Appendix: Monetary Policy Drivers of Bond and Equity Risks

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A Model Solution

Let \( \pi_t^* \) denote the central bank’s inflation target at time \( t \). We solve the model in terms of the output gap \( x_t \) and inflation and nominal interest rate gaps:

\[
\hat{\pi}_t = \pi_t - \pi_t^*, \quad \hat{i}_t = i_t - \pi_t^*.
\]

Denote the vector of state variables by:

\[
\hat{Y}_t = [x_t, \pi_t, i_t]'.
\]

We can re-write the model dynamics in terms of the state variables as:

\[
x_t = \rho^{-} x_{t-1} + \rho^{+} E_t x_{t+1} - \psi \left( E_t \hat{i}_t - E_t \hat{\pi}_{t+1} \right) + u_t^{IS},
\]

\[
\hat{\pi}_t = \rho^{-} \hat{\pi}_{t-1} + (1 - \rho^{+}) E_t \hat{\pi}_{t+1} + \lambda x_t - \rho^{\pi} u_t^* + u_t^{PC},
\]

\[
\hat{i}_t = \rho^{i} \hat{i}_{t-1} + (1 - \rho^{i}) \left[ \gamma x_t + \gamma^{\pi} x_t \right] + u_t^{MP},
\]

\[
\pi_t^* - \pi_{t-1}^* = u_t^*.
\]

Using \( E_t \hat{i}_t = \hat{i}_t - u_t^{MP} \), we can write the model as:

\[
0 = FE_t \hat{Y}_{t+1} + G \hat{Y}_t + H \hat{Y}_{t-1} + Mu_t.
\]

where

\[
F = \begin{bmatrix} \rho^{+} & \psi & 0 \\ 0 & (1 - \rho^{x}) & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]

\[
G = \begin{bmatrix} -1 & 0 & -\psi \\ \lambda & -1 & 0 \\ (1 - \rho^{i}) \gamma^{x} & (1 - \rho^{i}) \gamma^{\pi} & -1 \end{bmatrix},
\]

\[
H = \begin{bmatrix} \rho^{-} & 0 & 0 \\ 0 & \rho^{x} & 0 \\ 0 & 0 & \rho^{i} \end{bmatrix},
\]

\[
M = \begin{bmatrix} 1 & 0 & \psi \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\]

We guess a solution of the form:

\[
\hat{Y}_t = P \hat{Y}_{t-1} + Qu_t.
\]

\( P \) has to satisfy:

\[
FP^2 + GP + H = 0.
\]
Following Uhlig (1999), we first solve for the generalized eigenvectors and eigenvalues of $\Xi$ with respect to $\Delta$, where:

$$\Xi = \begin{bmatrix} -G & -H \\ I_3 & 0 \\ 0_3 & 3 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} F & 0_3 \\ 0_3 & I_3 \end{bmatrix}.$$  

(15)

(16)

For three generalized eigenvalues $\lambda_1, \lambda_2, \lambda_3$ with generalized eigenvectors $[\lambda_1 z_1', z_1']'$, $[\lambda_2 z_2', z_2']'$, $[\lambda_3 z_3', z_3']'$, a solution is given by

$$P = \Omega \Lambda \Omega^{-1},$$

where $\Lambda = diag(\lambda_1, \lambda_2, \lambda_3)$ and $\Omega = [z_1, z_2, z_3]$. Generalized eigenvalues are stable if their absolute value is $< 1$.

Let $e_k$ denotes the row vector with a 1 in position $k$ and zeros otherwise. $Q$ has to satisfy

$$Qe'_k = -[FP + G]^{-1}Me'_k \quad k = 1, 2, 4 \quad (18)$$

$$Qe'_3 = -G^{-1}Me'_3 \quad (19)$$

Provided that $G$ is nonsingular, $G \times Q \times e'_3 = -Me'_3 = -[\psi, 0, 1]'$ implies that

$$Q \times e'_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

i.e. the monetary policy shock has no contemporaneous effect on $x_t$ or $\hat{\pi}_t$.

### A.1 Equilibrium Selection and Properties

We are essentially solving a quadratic matrix equation, so picking a solution amounts to picking three out of six generalized eigenvalues. We only consider solutions that are real-valued, and have finite entries for $Q$. We also require the diagonal entries of $Q$ to be positive. This requirement imposes that the immediate impact of a positive IS shock on output is positive rather than negative.

We only consider dynamically stable solutions with all eigenvalues less than 1 in absolute value, yielding non-explosive solutions for the output gap, inflation gap and interest rate gap. When there are only three generalized eigenvalues with absolute values less than 1, there exists a unique dynamically stable solution. For the period 1 calibration, we have $\gamma^\pi < 1$ and there exist multiple real-valued, dynamically stable solutions. The period 2 and 3 calibrations have unique dynamically stable solutions.

We apply multiple equilibrium selection criteria, which have been proposed in the literature, to rule out “bubble” or unreasonable solutions. These different equilibrium
refinements are not identical, but coincide in many cases. Therefore, there exists a unique solution satisfying all criteria for a large part of our parameter space.

McCallum (1983) proposes to pick the minimum state variable solution. This solution has a minimum set of state variables and satisfies a continuity criterion. Unfortunately, Uhlig (1999) points out that implementing this criterion directly can be computationally demanding. We therefore follow Uhlig (1999) in picking the solution with the minimum absolute eigenvalues, which under certain conditions coincides with the minimum state variable solution (McCallum, 2004).

We also require that our solution is locally E-stable (Evans 1985, 1986, Evans and Honkapohja, 1994) as a plausible necessary, but not sufficient, condition. Local E-stability intuitively requires that the solution is learnable. If agents expectations deviate slightly from equilibrium dynamics, the system will return to an E-stable equilibrium under a simple revision rule.

Finally, we ensure uniqueness of our solution by requiring that it equals the forward solution of Cho and Moreno (2011). The forward solution is obtained by imposing a zero terminal condition. Expectations about shocks far in the future do not affect the current equilibrium. Viewed differently, if we assume that all state variables are constant from time $t+T$ onwards, we can solve for the time $t$ output gap, inflation gap, and interest rate gap recursively. The forward solution obtains by letting $T$ go to infinity. Bikbov and Chernov (2013) state in their appendix that they also use the forward solution of Cho and Moreno (2011).

Let $\text{vec}$ denote vectorization. Applying Proposition 1.3 of Fudenberg and Levine (1998, p.25) the E-stability condition translates into the requirement that the eigenvalues of the derivative
\[
\frac{\partial \text{vec}([FP + G]^{-1}H)}{\partial \text{vec}(P)}
\]
have eigenvalues with absolute values less than 1.

We implement the Cho and Moreno (2011) criterion by requiring that the following sequence $P_n, n = 0, 1, ...$ converges to $P$
\[
P_0 = 0_{3\times3}
\]
\[
P_{n+1} = -[FP_n + G]^{-1} \times H
\]
This sequence $P_n$ has at most one limit and therefore this selection criterion yields a unique solution.

A.2 Solving for SDF and Model Dynamics Simultaneously

We can solve for the matrices $P$ and $Q$ in terms of the model coefficients $\rho^x, \rho^{x'}, \psi, \rho^x, \lambda, \rho^i, \gamma^x, \gamma^\pi$. 

3
We now want to solve the model for a given slope coefficient of volatility with respect to the output gap \( b \) and the variance-covariance matrix \( \Sigma_u \). We therefore solve for the slope coefficients \( \rho^{x-} \) and \( \rho^{x+} \) in terms of the preference parameters and volatilities.

We have that

\[
\begin{align*}
\rho^{x-} &= \frac{\theta}{1 + \theta^*} \quad (23) \\
\rho^{x+} &= \frac{1}{1 + \theta^*} \quad (24) \\
\psi &= \frac{1}{\alpha(1 + \theta^*)} \quad (25) \\
\theta^* &= \theta - \alpha b \bar{\sigma}^2 / 2 \quad (26) \\
\bar{\sigma}^2 &= Q^M \Sigma_u Q^{M'} \quad (27) \\
Q^M &= e_1 Q - (1 + \theta^*) e_1 \quad (28)
\end{align*}
\]

\( \theta^* \) is therefore a fixed point:

\[
\theta^* = \theta - \frac{1}{2} \alpha b (e_1 Q - (1 + \theta^*) e_1) \Sigma_u (e_1 Q - (1 + \theta^*) e_1)' \quad (29)
\]

This fixed point therefore depends on the matrix \( Q \), which depends on the solution for state variable dynamics. It would therefore substantially complicate the solution if we wanted to hold \( b \) constant across sub periods.

### A.3 Bond Returns

We solve for nominal and real bond log return surprises in terms of the fundamental vector of shocks \( u_t \). We use the loglinear framework of Campbell and Ammer (1993) and do not impose the Expectations Hypothesis. We maintain our previous simplifying approximation that risk premia on one period nominal bonds equal zero. Risk premia on longer-term bonds are allowed to vary.

We write \( r_{n-1,t+1} \) for the real one-period return on a real \( n \)-period bond from time \( t \) to time \( t+1 \) and \( x r_{n-1,t+1} \) for the corresponding return in excess of \( r_t \). \( r^S_{n-1,t+1} \) denotes the nominal one-period return on a similar nominal bond and \( x r^S_{n-1,t+1} \) the corresponding excess return over \( i_t \). We use the identities:
\[ r^\$_{n-1,t+1} - E_t r^\$_{n-1,t+1} = - (E_{t+1} - E_t) \sum_{j=1}^{n-1} \left( \hat{i}_{t+j} + \pi^*_t \right) \]  
(30)

\[-(E_{t+1} - E_t) \sum_{j=1}^{n-1} x r^\$_{n-j-1,t+1+j} \]  
(31)

\[ r_{n-1,t+1} - E_t r_{n-1,t+1} = - (E_{t+1} - E_t) \sum_{j=1}^{n-1} r_{t+j} \]  
(32)

\[-(E_{t+1} - E_t) \sum_{j=1}^{n-1} x r_{n-j-1,t+1+j} \]  
(33)

We now derive recursive expressions for unexpected nominal and real bond returns. We guess the functional forms:

\[ E_t x r^\$_{n-1,t+1} = (1 - b x_t) b^\$ \]  
(34)

\[ E_t x r_{n-1,t+1} = (1 - b x_t) b^n \]  
(35)

The functional forms (34) and (35) hold for \( n = 1 \) with \( b^\$ = b^1 = 0 \). Assuming (34) and (35) for maturities less than \( n \), we can express (31) and (33) as:

\[-(E_{t+1} - E_t) \sum_{j=1}^{n-1} x r^\$_{n-j-1,t+1+j} = b \sum_{j=1}^{n-1} b^{^\$}_{n-j} e_1 P^{j-1} Qu_{t+1} \]  
(36)

\[-(E_{t+1} - E_t) \sum_{j=1}^{n-1} x r_{n-j-1,t+1+j} = b \sum_{j=1}^{n-1} b^n_{n-j} e_1 P^{j-1} Qu_{t+1} \]  
(37)

We can express (30) and (32) as:

\[-(E_{t+1} - E_t) \sum_{j=1}^{n-1} (\hat{i}_{t+j} + \pi^*_t) = -e_3 [I - P]^{-1} [I - P^{n-1}] Qu_{t+1} \]  
\[-(n-1)u^*_{t+1} \]  
(38)

\[-(E_{t+1} - E_t) \sum_{j=1}^{n-1} r_{t+j} = -(e_3 - e_2 P) [I - P]^{-1} [I - P^{n-1}] Qu_{t+1} \]  
(39)

Denoting

\[ S^\$,n = -(n-1)e_4 - e_3 [I - P]^{-1} [I - P^{n-1}] Q, \]  
(40)

\[ S^n = -(e_3 - e_2 P) [I - P]^{-1} [I - P^{n-1}] Q, \]  
(41)
we obtain:

\[
\begin{align*}
 r_{n-1,t+1} - E_t r_{n-1,t+1} &= S^{n-1} + b \sum_{j=1}^{n-1} b^{n-1-j} e_1 P^{j-1} Q \left( u_{t+1} \right), \\
 r_{n-1,t+1} - E_t r_{n-1,t+1} &= S^n + b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q \left( u_{t+1} \right).
\end{align*}
\]

(42)

(43)

The conditional expected return adjusted for Jensen’s inequality equals the conditional covariance between bond excess returns and marginal utility. It hence follows that:

\[
b^{n-1} = \alpha \left[ S^{n-1} + b \sum_{j=1}^{n-1} b^{n-1-j} e_1 P^{j-1} Q \right] \Sigma u Q'M' \] 

(44)

\[
-\frac{1}{2} \left[ S^n + b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q \right] \Sigma u \left[ S^{n-1} + b \sum_{j=1}^{n-1} b^{n-1-j} e_1 P^{j-1} Q \right]'.
\]

(45)

Similarly, we obtain the recursive expression:

\[
b^n = \alpha \left[ S^n + b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q \right] \Sigma u Q'M' \] 

(46)

\[
-\frac{1}{2} \left[ S^n + b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q \right] \Sigma u \left[ S^n + b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q \right]'.
\]

(47)

Up to a constant, log yields of nominal and real zero coupon bonds then equal:

\[
y^S_{n,t} = \frac{1}{n} \left[ \sum_{j=0}^{n-1} r^S_{n-j-1,t+1+j} \right] 
\]

(48)

\[
y^S_{n,t} = \frac{1}{n} E_t \sum_{j=0}^{n-1} i_{t+j} - \frac{1}{n} E_t \sum_{j=0}^{n-1} b^{n-j} X_{t+j} 
\]

(49)

\[
y^S_{n,t} = e_3 \left[ I - P \right]^{-1} \left[ I - P^n \right] - \frac{1}{n} \sum_{j=0}^{n-1} b^{n-j} e_1 P^j \hat{Y}_t
\]

(50)

\[
y^S_{n,t} = \left[ \frac{1}{n} (e_3 - e_2 P) \left[ I - P \right]^{-1} \left[ I - P^n \right] - \frac{1}{n} \sum_{j=0}^{n-1} b^{n-j} e_1 P^j \right] \hat{Y}_t
\]

(51)
We can then calculate the conditional slope of the term structure as follows:

\[ y_{n,t}^s - i_t = \frac{1}{n} E_t \sum_{j=0}^{n-1} \hat{i}_{t+j} - i_t + \frac{1}{n} E_t \sum_{j=0}^{n-1} b_{s,n-j} (1 - bx_{t+j}) \] (52)

\[ \hat{y}_{n,t}^s - i_t = \frac{1}{n} E_t \sum_{j=0}^{n-1} \hat{i}_{t+j} - i_t + \frac{1}{n} E_t \sum_{j=0}^{n-1} b_{s,n-j} (1 - bx_{t+j}) \] (53)

\[ \hat{y}_{n,t}^s - i_t = \left( \Gamma_{s,n} - e_3 \right) \hat{Y}_t + \frac{1}{n} \sum_{j=0}^{n-1} b_{s,n-j} (54) \]

With \( \hat{Y}_t \) mean zero, the average slope of the term structure and the average conditional expected bond excess return are:

\[ E \left( y_{n,t}^s - i_t \right) = \frac{1}{n} \sum_{j=0}^{n-1} b_{s,n-j} \] (55)

\[ E \left( E_t x_{r,t+1}^s + \frac{1}{2} Var_t(x_{r,t+1}^s) \right) = \alpha A^{s,n} \Sigma_u Q_{3} m \] (56)

### A.4 Stock Returns

Modeling stocks as a levered claim on the output gap \( x_t \), we assume that dividends are given by:

\[ d_t = \delta x_t. \] (57)

We interpret \( \delta \) as capturing a broad concept of leverage, including operational leverage.

We write \( r_{t+1}^c \) for the log stock return and \( x r_{t+1}^c \) for the log stock return in excess of \( r_t \). Following Campbell (1991) we decompose stock returns into dividend news, news about real interest rates, and news about future excess stock returns ignoring constants:

\[ r_{t+1}^c - E_t r_{t+1}^c = \delta (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta x_{t+1+j} + (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+j} \]

\[ - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j x r_{t+1+j} \] (58)

\( \rho \) is a loglinearization constant close to 1. Now guess the functional form:

\[ E_t x r_{t+1}^c = (1 - bx_t)b^e. \] (59)

Then:

\[ r_{t+1}^c - E_t r_{t+1}^c = (\kappa A^x + A^r) u_{t+1}, \] (60)
\[ A^x = e_1 (I - \rho P)^{-1} Q, \]  
\[ A^r = -\rho(e_3 - e_2 P)(I - \rho P)^{-1} Q, \]  
\[ \kappa = \delta(1 - \rho) + \rho b x. \]

We also write:
\[ A^e = (\kappa A^x + A^r). \]

\( \kappa A^x \) captures the stock returns’ exposure to long-term news about the output gap. \( A^r \) captures the exposure of stock returns to real interest rate news.

The conditional equity premium adjusted for Jensen’s inequality equals the conditional covariance of excess stock returns and marginal utility:
\[ E_t(xr_t e_{t+1}) + \frac{1}{2} Var_t(xr_t e_{t+1}) = \alpha Cov_t(r_t e_{t+1}, s_{t+1} + c_{t+1}) \]
\[ = \alpha A^e \Sigma u Q^{M'}(1 - b x_t) \]

The average conditional equity premium is then given by:
\[ E \left( E_t(xr_t e_{t+1}) + \frac{1}{2} Var_t(xr_t e_{t+1}) \right) = \alpha A^e \Sigma u Q^{M'} \]

It then follows that expected stock returns indeed take the hypothesized form, where \( \kappa \) is the positive root of the quadratic equation:
\[ 0 = \kappa^2 + \kappa \times 2 \left( \rho b \right)^{-1} - \alpha A^x \Sigma u Q^{M'} + A^x \Sigma A^r \]
\[ + \frac{-2\delta(1 - \rho)(\rho b)^{-1} + A^r \Sigma u A^r - 2\alpha A^r \Sigma u Q^{M'}}{A^x \Sigma A^r} \]

Applying the Campbell and Shiller (1988) approximate loglinear present value model to equity prices (ignoring constants), we obtain log dividend price ratio:
\[ d_t - p_t = -\delta E_t \sum_{j=0}^{\infty} \rho^j \Delta x_{t+1+j} + E_t \sum_{j=0}^{\infty} \rho^j (r_{t+1+j}^e - r_{t+j}) + E_t \sum_{j=0}^{\infty} \rho^j r_{t+j} \]
\[ = [\delta e_1 (I - P) - (b \times b^e)e_1 + e_3 - e_2 P] [I - \rho P]^{-1} \hat{Y}_t. \]

The model has implications for the relation between the log dividend price ratio and expected long-term excess stock returns. Denoting the k-period log equity return in excess of short-term real T-bills by \( xr_t e_{t+1+k} \):
\[ E_t(xr_t e_{t+1+k}) = -(b \times b^e)e_1 [I - P]^{-1} [I - P^k] \hat{Y}_t. \]
A.4.1 Bond-Stock Covariances

The conditional nominal and real bond-stock return covariances equal:

\[ \text{Cov}_t(r_{t+1}^e, r_{n,t+1}^s) = A^s_n \Sigma_u A^e (1 - bx_t) \]  
\[ \text{Cov}_t(r_{t+1}^e, r_{n-1,t+1}^s) = A^n \Sigma_u A^e (1 - bx_t) \]

(72)

(73)

The nominal bond return loadings \( A^s_n \), as defined in (42), contain a term \(- (n - 1) \times [0, 0, 0, 1]\) increasing linearly in bond duration and for long-term nominal bonds this is the dominant term. If a positive shock to the inflation target increases stock returns, this term contributes negatively to the nominal bond-stock covariance. If a positive shock to the inflation target decreases stock returns, this term contributes positively.

The variances of equity excess returns, nominal and real bond excess returns are:

\[ \text{Var}_t(r_{t+1}^e) = A^e \Sigma_u A^e (1 - bx_t) \]
\[ \text{Var}_t(r_{n,t+1}^s) = A^s_n \Sigma_u A^{s,n} (1 - bx_t) \]
\[ \text{Var}_t(r_{n-1,t+1}^s) = A^n \Sigma_u A^{n} (1 - bx_t) \]

(74)

(75)

(76)

The conditional stock market betas of nominal and real bonds are independent of \( x_t \) and given by:

\[ \beta_t(r_{n,t+1}^s) = \frac{A^s_n \Sigma_u A^e}{A^e \Sigma_u A^e} \]
\[ \beta_t(r_{n-1,t+1}^s) = \frac{A^n \Sigma_u A^e}{A^e \Sigma_u A^e} \]

(77)

(78)

A.5 Estimable VAR(1) in Output, Inflation, and Nominal Yields

While standard empirical measures are available for the output gap, we do not observe the interest rate and inflation gaps. We therefore cannot directly estimate the recursive law of motion (13). However, for a long-term bond maturity \( n \), we can estimate a VAR(1) in the vector:

\[ Y_t = \begin{bmatrix} x_t \\ \pi_t \\ i_t \\ y_{n,t}^s \end{bmatrix} \]
\[ = A \begin{bmatrix} Y_t^* \\ \pi_t^* \end{bmatrix}' \]

(79)

(80)
The model implies that:

\[ Y_{t+1} = P^Y Y_t + Q^Y u^Y_{t+1}. \]  \hspace{1cm} (81)

Here:

\[ A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ \Gamma^s,n & 1 \end{bmatrix}, \]  \hspace{1cm} (83)

\[ P^Y = A \begin{bmatrix} P \\ 0 \\ 1 \end{bmatrix} A^{-1}, \]  \hspace{1cm} (84)

\[ Q^Y = A \begin{bmatrix} Q \\ 0 & 0 & 0 & 1 \end{bmatrix}, \]  \hspace{1cm} (85)

\[ u^Y_t = u_t. \]  \hspace{1cm} (86)

provided that the inverse of \( A \) exists.

### B A Note on Units

Our empirical yields and returns are in annualized percent units. Log real dividends and the log output gap are in natural percent units. Our empirical units are analogous to those used by CGG. Our empirical coefficients in Table 4 in the main paper can therefore be compared directly to those in CGG.

However, the Campbell and Shiller (1988) loglinearizations, the expression for the equity premium (65) and expected bond returns (45) are expressed in natural units. We therefore solve the model in natural units and subsequently report scaled parameters and model moments reflecting our choice of empirical units. Let a superscript \( c \) denote natural units used for solving the calibrated model. Values with no superscript denote the parameters and variables corresponding to empirical units.

Our quantities in empirical units are related to quantities in calibration units according to: \( x_t = 100x^c_t, i_t = 400i^c_t, \pi_t = 400\pi^c_t, \) and \( y_t^{s,n} = 400y^{s,n}_t \) and \( \pi^*_t = 400\pi^*_t. \) We can therefore write the model as:

\[ x_t = \rho^{x-c} x_{t-1} + \rho^{x+c} E_{t-1} x_{t-1} - \frac{\psi^c}{4} (E_{t-1} i_t - E_{t-1} \pi_{t+1}) + 100 \times u^IS,c_t \]  \hspace{1cm} (87)

\[ \pi_t = \rho^{\pi-c} \pi_{t-1} + (1 - \rho^{\pi-c}) E_{t-1} \pi_{t+1} + 4\lambda^c x_t + 400 \times u^PC,c_t \]  \hspace{1cm} (88)

\[ i_t = \rho^{i-c} (i_{t-1} - \pi^*_t) + (1 - \rho^{i-c}) [4\gamma^{x,c} x_t + \gamma^{\pi,c} (\pi_t - \pi^*_t)] + \pi_t^* + 400 u^MP,c_t \]  \hspace{1cm} (89)

\[ \pi^*_t = \pi^*_{t-1} + 400 u^*_t \]  \hspace{1cm} (90)
Equations (87) through (90) imply relations between the empirical and calibration parameters:

\[
\begin{align*}
\rho^{x-} &= \rho^{x-c}, \quad \rho^{x+} = \rho^{x+c}, \quad \psi = \frac{\psi^c}{4} \tag{91} \\
\rho^\pi &= \rho^{\pi,c}, \quad \lambda = 4\lambda^c \tag{92} \\
\rho^i &= \rho^{i,c}, \quad \gamma^x = 4\gamma^{x,c}, \quad \gamma^\pi = \gamma^{\pi,c} \tag{93} \\
\bar{\sigma}^{IS} &= 100\bar{\sigma}^{IS,c}, \quad \bar{\sigma}^{PC} = 400\bar{\sigma}^{PC,c}, \quad \bar{\sigma}^{MP} = 400\bar{\sigma}^{MP,c}, \quad \bar{\sigma}^* = 400\bar{\sigma}^* \tag{94}
\end{align*}
\]

Fuhrer (1997) estimates a Phillips curve with both backward-looking and forward-looking components. Using inflation in annualized percent, and the log output gap in natural units, he finds a backward-looking component of 0.8, a forward-looking component of 0.2, and a weight on the output gap of 0.12. We can therefore compare the parameter \( \lambda \) in empirical units directly to the magnitudes in CGG, Fuhrer (1997), and Roberts (1995).

Yogo (2004) scales interest rates and inflation to quarterly units. Our calibrated values for \( \psi^c \) in natural units can therefore be compared directly to the estimated values in Yogo (2004). We therefore report the value \( \psi^c \) corresponding to natural units rather than \( \psi \) corresponding to empirical units throughout the paper.

We choose the leverage parameter \( \delta \) to match the relative volatilities of log real dividend growth and log output gap growth. We use four quarter growth rates to smooth out some of the more seasonal fluctuations. We consider four quarter log output growth \( \Delta x_t = x_t - x_{t-4} \). The standard deviation of this growth rate over the period 1960.Q1-2011.Q4 is 2.20%. Let \( d_t \) denote the sum of log S&P 500 real dividends. Monthly real S&P 500 dividends are from Robert Shiller's web site. These real dividends are deflated by the not seasonally adjusted CPI-U with a basis of 1982-84=100. We obtain quarterly dividends by summing the level real dividends within the quarter. The standard deviation 1960.Q1-2011.Q4 of the four quarter log dividend growth rate \( \Delta d_t = d_t - d_{t-4} \) equals 5.35%. Our model specifies dividends as a levered claim on the output gap with \( d_t = \delta x_t \). We therefore set the leverage parameter \( \delta \) to match the relative standard deviations of output and dividend growth. This gives \( \delta = 2.43 \).

Due to our choice of empirical units, we use a slightly different transformation from the transition matrix \( P^c \) of the state variables in natural unit to the transition matrix \( P^Y \) of the estimable VAR(1). We have the relation:

\[
Y_t = \left[ \begin{array}{c} x_t \\ \pi_t \\ i_t \\ y^S_{n,t} \end{array} \right] = A^c \left[ \begin{array}{c} \hat{x}_t^c, \hat{\pi}_t^c, \hat{i}_t^c, \hat{\pi}_t^c, \pi_t^c, \pi_t^c, \pi_t^c \end{array} \right]', \tag{95}
\]

\[
= A^c \left[ \begin{array}{c} \hat{x}_t^c, \hat{\pi}_t^c, \hat{i}_t^c, \hat{\pi}_t^c, \pi_t^c, \pi_t^c, \pi_t^c \end{array} \right]', \tag{96}
\]
where:

\[
A^c = \text{diag}(100, 400, 400, 400) \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ \Gamma^{S,n} & 1 \end{bmatrix},
\]

(97)

\[
P^Y = A^c \begin{bmatrix} P \\ 0 \\ 1 \end{bmatrix} A^{c-1},
\]

(98)

\[
Q^Y = A^c \begin{bmatrix} Q \\ 0 \\ 0 \\ 1 \end{bmatrix},
\]

(99)

\[
u_t^{c,Y} = \begin{bmatrix} u_t^{c,IS} \\ u_t^{c,PC} \\ u_t^{c,MP} \\ u_t^{c,*} \end{bmatrix}',
\]

(100)

\[
Y_t = P^Y Y_{t-1} + Q^Y u_{t+1}^{c,Y}.
\]

(101)

We report annualized percent standard deviations of equity and bond returns at the average output gap \(x_t = 0\). We calculate annualized standard deviations of equity and bond returns in percent at \(x_t = 0\):

\[
\text{Std}_t(r_{t+1}^e) = 200 \sqrt{A^e \Sigma_u A^{e'}},
\]

(102)

\[
\text{Std}_t(r_{n-1,t+1}^S) = 200 \sqrt{A^{S,n} \Sigma_u A^{S,n'}},
\]

(103)

\[
\text{Std}_t(r_{n-1,t+1}) = 200 \sqrt{A^n \Sigma_u A^{n'}}.
\]

(104)

We back out empirical shocks for each sub period separately. From the empirical series \(Y_t^{\text{emp}} = \begin{bmatrix} x_t^{\text{emp}} & \pi_t^{\text{emp}} & i_t^{\text{emp}} & y_t^{\text{emp},S,n} \end{bmatrix}'\), we back out fundamental shocks in empirical units:

\[
\begin{bmatrix} u_t^{IS} \\ u_t^{PC} \\ u_t^{MP} \\ u_t^{*} \end{bmatrix}' = \begin{bmatrix} 100u_t^{c,IS} \\ 400u_t^{c,PC} \\ 400u_t^{c,MP} \\ 400u_t^{c,*} \end{bmatrix}',
\]

(105)

\[
= \text{diag}(100, 400, 400, 400) \times Q^{Y-1} (Y_t^{\text{emp}} - P^Y Y_{t-1}^{\text{emp}}).
\]

(106)

We transform the parameter \(b\) into empirical units according to \(b = b^c/100\). Then \((1 - b^c x_t^c) = (1 - bx_t)\). We calculate the standard deviation of the volatility factor \((1 - bx_t)\) at \(x_t = 0\) according to \(b \sqrt{e_1 Q \Sigma_u Q' e_1'}\).
B.1 Partial Derivatives

We compute the partial derivative of the nominal bond beta with respect to \( \ln(\bar{\sigma}_k^u) \) as follows:

\[
\frac{\partial \beta^S}{\partial \ln(\bar{\sigma}_k^u)} = \frac{1}{A^e \Sigma_u A^{e'}} 2 A^{n_k} e_k^e e_k^u A^{e'k} \sigma_u^{k2} \quad (107)
\]

\[
\frac{\partial \text{Std}(r_{t+1}^e)}{\partial \ln(\bar{\sigma}_k^u)} = \frac{200 A^e e_k^e A^{e'} \sigma_u^{k2}}{\sqrt{A^e \Sigma_u A^{e'}}} \quad (108)
\]

\[
\frac{\partial \text{Std}(r_{n-1,t+1}^e)}{\partial \ln(\bar{\sigma}_k^u)} = \frac{200 A^{n_k} e_k^e A^{n'u} \sigma_u^{k2}}{\sqrt{A^{n_k} \Sigma_u A^{n'u}}} \quad (109)
\]

The partial derivatives for the nominal bond beta sum to two times the calibrated nominal bond beta for each sub period. The partial derivatives for the standard deviations of asset returns sum to the calibrated standard deviation of asset returns for each sub period.

C Details of Moment Fitting Procedure

We minimize the distance between model and empirical moments summed over all three sub-periods. We use a superscript \( p \) to denote period \( p \) moments and a hat to denote empirically estimated moments. Our objective function is:

\[
\text{Obj} = \sum_{p=1}^3 \left[ \left\| P^{Y,p} - \hat{P}^{Y,p} \right\|^2 + \left\| \text{diag}(Q^{Y,p} \Sigma_u Q^{Y,p}) - \text{diag}(\hat{Q}^{Y,p} \hat{\Sigma}_u Q^{Y,p}) \right\|^2 \right] + \frac{1}{10} \left( \text{Std}^p(r_{n-1,t+1}^e) - \text{Std}^p(\hat{r}_{n-1,t+1}^e) \right)^2 \quad (110)
\]

\[
+ \frac{1}{10} \left( \text{Std}^p(r_{t+1}^e) - \text{Std}^p(\hat{r}_{nt+1}^e) \right)^2 \quad (111)
\]

\[
+ (10 \times (\beta^p(r_{n-1,t+1}^e) - \hat{\beta}^p(\hat{r}_{n-1,t+1}^e))^2 \right) \quad (112)
\]

We optimize \( \text{Obj} \) over the following parameters: \( \rho^{x-}, \rho^{x+}, \bar{\sigma}^{IS,p}, \bar{\sigma}^{PC,p}, \bar{\sigma}^{MP,p}, \) and \( \bar{\sigma}^{x,p}, p = 1, 2, 3 \). We hold all other parameters constant at the values shown in Table 5 in the main paper.

In order to reduce the dimensionality of the minimization problem, we minimize \( \text{Obj} \) iteratively. First, we minimize with respect to the standard deviations of shocks while holding the Euler equation parameters \( \rho^{x+} \) and \( \rho^{x-} \) constant at initial guesses. Second, we minimize with respect to \( \rho^{x+} \) and \( \rho^{x-} \) while holding constant the standard deviations of shocks at their optimal values from the first step. Third, we minimize again with respect to the standard deviations of shocks holding constant \( \rho^{x+} \) and \( \rho^{x-} \) at their optimal values from the second step.
Step 1: Starting from an initial guess of $\rho^- = 0.4503$ and $\rho^+ = 0.6161$, we first minimize with respect to $\bar{\sigma}^{IS, p}, \bar{\sigma}^{PC, p}, \bar{\sigma}^{MP, p}$, and $\bar{\sigma}^{* p}$ holding $\rho^-$ and $\rho^+$ constant. Given $\rho^-$ and $\rho^+$, we can minimize with respect to $\bar{\sigma}^{IS, p}, \bar{\sigma}^{PC, p}, \bar{\sigma}^{MP, p}$, and $\bar{\sigma}^{* p}$ independently for each period $p$.

We use a simple and robust minimization procedure. We randomly draw 50000 parameter vectors. We draw $\bar{\sigma}^{IS, p}, \bar{\sigma}^{PC, p}, \bar{\sigma}^{MP, p}, \bar{\sigma}^{* p}$ from independent uniform distributions. Our support intervals for 1960.Q1-1979.Q2 are such that $[(\bar{\sigma}^{IS, 1}), (\bar{\sigma}^{PC, 1}), (\bar{\sigma}^{MP, 1}), (\bar{\sigma}^{* 1})] \in [0, 0, 0, 0] \times [0.8256, 2.2069, 2.2768, 0.7715]$. Our support intervals for 1979.Q3-1996.Q4 are such that $[(\bar{\sigma}^{IS, 2}), (\bar{\sigma}^{PC, 2}), (\bar{\sigma}^{MP, 2}), (\bar{\sigma}^{* 2})] \in [0, 0, 0, 0] \times [0.7800, 1.3338, 4.3310, 1.1228]$. Our support intervals for 1997.Q1-2011.Q4 are such that $[(\bar{\sigma}^{IS, 3}), (\bar{\sigma}^{PC, 3}), (\bar{\sigma}^{MP, 3}), (\bar{\sigma}^{* 3})] \in [0, 0, 0, 0] \times [0.6153, 1.8542, 0.8119, 1.0595]$. Minimizing with respect to the volatilities of shocks for each sub sample yields:

$$
\begin{pmatrix}
\bar{\sigma}^{IS, 1} & \bar{\sigma}^{IS, 2} & \bar{\sigma}^{IS, 3} \\
\bar{\sigma}^{PC, 1} & \bar{\sigma}^{PC, 2} & \bar{\sigma}^{PC, 3} \\
\bar{\sigma}^{MP, 1} & \bar{\sigma}^{MP, 2} & \bar{\sigma}^{MP, 3} \\
\bar{\sigma}^{* 1} & \bar{\sigma}^{* 2} & \bar{\sigma}^{* 3}
\end{pmatrix} = \begin{pmatrix}
0.38 & 0.54 & 0.34 \\
1.09 & 0.83 & 0.90 \\
1.23 & 1.93 & 0.38 \\
0.37 & 0.72 & 0.51
\end{pmatrix}
$$

(114)

Step 2: In the second step, we minimize with respect to $\rho^+$ and $\rho^-$ while holding the volatilities of shocks constant at the values shown in (114). We randomly draw 10000 draws from two independent uniform distributions $U_1 \in [0, 1]$ and $U_2 \in [0, 1]$ and set $\rho^- = 0.4253 + 0.05U_1$ and $\rho^+ = (1 - \rho^-) + 0.2 \times \rho^-U_2$, thereby ensuring that $\rho^+$ and $\rho^-$ sum to more than 1. We obtain minimizing parameter values $\rho^- = 0.4466$ and $\rho^+ = 0.6224$ agreeing with the initial guesses up to two significant digits. Figure A.1 shows the objective function $\text{Obj}$ against $U_1$ and $U_2$. Each dot corresponds to one combination of parameter values. Figure A.1 shows that the optimizing parameter values are in the middle of the ranges considered. We therefore are not at a boundary solution. Moreover, the optimal parameter values occur at a clear minimum, indicating that the parameters $\rho^+$ and $\rho^-$ are well identified.

Step 3: The third step is exactly the same as the first step, except that we hold $\rho^-$ and $\rho^+$ constant at their new values. Figure A.2 shows the objective function against the standard deviations of shocks for each sub sample period. If the volatilities of shocks are well identified, the lower envelopes of the scatter plots in Figure A.2 should have clear minima. It appears that the objective function exhibits clear minima with respect to each of the shock volatilities. Figure A.2 shows that all volatilities are in the interior of the intervals that we are optimizing over. This finding is reassuring in that it suggests that we are considering sufficiently large ranges.

The optimal volatilities of shocks are shown in Table 5 in the main paper. These optimal volatilities are close to the preliminary values (114). Moreover, the final values for $\rho^+$ and $\rho^-$ are very close to the initial guesses, indicating convergence of our algorithm.
D Additional Calibration Features

Table A.1 shows the matrix of slope coefficients for the quarterly VAR(1) in the log output gap, inflation, the Federal Funds rate, and the 5 year nominal yield both in the model and in the data. Table A.1 shows that the calibrated model can generate substantial persistence in the output gap, inflation, Fed Funds rate, and the long-term nominal yield, even though the output gap is somewhat less persistent than in the data. The off-diagonal elements are generally small and often close to zero.

Figure A.3 shows a time series of smoothed shocks backed out from our sub period calibrations. For each sub period, we back out the fundamental model shocks by inverting the relation $Y_{t+1} = P^Y Y_t + Q^Y u^Y_{t+1}$ and plugging in the empirical time series for the vector $Y_t$ and the model implied matrices $P^Y$ and $Q^Y$.

Tables A.2 and A.3 present an alternative calibration and are analogous to Tables 5 and 6 in the main paper. The alternative calibration fits the volatility of VAR(1) residual volatilities, and the volatilities of bond and stock returns, but not the nominal bond beta. Table A.2 shows that in the alternative calibration we obtain a lower volatility of PC shocks in the middle sub period. Consequently, the alternative calibration obtains a negative nominal bond beta in the second sub period instead of a positive nominal bond beta.
References


### Table A.1: Empirical and Model VAR(1) Matrices for Sub Periods

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Coeff. on Lagged Variables</td>
<td>Coeff. on Lagged Variables</td>
</tr>
<tr>
<td>Output Gap</td>
<td>0.74 -0.28 -0.05 0.33</td>
<td>0.97 -0.05 -0.20 0.19</td>
</tr>
<tr>
<td>Inflation Gap</td>
<td>0.32 0.84 -0.03 0.19</td>
<td>-0.04 0.42 0.37 0.42</td>
</tr>
<tr>
<td>Fed Funds Rate</td>
<td>0.23 0.17 0.53 0.30</td>
<td>0.17 0.04 0.54 0.47</td>
</tr>
<tr>
<td>Log Nom. Yield</td>
<td>0.07 -0.02 -0.06 1.08</td>
<td>0.03 0.08 0.02 0.83</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>Coeff. on Lagged Variables</td>
<td>Coeff. on Lagged Variables</td>
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<tr>
<td>Output Gap</td>
<td>0.73 -0.79 -0.08 0.87</td>
<td>0.90 -0.08 0.02 -0.07</td>
</tr>
<tr>
<td>Inflation Gap</td>
<td>0.32 0.54 -0.05 0.51</td>
<td>0.03 0.78 0.10 -0.04</td>
</tr>
<tr>
<td>Fed Funds Rate</td>
<td>0.23 1.03 0.47 -0.50</td>
<td>0.07 0.76 0.11 0.65</td>
</tr>
<tr>
<td>Log Nom. Yield</td>
<td>0.22 -0.20 -0.07 1.27</td>
<td>-0.05 0.22 -0.02 0.83</td>
</tr>
</tbody>
</table>

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<tbody>
<tr>
<td></td>
<td>Coeff. on Lagged Variables</td>
<td>Coeff. on Lagged Variables</td>
</tr>
<tr>
<td>Output Gap</td>
<td>0.47 -0.22 -0.16 0.39</td>
<td>0.93 0.04 0.03 0.13</td>
</tr>
<tr>
<td>Inflation Gap</td>
<td>0.17 0.87 -0.09 0.22</td>
<td>0.20 0.37 -0.17 -0.06</td>
</tr>
<tr>
<td>Fed Funds Rate</td>
<td>0.14 0.16 0.90 -0.06</td>
<td>0.00 0.14 0.68 0.47</td>
</tr>
<tr>
<td>Log Nom. Yield</td>
<td>-0.32 -0.10 -0.13 1.23</td>
<td>0.07 -0.06 0.06 0.75</td>
</tr>
</tbody>
</table>

$P^Y$ is the matrix of slope coefficients of a quarterly VAR(1) in the log output gap, inflation, Fed Funds rate, and 5 Year Nominal Yield.
Table A.2: Alternative Calibration Parameter Choices

Time-Invariant Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Log-Linearization Constant ( \rho )</td>
<td>0.99</td>
</tr>
<tr>
<td>Leverage ( \delta )</td>
<td>2.43</td>
</tr>
<tr>
<td>Preference Parameter ( \alpha )</td>
<td>30</td>
</tr>
<tr>
<td>Backward-Looking Comp. PC ( \rho^\pi )</td>
<td>0.80</td>
</tr>
<tr>
<td>Slope PC ( \lambda )</td>
<td>0.30</td>
</tr>
<tr>
<td>Forward-Looking Comp. IS ( \rho^{x+} )</td>
<td>0.62</td>
</tr>
<tr>
<td>Backward-Looking Comp. IS ( \rho^{x-} )</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Monetary Policy Rule

<table>
<thead>
<tr>
<th>Period</th>
<th>MP Coefficient Output</th>
<th>MP Coefficient Infl.</th>
<th>Backward-Looking Comp. MP</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.Q1-79.Q2</td>
<td>( \gamma^x ) 0.42</td>
<td>( \gamma^\pi ) 0.69</td>
<td>( \rho^i ) 0.56</td>
</tr>
<tr>
<td>79.Q3-96.Q4</td>
<td>-0.07 1.44</td>
<td>1.92</td>
<td></td>
</tr>
<tr>
<td>97.Q1-11.Q4</td>
<td>0.44</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Std. Shocks

<table>
<thead>
<tr>
<th>Shock Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. IS</td>
<td>( \tilde{\sigma}^{IS} ) 0.38</td>
</tr>
<tr>
<td>Std. PC shock</td>
<td>( \tilde{\sigma}^{PC} ) 1.02</td>
</tr>
<tr>
<td>Std. MP shock</td>
<td>( \tilde{\sigma}^{MP} ) 1.21</td>
</tr>
<tr>
<td>Std. infl. target shock</td>
<td>( \tilde{\sigma}^* ) 0.33</td>
</tr>
</tbody>
</table>

The alternative calibration puts no weight on the nominal bond beta in fitting the standard deviations of fundamental shocks. The time-invariant parameters and monetary policy rule parameters are identical to those in Table 5 in the main paper. The standard deviations of shocks differ from the calibration in Table 5 in the main paper.
We compare model and empirical moments for the alternative calibration. Alternative calibration parameters are specified in Table A.2. The alternative calibration puts no weight on the nominal bond beta in fitting the standard deviations of fundamental shocks.
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<tbody>
<tr>
<td>Equity Premium.</td>
<td>3.61</td>
<td>3.12</td>
<td>3.29</td>
<td>3.35</td>
<td>3.23</td>
<td>8.12</td>
<td>4.94</td>
<td>5.36</td>
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<tr>
<td>Nom. Bond Exc. Ret.</td>
<td>0.25</td>
<td>0.68</td>
<td>-0.50</td>
<td>0.18</td>
<td>0.01</td>
<td>2.31</td>
<td>2.97</td>
<td>1.64</td>
</tr>
<tr>
<td>$E (y_{5,t}^S - i_t)$</td>
<td>0.14</td>
<td>0.39</td>
<td>-0.03</td>
<td>0.18</td>
<td>0.74</td>
<td>1.32</td>
<td>1.14</td>
<td>1.05</td>
</tr>
<tr>
<td>Corr$(x_t, y_{5,t}^S)$</td>
<td>0.05</td>
<td>0.31</td>
<td>0.93</td>
<td>0.39</td>
<td>-0.62</td>
<td>-0.21</td>
<td>-0.53</td>
<td>-0.46</td>
</tr>
<tr>
<td>Corr$(x_t, i_t)$</td>
<td>-0.02</td>
<td>-0.16</td>
<td>-0.20</td>
<td>-0.12</td>
<td>0.22</td>
<td>-0.21</td>
<td>0.80</td>
<td>0.05</td>
</tr>
<tr>
<td>$x_{r_{5,t+1}}^S$ onto $(y_{5,t}^S - i_t)$</td>
<td>-0.08</td>
<td>-0.71</td>
<td>4.12</td>
<td>0.92</td>
<td>2.24</td>
<td>3.09</td>
<td>2.37</td>
<td>2.84*</td>
</tr>
<tr>
<td>$x_{r_{5,t+1}}^S$ onto $x_t$</td>
<td>-1.23</td>
<td>-3.56</td>
<td>5.27</td>
<td>-0.14</td>
<td>-0.72*</td>
<td>-0.17</td>
<td>-0.16</td>
<td>-0.47</td>
</tr>
<tr>
<td>$x_{r_{5,t+1}}^S$ onto $dp_t$</td>
<td>0.06</td>
<td>0.14</td>
<td>-0.30</td>
<td>-0.02</td>
<td>0.06</td>
<td>0.02</td>
<td>-0.03</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

The equity premium and the average nominal bond excess return show average returns in excess of a short-term bond adjusted for Jensen’s inequality. The last three rows show regression coefficients of log 5 year bond excess returns (Annualized, %) onto the slope of the yield curve (Annualized, %), the output gap (%), and the log dividend price ratio (%), respectively. The last three rows show * when the coefficient is significant at the 5% level with Newey-West standard errors with two lags.
We minimize the objection function with respect to $\rho^-$ and $\rho^+$ while holding constant the volatilities of shocks at the values shown in (114). We randomly draw 10000 draws from two independent uniform distributions $U_1 \in [0, 1]$ and $U_2 \in [0, 1]$ and set $\rho^- = 0.4253 + 0.05U_1$ and $\rho^+ = (1 - \rho^-) + 0.2 \times \rho^+ U_2$. The minimizing parameter values are indicated by circles. The objective function is the sum of squared differences between model and empirical moments. The considered moments are the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes.
We minimize the objection function with respect to $\bar{\sigma}^{IS,1}$, $\bar{\sigma}^{PC,1}$, $\bar{\sigma}^{MP,1}$, and $\bar{\sigma}^*$ while holding constant all time-invariant parameters at the values shown in Table 5 in the main paper. The minimizing parameter values are indicated by circles. The objective function is the sum of squared differences between model and empirical moments for that sub-period. The considered moments are the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes.
We minimize the objection function with respect to $\bar{\sigma}^{IS^2}$, $\bar{\sigma}^{PC^2}$, $\bar{\sigma}^{MP^2}$, and $\bar{\sigma}^{*^2}$ while holding constant all time-invariant parameters at the values shown in Table 5 in the main paper. The minimizing parameter values are indicated by circles. The objective function is the sum of squared differences between model and empirical moments for that sub-period. The considered moments are the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes.
Figure A.2: (Panel C) Minimizing with Respect to Shock Volatilities: 1997.Q1-2011.Q4

We minimize the objection function with respect to $\bar{\sigma}^{IS,3}$, $\bar{\sigma}^{PC,3}$, $\bar{\sigma}^{MP,3}$, and $\bar{\sigma}^{*,3}$ while holding constant all time-invariant parameters at the values shown in Table 5 in the main paper. The minimizing parameter values are indicated by circles. The objective function is the sum of squared differences between model and empirical moments for that sub-period. The considered moments are the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes.
Figure A.3: Time Series of Model Shocks

This figure plots the time series of smoothed IS, PC, MP and inflation target ($\pi^*$) shocks. IS shocks are in natural percent units, while PC, MP and inflation target shocks are in annualized percent units. The shocks are smoothed with a trailing exponentially-weighted moving average. The decay parameter equals 0.08 per quarter corresponding to a half life of 24 quarters.