

Figure A.1 Segregated balance sheet for the public sector.

Note that the cross-holdings of government debt and guarantees from monetary authorities to Government are in italics. Liquidity operations of the monetary authorities are not included.

or private institutions or inflate to cover potential shortfalls. Also, in some countries, banks may have deposits with the monetary authorities that receive a priority claim on foreign currency reserves that is higher than that of holders of local currency, which could be junior to claims on foreign currency for payment of external foreign-currency debt.

The priority of the claims can vary from country to country. In many cases, though, we can think of the guarantees to banks or other "too-important-to-fail entities" as senior claims. Also most governments find it easier to inflate or dilute local-currency debt in a distress situation before defaulting on foreign-currency debt. Thus a case can be made that foreign-currency debt is senior to local-currency debt. The government certainly may take the view that credit from the monetary authorities is the most junior obligation and many governments may or may not honor that claim. The credit from the monetary authorities is an asset on the side of the monetary authority partner and a liability of the government partner. Similarly, the financial guarantees to the government partner are an asset on its balance sheet and a liability of the monetary authority partner. When the balance sheets are combined these two items drop out. The segregated balance sheet above reduces to the combined balance sheet in Figure 3. CCA can be applied to the segregated or the combined balance sheets, the choice of which depends on the purposes of the analysis. The numeraire for the analysis is usually in a reserve or "hard" currency. In an emerging market country with a "soft" currency, the numeraire for the analysis is a "hard" reserve currency, e.g., US dollars. The distress barrier is related to the debt obligations in a "hard" currency. For developed economies with a "hard" currency the numeraire will be in the "hard" currency as well. If a developed country has debt denominated in a "soft" currency it is not considered senior debt.

On the sovereign balance sheet, sovereign local-currency debt can be issued in large amounts even it causes dilution in value. In this sense base money and local-currency debt are like "shares" on a sovereign balance sheet. Excessive issue of these "shares" can cause inflation and price changes similar to the case where excessive issue of corporate shares dilute existing holder's claims and reduce the price per share on a corporate balance sheet. Foreign-currency denominated debt is traded and primarily held internationally. An important aspect of foreign-currency debt is that sovereigns can run out of "hard" currency to repay foreign debt but sovereigns can, in principle, print money and need not run out of local currency to pay local-currency debt. Local-currency debt can, in many cases, be considered subordinated debt on the CCA sovereign balance sheet. Governments are more likely to restructure or dilute or inflate domestic local-currency debt before defaulting on foreign-currency debt.[32]

When the analysis is done for a developing country sovereign, the liabilities on the CCA balance sheet can be base money plus local-currency debt constituting a single liability. The exchange rate equals "one." Volatility comes not from the exchange rate but from the changing quantities of base money and local-currency debt and volatility in the value of the local-currency debt as interest rates change. Foreign-currency debt would be included with the local-currency debt on equal priority, unless special circumstances warrant.

Acknowledgments

The authors would like to thank Carlos Medeiros, Michael Gapen, Yingbin Xiao, Cheng Hoon Lim, Matt Jones, Sam Malone, and Joe Zou, for their useful discussions and comments on the ideas presented in the paper.

Notes

1 See Merton (1974, 1977, 1992, 1998).

2 See KMV (1999) for a summary of their methodology.

3 This section is based on Gray *et al.* (2002); Draghi *et al.* (2003); Gray (2002) in Gray (2003); Gapen *et al.* (2004).

4 The "actual" probability of default is sometimes called "true" probability of default.

5 See Merton (1992, pp. 334–343; 448–450).

6 See Merton (1977) and Merton and Bodie (1992).

7 The relevant probability distribution is the "risk-neutral" one and so the probabilities of default are the "risk-adjusted" ones and not the actual ones.

8 Buiter (1993) lays out the components on the combined government and central bank balance sheet. Allen *et al.* (2002) lay out the accounting balance sheets of various economic sectors (Gapen *et al.*, 2005).

9 The value of assets of an operating firm can be considered as the present value of stochastic future cash flow from income minus net new investment expenditures to create that income. For the public sector, the net fiscal asset is the present value of stochastic future fiscal flows from taxes and revenues minus non-discretionary expenditures.

10 Base money is also known as high-powered money or reserve money. As is the common practice, it is the main liability of the monetary authorities (IMF, 2000, Buiter, 1993, Blejer and Schumacher, 2000). Base money is "multiplied" by the banking system; the multipliers relate base money to M1, M2, etc. When a country joins a currency union (i.e., merges with another sovereign or dollarizes) base money is exchanged for foreign currency reserves.

11 This analytical combined balance sheet includes the monetary authority activities related to foreign currency reserves and "net domestic credit" to government but excludes the direct activities of the monetary authority with the banking sector, such as credit and liquidity support activities that do not go through the government balance sheet or affect foreign exchange reserves.

12 Although the strict theoretical condition in the Merton Model for default is that the value of assets is less than the required payments due on the debt, in the real world, default typically occurs at much higher asset values, either because of a material breach of a debt covenant or because assets cannot be sold to meet the payments ("inadequate liquidity") or because the sovereign decides to default and induce a debt renegotiation rather than sell assets. To capture these real-world conditions for default in the model, we specify a market value of total assets at which the sovereign will default. We call this level of assets that trigger default the "distress barrier." This barrier can be viewed as the present value of the promised payments discounted at the risk-free rate. The approach used in the KMV model sets the barrier level equal to the sum of the book value of short-term debt, promised interest payments for the next 12 months, and half of long-term debt (see Crouhy *et. al.* (2000), Saunders and Allen (2002) and KMV (1999, 2001)). In our numerical estimations of default here, we adopt the same measure for the distress barrier.

13 See Pettis (2001).

14 Support for viewing foreign-currency debt as senior can be found in the literature on "original sin" in Eichengreen *et al.* (2002). Support for modeling domestic currency liabilities as junior claims can be found in Sims (1999) who argues that local-currency debt has many similarities to equity issued by firms. He models domestic currency debt as "equity" and in this setting, domestic currency

debt becomes an important absorber of fiscal risk, just as equity is a cushion and risk absorber for firms. As long as there is some probability that the government will run a primary surplus in the future and/or will engage in the repurchase of domestic currency debt then such debt has value.

[15] An implied value refers to an estimate derived from other observed data. Techniques for using implied values are widely practiced in options pricing and financial engineering applications. See Bodie and Merton (1995).

[16] For a recently published book explaining these concepts applied to credit risk, see Crouhy *et al.* (2000).

[17] See Gray (2000, 2001).

[18] Xu and Ghezzi (2002) develop a stochastic debt sustainability model and show how it is related to the CCA model described in Gray *et al.* (2002) and this paper.

[19] See IMF GFSR (April 2006), Box 3.6 for sovereign CCA and impact of changes in debt structure.

[20] Also see Gray and Jones (2006) on sovereign and banking risk analysis applying contingent claims.

[21] See Gray *et al.* (forthcoming).

[22] This calculation used historical volatility, however comparisons across countries shows that implied volatility of the exchange rate derived from option prices produces more robust results.

[23] See Gray and Malone (forthcoming), for a detailed explanation of macro financial risk applications for investors, for policy makers, and the integration of CCA balance sheets with sophisticated macroeconomic models.

[24] See Gray and Malone (forthcoming).

[25] See Gray (2007).

[26] The MFRisk™models have been applied to numerous countries and use simulated shocks and policy adjustments to assess impact on risk indicators.

[27] One example, in Blejer and Schumacher (2000), includes the central bank balance sheet using forward contracts in currencies.

[28] See Merton (1999, 2002, 2005), Bodie and Merton (2002) and Draghi *et al.* (2003).

[29] See Gapen *et al.* (2005).

[30] See Buiter (1993).

[31] See Schaechter (2001).

[32] See Gray and Malone (forthcoming).

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