A: Additional Empirical Analysis

1. Present value calculation

Ship earnings are persistent at short horizons, but the correlation between earnings in year $t$ and year $t+k$ disappears for $k$ greater than or equal to 2 years. To construct a simple estimate of the present value of earnings, we assume that the ship owner enters a 1-year time charter. We forecast the gross earnings from a time charter 1 year later by regressing future gross earnings on current gross earnings: $\Pi_{t+1}^{g} = a + b \cdot \Pi_{t}^{g} + u_{t+1}$. Starting in 2 years, we assume that gross earnings equal the sample average of $5.4$ million. After the ship’s 15th year, we assume the owner receives 85% of the sample average earnings for the remainder of the ship’s life. Finally, when the ship reaches 25 years of age, we assume it is scrapped and the owner receives scrap value. This yields the following present value calculation:

$$PV_t = \frac{\Pi_t^{g}}{1+r} + \frac{\hat{\Pi}_{t+1}^{g}}{(1+r)^2} + \frac{\Pi_t^{g} \cdot (1-1/(1+r)^8)}{r \cdot (1+r)^2} + 0.85 \frac{\Pi_t^{g} \cdot (1-1/(1+r)^{10})}{r \cdot (1+r)^{10}} + \frac{\hat{\text{Scrap}}_{t+20}}{(1+r)^{20}},$$

(A1)

where $r$ denotes the discount rate.

2. Alternate measures of ship returns

a. Comparison of our return series with Stopford (2009)

Figure A1 compares an annual version of our return series (sampled in December) with the annual “return on shipping investments” series computed by Stopford (2009): the correlation between the two is 0.92.

---

1 In recent years, Clarkson provides an estimate of the scrap value. Scrap value is highly correlated with nominal steel prices, and thus when we are missing data on scrap value we estimate it from a projection of scrap value on steel prices.
b. Earnings and prices on Handymax and Capesize ships

Table A1 shows that we obtain similar results using estimates of earnings, prices, and returns on 32,000 DWT Handymax ships, 150,000 DWT Capesize ships, or the entire bulker fleet. Specifically, Table A1 shows that the key variables used in the paper (Panamax earnings, Panamax prices, and fleet-wide investment) also forecast the returns on Handymax ships, Capesize ships, and the entire bulker fleet. We also show that earnings and price variables based on Handymax ships, Capesize ships, and the overall fleet each forecast shipping returns. For instance, both Panamax prices (\(P_{\text{Pana}}\)) and Capesize prices (\(P_{\text{Cape}}\)) forecast Capesize returns. This is natural given the homogeneity of shipping capital: the average pairwise correlation between the four earnings series is 0.84, that between the four prices series is 0.96, and that between the four return series is 0.87.

Overall, Table A1 suggests that the return predictability we find on Panamax ships is representative of the overall bulk carrier fleet. However, returns on larger Capesize ships are somewhat more predictable that the rest of the fleet, while those on smaller Handmay ships are somewhat less predictable.

3. Time-series robustness

a. Sub-sample results

Table A2 below shows subsample results for our return forecasting regressions using \(X = \Pi, P,\) and \(\text{Inv}.\) We show full sample results, results dropping the 2006-2010 “super-cycle”, and results dividing the sample into two equal halves.
Table A1: Alternate Measure of Bulk Carrier Prices, Earnings, and Returns

Our construction of 32,000 DWT Handymax and 150,000 DWT Capesize earnings and prices follows our procedure for computing the analogous Panaxmax series. However, we assume that the daily operating costs for Handymax ships are $5,000 in constant 2011 dollars and those for Capesize ships are $8,000. To obtain an estimate of fleet-wide earnings and prices, we aggregate the Handymax, Panamax, and Capesize series using data on the composition of the fleet shown in Figure 1. We use earnings data for 30,000 DWT Handymax ships and price data for 32,000 Handymax ships. We use earnings data for 127,500 DWT Capesize ships and price data for 150,000 Capesize ships.

<table>
<thead>
<tr>
<th>Panel A: Forecasting Panamax returns using Panamax earnings and prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{xt+2} ) on Panamax</td>
</tr>
<tr>
<td>( X = \Pi_{\text{Pana}} ) \quad ( X = P_{\text{Pana}} ) \quad ( X = \text{Inv} )</td>
</tr>
<tr>
<td>( B )</td>
</tr>
<tr>
<td>( [t] )</td>
</tr>
<tr>
<td>( T )</td>
</tr>
<tr>
<td>( R^2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Forecasting alternate returns using Panamax earnings and prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{xt+2} ) on Handymax</td>
</tr>
<tr>
<td>( X = \Pi_{\text{Pana}} ) \quad ( X = P_{\text{Pana}} ) \quad ( X = \text{Inv} )</td>
</tr>
<tr>
<td>( X = \Pi_{\text{Hand}} ) \quad ( X = P_{\text{Hand}} ) \quad ( X = \text{Inv} )</td>
</tr>
<tr>
<td>( X = \Pi_{\text{Cape}} ) \quad ( X = P_{\text{Cape}} ) \quad ( X = \text{Inv} )</td>
</tr>
<tr>
<td>( X = \Pi_{\text{Fleet}} ) \quad ( X = P_{\text{Fleet}} ) \quad ( X = \text{Inv} )</td>
</tr>
<tr>
<td>( B )</td>
</tr>
<tr>
<td>( T )</td>
</tr>
<tr>
<td>( R^2 )</td>
</tr>
</tbody>
</table>

Dropping the super-cycle has only a moderate impact on the results. (This has the largest impact on the earnings regressions where the statistical significance is somewhat weaker even though the coefficient magnitudes are slightly larger). Turning to the first- versus second-half splits, we see that the results are typically significant in both the early and the late sub-sample and that the coefficients are fairly similar in magnitude. The only exception here is net investment which was a weaker signal of future returns in the latter half of the sample.

To explore the issue of sub-sample stability further, we report the \( t \)-statistics for the null hypothesis that the coefficients are the same in the first and second half of the sample, labeled as “\( t \).
stable” below. These tests suggest that the magnitude of return predictability has declined modestly since the first half of our sample. Why are the results for prices and earnings a bit stronger in the first half than in the second half? Prices and earnings were already high compared to the long-run average in 2005–2006, but then the industry received a second positive demand shock. This suggests that even though expected returns may have been low in 2006, realized returns in that year were high. Of course, realized returns ultimately more than caught up in 2008 and early 2009. In this sense, the regression was ultimately “correct”, but slightly off in the precise timing of the collapse of the bubble. Relatedly, we suspect that $\text{Inv}_t$ may be weaker predictor in the latter half of our sample, in part, because the congestion-induced increase in time-to-build delays during the “super-cycle” (Kalouptsidi (2014)) made it a noisier proxy for firms’ current investment plans.

### Table A2: Sub-sample Forecasting Results

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Forecasting 12-month returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>-0.020</td>
<td>-0.007</td>
<td>-2.941</td>
<td>-0.009</td>
<td>-0.008</td>
<td>-4.489</td>
<td>-0.019</td>
<td>-0.010</td>
<td>-5.901</td>
<td>-0.020</td>
<td>-0.006</td>
<td>-0.709</td>
</tr>
<tr>
<td>$T$</td>
<td>420</td>
<td>420</td>
<td>420</td>
<td>360</td>
<td>360</td>
<td>360</td>
<td>210</td>
<td>210</td>
<td>210</td>
<td>210</td>
<td>210</td>
<td>210</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.07</td>
<td>0.11</td>
<td>0.14</td>
<td>0.01</td>
<td>0.10</td>
<td>0.25</td>
<td>0.03</td>
<td>0.13</td>
<td>0.46</td>
<td>0.11</td>
<td>0.12</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Forecasting 24-month returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>-0.039</td>
<td>-0.012</td>
<td>-4.994</td>
<td>-0.048</td>
<td>-0.018</td>
<td>-5.828</td>
<td>-0.079</td>
<td>-0.024</td>
<td>-7.704</td>
<td>-0.027</td>
<td>-0.008</td>
<td>-0.971</td>
</tr>
<tr>
<td>$T$</td>
<td>408</td>
<td>408</td>
<td>408</td>
<td>360</td>
<td>360</td>
<td>360</td>
<td>204</td>
<td>204</td>
<td>204</td>
<td>204</td>
<td>204</td>
<td>204</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.13</td>
<td>0.18</td>
<td>0.14</td>
<td>0.09</td>
<td>0.19</td>
<td>0.18</td>
<td>0.21</td>
<td>0.34</td>
<td>0.34</td>
<td>0.12</td>
<td>0.13</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Next Figure A2 shows the distribution of return forecasting coefficients using $X = \Pi, P, \text{Inv}$ across all 20-year periods in our 35 year sample. Specifically, we show the coefficients for each 20-year period ending from 1995 to 2010. The key forecasting coefficients are negative in almost all of these 20-year subsamples and are often significantly negative even in these short samples.

Overall, these subsample results suggest that our findings are not solely due to a handful of outlying data points. Instead, return predictability appears to be a fairly robust feature of the data.

---

2 This $t$-statistic is obtained by estimated a full sample regression with a constant, a dummy for the second half of the sample, the covariate of interest, and the covariate interacted with the second half dummy. We report the $t$-statistic on this final interaction term.
Figure A2: Forecasting Coefficients for 20-year Samples Ending from 1995–2010

Forecasting 12-month returns

\[ X = \Pi \]

Forecasting 24-month returns

\[ X = \Pi \]

\[ X = P \]

\[ X = Inv \]
b. Overlapping returns and standard errors

Because we use monthly data and forecast 12-, 24-, and 36-month cumulative returns, it is important to account for the overlapping nature of the return observations. The regressions that forecast 12-month returns use a Newey-West window length of 18 months; the 24-month return regressions use a window of 36 months; the 36-month return regressions use a window of 54 months. This is in keeping with the rule of thumb that the window length should be $1.5 \times$ the forecasting horizon when working with overlapping monthly returns because the Newey-West estimator downweights higher order autocovariances (e.g., Cochrane and Piazzesi (2004)).

Table A3 provides comfort that our inferences here are robust. The first two rows show the results from the paper. The second set of rows shows what happens if we increase the number of Newey-West lags in our monthly forecasting regressions: here we use 30 lags for the 12-month regressions, 48 lags for the 24-month regressions, and 66 lags for the 36-month regressions. Adding more lags generally raises the $t$-statistics and suggests that we are being conservative. This is because adding more lags than is necessary when computing Newey-West standard errors often leads to larger $t$-statistics.\(^3\)

\(^3\) If the residuals are serially correlated, the probability of making a Type 1 error first falls as one increases the number of lags, before eventually rising as one adds further lags. Adding more lags reduces the bias of the variance estimator, but makes the estimator more volatile. Minimizing the probability of a Type I error roughly corresponds with minimizing the mean square error of our variance estimator—i.e., to balancing these two forces. See, for instance, Kiefer and Vogelsang (2005) or Newey and West (1994).
The remaining rows show the results if we sample the data annually. We show annual sampling in both June and December. Here we use robust standard errors when forecasting 1-year returns (there is no problem of overlapping returns here), a Newey-West window length of 2-years when forecasting 2-year returns, and a Newey-West window length of 3-years when forecasting 3-year returns. We also show Hansen-Hodrick standard errors that account for the 1 year of overlap when forecasting 2-year returns (2 years of overlap when forecasting 3-year returns). The overall take-away is that both our estimates and reported \( t \)-statistics appear to be robust.

c. Stambaugh bias

As shown by Stambaugh (1999), if we estimate

\[
\hat{r}_{t+1} = \alpha + \beta \cdot x_t + u_{t+1}
\]

using a time series of \( T \) observations where \( x_{t+1} = \theta + \rho \cdot x_t + v_{t+1} \), then the expected finite-sample bias is

\[
E[\hat{\beta} - \beta] = \frac{\sigma_{uv}}{\sigma_v^2} E[\hat{\rho} - \rho] = -\frac{\sigma_{uv}}{\sigma_v^2} \frac{1 + 3 \rho}{T} + O(1/T^2) \quad (A4)
\]

Thus, if \( \sigma_{uv}/\sigma_v^2 > 0 \)—as one would expect when forecasting returns with earnings, prices, or investment—Stambaugh’s effect would lead to a downward bias in finite samples. We use Amihud and Hurvich’s (2004) bias-adjusted estimator to see whether our results are being driven by Stambaugh bias. Table A4 shows raw (uncorrected) and bias-corrected estimates using annual data sampled in December. We present raw OLS estimates along with bias-corrected estimated. Note that \( Bias[\beta] = Bias[\rho] \times \sigma_{uv}/\sigma_v^2 \).

<table>
<thead>
<tr>
<th></th>
<th>( X = \text{Real Earnings} )</th>
<th>( X = \text{Used Ship Price} )</th>
<th>( X = \text{Net Investment} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1-yr 2-yr</td>
<td>1-yr 2-yr</td>
<td>1-yr 2-yr</td>
</tr>
<tr>
<td>Corrected ( \beta )</td>
<td>-0.025 -0.041</td>
<td>-0.007 -0.012</td>
<td>-2.700 -4.203</td>
</tr>
<tr>
<td>Corrected ( t )</td>
<td>[-1.76] [-2.32]</td>
<td>[-2.01] [-2.58]</td>
<td>[-2.02] [-1.99]</td>
</tr>
<tr>
<td>Raw ( \beta )</td>
<td>-0.027 -0.042</td>
<td>-0.009 -0.013</td>
<td>-2.801 -4.058</td>
</tr>
<tr>
<td>Raw ( t )</td>
<td>[-2.07] [-2.99]</td>
<td>[-2.68] [-3.00]</td>
<td>[-2.78] [-1.94]</td>
</tr>
<tr>
<td>Bias[( \beta )]</td>
<td>-0.002 -0.001</td>
<td>-0.001 -0.001</td>
<td>-0.101 0.146</td>
</tr>
<tr>
<td>Bias[( \rho )]</td>
<td>-0.044 -0.044</td>
<td>-0.067 -0.069</td>
<td>-0.078 -0.109</td>
</tr>
<tr>
<td>( \sigma_{uv}/\sigma_v^2 )</td>
<td>0.056 0.027</td>
<td>0.018 0.01</td>
<td>1.291 -1.332</td>
</tr>
<tr>
<td>Observations</td>
<td>35 34</td>
<td>35 34</td>
<td>35 34</td>
</tr>
</tbody>
</table>

Using this bias-adjusted estimator instead of the standard OLS estimator has almost no impact on the magnitude or significance of our estimated coefficients. Stambaugh is minimal.
because our forecasting variables are either not very persistent (in which case $\text{Bias}[\rho]$ is small), or when they are more persistent, innovations to our forecasting variables are not sufficiently correlated with returns (in which case $\sigma_{uv}/\sigma^2_v$ is small). In fact, most of the regressors have a high degree of mean reversion, especially when compared with the variables researchers have use to forecast aggregate stock returns.

\textit{d. Controlling for a time trends}

Finally, in Table A5 we include a time trend as a control in the regressions, estimating specifications of the form

$$rx_{t+k} = a + b \cdot X_t + c \cdot t + u_{t+k}. \quad (A5)$$

There is no good theoretical justification for including a time trend, but we do it to check that our results are not driven by some omitted slow-moving variable. Including a trend has little impact on the results except for the two order book variables (NetContracting and Orders) where the forecasting results are stronger when we control for the secular growth of the order book over time.
Table A5: Controlling for a Time Trend

Table A5 reports time-series forecasting regressions of the form

\[ r_{x,t+k} = a + b \cdot X_t + c \cdot t + u_{t+k}, \]

where \( r_{x,t+k} \) denotes the \( k \)-year log holding period excess return on a Panamax dry bulk ship. In Panel A, \( X \) alternately denotes real earnings \( \Pi \), the current real price of a 5-year used ship \( P \), or the earnings yield \( \Pi/P \). In Panel B, \( X \) alternately denotes net contracting activity over the past 12 months, the size of the order book, deliveries over the following 12 months, or demolitions over the past 12 months, each scaled by the current fleet size. In the rightmost set of columns, we forecast returns using net investment, \( \text{Inv}_t = \text{Deliveries}_{t+1} - \text{Demolitions}_t \). \( t \)-statistics are based on Newey-West (1987) standard errors allowing for serial correlation at up to \( 1.5 \times 12 \times k \) monthly lags—i.e., we allow for serial correlation at up 18, 36, and 54 month lags, respectively, when forecasting 1-, 2-, and 3-year returns.

Panel A: Forecasting Ship Returns Using Ship Earnings and Prices

<table>
<thead>
<tr>
<th></th>
<th>( X = \text{Real Earnings} \Pi )</th>
<th>( X = \text{Used Ship Price} \ P )</th>
<th>( X = \text{Earnings Yield} \Pi/P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>1-yr</td>
<td>2-yr</td>
<td>3-yr</td>
</tr>
<tr>
<td>( b )</td>
<td>-0.020</td>
<td>-0.040</td>
<td>-0.051</td>
</tr>
<tr>
<td>( c )</td>
<td>0.015</td>
<td>0.033</td>
<td>0.048</td>
</tr>
<tr>
<td>( [t] )</td>
<td>[0.32]</td>
<td>[0.42]</td>
<td>[0.46]</td>
</tr>
<tr>
<td>( T )</td>
<td>420</td>
<td>408</td>
<td>396</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.07</td>
<td>0.14</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Panel B: Forecasting Ship Returns Using Industry Investment

<table>
<thead>
<tr>
<th></th>
<th>( X = \text{NetContracting}_t )</th>
<th>( X = \text{Orders}_t )</th>
<th>( X = \text{Deliveries}_{t+1} )</th>
<th>( X = \text{Demolitions}_t )</th>
<th>( X = \text{Inv}_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>1-yr</td>
<td>2-yr</td>
<td>3-yr</td>
<td>1-yr</td>
<td>2-yr</td>
</tr>
<tr>
<td>( c )</td>
<td>0.236</td>
<td>0.543</td>
<td>1.093</td>
<td>0.527</td>
<td>0.977</td>
</tr>
<tr>
<td>( [t] )</td>
<td>[1.24]</td>
<td>[1.50]</td>
<td>[2.64]</td>
<td>[2.77]</td>
<td>[5.41]</td>
</tr>
<tr>
<td>( T )</td>
<td>169</td>
<td>157</td>
<td>145</td>
<td>180</td>
<td>168</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.19</td>
<td>0.31</td>
<td>0.59</td>
<td>0.30</td>
<td>0.51</td>
</tr>
</tbody>
</table>
B: Model Solution and Omitted Proofs

1. The optimization problem of the representative firm

We begin by deriving equation (13) in the main text. Each firm chooses current net investment to maximize the expected net present value of earnings. Each firm’s Bellman equation is

\[
J(q_t, A_t, Q_t) = \max_{i_t} \left\{ V(q_t, i_t, A_t, Q_t) + \frac{E_f \left[ J(q_t + i_t, A_{t+1}, Q_{t+1}) \right]}{1 + r} \right\}
\]

\[
= \sum_{j=0}^{\infty} \frac{E_f \left[ V(q_{t+j}, i_{t+j}, A_{t+j}, Q_{t+j}) \right]}{(1 + r)^j} \left| A_t, Q_t \right|
\]

(B1)

where \( r > 0 \) is the constant discount rate or required return used by firms. The first order condition for firm net investment is

\[
0 = -P_r - ki_t^* + \frac{1}{1 + r} E_f \left[ \frac{\partial J(q_t + i_t^*, A_{t+1}, Q_{t+1})}{\partial q_{t+1}} \right] \left| A_t, Q_t \right|
\]

(B2)

The Envelope Theorem implies that\(^4\)

\[
\frac{\partial J(q_t, A_t, Q_t)}{\partial q_t} = \Pi_t + \frac{1}{1 + r} E_f \left[ \frac{\partial J(q_t + i_t^*, A_{t+1}, Q_{t+1})}{\partial q_{t+1}} \right] \left| A_t, Q_t \right|
\]

(B3)

Assuming the standard “no bubbles” condition holds,\(^5\) we can iterate (B3) forward to obtain

\[
\frac{1}{1 + r} E_f \left[ \frac{\partial J(q_t + i_t^*, A_{t+1}, Q_{t+1})}{\partial q_{t+1}} \right] \left| A_t, Q_t \right| = \sum_{j=0}^{\infty} \frac{E_f \left[ \Pi_{t+j} \right] \left| A_t, Q_t \right|}{(1 + r)^j}
\]

(B4)

This shows that firm net investment is given by the familiar \( q \)-theory type investment equation

\[
i_t^* = \frac{P(A_t, Q_t) - P_r}{k},
\]

(B5)

\(^4\) To see this, suppose that \( i_t^* \) is the optimal policy action so that

\[
J(q_t, A_t, Q_t) = V(q_t, i_t^*, A_t, Q_t) + (1 + r)^{-1} E_f \left[ J(q_t + i_t^*, A_{t+1}, Q_{t+1}) \right] \left| A_t, Q_t \right|
\]

We then have

\[
\frac{\partial J(q_t, A_t, Q_t)}{\partial q_t} = \Pi_t + (1 + r)^{-1} E_f \left[ \frac{\partial J(q_t + i_t^*, A_{t+1}, Q_{t+1})}{\partial q_{t+1}} \right] \left| A_t, Q_t \right|
\]

\[
+ \left( -P_r - ki_t^* + (1 + r)^{-1} E_f \left[ \frac{\partial J(q_t + i_t^*, A_{t+1}, Q_{t+1})}{\partial q_{t+1}} \right] \left| A_t, Q_t \right| \right) \left( \frac{\partial i_t^*}{\partial q_t} \right).
\]

\(^5\) The “no bubbles” or “transversality” condition is \( \lim_{j \to \infty} \left( \frac{1}{1 + r} \right)^j E_f \left[ \Pi_{t+j} \right] | A_t, Q_t] = 0 \).
where \( P_r \) is the replacement cost of a ship and
\[
P(A_t, Q_t) = \frac{E_f \left[ \Pi_{t+1} + P(A_{t+1}, Q_{t+1}) \right]}{1 + r} = \sum_{j=1}^{\infty} \frac{E_f \left[ \Pi_{t+j} \right]}{(1 + r)^j} = \sum_{j=1}^{\infty} \frac{E_f \left[ A_{t+j} - BQ_{t+j} - C - \delta P_r \right]}{(1 + r)^j},
\]
is the market price of a ship.

The Bellman operator for this problem satisfies Blackwell’s Sufficient Conditions and is a Contraction Mapping. Therefore, the Contraction Mapping Theorem implies that there is a unique solution to the Bellman Equation. Thus, if we can guess and verify a solution to the Bellman equation, then this must be the unique solution. Specifically, using equations (B5) and (B6) it is easy to check that the following function solves the Bellman equation in (B1):
\[
J(q_t, A_t, Q_t) = q_t \left( \Pi_t + P(A_t, Q_t) \right) + \frac{1}{2k} \sum_{j=0}^{\infty} \frac{E_f \left[ \left( P(A_{t+j}, Q_{t+j}) - P_r \right)^2 \right]}{(1 + r)^j}.
\]
As in standard in \( q \)-theory, the value of a firm is the stock of current ships valued at current prices (assuming the firm does not adjust its capital stock) plus a term that reflects the value derived from dynamically adjusting its capital stock.

2. Model equilibrium

**Proposition 1 (Equilibrium investment and prices):** There exists a unique equilibrium such that the net investment of the representative firm is \( i^*_t = x^*_t + y^*_t A_t + z^*_t Q_t \) and equilibrium ship prices are \( P^*_t = P_r + kx^*_t + ky^*_t A_t + kz^*_t Q_t \). The two slope coefficients (i.e., \( y^*_t \) and \( z^*_t \)) are a function of five exogenous parameters: \( k, r, \rho, \theta, \) and \( B \). In addition to these five parameters, the intercept term (i.e., \( x^*_t \)) also depends on \( A, C, \delta, \) and \( P_r \).

Investment and ship prices are decreasing in the current fleet size (\( z^*_t < 0 \)). Furthermore, (i) investment and prices react more aggressively to the current fleet size when firms underestimate the competition (i.e., \( \partial z^*_t / \partial \theta > 0 \)); (ii) firms’ response to the current fleet size independent of the
perceived persistence of demand (i.e., $\partial z_i^* / \partial \rho_f = 0$); (iii) investment and prices react more aggressively to the current fleet size when the demand curve is more inelastic (i.e., $\partial z_i^* / \partial B < 0$); (iv) investment and prices react less aggressively to current fleet size when required returns are higher (i.e., $\partial z_i^* / \partial r > 0$); and (v) when adjustment costs are higher, investment reacts less aggressively to current fleet size, but prices react more aggressively (i.e., $\partial z_i^* / \partial k > 0$ and $\partial(z_i^* k) / \partial k = z_i^* + k \cdot (\partial z_i^* / \partial k) < 0$).

Investment and ship prices are increasing in current demand ($y_i^* > 0$). Furthermore, (i) investment and prices react more aggressively to current demand when firms underestimate the competition (i.e., $\partial y_i^* / \partial \theta < 0$); (ii) investment and prices react more aggressively to current demand when demand is more perceived to be persistent (i.e., $\partial y_i^* / \partial \rho_f > 0$); (iii) investment and prices react less aggressively to current demand when the demand curve is more inelastic (i.e., $\partial y_i^* / \partial B < 0$); (iv) investment and prices react less aggressively to current demand when required returns are higher (i.e., $\partial y_i^* / \partial r < 0$); and (v) when adjustment costs are higher, investment reacts less aggressively to current demand, but prices react more aggressively ($\partial y_i^* / \partial k < 0$ and $\partial(y_i^* k) / \partial k = y_i^* + k \cdot (\partial y_i^* / \partial k) > 0$).

**Proof:** We first solve for the equilibrium coefficients. We then prove the comparative statics discussed above and then characterize the system dynamics.

**Solve for the equilibrium coefficients:** We conjecture that

$$i_t = x_t + y_t A_t + z_t Q_t$$

$$P_t(A_t, Q_t) = P_r + kx_t + ky_t A_t + k z_t Q_t$$

(B8)

The equilibrium must satisfy

$$P_r + kx_t + ky_t A_t + k z_t Q_t$$

$$= \frac{E_t \left[ (A_{t+1} - B Q_{t+1} - C - \delta P_r) + (P_r + kx_t + ky_t A_t + k z_t Q_t) \right] A_t, Q_t}{1 + r}$$

$$= \frac{((1 - \rho_f) A + \rho_f A_t - B (Q + \theta(x_t + y_t A_t + z_t Q_t)) - C - \delta P_r)}{(1 - \rho_f) A + \rho_f A_t - B (Q + \theta(x_t + y_t A_t + z_t Q_t)) + k z_t (Q + \theta(x_t + y_t A_t + z_t Q_t))}$$

(B9)
Both the left-hand and right-hand sides are linear in $A_t$ and $Q_t$. Thus, by matching coefficients in (B9), we can then solve for the fixed-point values of $x_i$, $y_i$, and $z_i$.

Specifically, matching coefficients on $Q_t$ shows that the equilibrium value of $z_i$ satisfies

$$0 = f(z_i^*)$$

where

$$f(z) = -k\theta \cdot z^2 + (kr + B\theta) \cdot z + B.$$  \hspace{1cm} (B10)

We want the negative root of the quadratic in (B10), which is

$$z_i^* = \frac{kr + B\theta}{2k\theta} - \sqrt{\left(\frac{kr + B\theta}{2k\theta}\right)^2 + \frac{B}{k\theta}},$$  \hspace{1cm} (B11)

when $\theta > 0$ and $z_i^* = -B/(kr)$ when $\theta = 0$. Given this solution for $z_i^*$, matching coefficients on $A_t$ and the constant shows that the equilibrium values of $y_i^*$ and $x_i$ are given by

$$y_i^* = \frac{\rho_i}{k(1 - \rho_i) + kr + \theta(B - kz_i^*)} = \frac{\rho_i}{k(1 - \rho_i) - B / z_i^*} > 0,$$  \hspace{1cm} (B12)

and

$$x_i^* = \frac{\overline{A}(1 - \rho_i) + \overline{A}(\rho_i + y_i^*/B / z_i^*) - C - (r + \delta)P_r}{-B / z_i^*} = -y_i^*\overline{A} - z_i^*Q^*.$$  \hspace{1cm} (B13)

**Uniqueness of the equilibrium:** We now show that the linear equilibrium described in Proposition 1 is the unique stationary equilibrium of the model. To show this is the case, we conjecture an alternate stationary equilibrium of the form

$$i_t = x_t + y_tA_t + z_tQ_t + g(A_t,Q_t)$$

$$P_i(A_t,Q_t) = P_r + kx_t + ky_tA_t + kz_tQ_t + kg(A_t,Q_t),$$  \hspace{1cm} (B14)

where $g(A_t,Q_t)$ is an arbitrary function of the state variables. Proceeding as above, we again obtain conditions (B11), (B12), and (B13) as well as a functional equation characterizing $g(A_t,Q_t)$:

$$g(A_t,Q_t) = \frac{1}{1 - B / (kz_i^*)} E_t[g(A_{t+1},Q_{t+1}) | A_t,Q_t].$$  \hspace{1cm} (B15)

Iterating on (B15) and making use of the law of iterated expectations, we have

$$g(A_t,Q_t) = \lim_{k \to \infty} \left(\frac{1}{1 - B / (kz_i^*)}\right)^k \cdot \lim_{k \to \infty} E_t[g(A_{t+k},Q_{t+k}) | A_t,Q_t].$$  \hspace{1cm} (B16)
Since the conjectured equilibrium is stationary, we have \( \lim_{t \to \infty} E_t[g(A_{i+k}, Q_{i+k}) | A_i, Q_i] = c \) for some constant \( c \) irrespective of the initial values of \( A_t \) and \( Q_t \). Thus, since \( 1 - B / (kz_i) > 1 \) equation (B16) shows that we must have \( g(A_t, Q_t) = 0 \) in any stationary equilibrium.

**Comparative statics for \( z_i^* \):** Before proceeding we first show that

\[
\theta z_i^* + 1 > 0. \tag{B17}
\]

Using equation (B11), one can show that equation (B17) is equivalent to

\[
\frac{k(1+r)+B\theta}{2k\theta} + \frac{1}{2\theta} > \sqrt{\left(\frac{k(1+r)+B\theta}{2k\theta} - \frac{1}{2\theta}\right)^2 + \frac{B}{k\theta}}.
\]

This holds since \( 4XY > Z \iff X+Y > \sqrt{(X-Y)^2+Z} \) and using \( X = (k(1+r)+B\theta) / (2k\theta) \), \( Y = 1/(2\theta) \), and \( Z = B / (k\theta) \), we have

\[
\frac{1+r}{\theta^2} + \frac{B}{k\theta} = 4XY > Z = \frac{B}{k\theta}.
\]

With (B17) in hand, we now proceed to the comparative statics for \( z_i^* \). Since \( 0 = a \cdot z_i^* + b \cdot z_i^* + c \) where \( a = -k\theta < 0 \), \( b = kr + B\theta > 0 \), and \( c = B \). Thus, by the Implicit Function theorem, we have

\[
\frac{\partial z_i^*}{\partial \phi} = z_i^* \cdot \frac{\partial a}{\partial \phi} + z_i^* \cdot \frac{\partial b}{\partial \phi} + \frac{\partial c}{\partial \phi} = -\left( z_i^* \cdot \frac{\partial a}{\partial \phi} + z_i^* \cdot \frac{\partial b}{\partial \phi} + \frac{\partial c}{\partial \phi} \right), \tag{B18}
\]

for any primitive model parameter. Thus, we have:

- \( \partial z_i^* / \partial \theta \propto -\left( - z_i^* \cdot k + z_i^* \cdot B \right) > 0 \);
- \( \partial z_i^* / \partial \rho_j = 0 \);
- \( \partial z_i^* / \partial B = -\left( \theta z_i^* + 1 \right) < 0 \);
- \( \partial z_i^* / \partial r \propto -kz_i^* > 0 \);
- \( \partial z_i^* / \partial k = -\left( - z_i^* \cdot \theta + z_i^* \cdot r \right) > 0 \);
- \( \partial (kz_i^*) / \partial k = z_i^* \cdot k \cdot (\partial z_i^* / \partial k) = \frac{\theta z_i^* (B - kz_i^*)}{-2k\theta z_i^* + kr + B\theta} < 0 \);
Comparative statics for $y^*_i$: Recall that 

$$y^*_i = \frac{\rho_f}{k(1-\rho_f) + kr + \theta(B - k z^*_i)} = \frac{\rho_f}{k(1-\rho_f) - B / z^*_i}.$$ 

Thus, we have

- $\partial y^*_i / \partial \theta < 0$: Obvious since $\partial z^*_i / \partial \theta > 0$;
- $\partial y^*_i / \partial \rho_r > 0$;
- $\partial y^*_i / \partial B < 0$: Obvious since $\partial z^*_i / \partial B < 0$;
- $\partial y^*_i / \partial r < 0$: Obvious since $\partial z^*_i / \partial r > 0$;
- $\partial y^*_i / \partial k < 0$: Obvious since $\partial(kz^*_i) / \partial k < 0$;
- $\partial(ky^*_i) / \partial k = y^*_i + k \cdot (\partial y^*_i / \partial k) > 0$: Note that $ky^*_i = \frac{\rho_f}{(1-\rho_f) - B / (kz^*_i)}$. Since $\partial(kz^*_i) / \partial k < 0$, the denominator is decreasing in $k$, which shows that $\partial(ky^*_i) / \partial k > 0$.

3. System dynamics and steady-state distribution

The true system dynamics perceived by the econometrician can be summarized using a vector auto-regression:

$$\begin{bmatrix} A_{t+1} \\ Q_{t+1} \end{bmatrix} = \begin{bmatrix} (1-\rho_0)\bar{A} \\ x^*_i \end{bmatrix} + \begin{bmatrix} \rho_0 & 0 \\ y^*_i & 1 + z^*_i \end{bmatrix} \begin{bmatrix} A_t \\ Q_t \end{bmatrix} + \begin{bmatrix} \epsilon_{t+1} \\ 0 \end{bmatrix}. \quad (B19)$$

Therefore, the true steady state is given by

$$\begin{bmatrix} A' \\ Q' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \rho_0 & 0 \\ y^*_i & 1 + z^*_i \end{bmatrix} \begin{bmatrix} \bar{A} \\ x^*_i + y^*_i \bar{A} \\ z^*_i \end{bmatrix} = \begin{bmatrix} \bar{A} \\ \frac{\bar{A} - (r+\delta)P} {B} \end{bmatrix}, \quad (B20)$$

assuming that $\rho_0 \neq 1$ so the matrix is invertible.

The system dynamics are governed by sign and magnitude of the second eigenvalue of the $2 \times 2$ matrix in (B19). Specifically, the two eigenvalues are $\lambda_1 = \rho_0$ and $\lambda_2 = 1 + z^*_i$. Specifically, the dynamics are oscillatory if $\lambda_2 < 0$ and non-oscillatory if $\lambda_2 > 0$; and the system has convergent dynamics, which return it to the steady state if $|\lambda_2| < 1$ and divergent dynamics if $|\lambda_2| > 1$.

Above we showed that $0 < 1 + \theta z^*_i < 1$. Thus, in the rational expectation case ($\theta = 1$) we always have non-oscillatory and convergent dynamics. When $0 \leq \theta < 1$, we can have either non-
oscillatory dynamics about the steady-state if $1 + z_i^* > 0$ or oscillatory dynamics if $1 + z_i^* < 0$. When $0 \leq \theta < 1$, we can have $1 + z_i^* < -1$, corresponding to divergent, oscillatory dynamics if (i) $\theta$ is sufficient close to 0 and (ii) $B$ is sufficiently large or $k$ is sufficiently small. Obviously, in our model simulations and estimation we focus on the empirically relevant case with convergent dynamics.

We can rewrite equation (B19) as

$$
\begin{bmatrix}
A_{t+1} - \bar{A} \\
Q_{t+1} - \bar{Q}'
\end{bmatrix} =
\begin{bmatrix}
\rho_0 & 0 \\
y_i^* & 1 + z_i^*
\end{bmatrix}
\begin{bmatrix}
A_t - \bar{A} \\
Q_t - \bar{Q}'
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_{t+1}
\end{bmatrix}.
$$

Taking variances of both sides of (B21), the variance of the system about the steady-state satisfies

$$
\begin{bmatrix}
\sigma^2_{A,0} & \sigma_{AQ,0} \\
\sigma_{AQ,0} & \sigma^2_{Q,0}
\end{bmatrix} =
\begin{bmatrix}
\rho_0^2 & 0 \\
y_i^* & 1 + z_i^*
\end{bmatrix}
\begin{bmatrix}
\sigma^2_{A,0} & \sigma_{AQ,0} \\
\sigma_{AQ,0} & \sigma^2_{Q,0}
\end{bmatrix}
\begin{bmatrix}
\rho_0 & y_i^* \\
0 & 1 + z_i^*
\end{bmatrix} +
\begin{bmatrix}
\sigma^2_{\varepsilon}
\end{bmatrix}.
$$

Solving (B22) for the 3-unknown parameters, we obtain

$$
\sigma^2_{A,0} = \rho_0^2 \sigma^2_{A,0} + \sigma^2_{\varepsilon} \Rightarrow \sigma^2_{A,0} = \frac{\sigma^2_{\varepsilon}}{1 - \rho_0^2} > 0,
$$

$$
\sigma_{AQ,0} = \rho_0 (y_i^* \sigma^2_{A,0} + (1 + z_i^*) \sigma_{AQ,0}) \Rightarrow \sigma_{AQ,0} = \frac{\rho_0 y_i^* \sigma^2_{A,0}}{1 - \rho_0 (1 + z_i^*)} > 0,
$$

and

$$
\sigma^2_{Q,0} = (y_i^*)^2 \sigma^2_{A,0} + (1 + z_i^*)^2 \sigma^2_{Q,0} + 2y_i^* (1 + z_i^*) \sigma_{AQ,0}
$$

$$
\Rightarrow \sigma^2_{Q,0} = \frac{(y_i^*)^2 \sigma^2_{A,0} + 1 + \rho_0 (1 + z_i^*)}{1 - (1 + z_i^*)^2} > 0,
$$

since $(1 + z_i^*) > -1$ in any stationary distribution induced by the model.

Straightforward algebra shows that the variance of earnings is

$$
\sigma^2_{\Pi,0} = \sigma^2_{A,0} + B^2 \sigma^2_{Q,0} - 2B \sigma_{AQ,0}
$$

$$
= \frac{\sigma^2_{A,0}}{1 - \rho_0 (1 + z_i^*)} \left( (1 - \rho_0 (1 + z_i^* + 2B y_i^*)) + B^2 (y_i^*)^2 \frac{1 + \rho_0 (1 + z_i^*)}{1 - (1 + z_i^*)^2} \right),
$$

and the variance of prices is

$$
\sigma^2_{P,0} = (k y_i^*)^2 \sigma^2_{A,0} + (k z_i^*)^2 \sigma^2_{Q,0} + 2(k y_i^*) (k z_i^*) \sigma_{AQ,0} = \frac{2(k y_i^*)^2 \sigma^2_{A,0}}{1 + (1 + z_i^*)} \frac{1 - \rho_0 (1 + z_i^*)}{1 - \rho_0 (1 + z_i^*)}
$$

We can also use (B21) to characterize the path and auto-covariance of earnings in the model. Specifically, we have
\[ E_0[\Pi_{t+j} | A_t, Q_t] = \Pi^* + \left[1 - B\right] \begin{bmatrix} \rho_0 & 0 \\ -\frac{\rho_0}{1 + z_i^*} & \frac{A_t - \bar{A}}{Q_t - Q^*} \end{bmatrix} \]

\[ = \Pi^* + \rho_0^j (A_t - \bar{A}) - B y_j^* \left( \sum_{t=0}^{j-1} \rho_0^{j+t-1} (1 + z_i^*)^t \right) (A_t - \bar{A}) - B(1 + z_i^*)^t (Q_t - Q^*) \]

\[ = \Pi^* + \left( \rho_0^j - B y_j^* \frac{\rho_0^j - (1 + z_i^*)^j}{\rho_0 - (1 + z_i^*)^j} \right) (A_t - \bar{A}) - B(1 + z_i^*)^t (Q_t - Q^*) \]

Thus, the auto-covariance of ship earnings is

\[ \text{Cov}_0[\Pi_{t+j}, \Pi_t] = \text{Cov}_0 \left[ \left( \rho_0^j - B y_j^* \frac{\rho_0^j - (1 + z_i^*)^j}{\rho_0 - (1 + z_i^*)^j} \right) (A_t - \bar{A}) - B(1 + z_i^*)^t (Q_t - Q^*), (A_t - \bar{A}) - B(Q_t - Q^*) \right] \]

\[ = \left( \rho_0^j - B y_j^* \frac{\rho_0^j - (1 + z_i^*)^j}{\rho_0 - (1 + z_i^*)^j} \right) \left( \sigma_{x,0}^2 - B \sigma_{AQ,0} \right) - B(1 + z_i^*)^t \left( \sigma_{AQ,0} - B \sigma_{Q,0}^2 \right) \]

\[ = \left( \rho_0^j - B y_j^* \frac{\rho_0^j - (1 + z_i^*)^j}{\rho_0 - (1 + z_i^*)^j} \right) \left( \sigma_{A1,0} \right) - B(1 + z_i^*)^t \left( \sigma_{Q1,0} \right) \]

And the auto-correlations of earnings are given by

\[ \text{Corr}_0[\Pi_{t+j}, \Pi_t] = \text{Corr}_0 \left[ \left( \rho_0^j - B y_j^* \frac{\rho_0^j - (1 + z_i^*)^j}{\rho_0 - (1 + z_i^*)^j} \right) \left( \frac{\sigma_{A1,0}}{\sigma_{Q1,0}} \right), (A_t - \bar{A}) - B(Q_t - Q^*) \right] \]

\[ = \left( \rho_0^j - B y_j^* \frac{\rho_0^j - (1 + z_i^*)^j}{\rho_0 - (1 + z_i^*)^j} \right) \left( \frac{\sigma_{A1,0}}{\sigma_{Q1,0}} \right) - B(1 + z_i^*)^t \left( \frac{\sigma_{Q1,0}}{\sigma_{Q,0}^2} \right) \]

4. System dynamics and steady-state distribution perceived by firms

The system dynamics perceived by firms are

\[ \begin{bmatrix} A_{t+1} \\ Q_{t+1} \end{bmatrix} = \begin{bmatrix} (1 - \rho_f) \bar{A} \\ \theta \bar{y}^*_i 1 + \theta z_i^* \end{bmatrix} \begin{bmatrix} A_t \\ Q_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}, \]

so the perceived steady state is given by

\[ \begin{bmatrix} A^* \\ Q^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \left( \begin{bmatrix} 0 & \rho_f \\ \rho_f & \rho_0 - (1 + z_i^*)^j \end{bmatrix} \right)^{-1} \begin{bmatrix} (1 - \rho_f) \bar{A} \\ \theta \bar{y}^*_i 1 + \theta z_i^* \end{bmatrix} = \begin{bmatrix} \bar{A} \\ \theta \bar{y}^*_i 1 + \theta z_i^* \end{bmatrix}. \]

Thus, firms perceive the same steady-state as the econometrician. However, since \( 0 < 1 + \theta z_i^* < 1 \), firms expect the dynamics to be convergent and non-oscillatory.

We can rewrite equation (B30) as

\[ \begin{bmatrix} A_{t+1} - \bar{A} \\ Q_{t+1} - Q^* \end{bmatrix} = \begin{bmatrix} \rho_f & 0 \\ \theta \bar{y}^*_i 1 + \theta z_i^* \end{bmatrix} \begin{bmatrix} A_t - \bar{A} \\ Q_t - Q^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}. \]

Taking variances of (B32), the perceived variance of the system about the steady-state is
\[
\begin{bmatrix}
\sigma_{A, f}^2 & \sigma_{AQ, f} \\
\sigma_{AQ, f} & \sigma_{Q, f}^2
\end{bmatrix}
= \begin{bmatrix}
\rho_f & 0 \\
\theta y_i^* & 1 + \theta z_i^*
\end{bmatrix}
\begin{bmatrix}
\sigma_{A, f}^2 & \sigma_{AQ, f} \\
\sigma_{AQ, f} & \sigma_{Q, f}^2
\end{bmatrix}
\begin{bmatrix}
\rho_f & 0 \\
\theta y_i^* & 1 + \theta z_i^*
\end{bmatrix}
+ \begin{bmatrix}
\sigma_z^2 & 0 \\
0 & 0
\end{bmatrix}.
\] (B33)

Thus, we obtain
\[
\sigma_{A, f}^2 = \frac{\sigma_z^2}{1 - \rho_f^2} > 0,
\] (B34)
\[
\sigma_{AQ, f} = \frac{\rho_f \theta y_i^* \sigma_{A, f}^2}{1 - \rho_f^2 (1 + \theta z_i^*)^2} > 0,
\] (B35)
and
\[
\sigma_{Q, f}^2 = \frac{(\theta y_i^*)^2 \sigma_{A, f}^2}{1 - (1 + \theta z_i^*)^2} \left\{ (1 - \rho_f^2 (1 + \theta z_i^*)^2) + B^2 (\theta y_i^*)^2 \right\},
\] (B36)

The perceived variance of earnings is
\[
\sigma_{\Pi, f}^2 = \sigma_{A, f}^2 + B^2 \sigma_{Q, f}^2 - 2B \sigma_{AQ, f}
= \sigma_{A, f}^2 + \frac{(k \theta y_i^*)^2 \sigma_{A, f}^2}{1 - (1 + \theta z_i^*)^2} \left( 1 - \rho_f^2 (1 + \theta z_i^*)^2 \right),
\] (B37)

and the perceived variance of prices is
\[
\sigma_{\bar{P}, f}^2 = (k \theta y_i^*)^2 \sigma_{A, f}^2 + (k \theta z_i^*)^2 \sigma_{Q, f}^2 + 2(k \theta y_i^*) (k \theta z_i^*) \sigma_{AQ, f} = \frac{2 (k \theta y_i^*)^2 \sigma_{A, f}^2}{1 - \rho_f^2 (1 + \theta z_i^*)^2}.
\] (B38)

We can use (B32) to characterize the path of earnings expected perceived by firms.
\[
E_j[\Pi_{t+j} \mid A_t, Q_j] = \Pi^* + \left( \rho_f - B \theta y_i^* \frac{(1 + \theta z_i^*)}{\rho_f - (1 + \theta z_i^*)} \right) (A_t - A) - B (1 + \theta z_i^* \gamma (Q_i - Q^*).)
\] (B39)

Thus, the perceived auto-covariance of ship earnings is
\[
\text{Cov}_j[\Pi_{t+j}, \Pi_t] = \left( \rho_f - B \theta y_i^* \frac{(1 + \theta z_i^*)}{\rho_f - (1 + \theta z_i^*)} \right) \left( \sigma_{A, f}^2 - B \sigma_{AQ, f} \right) - B (1 + \theta z_i^*) \left( \sigma_{AQ, f} - B \sigma_{Q, f}^2 \right)
\] (B40)

And the auto-correlations of earnings perceived by firms are given by
\[
\text{Corr}_j[\Pi_{t+j}, \Pi_t] = \left( \rho_f - B \theta y_i^* \frac{(1 + \theta z_i^*)}{\rho_f - (1 + \theta z_i^*)} \right) \left( \frac{\sigma_{A, f}^2}{\sigma_{\Pi, f}^2} \right) - B (1 + \theta z_i^*) \left( \frac{\sigma_{AQ, f}}{\sigma_{\Pi, f}^2} \right).
\]

5. **Observables are invariant in B/k**

Note that
only depends on \( B/k \). Furthermore, \( kx_i^* \) and \( ky_i^* \) only depend on \( B/k \). Thus, if we change \( B \) and \( k \) proportionately holding \( B/k \) constant, it is easy to see that this has a proportional effect on \( I_t \) and \( Q_t \), but has no effect on \( I_t / Q_t \), \( \Pi_t \), \( P_t \), or \( R_{t+1} \), and thus no effect on any of the data statistics used in our indirect inference estimation procedure.

It is also straightforward to see that a change in \( A \) or \( C \) has an additive effect on the fleet size \( Q_t \), but has no effect on \( I_t \), \( \Pi_t \), \( P_t \), or \( R_{t+1} \). As a result, a change in these parameters has a small impact on \( I_t / Q_t \).

6. A useful limiting case

It is easy to show that the cobweb model is a limiting case of our model when firms completely neglect the competition (\( \theta = 0 \)) and radically over-extrapolate demand (\( \rho_f = 1 \)). To clearly isolate the role of competition neglect, here we consider the more general case where capital is infinitely lived and shifts in demand are permanent and deterministic, i.e., where \( \rho_f = \rho_o = 1 \) and \( \delta = \sigma_e = C = 0 \). Since \( \rho_f = \rho_o = 1 \), there is no scope for over-extrapolating demand.

In this case, \( z_i^* \) is given in equation (B11) and it is also easy to see that \( y_i^* = -z_i^* / B \) and \( x_i^* = z_i^* (rP_r) / B \), which implies that prices are \( P_t = P_r - (kz_i^* / B)(\Pi_i - rP_r) \) and investment is \( I_i = -(z_i^* / B)(\Pi_i - rP_r) \). Given an initial level of demand \( A_0 \), the steady-state fleet size is \( Q^*(A_0) = (A_0 - rP_r) / B \), and steady-state earnings are \( \Pi^* = rP_r \). In the limiting case where \( \theta = 0 \), we have \( z_i^* = -B / (kr) \) which implies \( P_t = \Pi_i / r \) and \( I_i = (\Pi_i / r - P_r) / k \).

Now suppose there is an unexpected shock at \( t = 0 \) that permanently raises demand to \( A_0 + \varepsilon \). The new steady-state fleet size is \( \bar{Q}^*(A_0 + \varepsilon) = (A_0 + \varepsilon - rP_r) / B \), and the steady-state rental rate and ship price are unchanged. Following the initial shock, the aggregate fleet size \( (Q_i) \) and earnings \( (\Pi_i) \) evolve according to

\[
Q_{t+1} = Q_t + \frac{I_i}{(\Pi_i - rP_r)} (\Pi_i - rP_r) \quad \text{and} \quad \Pi_{t+1} = A_0 + \varepsilon - BQ_{t+1}.
\]
Since $z_i^* < 0$, investment is positive when earnings are above the steady-state. Iterating on (B41a), we can show that

$$Q_t = Q^*(Q_0) - \varepsilon(z_i^*/B) \left[ \sum_{j=0}^{t-1} (1 + z_j^*) \right]$$

and

$$\Pi_t = rP_r + \varepsilon \left[ 1 + z_i^* \sum_{j=0}^{t-1} (1 + z_j^*) \right].$$

(B41b)

Thus, if $|1 + z_i^*| < 1$, the system converges to its new steady-state following the shock—i.e.,

$$\lim_{t \to \infty} Q_t = Q^*(Q_0 + \varepsilon)$$

and

$$\lim_{t \to \infty} \Pi_t = rP_r.$$

Figure B1 uses this simplified version of the model to contrast industry dynamics in three cases: the cobweb model ($\theta = 0$), rational expectations ($\theta = 1$), and partial competition neglect ($0 < \theta < 1$). In Figure B1, the sequence of equilibrium $(Q_t, \Pi_t)$ pairs are marked with dots. Vertical movements in the figure show the determination of spot earnings $\Pi_t$ given the current fleet size, $Q_t$. These movements are dictated by the demand curve (earnings and lease rates are the same in this case). Lateral movements in the figure depict firms’ investment response to current earnings. The lateral movements show the earnings that each firm expects to prevail next period and, given those expectations, the quantity that each firm chooses to supply. When firms suffer from competition neglect, actual earnings differ from expected earnings because actual industry investment differs from the industry investment that firms had expected.

Panel A illustrates Kaldor’s (1938) cobweb model in which firms choose the quantity to supply in period $t+1$ under the naïve assumption that there will be zero competitive supply response (i.e., $\theta = 0$), so earnings will always be the same as they were in period $t$. Specifically, when competition neglect is complete, $I_t^* = k^{-1}(\Pi_t/r - P_r)$, so the lateral movements in Panel A are perfectly horizontal, and the adjustment process traces out the cobweb-like pattern in price versus quantity space. These cobweb dynamics can be contrasted with the rational expectations equilibration process that obtains when $\theta \to 1$. With positive adjustment costs, earnings and ship prices must remain above their steady state levels for several periods to induce firms to invest. However, since expected earnings are the same as actual earnings under rational expectations, the adjustment process traces out a straight line.

Panel B of Figure B1 illustrates the case where competition neglect is severe but not complete (i.e., $\theta$ is close to zero). While firms correctly anticipate the directional change in earnings, they underestimate the magnitude of that change following a demand shock. This
generates the dampened cobweb-like pattern depicted in Panel B of Figure B1. Specifically, the lateral movements are diagonal, reflecting the fact that firms partially anticipate the competitive response.

As $\theta$ rises, individual firms increasingly recognize how competitors are likely to respond, so industry investment becomes less sensitive to deviations of earnings from their steady state. For small enough values of $\theta$, the dynamics can be oscillatory as in Panel B. As $\theta$ approaches 1, the dynamics become non-oscillatory—i.e., industry fleet size steadily rises to the new steady state, and earnings steadily fall back to $rP_r$. This is shown in Panel C, which compares the dynamics under moderate competition neglect with those under rational expectations. While the dynamics are not oscillatory in this case, there is still excess volatility in ship prices and investment.

In each of the scenarios described above, we can compute the return from owning and operating a ship. The return between time $t$ and $t+1$ along the equilibrium path following the initial shock is given by

$$\Pi_{t+1} + P_{t+1} = (1 + r) - (1 - \theta) \frac{(B - kz^*_i)( -z^*_i / B)(\Pi_t - rP_r)}{P_r + k(-z^*_i / B)(\Pi_t - rP_r)}.$$  

Since this example is deterministic, realized returns and the returns expected by the econometrician are the same. In the rational expectations case ($\theta = 1$), expected returns $R_{t+1}$ equal required returns $r$, irrespective of $\Pi_t$. In contrast, when firms suffer from competition neglect ($\theta < 1$), expected returns are less than required returns when current earnings are above their steady state—i.e., if $\Pi_t > rP_r$, then $R_{t+1} < r$—and vice versa. And since earnings, prices, and investment all contain the same information in this simple case, analogous statements hold for used prices and investment—i.e., $\Pi_t > rP_r \iff P_i > P_r \iff I_i > 0 \iff R_{t+1} < r$.

The intuition for these results is natural. In the rational expectations equilibrium, firms expect any earnings in excess of the steady-state level to be precisely offset by capital losses from holding ships. However, when firms suffer from competition neglect, investment overreacts to deviations of earnings from the steady state. Because firms are surprised by the industry supply response, future realized returns are below required returns.
Figure B1: Cobweb Dynamics
This figure illustrates model dynamics in the case where \(\delta = C = 0\) and \(\rho_f = \rho_0 = 1\) following a shock to demand.

Panel A: Complete competition neglect (\(\theta = 0\)) versus rational expectations (\(\theta = 1\))

Panel B: Severe competition neglect versus rational expectations

Panel C: Moderate competition neglect versus rational expectations
7. Equilibrium expected returns

Expected returns will generally not equal firms’ required returns. Specifically, equation (B9) insures that

\[ E_f [1 + R_{t+1} | A_t, Q_t] = \frac{E_f \left[ (A_{t+1} - BQ_{t+1} - C - \delta P_r) + \left( P_r + k(x^*_i + y^*_i A_{t+1} + z^*_i Q_{t+1}) \right) \right] | A_t, Q_t}{P_r + kx^*_i + ky^*_i A_t + kz^*_i Q_t} \]

\begin{align*}
&= \frac{\left( P_r (1 - \delta) + kx^*_i - C \right) + \left( (1 + ky^*_i)(1 - \rho_f)A + \rho_f A_t \right)}{P_r + kx^*_i + ky^*_i A_t + kz^*_i Q_t} \\
&= 1 + r,
\end{align*}

by construction. In words, the representative firm expects that holding period returns will equal the required return on capital, \( r \). However, the true expected return perceived by the econometrician—who does not suffer from competition neglect and does not overestimate the persistence of demand shocks—will generally differ from \( 1 + r \). Specifically, we have

\[ E_0 [1 + R_{t+1} | A_t, Q_t] = \frac{E_0 \left[ (A_{t+1} - BQ_{t+1} - C - \delta P_r) + \left( P_r + k(x^*_i + y^*_i A_{t+1} + z^*_i Q_{t+1}) \right) \right] | A_t, Q_t}{P_r + kx^*_i + ky^*_i A_t + kz^*_i Q_t} \]

\begin{align*}
&= \frac{\left( P_r (1 - \delta) + kx^*_i - C \right) + \left( (1 + ky^*_i)(1 - \rho_0)A + \rho_0 A_t \right)}{P_r + kx^*_i + ky^*_i A_t + kz^*_i Q_t} \\
&= 1 + r,
\end{align*}

Subtracting (B42) from (B43) shows that

\[ E_0 [R_{t+1} | A_t, Q_t] = r - \frac{(1 - \theta)(B - kz^*_i)(x^*_i + y^*_i A_t + z^*_i Q_t) + (\rho_f - \rho_0)(1 + ky^*_i)(A_t - \bar{A})}{P_r + kx^*_i + ky^*_i A_t + kz^*_i Q_t} \]

\[ (B44) \]

Equation (B44) gives the general expression for expected returns when \( \theta \neq 1 \) and \( \rho_f \neq \rho_0 \).

Although (B44) shows that expected returns can be decomposed into a term that vanishes when there is full competition awareness (\( \theta = 1 \)) and a term that vanishes when there is no demand over-extrapolation (\( \rho_f = \rho_0 \)), these two biases do interact in our model. Specifically, since \( \frac{\partial y^*_i}{\partial \theta} < 0 \)
and \( \partial y_i^* / \partial \rho_f > 0 \), demand over-extrapolation naturally amplifies the return predictability due to competition neglect and vice versa.

Since the latent demand process, \( A_n \), is not readily observable, it is useful to recast equation (B44) in terms of observables, namely, industry net investment \((I_t)\) and net earnings \((\Pi_t)\), which contain the same information as \( A_t \) and \( Q_t \). Note that

\[
\begin{bmatrix}
\Pi_t + (C + \delta P_r)
\hline
I_t - x_i^*
\end{bmatrix}
= \begin{bmatrix}
1 & -B
\end{bmatrix}
\begin{bmatrix}
A_t
\hline
Q_t
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
A_t
\hline
Q_t
\end{bmatrix}
= \begin{bmatrix}
1
\end{bmatrix}
\begin{bmatrix}
B(I_t - x_i^*) + z_i^* \Pi_t + z_i^*(C + \delta P_r)
\hline
(I_t - x_i^*) - y_i^* \Pi_t - y_i^*(C + \delta P_r)
\end{bmatrix}.
\]

(B12) implies \( z_i^* + By_i^* = (1 + ky_i^*)z_i^*(1 - \rho_f) \). Thus, using our expression for \( A_t \) and (B13), yields

\[
(1 + ky_i^*)(A_t - \bar{A}) = \frac{1 + ky_i^*}{z_i^* + By_i^*} \left( B(I_t - x_i^*) + z_i^* \Pi_t + z_i^*(C + \delta P_r) - \bar{A}(z_i^* + By_i^*) \right)
= \frac{1}{(1 - \rho_f)} \left( (B / z_i^*)I_t + \Pi_t - \left( (B / z_i^*)x_i^* + \bar{A}(1 - \rho_f)(1 + ky_i^*) - (C + \delta P_r) \right) \right)
= \frac{1}{(1 - \rho_f)} \left( (B / z_i^*)I_t + (\Pi_t - rP_r) \right)
= \frac{1}{(1 - \rho_f)} \left( (B / z_i^*)I_t + (\Pi_t - \Pi^*) \right).
\]

Therefore, we obtain

\[
E_0[R_{t+1} \mid I_t^*, \Pi_t] = r - (1 - \theta) \left( \frac{(B - k z_i^*)I_t}{P_r + k l_t} \right) - \frac{(\rho_f - \rho_0)}{1 - \rho_f} \left[ \frac{(\Pi_t - \Pi^*) + (B / z_i^*)I_t}{P_r + k l_t} \right].
\]

**Proposition 2 (Forecasting regressions):** Consider a forecasting regression of returns in a neighborhood of the steady-state.

(a) Consider a multivariate regression of returns on demand \((A_t)\) and fleet size \((Q_t)\). If \( \theta < 1 \) or \( \rho_0 < \rho_f \), then \( \partial E_0[R_{t+1} \mid A_t, Q_t] / \partial A_t < 0 \). If \( \theta < 1 \), then \( \partial E_0[R_{t+1} \mid A_t, Q_t] / \partial Q_t > 0 \).

(b) If \( \theta < 1 \) or \( \rho_0 < \rho_f \), then investment \((I_t)\), prices \((P_t)\), and profits \((\Pi_t)\) will each negatively forecast returns in a univariate regression.

(c) Consider a multivariate regression of returns on investment \((I_t)\) and profits \((\Pi_t)\).
(i) If there is competition neglect but no demand over-extrapolation \((\theta < 1 \text{ and } \rho_f = \rho_0)\), then 
\[ \frac{\partial E_0[R_{t+1} | \Pi_t, I_t]}{\partial I_t} < 0 \text{ and } \frac{\partial E_0[R_{t+1} | \Pi_t, I_t]}{\partial \Pi_t} = 0; \]

(ii) If there is demand over-extrapolation but no competition neglect \((\theta = 1 \text{ and } \rho_f > \rho_0)\), then 
\[ \frac{\partial E_0[R_{t+1} | \Pi_t, I_t]}{\partial \Pi_t} < 0 \text{ and } \frac{\partial E_0[R_{t+1} | \Pi_t, I_t]}{\partial I_t} > 0; \]

(iii) If there is both competition neglect and over-extrapolation, (i.e., \(\theta < 1 \text{ and } \rho_f > \rho_0\)), then we always have 
\[ \frac{\partial E_0[R_{t+1} | \Pi_t, I_t]}{\partial \Pi_t} < 0. \] Furthermore, if competition neglect is relatively important in the sense that 
\[ (\rho_f - \rho_0)/(1 - \rho_f)\left(\frac{1}{z^*_f(B - k^*_f)}\right) < (1 - \theta), \]
then 
\[ \frac{\partial E_0[R_{t+1} | \Pi_t, I_t]}{\partial I_t} < 0. \] Otherwise, if competition neglect is relatively unimportant, then 
\[ \frac{\partial E_0[R_{t+1} | \Pi_t, I_t]}{\partial I_t} > 0. \]

**Proof:** We have
\[
\frac{\partial E_0[R_{t+1} | A_t, Q_t]}{\partial A_t} = -(1 - \theta)P_r^{-1}y^*_i(B - k^*_i) \left[ \frac{(P_r)^2}{(P_r + k^*_i(A_t - \bar{A}) + k^*_i(Q_t - Q^*))^2} \right] \]
\[-(\rho_f - \rho_0)P_r^{-1}(1 + k^*_i) \left[ \frac{P(P_r + k^*_i(Q_t - Q^*))}{(P_r + k^*_i(A_t - \bar{A}) + k^*_i(Q_t - Q^*))^2} \right] \]
\[= -(1 - \theta)P_r^{-1}y^*_i(B - k^*_i) - (\rho_f - \rho_0)P_r^{-1}(1 + k^*_i) < 0, \] \tag{B46}

and
\[
\frac{\partial E_0[R_{t+1} | A_t, Q_t]}{\partial Q_t} = -P_r^{-1}(1 - \theta)z^*_i(B - k^*_i) \left[ \frac{P_r^2}{(P_r + k^*_i(A_t - \bar{A}) + k^*_i(Q_t - Q^*))^2} \right] \]
\[+ (\rho_f - \rho_0)P_r^{-1}(1 + k^*_i)k^*_i \left[ \frac{P_r^2(A_t - \bar{A})}{(P_r + k^*_i(A_t - \bar{A}) + k^*_i(Q_t - Q^*))^2} \right] \]
\[= -(1 - \theta)P_r^{-1}z^*_i(B - k^*_i) > 0. \] \tag{B47}

Part (a) follows from inspecting (B46) and (B47).
These approximations are not substantive in nature. Consider equation (B46) above. Using our parameter estimates, Figure B2 shows the actual expected returns implied by the model versus the linear approximation as a function of the demand state variable $A_t$ at $Q_t = Q^*$. We show this for those values of demand that would arise 95% of the time conditional on $Q_t = Q^*$. As shown below, the linear approximation is highly accurate for values of demand that are likely to arise in equilibrium. Nonetheless, we do not use this approximation when we use the model to simulate the data for our indirect inference estimator.\(^6\)

![Figure B2: Accuracy of Return Approximations](image)

To prove part (b), we approximate expected returns as

$$E_0 [1 + R_{t+1} | A_t, Q_t] \approx r + P_r^{-1} \left[-(1 - \theta) y_i^* (B - k z_i^*) - (\rho_f - \rho_0) (1 + k y_i^*) \right] (A_t - \bar{A})$$

$$+ P_r^{-1} \left[-(1 - \theta) z_i^* (B - k z_i^*) \right] (Q_t - Q^*).$$

(B48)

Therefore,

$$Cov[R_{t+1}, I_t] = Cov[E_0 [R_{t+1} | A_t, Q_t], I_t]$$

$$\approx P_r^{-1} (1 - \theta) (B - k z_i^*) [(y_i^*)^2 \sigma_A^2 + (z_i^*)^2 \sigma_Q^2 + 2 y_i^* z_i^* \sigma_{AQ}]$$

$$- P_r^{-1} (\rho_f - \rho_0) (1 + k y_i^*) [y_i^* \sigma_A^2 + z_i^* \sigma_{AQ}]$$

(B49) is negative since

$$[(y_i^*)^2 \sigma_A^2 + (z_i^*)^2 \sigma_Q^2 + 2 y_i^* z_i^* \sigma_{AQ}] = Var[I_t] > 0$$

and

$$y_i^* \sigma_A^2 + z_i^* \sigma_{AQ} = y_i^* \sigma_A^2 [(1 - \rho_0) / (1 - \rho_0 (1 + z_i^*))] > 0.$$  

Similarly, we have

\(^6\) However, we have checked and we obtain nearly identical parameter estimates if, instead of using simulation to compute the means, variances, and forecasting relationships implied by the model, we used linear approximations of this sort to compute the quantities analytically using approximate closed-form expressions.
$$\text{Cov}(R_{t+1}, \Pi_i) = \text{Cov}(E_0[R_{t+1} | A_t, Q_i], \Pi_i)$$

\[\approx -P_r^{-1}(1-\theta)(B-kz_i^*)[y_i^*\sigma_A^2 - Bz_i^*y_i^* - (By_i^* - z_i^*)\sigma_Q] \]

\[\approx -P_r^{-1}(\rho_f - \rho_o)(1 + ky_i^*)[\sigma_A^2 - B\sigma_Q]. \tag{B50} \]

(B50) is negative since algebra shows \([y_i^*\sigma_A^2 - Bz_i^*y_i^* - (By_i^* - z_i^*)\sigma_Q]\) and \([\sigma_A^2 - B\sigma_Q]\) > 0.

To prove part (c), note that

\[
\frac{\partial E_0[R_{t+1} | I_t, \Pi_i]}{\partial I_t} = -(1-\theta)(B-kz_i^*) \cdot \frac{P_r}{(P_r + kl_t)^2} - \frac{(\rho_f - \rho_o)}{1 - \rho_f} \left[ -\frac{(B/z_i^*)P_r - k(\Pi_t - \Pi_i^*)}{(P_r + kl_t)^2} \right],
\]

and

\[
\frac{\partial E_0[R_{t+1} | I_t, \Pi_i]}{\partial \Pi_t} = -\frac{(\rho_f - \rho_o)}{1 - \rho_f} \frac{1}{P_r + kl_t}. \]

Thus, in a neighborhood of the steady state, \((I_t, \Pi_t) = (0, \Pi_i^*)\), we have

\[
\frac{\partial E_0[R_{t+1} | I_t, \Pi_i]}{\partial I_t} \approx \frac{\partial E_0[R_{t+1} | 0, \Pi_i^*]}{\partial I_t} = -(1-\theta)P_r^{-1}(B-kz_i^*) - \frac{\rho_f - \rho_o}{1 - \rho_f} P_r^{-1}(B/z_i^*). \tag{B51}
\]

and

\[
\frac{\partial E_0[R_{t+1} | I_t^N, \Pi_i]}{\partial \Pi_t} \approx \frac{\partial E_0[R_{t+1} | 0, \Pi_i^*]}{\partial \Pi_t} = -\frac{\rho_f - \rho_o}{1 - \rho_f} P_r^{-1}. \tag{B52}
\]

Part (c) follows from inspecting (B51) and (B52).

Proposition B1 characterizes how these predictability results vary with the underlying model parameters. Specifically, we take comparative statics on the forecasting results near the model’s steady-state. However, when computing these comparative statics, we allow the steady-state of the model to change where relevant. These results are discussed in the text, but are stated and proven formally here for the sake of completeness.

Proposition B1 (The role of competition neglect, demand over-extrapolation, inelastic demand, and elastic supply): (a) Return predictability becomes stronger when competition neglect is more severe (i.e., \(\partial^2 E_0[R_{t+1} | A_t, Q_t] / \partial A_t \partial(1-\theta) < 0\) and \(\partial^2 E_0[1 + R_{t+1} | A_t, Q_t] / \partial Q_t \partial(1-\theta) > 0\)) or demand over-extrapolation is more severe (i.e., \(\partial^2 E_0[R_{t+1} | A_t, Q_t] / \partial A_t \partial \rho_f < 0\), but \(\partial^2 E_0[R_{t+1} | A_t, Q_t] / \partial Q_t \partial \rho_f = 0\).
(b) The predictability due to competition neglect becomes stronger when demand is more inelastic and weaker when supply is more inelastic. Formally, when \( \theta < 1 \) and \( \rho_f = \rho_b \),
\[
\partial^2 E_o[R_{t+1} \mid A_t, Q_t] / \partial A_t \partial B < 0, \quad \partial^2 E_o[R_{t+1} \mid A_t, Q_t] / \partial Q_t \partial B > 0, \quad \partial^2 E_o[R_{t+1} \mid A_t, Q_t] / \partial A_t \partial k > 0, \text{ and } \partial^2 E_o[R_{t+1} \mid A_t, Q_t] / \partial Q_t \partial k < 0. 
\]

(c) The predictability due to demand over-extrapolation becomes weaker when demand is more inelastic and stronger when supply is more inelastic. Formally, when \( \theta = 1 \) and \( \rho_f > \rho_b \),
\[
\partial^2 E_o[R_{t+1} \mid A_t, Q_t] / \partial A_t \partial B > 0 \text{ and } \partial^2 E_o[R_{t+1} \mid A_t, Q_t] / \partial A_t \partial k < 0. 
\]

(d) In a multivariate regression of returns on earnings and investment, the coefficient on earnings becomes more negative when demand extrapolation is more severe (i.e., \( \partial^2 E_o[R_{t+1} \mid \Pi_t, I_t] / \partial \Pi_t \partial \rho_f < 0 \)); the coefficient on investment falls when competition neglect is more severe and rises when demand extrapolation is more severe (i.e., \( \partial^2 E_o[R_{t+1} \mid \Pi_t, I_t] / \partial I_t \partial (1 - \theta) < 0 \) and \( \partial^2 E_o[R_{t+1} \mid \Pi_t, I_t] / \partial I_t \partial \rho_f > 0 \)). Finally, when competition neglect is relatively important, the coefficient on investment becomes more negative when either demand or supply is more inelastic.

**Proof:** Part (a) follows from differentiating (B46) and (B47). Specifically, we have
\[
\frac{\partial^2 E_o[R_{t+1} \mid A_t, Q_t]}{\partial A_t \partial \theta} = P_r^{-1} y_i^* (B - kz_i^*) - (1 - \theta) P_r^{-1} (B - kz_i^*) \frac{\partial y_i^*}{\partial \theta} + (1 - \theta) P_r^{-1} k y_i^* \frac{\partial y_i^*}{\partial \theta} - (\rho_f - \rho_o) P_r^{-1} k \frac{\partial y_i^*}{\partial \theta} > 0,
\]
\[
\frac{\partial^2 E_o[R_{t+1} \mid A_t, Q_t]}{\partial A_t \partial \rho_f} = -(1 - \theta) P_r^{-1} (B - kz_i^*) \frac{\partial y_i^*}{\partial \rho_f} - P_r^{-1} (1 + k y_i^*) - (\rho_f - \rho_o) P_r^{-1} k \frac{\partial y_i^*}{\partial \rho_f} > 0,
\]
\[
\frac{\partial^2 E_o[R_{t+1} \mid A_t, Q_t]}{\partial Q_t \partial \theta} = P_r^{-1} z_i^* (B - kz_i^*) - (1 - \theta) P_r^{-1} (B - 2kz_i^*) \frac{\partial z_i^*}{\partial \theta} < 0,
\]
and
\[
\frac{\partial^2 E_o[R_{t+1} \mid A_t, Q_t]}{\partial Q_t \partial \rho_f} = 0.
\]

Part (b) follows from differentiating (B46) and (B47) after setting \( \rho_f = \rho_o \). Specifically,
\[
\frac{\partial^2 E_0[R_{n1} | A_i, Q]}{\partial A_i \partial B} = -(1-\theta) P^{-1}_r \left[ y_i^* \left( 1 - k \frac{\partial z_i^*}{\partial B} \right) + (B - k z_i^*) \frac{\partial y_i^*}{\partial B} \right] = -(1-\theta) P^{-1}_r \left[ 1 - k \frac{\partial z_i^*}{\partial B} \right] (y_i^*)^2 k (1 - \rho_f + r) / \rho_f < 0,
\]

\[
\frac{\partial^2 E_0[R_{n1} | A_i, Q]}{\partial Q_i \partial B} = -(1-\theta) P^{-1}_r \left[ (B - k z_i^*) \frac{\partial z_i^*}{\partial B} + z_i^* \left( 1 - k \frac{\partial z_i^*}{\partial B} \right) \right] > 0,
\]

\[
\frac{\partial^2 E_0[R_{n1} | A_i, Q]}{\partial A_i \partial k} = (1-\theta) P^{-1}_r \left[ y_i^* \frac{\partial (k z_i^*)}{\partial k} - (B - k z_i^*) \frac{\partial y_i^*}{\partial k} \right] = (1-\theta) P^{-1}_r \left[ \left( y_i^* \right)^2 \frac{1 - \rho_f + r}{\rho_f} \frac{B}{kr + \theta (B - 2k z_i^*)} \right] > 0,
\]

and

\[
\frac{\partial^2 E_0[R_{n1} | A_i, Q]}{\partial Q_i \partial k} = -(1-\theta) P^{-1}_r \left[ (B - k z_i^*) \frac{\partial z_i^*}{\partial k} - z_i^* \frac{\partial (k z_i^*)}{\partial k} \right] = -(1-\theta) P^{-1}_r \left[ -\frac{\partial z_i^*}{\partial k} \frac{(B - k z_i^*)}{kr + \theta (B - 2k z_i^*)} \right] < 0.
\]

Part (c) follows from differentiation (B46) after setting \( \theta = 1 \). Specifically, we have

\[
\frac{\partial^2 E_0[R_{n1} | A_i, Q]}{\partial A_i \partial k} \approx -(\rho_f - \rho_0) P^{-1}_r \frac{\partial (k y_i^*)}{\partial k} < 0,
\]

and

\[
\frac{\partial^2 E_0[R_{n1} | A_i, Q]}{\partial A_i \partial B} \approx -(\rho_f - \rho_0) P^{-1}_r k \frac{\partial y_i^*}{\partial B} > 0.
\]

Finally, part (d) follows from differentiating (B51) and (B52). Specifically, we have

\[
\frac{\partial^2 E_0[R_{n1} | I_i, \Pi_i]}{\partial I_i \partial \theta} = P^{-1}_r (B - k z_i^*) + (1-\theta) P^{-1}_r k \frac{\partial z_i^*}{\partial \theta} + \frac{\rho_f - \rho_0}{1 - \rho_f} P^{-1}_r \frac{B}{(z_i^*)^2} \frac{\partial z_i^*}{\partial \theta} > 0,
\]

\[
\frac{\partial^2 E_0[R_{n1} | I_i, \Pi_i]}{\partial I_i \partial \rho_f} = -\frac{1 - \rho_0}{(1 - \rho_f)^2} P^{-1}_r (B / z_i^*) > 0,
\]

and

\[
\frac{\partial^2 E_0[R_{n1} | I_i, \Pi_i]}{\partial \Pi_i \partial \rho_f} = -\frac{1 - \rho_0}{(1 - \rho_f)^2} P^{-1}_r < 0.
\]
Finally, differentiating (B51) after setting $\rho_f = \rho_0$, we have

$$\frac{\partial^2 E_0[R_{t+1} \mid I_t, \Pi_t]}{\partial I_t \partial B} = -(1 - \theta)P_r^{-1} \left(1 - k\frac{\partial z_t^*}{\partial B}\right) < 0,$$

and

$$\frac{\partial^2 E_0[R_{t+1} \mid I_t, \Pi_t]}{\partial I_t \partial k} = (1 - \theta)P_r^{-1} \frac{\partial (kz_t^*)}{\partial k} < 0.$$

8. Model extensions

We have made a number of assumptions to keep the model tractable so as to transparently model the logic of competition neglect and demand-extrapolation. Here we discuss a number of possible extensions that we have explored.

a. Time-varying replacement costs of ships

We have also assumed that ships have a constant replacement cost of $P_r$. It is straightforward to extend the model so ship replacement costs follow an AR(1) process. In that case, firm investment and ship prices would be linear functions of the three state variables: the time-varying replacement cost, $P_{r,t}$, as well as $A_t$ and $Q_t$. Investment would be decreasing in $P_{r,t}$ and ship prices would be increasing in $P_{r,t}$. (Both prices and investment would be increasing in $A_t$ and decreasing in $Q_t$ as in our baseline model).\(^7\)

Relatedly, we have assumed that demand follows a stationary process. We could easily add a deterministic time trend to demand and instead assume that deviations of demand from trend are stationary.

b. Adding firms with unbiased beliefs

We could add sophisticated firms who hold unbiased expectations to our model. This would have the effect of dampening the boom-bust cycles and attenuating the predictability of shipping returns. Specifically, consider an extension in which unbiased firms are treated similarly to biased firms.

---

\(^7\) Thus, if we had time-series data on the real price of raw materials used to construct a ship (this is not the same as the price of a new ship which would reflect a potentially time-varying mark-up over cost) we could extend the model and consider this extension in our estimation.
firms: they are allowed to operate ships, invest by purchasing ships in the new build market, and to disinvest by scrapping ships. These unbiased firms will endogenously choose to lean again the wind, investing less (more) when biased firms are over-investing (under-investing). This contrarian investment response will reduce the magnitude of return predictability because it impacts tomorrow’s fleet size and hence tomorrow’s earnings. The degree of dampening would be a function of the number of sophisticated firms and their investment adjustment costs (i.e., adjustment costs would play an analogous role to arbitrageur risk aversion in behavioral finance models).

\[ \text{c. Irreversible investment and asymmetric adjustment} \]

Finally, we can introduce a wedge between the scrap value realized when \( i_t < 0 \) and the replacement cost of a new ship incurred when \( i_t > 0 \), so that firms optimally choose \( i_t = 0 \) for a non-degenerate set of values for \( A_t \) and \( Q_t \). We have also assumed adjustment costs are symmetric. Obviously, any differences between the costs of increasing or decreasing the capital stock would generate asymmetries in the speed of adjustment to positive or negative demand shocks. However, the key propositions that we emphasize in the paper continue to hold in a more general version of the model that incorporates investment asymmetries. The cost of this generality is that we are no longer able to solve the model in closed form.

To introduce irreversibility and asymmetry, suppose that per-period firm profits are

\[
V(q_t, i_t, A_t, Q_t) = q_t \Pi(A_t, Q_t) - (P_r^+ 1'[i_t] + P_r^- 1'[-i_t]) \cdot i_t - (k^+ 1'[i_t] + k^- 1'[-i_t]) \cdot i_t^2 / 2,
\]

where \( \Pi(A_t, Q_t) = A_t - BQ_t - C - \delta P_r \) is current ship earnings, \( 1'[i_t] \) is an indicator for \( i_t > 0 \), and \( 1'[-i_t] \) is an indicator for \( i_t < 0 \). We allow the new build price to exceed the scrap price (\( P_r^+ > P_r^- \)). A simple way to capture the idea that the stock of ships will be stickier down than up is to assume that the adjustment costs associated with disinvestment to exceed those associated with investment, i.e., \( k^+ < k^- \). Relatedly, a rough way to capture Kalouptsidi’s (2014) idea that time-to-build delays increase during investment booms is to make the opposite assumption, i.e., \( k^+ > k^- \).

Under these more general assumptions, the optimal investment of the representative firm is

\[
i_t^* = \frac{P(A_t, Q_t) - P_r^+}{k^+} \cdot 1'[P(A_t, Q_t) \geq P_r^+] + \frac{P(A_t, Q_t) - P_r^-}{k^-} \cdot 1'[P(A_t, Q_t) \leq P_r^-]
\]

where
is the present value of future profits that firms expect from a unit of installed capital. This conclusion is a special case of the general q-theory results in Abel and Eberley (1994). Naturally, any difference between new-build and scrapping prices \( P_r^+ > P_r^- \) generates an inaction region where firms neither actively invest nor actively disinvest. And if the adjustment costs associated with disinvestment exceed those associated with investment \( k^+ < k^- \), investment responds more aggressively to high ship prices than to low ship prices. By contrast, if \( k^+ > k^- \) because time-to-build delays lengthen when ordering it high, then firm investment will be concave in used ship prices. The relationship between investment and prices for the case where \( k^+ < k^- \) and \( P_r^+ = P_r^- \) is shown in Figure B2 below.

**Figure B2: Net Investment Versus Price when \( k^+ < k^- \) and \( P_r^+ = P_r^- \).**

Given these investment policies, we can solve for equilibrium prices. The law of motion for \( Q_t, \text{perceived} \) by firms who suffer from competition neglect is

\[
Q_{t+1} = Q_t + \theta \cdot \frac{P(A_t, Q_t) - P_r^+}{k^+} \cdot 1^+[P(A_t, Q_t) \geq P_r^+] + \theta \cdot \frac{P(A_t, Q_t) - P_r^-}{k^-} \cdot 1^+[P(A_t, Q_t) \leq P_r^-].
\]

By contrast, the true law of motion sets \( \theta = 1 \) in the above equation. As in our baseline model, firms may also believe that the exogenous demand process \( A_t \) is more persistent than it truly is. Thus, the equilibrium price satisfies the following functional equation.
\[
\begin{align*}
P(A_t, Q_t) &= \frac{E \left[ \prod_{t} (A_{t+1}, Q_{t+1}) + P(A_{t+1}, Q_{t+1}) \mid A_t, Q_t \right]}{1+r} \\
&= \frac{\left( (1-\rho_t) \bar{A} + \rho_t A_t \right) - B \left( Q_t + \theta \left( \frac{P(A_t, Q_t) - P^+}{k^+} 1 \left[ P(A_t, Q_t) \geq P^+ \right] + \frac{P(A_t, Q_t) - P^-}{k^-} 1 \left[ P(A_t, Q_t) \leq P^- \right] \right) \right)}{1+r} \\
&+ \frac{E \left[ P \left( (1-\rho_t) \bar{A} + \rho_t A_t + \epsilon_{t+1} Q_t + \theta \left( \frac{P(A_t, Q_t) - P^+}{k^+} 1 \left[ P(A_t, Q_t) \geq P^+ \right] + \frac{P(A_t, Q_t) - P^-}{k^-} 1 \left[ P(A_t, Q_t) \leq P^- \right] \right) \right]}{1+r} \\
&= \frac{1}{1+r} \left[ \left( 1-\rho_t \right) \bar{A} + \rho_t A_t - B \left( Q_t + \theta \left( \frac{P(A_t, Q_t) - P^+}{k^+} 1 \left[ P(A_t, Q_t) \geq P^+ \right] + \frac{P(A_t, Q_t) - P^-}{k^-} 1 \left[ P(A_t, Q_t) \leq P^- \right] \right) \right) \right] - \delta P,
\end{align*}
\]

where the expectation is over the distribution of \( \epsilon_{t+1} \). Using standard discrete grid approximation techniques (Tauchen (1986)), we can numerically solve for the equilibrium price function by iterating on the above mapping.

Given the non-linear response of investment to price when either \( k^+ \neq k^- \) or \( P_r^+ > P_r^- \), we find that the equilibrium price function is a non-linear function of the state variables \((A_t, Q_t)\). For instance, if \( k^+ < k^- \) and \( P_r^+ = P_r^- \), periods of below normal profits are expected to last longer than periods of above normal profits due to the asymmetry in firm’s endogenous supply response. This implies that ship prices will be a concave function of \((A_t, Q_t)\) and that prices will be uniformly lower than they are with symmetric adjustment costs. However, equilibrium ships prices are still strictly increasing in current demand \((A_t)\) and strictly decreasing in current fleet size \((Q_t)\). The case where \( k^+ < k^- \) and \( P_r^+ = P_r^- \) is illustrated below.

**Figure B3: Prices versus Demand and Supply when \( k^+ < k^- \) and \( P_r^+ = P_r^- \).**

By construction, firms believe that the expected return to investing in ships is constant and equal to \( r \). By contrast, an unbiased econometrician who knows the true laws of motion for both demand and the fleet size, expects returns to differ from \( r \). The expected future payoff from purchasing a ship is
\[ E_0 \left[ \Pi(A_{t+1}, Q_{t+1}) + P(A_{t+1}, Q_{t+1}) | A_t, Q_t \right] - (1 + r)P(A_t, Q_t) \\
= - \left( E_f \left[ \Pi(A_{t+1}, Q_{t+1}) | A_t, Q_t \right] - E_0 \left[ \Pi(A_{t+1}, Q_{t+1}) | A_t, Q_t \right] \right) \\
- \left( E_f \left[ P(A_{t+1}, Q_{t+1}) | A_t, Q_t \right] - E_0 \left[ P(A_{t+1}, Q_{t+1}) | A_t, Q_t \right] \right). \]

If \( \theta = 1 \) and \( \rho_0 = \rho_f \), this expected payoff is always zero. By contrast, this expected payoff is weakly decreasing in \( A_t \) if either \( \theta < 1 \) or \( \rho_0 < \rho_f \) and weakly increasing in \( Q_t \) if \( \theta < 1 \). Furthermore, the expected payoff will be strictly decreasing in \( A_t \) if \( \rho_0 < \rho_f \) or \( \theta < 1 \) and \( \tilde{t}^* \neq 0 \) and strictly increasing in \( Q_t \) if \( \theta < 1 \) and \( \tilde{t}^* \neq 0 \). \(^8\)

Thus, since

\[ E_0 \left[ R_{t+1} | A_t, Q_t \right] = r + \frac{E_0 \left[ \Pi(A_{t+1}, Q_{t+1}) + P(A_{t+1}, Q_{t+1}) | A_t, Q_t \right] - (1 + r)P(A_t, Q_t)}{P(A_t, Q_t)} \]

expected returns will inherit these same properties in a region of the steady state. In this way, our main return predictability results, namely, that \( \partial E_0 \left[ R_{t+1} | A_t, Q_t \right] / \partial A_t < 0 \) and \( \partial E_0 \left[ R_{t+1} | A_t, Q_t \right] / \partial Q_t > 0 \), are unchanged in this more general version of the model.

Adding investment asymmetries to our model can also generate skewed price changes. Specifically, if investment is a convex function of prices (i.e., if \( k^+ < k^- \) and \( P^+_t = P^-_r \)), equilibrium prices will be a concave function of \( (A_t, Q_t) \) as shown above. As a result, price changes will be negatively skewed even if the primitive demand shocks \( (\tilde{e}_{t+1}) \) are drawn from a symmetric distribution. Intuitively, because investment asymmetries mean that supply gluts are expected to persist for longer than supply shortages, a negative demand shock leads to a larger drop in prices than an positive demand shock of the same magnitude. However, this negative return skewness would arise whether or not firms have biased expectations.

\(^8\) To see this, recall that \( \tilde{t}^* = I(A_t, Q_t) \) is weakly increasing in \( A_t \) and weakly decreasing in \( Q_t \); similarly, \( P(A_t, Q_t) \) is strictly increasing in \( A_t \) and strictly decreasing in \( Q_t \). Now write

\[ E_0 \left[ \Pi(A_{t+1}, Q_{t+1}) + P(A_{t+1}, Q_{t+1}) | A_t, Q_t \right] - (1 + r)P(A_t, Q_t) \\
= - \left( (\rho_f - \rho_f)(A_t - \tilde{A}) + B(1 - \theta) \cdot I(A_t, Q_t) \right) \\
- \left( E \left[ P(A + \rho_f (A - \tilde{A}) + e_{t+1}, Q + \theta \cdot I(A_t, Q_t)) - P(A + \rho_f (A - \tilde{A}) + e_{t+1}, Q + \theta \cdot I(A_t, Q_t)) \right] \right) \\
- \left( E \left[ P(A + \rho_f (A - \tilde{A}) + e_{t+1}, Q + \theta \cdot I(A_t, Q_t)) - P(A + \rho_f (A - \tilde{A}) + e_{t+1}, Q + I(A_t, Q_t)) \right] \right). \]

Each of these three terms is weakly decreasing in \( A_t \) if either \( \theta < 1 \) or \( \rho_0 < \rho_f \) and weakly increasing in \( Q_t \) if \( \theta < 1 \). Furthermore, the first and second terms are strictly increasing in \( A_t \) if \( \rho_0 < \rho_f \) and the first and third terms are strictly increasing in \( A_t \) and strictly increasing in \( Q_t \) outside of the investment inaction region if \( \theta < 1 \).
C: Indirect Inference Procedure

We use indirect inference to estimate the parameters of our model of industry cycles. We are interested in a $L \times 1$ vector of model parameters $\Theta$. Assume that we have $M \geq L$ data statistics of the form

$$\hat{\beta}_T,$$

where the $\hat{\beta}_T$ are functions of our sample of $T$ time-series observations (e.g., time-series means, time-series variances, time-series regression coefficients, etc.) and the $\beta(\Theta)$ are the corresponding functions of our simulated time-series data. By simulating a sufficiently long time series we can eliminate simulation noise. We can thus regard the simulated statistics as deterministic and continuously differentiable function of the unknown parameters $\beta(\Theta)$.

Now define the estimator

$$\hat{\Theta}_T = \arg\min_{\Theta} (\hat{\beta}_T - \beta(\Theta))' W (\hat{\beta}_T - \beta(\Theta))$$

(C2)

If we assume

- **Compactness**: $\theta_0 \in \text{int}(\Theta)$ where $\Theta$ is a compact subset of $\mathbb{R}^L$.
- **Identification**: $\beta(\Theta) = \lim_{T \to \infty} E[\hat{\beta}_T]$ implies $\Theta = \theta_0$
- **Limiting Behavior**: A Central Limit Theorem implies that $\sqrt{T} (\hat{\beta}_T - \beta(\Theta_0)) \overset{d}{\to} N(0, S)$
- **Full Rank**: $\Gamma(\Theta) = D_{\theta} \beta(\Theta)$ exists, has full rank $L$, and is a continuous function of $\Theta$.

Then we will have

$$\hat{\Theta}_T \overset{p}{\to} \theta_0,$$

(C3)

i.e., our estimator will be consistent for the true population parameter of interest.

We now prove asymptotic normality. The first order condition for $\hat{\Theta}_T$ is

$$0 = -(\Gamma(\hat{\Theta}_T))' W (\hat{\beta}_T - \beta(\hat{\Theta}_T)).$$

(C4)

By the intermediate value theorem, this implies

$$0 = -(\Gamma(\hat{\Theta}_T))' W (\hat{\beta}_T - \beta(\Theta_0) - \Gamma(\hat{\Theta}_T)(\hat{\Theta}_T - \Theta_0))$$

$$\Rightarrow \sqrt{T} (\hat{\Theta}_T - \Theta_0) = [(\Gamma(\hat{\Theta}_T))' W \Gamma(\hat{\Theta}_T)]^{-1} (\Gamma(\hat{\Theta}_T))' W \sqrt{T} (\hat{\beta}_T - \beta(\Theta_0)),$$

(C5)
for some \( \bar{\theta}_T \) that is a convex combination of \( \hat{\theta}_T \) and \( \theta_0 \). Thus, since \( \hat{\theta}_T \xrightarrow{p} \theta_0 \) and since \( \Gamma(\theta) = D_\theta \beta(\theta) \) is continuous, Slutsky’s Theorem implies that
\[
\sqrt{T}(\hat{\theta}_T - \theta_0) \xrightarrow{d} N(0, (\Gamma'W\Gamma)^{-1}\Gamma'WSW\Gamma(\Gamma'W\Gamma)^{-1}), \tag{C6}
\]
where \( \Gamma = D_\theta \beta(\theta_0) \).

It is easy to show that asymptotically efficient estimates can be obtained by using \( W = \hat{S}^{-1} \) where \( \hat{S} \xrightarrow{p} S \). In that case, equation (C6) implies that
\[
\sqrt{T}(\hat{\theta}_T - \theta_0) \xrightarrow{d} N(0, (\Gamma'S^{-1}\Gamma)^{-1}), \tag{C7}
\]
which has the smallest asymptotic variance in the class of estimators defined by (C2).

However, we do not use \( W = \hat{S}^{-1} \) since we worry that, while improving asymptotic efficiency in theory, this is likely to lead to uninteresting and potentially strange results in practice. As in many applications, our estimated \( S \) matrix is nearly singular because some of our time-series statistics are highly correlated with each other. As a result, using \( W = \hat{S}^{-1} \) in our indirect inference estimator would place a huge amount of weight on minimizing linear combinations of the moments that are estimated with suspiciously high precision in the data. However, these precisely estimated linear combinations are not necessarily the most economically interesting. See Cochrane (2005) for a further discussion of the benefits of using simple pre-specified weighting matrices instead.

Instead, we use \( W = [\text{diag}(\hat{S})]^{-1} \) so our estimator is \( \hat{\theta}_T = \arg\min_\theta \sum_{m=1}^{M} [(\hat{\beta}_m - \hat{\beta}_m(\theta)) / \hat{s}_m]^2 \)
where \( \hat{s}_m = \sqrt{[\hat{S}]_{mm}} \) —i.e., we weight statistics inversely by their estimated variances. This choice for \( W \) puts all the moments in common (unit free) terms and accomplishes the basic efficiency goal of assigning greater weight to statistics that are estimated more precisely while avoiding the pitfalls of \( W = \hat{S}^{-1} \) that arise when \( S \) is nearly singular.

Our estimator for the variance of the indirect inference estimator takes the form
\[
T^{-1}\hat{V}(\hat{\theta}_T) = T^{-1}(\hat{\Gamma}'W\hat{\Gamma})^{-1}\hat{\Gamma}'WSW\hat{\Gamma}(\hat{\Gamma}'W\hat{\Gamma})^{-1}, \tag{C9}
\]
where \( \hat{\Gamma} = D_\theta \beta(\hat{\theta}_T) \) is a consistent estimator of \( \Gamma \), and \( \hat{S} \) is a consistent estimator of \( S \).
Intuitively, an auxiliary parameter $\beta_m(\theta_0)$ is informative about a parameter $l$ if the partial derivative of the moment with respect to the parameter is large in absolute magnitude—i.e., if $\Gamma_{ml} = \partial \beta_m(\theta_0) / \partial \theta_l$ is large in magnitude. If we have lots of uninformative moments, then $\Gamma' W \Gamma$ will be an ill-conditioned matrix with lots of near-zero values. Consequently, $(\Gamma' W \Gamma)^{-1}$ will be very large in magnitude, implying that the standard errors on our SMM estimates will be very large—i.e., they will be estimated quite imprecisely.

More formally, we follow Gentzkow and Shapiro (2013) and examine the elements of the influence matrix, $\Lambda$, and the scaled-influence matrix, $\Lambda_{\text{SCALE}}$. The elements of the influence matrix are simply the partial derivatives of our estimator with respect to the sample data statistics, evaluated at the true parameter vector. The elements of the scaled influence matrix are a natural unit-free measure of influence: $[\Lambda_{\text{SCALE}}]_{lm}$ is the standard deviation response of model parameter $l$ to a one standard deviation increase in data statistic $m$. Thus, Gentzkow and Shapiro (2013) argue that examining $\Lambda_{\text{SCALE}}$ is a natural way to assess sources of identification in non-linear models such as ours. Since indirect inference is a minimum distance estimator, we have $\Lambda = (\Gamma' W \Gamma)^{-1} \Gamma' W$ and $\Lambda_{\text{SCALE}} = [\text{diag}(V)]^{-1/2} (\Gamma' W \Gamma)^{-1} \Gamma' W [\text{diag}(S)]^{1/2}$ where $V = \Lambda S \Lambda'$ is the asymptotic variance of our estimator. Thus, $\Lambda$ incorporates information both about how the simulated data statistics depend on the model parameters ($\Gamma$) as well as information about how the sample data statistics are weighted by the estimator ($W$).

To obtain a consistent estimator for $S$, we use a system OLS approach (or seemingly-unrelated regression framework) using Newey-West (1987) standard errors that account for the serial correlation of residuals to estimate the joint variance of the sample data statistics. Specifically, we can interpret our data statistics as consisting of a system of linear equations

$$y_t = X_t \beta + \epsilon_t$$

where

$$y_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{M,t} \end{bmatrix}, \quad X_t = \begin{bmatrix} x_{1,t}' & 0' & \cdots & 0' \\ 0' & x_{2,t}' & \cdots & 0' \\ \vdots & \vdots & \ddots & \vdots \\ 0' & 0' & \cdots & x_{M,t}' \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_M \end{bmatrix}, \quad \epsilon_t = \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \vdots \\ \epsilon_{M,t} \end{bmatrix}$$

The system OLS estimator for $\beta$ is

A-37
\[ \hat{\beta}_r = \left( \sum_{i=1}^{T} X'_i X_i \right)^{-1} \left( \sum_{i=1}^{T} X'_i y_i \right), \]  

(C12)

and, letting \( e_t = y_t - X_t b_{OLS} \), the Newey-West (1987) style variance estimator for \( \hat{\beta}_r \) is

\[ T^{-1} \hat{\Sigma}_{NW} = \frac{T}{T-L} \left( \sum_{i=1}^{T} X'_i X_i \right)^{-1} \left[ \sum_{i=1}^{T} e'_i e_i + \sum_{j=1}^{J} \left( 1 - \frac{j}{J+1} \right) \sum_{i=1}^{T} \left( X'_i e'_{i+j} + X'_{i+j} e_i \right) \right] \left( \sum_{i=1}^{T} X'_i X_i \right)^{-1} \]  

(C13)

We use (C13) allowing for serial correlation of residuals at up to \( J = 36 \) months.

**D: Additional Structural Estimation Results**

**D.1. Robustness with respect to the our assumption about \( \bar{A} \)**

Table D1 provides robustness on our assumption on the \( \bar{A} \) parameter which determines the scale of demand and the equilibrium fleet size. For brevity, we show only the unrestricted estimation, although the message is the same in the restricted specifications.

**Table D1: Robustness on \( \bar{A} \) assumption**

This table provides robustness on the \( \bar{A} \) parameter which determines the scale of demand and the equilibrium fleet size. Specifically, we show the results for the unrestricted estimates corresponding to different levels of \( \bar{A} \).

<table>
<thead>
<tr>
<th>Baseline: ( \bar{A} = 50 )</th>
<th>Smaller: ( \bar{A} = 25 )</th>
<th>Larger: ( \bar{A} = 75 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - ( \theta )</td>
<td>b [t]</td>
<td>b [t]</td>
</tr>
<tr>
<td>0.454 [3.16]</td>
<td>0.626 [3.12]</td>
<td>0.395 [3.24]</td>
</tr>
<tr>
<td>( \rho_1 - \rho_0 )</td>
<td>b [t]</td>
<td>b [t]</td>
</tr>
<tr>
<td>0.098 [2.72]</td>
<td>0.111 [2.60]</td>
<td>0.086 [2.79]</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>b [t]</td>
<td>b [t]</td>
</tr>
<tr>
<td>0.600 [12.69]</td>
<td>0.550 [13.07]</td>
<td>0.638 [12.00]</td>
</tr>
<tr>
<td>( \sigma_e )</td>
<td>b [t]</td>
<td>b [t]</td>
</tr>
<tr>
<td>4.335 [5.52]</td>
<td>4.242 [5.50]</td>
<td>4.404 [5.54]</td>
</tr>
<tr>
<td>( k )</td>
<td>b [t]</td>
<td>b [t]</td>
</tr>
<tr>
<td>( r )</td>
<td>b [t]</td>
<td>b [t]</td>
</tr>
<tr>
<td>0.110 [8.81]</td>
<td>0.109 [8.82]</td>
<td>0.111 [8.79]</td>
</tr>
<tr>
<td>( P_r )</td>
<td>b [t]</td>
<td>b [t]</td>
</tr>
<tr>
<td>35.226 [7.31]</td>
<td>35.500 [7.25]</td>
<td>34.938 [7.34]</td>
</tr>
</tbody>
</table>

As shown above, the choice of \( \bar{A} \) really only affects our estimate of \( k \)—which controls the elasticity of firm’s investment response. The reason is that our estimate of \( k \) is highly sensitive to \( \beta(I_t/Q_t | P_t) \), the slope from a regression of \( I_t/Q_t \) on \( P_t \). Since

\[ \beta(I_t/Q_t | P_t) \approx \frac{1}{kQ_t} = \frac{1}{k \left( \bar{A} - C - (r + \delta)P_t \right) / B}, \]
a higher assumed level for $\bar{A}$ translates into a lower estimate of $k$. Fortunately, the key behavioral parameters we emphasize, namely $(1-\theta)$ and $(\rho_f - \rho_0)$, are not very sensitive to moderate changes in $k$ induced by choosing different values of $\bar{A}$.

**D.2. Model-implied impulse response functions**

Figures D1, D2, and D3 show the model-implied impulse response functions allowing for competition neglect only, allowing for demand over-extrapolation only, and allowing for both biases, respectively.
Figure D1
Model-Implied Impulse Response Functions: Competition Neglect Only

This figure shows the model-implied impulse response functions following a one-time shock to demand. The figure corresponds to the estimates in column (2) of Table VI, which allows for competition neglect ($\theta < 1$), but does not allow for demand over extrapolation (we impose $\rho_f = \rho_0$). Following an eight unit demand shock at $t = 1$, the figures contrast the impulse response under rational expectations (imposing $\theta = 1$) with the impulse response anticipated by firms who suffer from competition neglect and the actual impulse respond under competition neglect.
This figure shows the model-implied impulse response functions following a one-time shock to demand. The figure corresponds to the estimates in column (3) of Table VI which allows for demand over extrapolation ($\rho_f > \rho_0$), but does not allow for competition neglect (we impose $\theta = 1$). Following an eight unit demand shock at $t = 1$, the figures contrast the impulse response under rational expectations ($\rho_f = \rho_0$) with the impulse response anticipated by firms who suffer over-extrapolation demand and the actual impulse response under demand over-extrapolation.
This figure shows the model-implied impulse response functions following a one-time shock to demand. The figures corresponds to the estimates in column (4) of Table VI which allows for both competition neglect ($\theta < 1$) and demand over extrapolation ($\rho_f > \rho_0$). Following an eight unit demand shock at $t = 1$, the figures contrast the impulse response under rational expectations ($\rho_f = \rho_0$ and $\theta = 1$) with the impulse response anticipated by firms who suffer from both biases and the actual impulse response when firms suffer from both biases.