Coordination and Signal Design: The Electronic Mail Game in Asymmetric and Multiplayer Settings

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Abstract

We examine the nature of the coordination failure in Rubinstein’s Electronic Mail Game, a classic game-theoretic paradox in which parties are unable to coordinate on a mutually beneficial action, and we propose a principle for designing more successful signaling protocols. When signaling technology is such that signals fail to reach their destination with small probability, it is important that the signaling protocol be designed so that when there is signal failure, the parties largely agree on where the failure took place. This agreement over information in turn allows the parties to coordinate their actions.

To illustrate this principle, we show that if one party is able to send a stronger signal than the other, the asymmetry leads to agreement over the source of signal failure, and hence permits coordination. We then extend the model to multiplayer settings. If a central party is communicating separately with multiple parties, or if a central party is communicating with a group via correlated signals, an asymmetry in signal strength arises naturally from the many-to-one nature of the model and the coordination problem is again resolved.

Keywords: Electronic Mail Game, Signaling, Asymmetry, Multiplayer

JEL Class: D83, C72

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1 Introduction

Both logic and basic human experience suggest that increased communication provides a means for parties to better communicate. Indeed, in businesses, team athletics, military operations, and countless other areas, shared information is vital for success. In particular, when two or more parties are trying to coordinate to take advantage of a mutually beneficial opportunity but lack common knowledge, it would seem natural that any shared information about the opportunity would allow for better coordination.

Yet Rubinstein [7] and others have provided examples where despite detailed communication, coordination fails. Fundamental to these examples is the specific form of communication used. Paradoxically, some arrangements that involve less accurate sharing of information allow parties to coordinate, while more accurate arrangements do not. In this paper, we argue that the design of the communication arrangement is just as important for coordination as the accuracy of the information transmitted. Specifically, we propose a general principle for coordination games in which communication takes place via a signaling technology in which signals fail to reach their destination with small probability. In such arrangements, it is important that the signaling protocol be designed so that when there is signal failure, the parties largely agree on where the failure took place. This agreement over information received by the parties translates to agreement over expectations about which actions will be played, which in turn allows the parties to coordinate.

One way to achieve this agreement over the source of signal failure is via asymmetric signal strength. Suppose there are multiple signals transmitted in a communication arrangement, but a signal over one of the paths is more likely to fail than are others. Then when a signal does in fact fail, the parties place higher probability on this weaker signal being the source of failure. In the setting of the Electronic Mail Game, we will see that introducing this asymmetry allows for the parties to coordinate, while in symmetric arrangements, even the most accurate of signals does not.

This principle also applies in multiplayer settings. In a hub-and-spoke version of the Electronic Mail game, at the surface, any of a variety of signaling protocols seem to be logical communication choices. However, we will show that some of the protocols lead to coordination, while others do not. In the successful protocols, the signal design translates the many-to-one nature of the model to a source of asymmetry, so that just as in the two player asymmetric game, players agree on the
To illustrate the difficulty of coordinating action in settings with imperfect communication technology, we describe the classic “coordinated-attack problem” (see for example Gray [3]), which is the basis for the Electronic Mail Game.

Two generals sit on opposite sides of an enemy. If both generals attack, victory is certain. However, if only one general attacks, his army suffers devastating losses. The two generals are not in direct contact with each other and can only communicate by sending a messenger through the enemy’s position to the other side. One general sends a messenger proposing an attack, but there is a small probability that the messenger will be captured and will not make it through the enemy’s lines. Since the first general cannot be assured of the messenger’s success, the second general sends a messenger back, confirming the receipt of the message. But still the generals cannot be certain of a coordinated attack, because the second general does not know if the first general received the confirmation. The first general again sends a messenger, confirming the receipt of the confirmation. The generals continue to send messengers back and forth until some messenger fails to make it through. Because only a finite number of messages can be relayed before a messenger is captured, the proposal of an attack can never be common knowledge. For the proposal to be common knowledge, it must be true that for all integers $n > 0$,

$$(\text{everybody knows that})^n \text{ an attack has been proposed.}$$

While the attack proposal never will become common knowledge, perhaps the generals may eventually determine that enough information has been relayed to justify attacking, even when the final messenger fails to come through. Intuition suggests that after a large number of messages have been relayed, the generals should each be relatively sure that the other will coordinate on the proposed attack. However, Rubinstein’s Electronic Mail Game (Section 2) provides an example where regardless of the number of messages received, coordination is never possible.

Paradoxically, if the generals’ options are limited so that only the initial messenger can be sent, coordination is possible. The first general sends the messenger, evaluates the expected benefit from attacking (which should be high if the probability of the messenger getting through is high, and the second general always attacks upon receipt of the message), and compares this to the
value of the no attack option. This limited signaling protocol can allow for coordination while the more complete back-and-forth signaling protocol suggested above precludes it.

This one-way signal is precisely the extreme version of the asymmetry we will look at in Section 2, and the rationale behind the coordination in this simple case is similar to that in the asymmetric and multiplayer settings. Both parties agree on the likely source of signal failure - that the signal made it though to the other side and did not make it back (the probability of coming back is zero!) Hence, they can coordinate on attacking.\(^2\)

Several variations of Rubinstein’s model have been examined in the literature, and in most cases, these papers focus on explaining the central paradox by questioning the model’s underlying assumptions. Binmore and Samuelson [1] examine costly and voluntary signaling. Morris [6] introduces a time element, and also examines multiplayer settings where certain subsets of the players meet together and exchange information. Dimitri [2] considers a version where a third party learns the state of nature and informs both generals with noise. De Jaegher [5] examines a model in which message confirmations are strategic, rather than automatic. These papers, as well as several others in the literature, address “unrealistic” elements of Rubinstein’s game and resolve the paradox by modifying certain assumptions. In contrast, rather than examining the assumptions, we accept that poor signaling protocol choice can indeed lead to mis-coordination. We focus on the importance of design, and demonstrate how designing protocols that adhere to the principle of agreement over fault can lead to behavior that permits coordination.

After reviewing the Electronic Mail Game in Section 2, in Section 3 of this paper we demonstrate how an asymmetric signaling protocol allows for coordination, and why this is the case. In Section 4 we show that Rubinstein’s “no coordination result” extends to a multiplayer game involving a central party and \(n\) peripheral parties, while in Sections 5 and 6 we examine multiplayer versions of the game where coordination is possible, first when the central party coordinates separately with each of the \(n\) peripheral parties, and then when the central party communicates with the peripheral parties using correlated signals. Section 7 concludes.

\(^2\)The first general attacks if the messenger is sent, and the second general attacks if a message is received. Note that in this example, as in all examples, there will be some probability of mis-coordination, which occurs when the stronger message (in this case the only message) fails to make it through.
Rubinstein’s Electronic Mail Game

The Electronic Mail Game, due to Rubinstein [7], is a game-theoretic representation of the coordinated-attack problem.

Two players, 1 and 2, are playing a coordination game. There are two possible states of the world, $a$ and $b$. States $a$ and $b$ occur with probability $1 - p$ and $p$ respectively, where $p < 1/2$. Thus, $a$ is the more likely state. For each player, there are two possible actions, $A$ and $B$. The payoff matrices for states $a$ and $b$ are as follows:

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Notice that if the players are able to coordinate their actions and the state is “right,” each receives payoff $M$. Coordination when the state is “wrong” yields a payoff of 0. It is lack of coordination that is costly. If the players fail to coordinate, it is the player who played $B$ who must pay $L$, where $L > M$. Thus, playing $B$ is risky if a player is unsure that his opponent will also play $B$.

If no player has any information about the state, the best outcome the players can achieve is a payoff for each of $(1 - p)M$, achieved when both players choose $A$.

But in this game, player 1 learns the state of nature. An automatic communication system is in place that works as follows: If the state of nature is $a$, no communication takes place. If the state of nature is $b$, an email message (which we also refer to as signal) is automatically sent from player 1 to player 2 notifying him of this. However, the line of communication is flawed. There is an $\varepsilon > 0$ chance that any particular email message does not get through. To verify the receipt of messages, both players’ email systems are set up to automatically send email confirmations when any message (including any confirmation) is received. Thus, when the state of the world is $b$, confirmations are automatically sent back and forth between the players until some confirmation.
does not make it through. At this point, communication between the players stops, and each player’s screen displays the number of messages his machine has sent. Let $T_i$ indicate the number on player $i$’s screen.

When communication ends, each player must decide between actions $A$ and $B$. Notice that the email process is automatic, so this decision is the only strategic choice made by the players. A strategy for player $i$ consists of a choice from $\{A, B\}$ for each value of $T_i$. Let $S_i(t)$ indicate player $i$’s action if $t$ messages have been sent.

Notice that when the state is $b$, the successful receipt of the first message by player 2 is described by the statement $K_2K_1b$, that is, 2 knows that 1 knows that the state is $b$. The receipt of a further message by 1 is described by $K_1K_2K_1b$, and so on. Because only a finite number of messages will ever be sent, the state of nature will never become common knowledge.

When the communication ends, player 1 faces uncertainty when his machine displays any $T_1 > 0$. In this game of incomplete information, we use the approach due to Harsanyi [4]. Player 1 is unsure if communication stopped because player 2 did not receive his final message, in which case $T_2 = T_1 - 1$, or if player 2 did receive player 1’s final message and the confirmation didn’t make it back, in which case $T_2 = T_1$. If player 2 were planning to choose different actions under these two circumstances, then the relative likelihoods of these circumstances becomes crucial. Given that player 1 has sent $T_1 > 0$ messages, he assigns the following conditional probabilities:

$$T_2 = \begin{cases} 
  T_1 - 1 \text{ with probability } \frac{1}{2-\varepsilon}, \\
  T_1 \text{ with probability } \frac{1-\varepsilon}{2-\varepsilon}.
\end{cases}$$

Thus, it is slightly more likely that player 2 did not receive the final message.

Player 2 likewise calculates conditional probabilities. Notice that when player 2 sees $T_2 = 0$, the uncertainty is whether the first message from 1 failed to make it through, or if the state of nature is actually $a$, in which case no message was ever sent.

Intuitively, when values of $T_1$ are large, one might think that because so many confirmations have been sent regarding the state of nature being $b$, $B$ would be a safe action. Yet Rubinstein shows the following remarkable result:

**Proposition 1.** There is only one Nash Equilibrium in which player 1 plays $A$ in the state of
nature a. In this equilibrium, the players play A independently of the number of messages sent.

Thus, despite the potentially many confirmations and depth of knowledge of this situation, the players are never able to coordinate on the action B in state b. (Note that the theorem excludes the uninteresting case of coordination on B independently of state.) So surprisingly, as a means of promoting coordination, this elaborate communication system is no better than no communication at all.

Why is this the case? How might we alter the communication so that the players can coordinate? These questions are addressed in the remainder of the paper.

3 Asymmetric Signal Strength

In the introduction, we mentioned the intuitive “cutoff” strategies to address the coordination problem. We define a cutoff strategy with cutoff N as follows: If the number of messages sent is greater than some integer N, play B; otherwise play A. Why is there no Nash Equilibrium in cutoff strategies? Roughly stated, no player wishes to have the earlier cutoff.

Let us suppose that player 2 has a cutoff of $N_2$; that is, $S_2(T_2) = B \quad \forall \ T_2 \geq N_2$ and $S_2(T_2) = A \quad \forall \ T_2 < N_2$. Then player 1’s best response is to use a cutoff strategy with cutoff $N_1 = N_2 + 1$. Why is this? When $T_1 \geq N_2 + 1$ or $T_1 < N_2$, there is no uncertainty about player 2’s action, so player 1 will play B and A in these cases respectively.

When $T_1 = N_2$, that is, under the contingency that player 1 has sent exactly $N_2$ messages, he is unsure if player 2 received his final message or if it was lost en route. He assigns conditional probability $\frac{1}{2-\epsilon} \ (\approx 1/2)$ to player 2 not having received the final message (and thus playing A) and conditional probability $\frac{1-\epsilon}{2-\epsilon} \ (\approx 1/2)$ to player 2 having received the final message (and thus playing B.) Player 1 choosing B in this contingency would yield payoffs of $M$ and $-L$ each about half the time, and since $L > M$, the safe, zero payoff action A is preferable.

Thus, player 1 wishes to have a later cutoff than 2. An analogous argument shows that player 2 wishes to have a later cutoff than player 1, so no cutoff Nash Equilibrium is possible. However, if the communication structure were such that one of the players preferred a later cutoff while the
In Rubinstein’s communication arrangement, whenever communication stops and there is uncertainty about which action the opponent will play, the symmetry of the signal failure rates and the low probability of signal failure lead each player to believe that there is approximately a 50-50 split as to whether the other player received the final message. Each reacts to this scenario by choosing the safe action \( A \). If, however, a player believes that it is more likely that the opponent did receive the final message, then in this uncertain contingency, \( B \) would be the best action. If both players held these beliefs, then each player would prefer the earlier cutoff, and we would be no better off than before. Our goal is to create a setting where only one of the players prefers the earlier cutoff.

Therefore, let us consider the situation where player 1’s signals break down with lower likelihood than those of player 2; that is a signal from 1 is \textit{stronger} than a signal from 2. Let \( \mu \) be the probability that a message sent by player 1 does not make it through, while \( \varepsilon > \mu \) is the probability that a message sent by 2 is not received. Call \( 1 - \mu \) and \( 1 - \varepsilon \) the \textit{signal strengths} of 1 and 2, respectively.
respectively. We refer to this game as the Asymmetric Electronic Mail Game. We have the following proposition:

**Proposition 2.** *In the Asymmetric Electronic Mail Game, if \( \varepsilon > \frac{\mu \cdot L}{1 - \mu M} \), then there exist Nash equilibria where players 1 and 2 use cutoff strategies with \( N_1 = N_2 > 0 \).*

Surprisingly, if we compare the situation in which the signaling strengths are equal to the situation in which one party’s signaling strength is greatly reduced, the latter represents a less accurate form of information sharing. Yet only in the second case is coordination possible. Why is this the case? It should be observed that the high number of messages sent resulting from the accurate signal will still not solve the coordination problem. Despite the accurate signal, at some point a message will be lost. Coordination at this point is essential. The asymmetric signaling arrangement allows for this, while the symmetric arrangement does not.

Crucially, it is due to the agreement by the parties over where the signal likely broke down that allows them to coordinate. To see this, suppose that after signals have been sent back and forth, player 1 sees \( N \) on his screen; that is, he has sent exactly \( N \) messages. Since 1’s signal is stronger, he finds it likely that player 2 has also sent \( N \) messages - he believes this to have happened with probability \( \frac{(1 - \mu)^\varepsilon}{\mu + (1 - \mu)^\varepsilon} \). Furthermore, when 2 has sent exactly \( N \) messages, by virtue of his weaker signal, he finds it likely that 1 has also sent \( N \) signals. This he believes to have happened with probability \( \frac{\varepsilon}{\varepsilon + (1 - \varepsilon)\mu} \). Hence, in equilibrium, the players are happy to play a strategy in which they coordinate on the same action upon receipt of \( N \) messages, for every \( N \). That is, it is exactly this ex-ante agreement over information held that allows the players to coordinate actions in equilibrium.

Notice that while Proposition 2 describes equilibria that involve coordination, there will still be some instances of mis-coordination, namely when the signal breaks down in the stronger direction, precisely before player 2’s cutoff.\(^3\) But the crucial argument here is that even in this scenario, the players believe they agree on the source of signal failure and hence can coordinate in equilibrium.

Notice also that the asymmetry that yields the Nash Equilibria need not be present in every message sent. For example, it is enough that any pair of consecutive signals be sufficiently different

\(^3\) Using an argument similar to a proof in Gray \([3]\), it can be shown that in the class of signaling mechanisms that involve a finite number (in expectation) of imperfect signals, no mechanism can achieve perfect coordination in both states of the world.
in strength to allow for existence of a cutoff equilibrium, as described in the following Corollary:

**Corollary 3.** Let the $N$th messages sent by players 1 and 2 reach their destinations with probability $\mu$ and $\varepsilon$ respectively where $\varepsilon > \frac{\mu L}{1-\mu M}$. Then there is a cutoff Nash Equilibrium where $N_1 = N_2 = N$.

The logical extreme of this situation is $\varepsilon = 1$. This describes the situation in which player 2 is unable to send the $N$th message; that is, if player 2 receives his $N$th message, communication stops. This scenario is mentioned in Rubinstein (1989) and is a special case of the Asymmetric Electronic Mail Game.

**4 The Multiplayer Electronic Mail Game**

We now turn to coordination among three or more parties. We consider a centralized, hierarchical structure (Figure 2) where one party sends email to $n$ peripheral parties, who send email back to the central party, and so forth. Following the language of the coordinated-attack problem, this setup can be likened to a general and his $n$ lieutenants.

![Figure 2: Signaling Structure in the Multiplayer Setting](image)

To create an information structure parallel to that in Rubinstein's original game, we assume that the central party, who we shall call player 1, learns of the state of nature, either $a$ or $b$. If the state of nature is $b$, an email message (signal) is automatically sent to each of the $n$ peripheral players.
Any particular message arrives at its destination with probability \(1 - \varepsilon\), and the probabilities are independent. Whenever any peripheral party receives a message, a confirmation is automatically sent back to the central party. These messages again are independent and fail with probability \(\varepsilon\). Whenever the central party receives confirmations from all peripheral parties, confirmations are sent back to all. If, however, the central party receives only an incomplete subset of the \(n\) possible confirmations, confirmations are sent back to none.

Note that in terms of levels of knowledge, this information structure preserves the spirit of the original game. That is, when a player’s machine displays \(T_i = N\), then \(K_i(K_E)^{N-1}b\).

The peripheral players choose actions from \(\{A, B\}\) based on the number \(T_i\) of messages their machine has sent. The central party also chooses an action from \(\{A, B\}\), but this action is contingent on the number of messages his machine has sent, and the number \(m < n\) of messages he receives in the final communication. Note that it is exactly when \(m < n\) that communication stops. For \(i = 2 \ldots n+1\), let \(S_i(T_i) \in \{A, B\}\) indicate player \(i\)’s action when his machine has sent \(T_i\) messages. Let \(S_1(T_1, m)\) indicate player 1’s action when his machine has sent \(T_1\) messages and \(m < n\) messages triggered the communication breakdown. Note that \(T_1 = 0\) is a special case. When the state of nature is \(a\), player 1’s action is not contingent on the number of failed messages triggering a breakdown. His action under these circumstances is given by \(S_1(0, *) \in \{A, B\}\).

We also create a payoff structure parallel to that in Rubinstein’s original game. If all players coordinate and the state matches the action, then all receive payoff \(M\). If they coordinate but the state is “wrong,” then all receive payoff 0. If they fail to coordinate, then the players who chose \(A\) receive a payoff of 0, while the players who chose \(B\) receive a payoff of \(-L\). Thus, it is again risky to play \(B\) unless you are confident that your opponents will also all play \(B\).

We refer to this setting as the Multiplayer Electronic Mail Game. Here we have the following result:

**Proposition 4.** In the Multiplayer Electronic Mail Game, there is only one Nash equilibrium in which player 1 plays \(A\) in the state of nature \(a\). In this equilibrium, the players play \(A\) independently of the number of messages sent.

Both the paradox and the explanation for the paradox extend to this version of the multiplayer game. When all parties have large values for \(T\) displayed on their machines, everyone has a
great depth of knowledge of the state of nature. Nevertheless, no cutoff point is acceptable for an equilibrium, because each player would wish to lower his cutoff point in response. When the central party receives \( n - 1 \) signals, the most likely end state, he will find it nearly equally likely that the peripheral player from whom no signal arrived, did or did not send the final signal. Hence, at the cutoff, the central party will respond conservatively by playing \( A \). The peripheral player, upon failure to receive a signal, will assume that it was one of the other \( n - 1 \) parties’ signals that failed, and so in response to the uncertain action of the central party, he will also prefer the conservative action \( A \). Both parties perceive their signals as ‘weaker,’ prefer the later cutoff, and hence no equilibrium involving coordination on \( B \) in state \( b \) and \( A \) in state \( a \) exists.

In stark contrast to this result, in the following two sections we will describe how altering first the signaling protocol, and then the payoff structure can restore the coordination result of the Asymmetric Electronic Mail Game. In both cases, the signaling structure generates an asymmetry in signal strength that allows the parties to agree on the source of signal failure, and hence allows the parties to coordinate.

5 Multiplayer Electronic Mail Game: Correlated Signals

Until now we have assumed that messages are independent. For many signaling technologies, this is a plausible assumption. If the general’s messengers all set out separately, we might expect no correlation in their success rates. But for any technology where the failure stems from the signal source, rather than from the channels, we might expect correlation in success rates. That is, the central party’s signal will either be effective and reach all peripheral parties, or will be ineffective and reach none.\(^4\)

In this section, we consider the multiparty model of Section 4, but with correlated signals. We preserve the independence of the messages sent from the peripheral parties to the central party, reflecting that return signals emanate from separate sources, so their success rates should not be correlated. Specifically, when player 1 sends a message - either the original message, or any

\(^4\)There are several scenarios where this is plausible. Signals might fail to go out based on failure of the sending technology, such as a faulty router, radio antenna, etc. Or else the central party might be sending encoded signals where the encoding technology fails with probability \( \varepsilon \) in a manner such that no recipient is able to decode and receive the message.
confirmation - the message will reach all parties with probability \(1 - \varepsilon\) and will reach no parties with probability \(\varepsilon\). Messages from peripheral players independently fail to reach their destination each with probability \(\varepsilon\). Again, a confirmation is sent by player 1 only when the messages from all \(n\) peripheral parties have been received. If 1 receives an incomplete subset of the \(n\) messages, no confirmation is sent and communication stops. We refer to this setting as the Multiplayer Electronic Mail Game with Correlated Signals.

Under this signaling arrangement, the players are able to coordinate actions using strategies akin to the cutoff strategies in the Asymmetric Electronic Mail Game. For player 1, define a cutoff strategy with cutoff \(t\) and threshold \(\bar{m}\) as follows:

\[
S_1(T_1, m) = \begin{cases} 
B & \text{for } T_1 \geq t + 1 \\
B & \text{for } T_1 = t, m \geq \bar{m} \\
A & \text{for } T_1 = t, m < \bar{m} \\
A & \text{for } T_1 < t.
\end{cases}
\]

That is, provided more than \(t\) messages are sent, or if exactly \(t\) messages are sent, and at least \(\bar{m}\) messages were received in the final round of communication, 1 plays \(B\). Otherwise 1 plays \(A\). Proposition 5 states that in the Multiplayer Electronic Mail Game with Correlated Signals, players can coordinate using cutoff strategies:

**Proposition 5.** In the Multiplayer Electronic Mail Game with Correlated Signals, \(\forall t > 0\), there exists a Nash Equilibrium where each peripheral player \(i\) uses a cutoff strategy with cutoff \(t\), and player 1 uses a cutoff strategy with cutoff \(t\) and threshold \(1\).

Why are the players now able to coordinate on action \(B\), but were unable to do so in the multiplayer setting of Section 4? Consider the uncertainty player \(i\) faces when his machine has sent \(T_i\) messages. He knows that the communication breakdown could have resulted from one or more peripheral players’ signals not making it through (including possibly his own), and that it is much less likely that all \(n\) signals made it through and in fact it was player 1’s confirmation that was lost. Thus, in a sense player \(i\) is sending a weaker signal, yielding a form of asymmetry. On the other hand consider player 1’s decision when his machine has sent \(T_i\) messages and has received at least one confirmation in the final round. Because 1’s signals are correlated, 1 can be sure that his signal made it through to at least one player, and hence to all other players - a
perfectly strong signal. Knowing his counterparts received the final message, he can safely play the risky action $B$, and coordinate with the other players.

6 Multiplayer Electronic Mail Game: Pairwise Coordination

We now examine another modified multiplayer setting. Continuing with the general/lieutenant analogy, consider a general who must take an action and would like to coordinate with his $n$ lieutenants. His action may coincide with that of some lieutenants but not with that of others. The successful coordination with some of the lieutenants yields gains, while the failed coordination with the others may incur losses. The merit of the action can then be assessed as the gains net of the losses.

From the multiplayer framework introduced in Section 4, we preserve the information structure and signaling protocol, altering only the payoffs. In this setting, we have $n$ separate pairs coordinating, each involving the central party and one of the $n$ peripheral parties. The payoffs to each pair are the same as those in the original Rubinstein two player game. Specifically, consider the actions chosen by player 1 (the central party) and peripheral player $i$. If the players coordinate and the action matches the state, each receives payoff $M$. If they coordinate and the state is “wrong”, then each receives 0. If they fail to coordinate, then the player who chose $A$ receives 0, while the player who chose $B$ receives $-L$. For player $i$, this is all he will receive. Player 1’s total payoff is the sum of the payoffs from each pairwise coordination effort. We refer to this setting as the Multiplayer Electronic Mail Game with Pairwise Coordination.

Notice that now player 1 may be willing to take the action $B$ if he expects only a subset of the $n$ peripheral players to play $B$. The peripheral players, however, will only play $B$ if it is likely that player 1 will also play $B$.

To prove the result in this section, we need two technical conditions.

*Condition 1.* $M(n - 1) - L > 0$

Condition 1 has the following interpretation: If player 1 coordinates with all the peripheral
players except one, this yields a positive payoff. This represents the plausible restriction that the loss \( L \) is not too large relative to \( M \). Notice that regardless of \( M \) and \( L \), choosing \( n \) large enough satisfies the condition.

**Condition 2.** \( \frac{\varepsilon(1-\varepsilon)^n + \varepsilon(1-\varepsilon)^{n-1}}{1-(1-\varepsilon)^{n+1}} > \frac{L}{L+M} \)

Condition 2 describes that from the perspective a peripheral player, when communication breaks down, it is sufficiently unlikely that multiple messages were lost in the final communication. This holds, for example, when \( \varepsilon \) is sufficiently small.

We have the following proposition:

**Proposition 6.** In the Multiplayer Electronic Mail Game with Pairwise Coordination, \( \exists \bar{m} < n \) such that \( \forall t > 0 \), there exists a cutoff Nash equilibrium in which players 2 . . . n use a cutoff strategy with cutoff \( t \), and player 1 uses a cutoff strategy with cutoff \( t \) and threshold \( \bar{m} \).

We outline specifically the players’ strategies. For player \( i \), a cutoff strategy with cutoff \( t \) is described by

\[
S_i(T_i) = \begin{cases} 
B & \text{for } T_i \geq t \\
A & \text{for } T_i < t. 
\end{cases}
\]

Player 1’s cutoff strategy with cutoff \( t \) and threshold \( \bar{m} \) is described by

\[
S_1(T_1, m) = \begin{cases} 
B & \text{for } T_1 \geq t + 1 \\
B & \text{for } T_1 = t, m \geq \bar{m} \\
A & \text{for } T_1 = t, m < \bar{m} \\
A & \text{for } T_1 < t. 
\end{cases}
\]

Again we must ask why the players can coordinate on playing \( B \) here but not in the multiplayer setting of Section 4? When communication stops, the peripheral player is uncertain if his final message made it through. However, he knows that it is highly likely that most of the final messages from the peripheral players to the central player did in fact made it through. Given that at least one message failed, it is highly unlikely that two messages failed. Hence, in a sense, the peripheral players send a “stronger” signal than the outgoing signal from the central party, because multiple
signals need to break down for the central party to miss his threshold. Upon sending \( t \) messages, each peripheral player \( i \) is confident that enough messages have gotten through to trigger an attack by player 1, so the peripheral players are content with an earlier cutoff. Just as in section 3, it is the asymmetry that causes the peripheral parties to prefer this earlier cutoff (and the central party to prefer a later cutoff), thus permitting coordination in equilibrium.

7 Conclusion

We have examined Rubinstein’s Electronic Mail Game and extended it to several settings. But more importantly, we have examined the source of coordination failure, and have proposed a general principle for designing signal protocols to avoid this failure. Namely, for signaling technologies in which signals fail with small but positive probability, protocols in which parties largely agree on the source of signal failure permit coordination in equilibrium. One way to achieve this agreement is to focus on protocols that involve asymmetric signal strength. Importantly, asymmetric signal strength can resolve the coordination problem where highly accurate yet symmetric signals fail. We illustrate this principle in the Asymmetric Electronic Mail Game, where we establish conditions for the existence of cutoff Nash Equilibria. In a multiplayer setting, we showed that the counter-intuitive, no-coordination result extends and that logic similar to that in the two player setting explains this paradox. However, the Multiplayer Electronic Mail Game with Correlated Signals and Multiplayer Electronic Mail Game with Pairwise Coordination introduce a form of asymmetry that, in a manner analogous to that of the two player game, again permits coordination. We emphasize that while accuracy of signal technology can be helpful in ensuring coordination, equal or greater care should be given to the choice of communication arrangement, as subtle differences may determine whether coordination among the parties is achievable, or whether there can be no coordination at all.

8 Appendix

Proof of Proposition 2:

Suppose player 2 uses a cutoff strategy with \( N_2 > 0 \). That is, \( S_2(T_2) = B \quad \forall \quad T_2 \geq N_2 \) and
\( S_2(T_2) = A \ \forall \ T_2 < N_2 \). In the contingencies \( T_1 < N_2 \), player 2 will certainly take action \( A \), so player 1 must take action \( A \). That is, \( S_1(T_1) = A \ \forall \ T_1 < N_2 \). In the contingencies \( T_1 > N_2 \), player 2 will certainly play \( B \), so we must have \( S_1(T_1) = B \ \forall \ T_1 > N_2 \). If \( T_1 = N_2 \), then in this contingency, player 1 assigns conditional probability \( \frac{\mu}{\mu + (1-\mu)\varepsilon} \) to \( T_2 = N_2 - 1 \) and conditional probability \( \frac{(1-\mu)\varepsilon}{\mu + (1-\mu)\varepsilon} \) to \( T_2 = N_2 \). Thus, playing \( B \) will yield a payoff for player 1 of \( \frac{\mu}{\mu + (1-\mu)\varepsilon} (-L) + \frac{(1-\mu)\varepsilon}{\mu + (1-\mu)\varepsilon} (M) \). This is better than the safe, zero payoff from playing \( A \) exactly when \( \varepsilon > \frac{\mu}{1-\mu} \frac{L}{M} \). Thus, player 1’s best response to a cutoff strategy with cutoff \( N_2 \) is to play a cutoff strategy with cutoff \( N_1 = N_2 \).

Suppose player 1 uses a cutoff strategy with \( N_1 > 0 \). In the contingencies \( T_2 < N_1 - 1 \), player 1 will certainly take action \( A \) (refer to Figure 1 to verify this), so player 2 must take action \( A \). That is, \( S_2(T_2) = A \ \forall \ T_2 < N_1 - 1 \). In the contingencies \( T_2 \geq N_1 \), player 1 will certainly play \( B \), so we must have \( S_2(T_2) = B \ \forall \ T_2 \geq N_1 \). If \( T_2 = N_1 - 1 \), then in this contingency, player 2 assigns conditional probability \( \frac{\varepsilon}{\varepsilon + (1-\varepsilon)\mu} \) to \( T_1 = N_1 - 1 \) and conditional probability \( \frac{(1-\varepsilon)\mu}{\varepsilon + (1-\varepsilon)\mu} \) to \( T_1 = N_1 \). Thus, playing \( B \) will yield a payoff for player 1 of \( \frac{\varepsilon}{\varepsilon + (1-\varepsilon)\mu} (-L) + \frac{(1-\varepsilon)\mu}{\varepsilon + (1-\varepsilon)\mu} (M) \). This is better than the safe, zero payoff from playing \( A \) exactly when \( \mu > \frac{\varepsilon}{1-\varepsilon} \frac{L}{M} \). But since \( \varepsilon > \mu \), this is never the case. Therefore \( S_2(N_1 - 1) = A \). Hence player 2’s best response to a cutoff strategy with cutoff \( N_1 \) is to play a cutoff strategy with cutoff \( N_2 = N_1 \). \( \square \)

Proof of Proposition 4:

Let \((S_1, \ldots, S_{n+1})\) be a Nash Equilibrium such that \( S_1(0, \ast) = A \). We will show by induction on \( t \) that \( S_1(t, m) = S_i(t) = A \ \forall t, m = 0 \ldots n - 1, i = 2 \ldots n + 1 \).

If peripheral player \( i \) has \( T_i = 0 \), then he did not receive any message. It could be that the state of nature is \( a \) and no message was sent (this occurs with probability \( 1 - p \)), or that a message was sent and was lost en route (probability \( p\varepsilon \)). In the first case, by assumption, player 1 plays \( A \). If player \( i \) plays \( A \), then regardless of what \( S_1(1, m) \) is, his expected payoff is at least \( \frac{M(1-p)}{1-p+p\varepsilon} \). If he plays \( B \), his payoff is at most \( \frac{L(1-p)+Mp\varepsilon}{1-p+p\varepsilon} \). Comparing these, we see that it is optimal for player \( i \) to play \( A \); that is, \( S_i(0) = A \ \forall i = 2 \ldots n + 1 \).

Now suppose that we have shown that in equilibrium, for all \( T < t \), \( S_1(T, m) = S_i(T) = A \ \forall m = 0 \ldots n - 1, i = 2 \ldots n + 1 \). Assume \( T_1 = t \). Then player 1 has sent \( t \) messages, but did not receive a confirmation from one or more players. Let \( i \) designate one of these players. Player
1 assigns the following conditional probabilities:

\[
T_i = \begin{cases} 
  t - 1 & \text{with probability } \frac{1}{2-\varepsilon} \\
  t & \text{with probability } \frac{1-\varepsilon}{2-\varepsilon}
\end{cases}
\]

When \( T_i = t - 1 \), \( i \) plays \( A \). Thus, if player 1 chooses \( B \), he gets at most \( M \frac{1-\varepsilon}{2-\varepsilon} - L \frac{1-\varepsilon}{2-\varepsilon} \). If he chooses \( A \), he receives 0. Because \( L > M \) and \( \frac{1}{2-\varepsilon} > \frac{1-\varepsilon}{2-\varepsilon} \), his best option is \( A \). Thus, \( S_1(t, m) = A \ \forall \ m = 0 \ldots n - 1 \).

Assume \( T_i = t \) for some peripheral player \( i \). Then player \( i \) is uncertain if player 1 received all \( n \) messages (\( T_1 = t + 1 \)), or if at least one message failed to make it through (\( T_1 = t \)). Player \( i \) assigns the following conditional probabilities:

\[
T_1 = \begin{cases} 
  t & \text{with probability } z \equiv \frac{1-(1-\varepsilon)^n}{1-(1-\varepsilon)^n + \varepsilon(1-\varepsilon)^n} \approx \frac{n}{n+1} \\
  t + 1 & \text{with probability } 1 - z 
\end{cases}
\]

When \( T_1 = t \), 1 plays \( A \). Then regardless of what 1 does when \( T_1 = t + 1 \), if \( i \) chooses \( B \), his payoff is at most \(-Lz + M(1 - z)\). If he chooses \( A \), his payoff is 0. Because \( L > M \) and \( z > \frac{1}{2} \), \( A \) is his best option. That is, \( S_i(t) = A \ \forall \ i = 2 \ldots n + 1 \). □

**Proof of Proposition 5:**

We show first that player 1’s strategy is a best response to those of players 2\ldots n + 1.

Suppose player 1’s machine has sent \( T_1 \) messages, with \( m < n \) messages received in the final communication. If \( T_1 > t \), then \( T_i \geq t \) for each \( i \), so the peripheral players all play \( B \). Thus, \( B \) is the optimal action. If \( T_1 < t \), then \( T_i < t \) for each \( i \), so the peripheral players play \( A \). Thus, \( A \) is 1’s best choice. Now suppose \( T_1 = t \). If \( m > 0 \), then player 1 can be sure that his final signal did in fact make it out to all the peripheral parties, because if it did not, there could not have been \( m > 0 \) confirmations. Therefore, 1 is certain that \( T_i = t \) for each \( i \). Thus, the peripheral parties all play \( B \), so \( B \) is 1’s optimal action. If \( m = 0 \), then 1 is uncertain if his message failed to reach the peripheral parties, or if it did reach the peripheral parties, but each confirmation failed independently (a highly unlikely event). He assigns conditional probability \( z \equiv \frac{\varepsilon}{\varepsilon + (1-\varepsilon)^n} \) to the first possibility, and \( 1 - z \) to the second. Playing \( B \) yields payoff \( M(1 - z) - Lz \). Because \( z > \frac{1}{2} \),
this payoff is negative, so A is his best option.

We now show that player i’s strategy is a best response to the strategies of the others.

Suppose first that $T_i > t$. Then $T_1 > t$, so player 1 will play B. Thus, it is optimal for i to play B. If $T_i < t$, then it must be that either $T_1 < t$, or $T_1 = t$ and $m = 0$. In either of these cases, player 1 chooses A, so it is optimal for i to choose A.

Now suppose that $T_i = t$. In this scenario, the only way player 1 could play A is if player i’s final signal failed to make it through to 1, and also every other peripheral players’ signal independently failed to make it through to 1. Then player 1 would face $T_1 = t, m = 0$ and choose A. The conditional probability of this event is $q \equiv \frac{\varepsilon^n}{1 - (1 - \varepsilon)^m + (1 - \varepsilon)^n} \approx 0$. If player i chooses B, his payoff is $M(1 - q) - Lq > 0$, so B is better than the safe option A. Thus player i’s best response is a cutoff strategy with cutoff $t$. □

Proof of Proposition 6:

We first find a threshold $\hat{m}$ such that $\forall t$, cutoff strategy with cutoff $t$ and threshold $\hat{m}$ is a best response for player 1 to the strategies of players $2 \ldots n + 1$.

Suppose player 1’s machine displays $T_1$, with $m < n$ messages received in the final communication. If $T_1 \geq t + 1$, then player 1 can be sure that each peripheral player i has $T_i \geq t$, so that each will choose action B. Thus B is the best response. If $T_1 < t$, then each peripheral player i has $T_i < t$, so that each will choose action A. Here, A is the best response.

Now suppose that $T_1 = t$. In this case, since 1 has received $m$ confirmations, he is certain that at least $m$ peripheral players have $T_i = t$. For each of the $n - m$ remaining players, player 1 is unsure if his final communication was lost en route, or if it was received and the confirmation was lost. He assigns conditional probabilities $\frac{1}{2 - \varepsilon}$ and $\frac{1 - \varepsilon}{2 - \varepsilon}$ to these two possibilities. Thus, playing B will yield payoff $mM + (n - m)[-L\frac{1}{2 - \varepsilon} + M\frac{1 - \varepsilon}{2 - \varepsilon}]$. This payoff is increasing and linear in $m$. When $m = 0$, the payoff is negative and by condition 1, the payoff is positive when $m = n - 1$. Hence, setting $\bar{m} \equiv n\left[1 - \frac{(2 - \varepsilon)M}{M + L}\right]$, the value of $m$ for which this payoff is zero, we must have $\bar{m} \in (0, n - 1)$. Let $\bar{m}$ be $\lceil \bar{m} \rceil$. Then when $m \geq \bar{m}$, B is the best action, and when $m < \bar{m}$, the zero payoff action A is preferable.

Thus, cutoff strategy with cutoff $t$ and threshold $\bar{m}$ is the best response to $S_i, i = 2 \ldots n + 1$. 
We now show that peripheral player $i$’s strategy is a best response to the strategies of the others, where 1 uses a cutoff strategy with cutoff $t$ and threshold $\bar{m}$.

If $T_i \geq t + 1$, then $T_i \geq t + 1$, so player 1 will play $B$. Thus, $i$’s best response is $B$. If $T_i < t − 1$, then player 1 has $T_i < t$ and will play $A$. Thus, $A$ is the best response. If $T_i = t − 1$, then the only way player 1 could have $T_i = t$ is if $i$’s $(t − 1)$st message made it through, all other players’ messages made it through, and it was the confirmation sent from 1 to $i$ that did not make it back. Thus, player $i$ assigns conditional probabilities $z \equiv \frac{\varepsilon(1−\varepsilon)^n}{\varepsilon(1−\varepsilon)^n+1−(1−\varepsilon)^n}$ to $T_i = t$ and $1 − z$ to $T_i = t − 1$. When $T_i = t − 1$, player 1 chooses $A$. Thus, player $i$ choosing $B$ in this scenario yields a payoff of at most $Mz − L(1−z)$. But $z < \frac{1}{2}$, so the safe, zero payoff choice $A$ is preferred when $T_i = t − 1$.

Suppose $T_i = t$. Let $q$ be the conditional probability $i$ assigns to 1 playing $B$. Player 1 chooses $B$ in the unlikely event that $T_1 = t + 1$, or in the highly likely event that $T_1 = t$ and $m \geq \bar{m}$. The first event occurs with conditional probability $\frac{\varepsilon(1−\varepsilon)^n}{\varepsilon(1−\varepsilon)^n+1−(1−\varepsilon)^n}$, and the second occurs with conditional probability $\frac{\sum_{k=1}^{n-\bar{m}} \binom{n}{k}(1−\varepsilon)^{n-k}\varepsilon^k}{\varepsilon(1−\varepsilon)^n+1−(1−\varepsilon)^n}$. Note that the summation is a result of adding the probabilities that $m = n − 1, m = n − 2, \ldots, m = \bar{m}$. Observe that $q > r \equiv \frac{\varepsilon(1−\varepsilon)^n}{\varepsilon(1−\varepsilon)^n+1−(1−\varepsilon)^n} + \frac{n\varepsilon(1−\varepsilon)^{n-1}}{\varepsilon(1−\varepsilon)^n+1−(1−\varepsilon)^n}$, which is the probability that $T_1 = t + 1$ or $m = n − 1$. Condition 2 states that $r > \frac{L}{L+M}$. Playing $B$ yields expected payoff $Mq − L(1−q) > Mr − L(1−r) > 0$ where the second inequality follows from condition 2. Thus, when $T_i = t$, $B$ is the best action. Thus, $i$’s best response is a cutoff strategy with cutoff $t$. □

References


