A Appendix

A.1 Derivations

A.1.1 State Variables

The state variables in the model follow the processes:

$$\begin{aligned} -m_{t+1} &= x_t + \frac{1}{2} z_t^2 \sigma_m^2 + z_t \varepsilon_{m,t+1} \\ x_{t+1} &= \mu_x (1 - \phi_x) + \phi_x x_{t+1} + \varepsilon_{x,t+1} \\ z_{t+1} &= \mu_z (1 - \phi_z) + \phi_z z_{t+1} + \varepsilon_{z,t+1} \end{aligned}$$

$$\begin{aligned} \pi_{t+1} &= \lambda_t + \xi_t + \frac{1}{2} \psi_t^2 \sigma_\pi^2 + \psi_t \varepsilon_{\pi,t+1} \\ \lambda_{t+1} &= \lambda_t + \psi_t \varepsilon_{\lambda,t+1} + \varepsilon_{\Lambda,t+1} \\ \xi_{t+1} &= \phi_\xi \xi_t + \psi_t \varepsilon_{\xi,t+1} \\ \psi_{t+1} &= \mu_\psi \left(1 - \phi_\psi\right) + \phi_\psi \psi_{t+1} + \varepsilon_{\psi,t+1} \end{aligned}$$

A.1.2 Pricing Equations

Real Term Structure The price of a single-period zero-coupon real bond satisfies

$$P_{1,t} = E_t \left[\exp\left\{m_{t+1}\right\}\right] = -x_t - \frac{1}{2} z_t^2 \sigma_m^2 + \frac{1}{2} z_t^2 \sigma_m^2 = -x_t$$

We conjecture that the price function is exponential affine in x_t and z_t with the form

$$P_{n,t} = A_n + B_{x,n} x_t + B_{z,n} z_t.$$

The standard pricing equation implies

$$\begin{aligned} P_{n,t} &= E_t \left[\exp\left\{p_{n-1,t+1} + m_{t+1}\right\}\right] = E_t \left[\exp\left\{A_{n-1} + B_{x,n-1} x_{t+1} + B_{z,n-1} z_{t+1} - x_t - \frac{1}{2} z_t^2 \sigma_m^2 + z_t \varepsilon_{m,t+1}\right\}\right] \\ &= A_{n-1} + B_{x,n-1} \left(\left(1 - \phi_x\right) \mu_x + \phi_x x_t\right) + B_{z,n-1} \left(\left(1 - \phi_z\right) \mu_z + \phi_z z_t\right) - x_t - \frac{1}{2} z_t^2 \sigma_m^2 \\ &\quad + \frac{1}{2} B_{x,n-1}^2 \sigma_x^2 + \frac{1}{2} B_{z,n-1}^2 \sigma_z^2 + \frac{1}{2} z_t^2 \sigma_m^2 + B_{x,n-1} B_{z,n-1} \sigma_{xz} - B_{x,n-1} \sigma_{xm} z_t - B_{z,n-1} \sigma_{zm} z_t \end{aligned}$$

since the shocks are conditionally jointly normal. Equating the coefficients implies that

$$\begin{aligned}
A_n &= A_{n-1} + B_{x,n-1}\left(1-\phi_x\right)\mu_x + B_{z,n-1}\left(1-\phi_z\right)\mu_z + \frac{1}{2}B_{x,n-1}^2\sigma_x^2 + \frac{1}{2}B_{z,n-1}^2\sigma_z^2 + B_{x,n-1}B_{z,n-1}\sigma_{xz} \\
B_{x,n} &= B_{x,n-1}\phi_x - 1 \\
B_{z,n} &= B_{z,n-1}\phi_z - B_{x,n-1}\sigma_{xm} - B_{z,n-1}\sigma_{zm}
\end{aligned}$$

where $B_{x,1} = -1$ and $B_{z,1} = 0$.

Nominal Term Structure The price of a single-period zero-coupon nominal bond satisfies

$$\begin{aligned}
P_{1,t}^{\$} &= E_t\left[\exp\left\{m_{t+1} - \pi_{t+1}\right\}\right] \\
&= E_t\left[\exp\left\{-x_t - \frac{1}{2}z_t^2\sigma_m^2 - z_t\varepsilon_{m,t+1} - \lambda_t - \xi_t - \frac{1}{2}\psi_t^2\sigma_\pi^2 - \psi_t\varepsilon_{\pi,t+1}\right\}\right] \\
&= \exp\left\{-x_t - \frac{1}{2}z_t^2\sigma_m^2 - \lambda_t - \xi_t - \frac{1}{2}\psi_t^2\sigma_\pi^2 + \frac{1}{2}z_t^2\sigma_m^2 + \frac{1}{2}\psi_t^2\sigma_\pi^2 + z_t\psi_t\sigma_{m\pi}\right\} \\
&= \exp\left\{-x_t - \lambda_t - \xi_t + z_t\psi_t\sigma_{m\pi}\right\}
\end{aligned}$$

where the last equality follows from the joint conditional normality of $z_t\varepsilon_{m,t+1}$ and $\psi_t\varepsilon_{\pi,t+1}$.

We now guess that the price function is exponential linear-quadratic in the state variables with the following form:

$$P_{n,t}^{\$} = \exp\left\{A_n^{\$} + B_{x,n}^{\$}x_t + B_{z,n}^{\$}z_t + B_{\lambda,n}^{\$}\lambda_t + B_{\xi,n}^{\$}\xi_t + B_{\psi,n}^{\$}\psi_t + C_{z,n}^{\$}z_t^2 + C_{\psi,n}^{\$}\psi_t^2 + C_{z\psi,n}^{\$}z_t\psi_t\right\}$$

The standard pricing equation then implies

$$\begin{aligned}
P_{n,t}^{\$} &= E_t\left[\exp\left\{p_{n-1,t+1}^{\$} + m_{t+1} - \pi_{t+1}\right\}\right] \\
&= E_t\left[\exp\left\{\begin{array}{c} A_{n-1}^{\$} + B_{x,n-1}^{\$}x_{t+1} + B_{z,n-1}^{\$}z_{t+1} + B_{\lambda,n-1}^{\$}\lambda_{t+1} + B_{\xi,n-1}^{\$}\xi_{t+1} + B_{\psi,n-1}^{\$}\psi_{t+1} \\ +C_{z,n-1}^{\$}z_{t+1}^2 + C_{\psi,n-1}^{\$}\psi_{t+1}^2 + C_{z\psi,n-1}^{\$}z_{t+1}\psi_{t+1} \\ -x_t - \frac{1}{2}z_t^2\sigma_m^2 - z_t\varepsilon_{m,t+1} - \lambda_t - \xi_t - \frac{1}{2}\psi_t^2\sigma_\pi^2 - \psi_t\varepsilon_{\pi,t+1} \end{array}\right\}\right] \\
&= \exp\left\{\begin{array}{c} A_{n-1}^{\$} + B_{x,n-1}^{\$}\left(\mu_x\left(1-\phi_x\right) + \phi_x x_t\right) + B_{z,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right) + B_{\lambda,n-1}^{\$}\left(\mu_\lambda + \lambda_t\right) + B_{\xi,n-1}^{\$}\phi_\xi\xi_t + B_{\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right) \\ +C_{z,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right)^2 + C_{\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right)^2 + C_{z\psi,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right)\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right) \\ -x_t - \frac{1}{2}z_t^2\sigma_m^2 - \lambda_t - \xi_t - \frac{1}{2}\psi_t^2\sigma_\pi^2 \end{array}\right\} \\
&\quad \times E_t\left[\exp\left\{\mathbf{d}_1'\boldsymbol{\omega}_{t+1} + \boldsymbol{\omega}_{t+1}'\mathbf{D}_2\boldsymbol{\omega}_{t+1}\right\}\right]
\end{aligned}$$

where $\boldsymbol{\omega}'_{t+1} = (\varepsilon_{\Lambda,t+1}, \varepsilon_{\lambda,t+1}, \varepsilon_{m,t+1}, \varepsilon_{\pi,t+1}, \varepsilon_{x,t+1}, \varepsilon_{\xi,t+1}, \varepsilon_{z,t+1}, \varepsilon_{\psi,t+1}) \tilde{} N(0, \boldsymbol{\Sigma}_\omega)$,

$$\mathbf{d}_1 = \begin{pmatrix} B^{\$}_{\lambda,n-1} \\ B^{\$}_{\lambda,n-1}\psi_t \\ -z_t \\ -\psi_t \\ B^{\$}_{x,n-1} \\ B^{\$}_{\xi,n-1}\psi_t \\ B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z(1-\phi_z) + \phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi(1-\phi_\psi) + \phi_\psi\psi_t\right) \\ B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi(1-\phi_\psi) + \phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z(1-\phi_z) + \phi_z z_t\right) \end{pmatrix}$$

$$\mathbf{D}_2 = \begin{pmatrix} 0 & \cdots & & 0 \\ & & & \vdots \\ \vdots & \ddots & & \\ & & C^{\$}_{z,n-1} & \frac{1}{2}C^{\$}_{z\psi,n-1} \\ 0 & \cdots & \frac{1}{2}C^{\$}_{z\psi,n-1} & C^{\$}_{\psi,n-1} \end{pmatrix}$$

Following Campbell, Chan, and Viceira (2003), we complete the square to calculate

$$\begin{aligned} E_t\left[\exp\left\{\mathbf{d}'_1\boldsymbol{\omega}_{t+1} + \boldsymbol{\omega}'_{t+1}\mathbf{D}_2\boldsymbol{\omega}_{t+1}\right\}\right] &= \frac{|\boldsymbol{\Sigma}_\omega|^{-1/2}}{\left|\boldsymbol{\Sigma}_\omega^{-1} - 2\mathbf{D}_2\right|^{1/2}} \exp\left\{\frac{1}{2}\mathbf{d}_1\left(\boldsymbol{\Sigma}_\omega^{-1} - 2\mathbf{D}_2\right)^{-1}\mathbf{d}'_1\right\} \\ &= \exp\left\{-\frac{1}{2}\log|\boldsymbol{\Sigma}_\omega| + \frac{1}{2}\log|\mathbf{G}| + \frac{1}{2}\mathbf{d}_1\mathbf{G}\mathbf{d}'_1\right\} \end{aligned}$$

where $\mathbf{G} = \left(\boldsymbol{\Sigma}_\omega^{-1} - 2\mathbf{D}_2\right)^{-1}$. Let g_{ij} be the ij-th element of $\mathbf{G}$. Then expanding and collecting terms gives

$$
p^{\$}_{n,t} = \left[\begin{array}{c}
A^{\$}_{n-1} + B^{\$}_{x,n-1}\left(\mu_x\left(1-\phi_x\right)+\phi_x x_t\right) + B^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right) + B^{\$}_{\lambda,n-1}\lambda_t + B^{\$}_{\xi,n-1}\phi_\xi \xi_t + B^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right) \\
+C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)^2 + C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)^2 + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right) \\
-x_t - \frac{1}{2}z_t^2\sigma_m^2 - \lambda_t - \xi_t - \frac{1}{2}\psi_t^2\sigma_\pi^2 - \frac{1}{2}\log|\mathbf{\Sigma}_\omega| + \frac{1}{2}\log|\mathbf{G}| \\
+\frac{1}{2}g_{11}B^{\$2}_{\lambda,n-1} + \frac{1}{2}g_{22}B^{\$2}_{\lambda,n-1}\psi_t^2 + \frac{1}{2}g_{33}z_t^2 + \frac{1}{2}g_{44}\psi_t^2 + \frac{1}{2}g_{55}B^{\$2}_{x,n-1} + \frac{1}{2}g_{66}B^{\$2}_{\xi,n-1}\psi_t^2 \\
+\frac{1}{2}g_{77}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)^2 \\
+\frac{1}{2}g_{88}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)^2 \\
+g_{12}B^{\$2}_{\lambda,n-1}\psi_t - g_{13}B^{\$}_{\lambda,n-1}z_t - g_{14}B^{\$}_{\lambda,n-1}\psi_t + g_{15}B^{\$}_{\lambda,n-1}B^{\$}_{x,n-1} + g_{16}B^{\$}_{\lambda,n-1}B^{\$}_{\xi,n-1}\psi_t \\
+g_{17}B^{\$}_{\lambda,n-1}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right) \\
+g_{18}B^{\$}_{\lambda,n-1}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right) \\
-g_{23}B^{\$}_{\lambda,n-1}z_t\psi_t - g_{24}B^{\$}_{\lambda,n-1}\psi_t^2 + g_{25}B^{\$}_{\lambda,n-1}B^{\$}_{x,n-1}\psi_t + g_{26}B^{\$}_{\lambda,n-1}B^{\$}_{\xi,n-1}\psi_t^2 \\
+g_{27}B^{\$}_{\lambda,n-1}\psi_t\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right) \\
+g_{28}B^{\$}_{\lambda,n-1}\psi_t\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right) \\
+g_{34}z_t\psi_t - g_{35}B^{\$}_{x,n-1}z_t - g_{36}B^{\$}_{\xi,n-1}z_t\psi_t \\
-g_{37}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)z_t \\
-g_{38}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)z_t \\
-g_{45}B^{\$}_{x,n-1}\psi_t - g_{46}B^{\$}_{\xi,n-1}\psi_t^2 \\
-g_{47}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)\psi_t \\
-g_{48}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)\psi_t \\
+g_{56}B^{\$}_{x,n-1}B^{\$}_{\xi,n-1}\psi_t \\
+g_{57}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)B^{\$}_{x,n-1} \\
+g_{58}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)B^{\$}_{x,n-1} \\
+g_{67}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)B^{\$}_{\xi,n-1}\psi_t \\
+g_{68}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)B^{\$}_{\xi,n-1}\psi_t \\
+g_{78}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right) \\
\times\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)
\end{array} \right]
$$

Thus, the coefficients of the pricing equation satisfy

$$
\begin{aligned}
A_n^{\$} &= \left[\begin{array}{c}
A_{n-1}^{\$} + B_{x,n-1}^{\$}\mu_x\left(1-\phi_x\right) + B_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + B_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) \\
+C_{z,n-1}^{\$}\mu_z^2\left(1-\phi_z\right)^2 + C_{\psi,n-1}^{\$}\mu_\psi^2\left(1-\phi_\psi\right)^2 + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\mu_\psi\left(1-\phi_\psi\right) - \frac{1}{2}\log\left|\boldsymbol{\Sigma}_\omega\right| + \frac{1}{2}\log\left|\mathbf{G}\right| \\
+\frac{1}{2}g_{11}B_{\lambda,n-1}^{\$2} + \frac{1}{2}g_{55}B_{x,n-1}^{\$2} \\
+\frac{1}{2}g_{77}\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)^2 \\
+\frac{1}{2}g_{88}\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)^2 \\
+g_{15}B_{\lambda,n-1}^{\$}B_{x,n-1}^{\$} + g_{17}\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)B_{\lambda,n-1}^{\$} \\
+g_{18}\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)B_{\lambda,n-1}^{\$} \\
+g_{57}\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)B_{x,n-1}^{\$} \\
+g_{58}\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)B_{x,n-1}^{\$} \\
+g_{78}\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)
\end{array}\right] \\
B_{x,n}^{\$} &= B_{x,n-1}^{\$}\phi_x - 1
\end{aligned}
$$

$$
\begin{aligned}
B_{z,n}^{\$} &= \left[\begin{array}{c}
\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)\phi_z - g_{35}B_{x,n-1}^{\$} - g_{13}B_{\lambda,n-1}^{\$} \\
+2g_{77}\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)C_{z,n-1}^{\$}\phi_z \\
+g_{88}\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)C_{z\psi,n-1}^{\$}\phi_z \\
+2g_{17}B_{\lambda,n-1}^{\$}C_{z,n-1}^{\$}\phi_z + g_{18}B_{\lambda,n-1}^{\$}C_{z\psi,n-1}^{\$}\phi_z \\
-g_{37}\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right) \\
-g_{38}\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right) \\
+2g_{57}B_{x,n-1}^{\$}C_{z,n-1}^{\$}\phi_z + g_{58}B_{x,n-1}^{\$}C_{z\psi,n-1}^{\$}\phi_z \\
+g_{78}\left(\begin{array}{c}
2C_{z,n-1}^{\$}\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right) + \\
C_{z\psi,n-1}^{\$}\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)
\end{array}\right)\phi_z
\end{array}\right] \\
B_{\lambda,n}^{\$} &= B_{\lambda,n-1}^{\$} - 1 \\
B_{\xi,n}^{\$} &= B_{\xi,n-1}^{\$}\phi_\xi - 1
\end{aligned}
$$

$$
B^{\$}_{\psi,n} = \left[\begin{array}{c}
g_{12}B^{\$2}_{\lambda,n-1} - g_{14}B^{\$}_{\lambda,n-1} + g_{16}B^{\$}_{\lambda,n-1}B^{\$}_{\xi,n-1} + g_{17}B^{\$}_{\lambda,n-1}C^{\$}_{z\psi,n-1}\phi_\psi + 2g_{18}B^{\$}_{\lambda,n-1}C^{\$}_{\psi,n-1}\phi_\psi \\
+g_{27}B^{\$}_{\lambda,n-1}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right) \\
+g_{28}B^{\$}_{\lambda,n-1}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right) \\
+g_{25}B^{\$}_{\lambda,n-1}B^{\$}_{x,n-1} + \left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)\phi_\psi \\
+g_{77}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right)C^{\$}_{z\psi,n-1}\phi_\psi \\
+2g_{88}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)C^{\$}_{\psi,n-1}\phi_\psi \\
-g_{45}B^{\$}_{x,n-1} + g_{56}B^{\$}_{x,n-1}B^{\$}_{\xi,n-1} + g_{57}B^{\$}_{x,n-1}C^{\$}_{z\psi,n-1}\phi_\psi + 2g_{58}B^{\$}_{x,n-1}C^{\$}_{\psi,n-1}\phi_\psi \\
-g_{47}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right) \\
-g_{48}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right) \\
+g_{67}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right)B^{\$}_{\xi,n-1} \\
+g_{68}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)B^{\$}_{\xi,n-1} \\
+g_{78}\left(\begin{array}{c} 2\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right)C^{\$}_{\psi,n-1} + \\ \left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)C^{\$}_{z\psi,n-1} \end{array} \right)\phi_\psi
\end{array} \right]
$$

$$
\begin{aligned}
C^{\$}_{z,n} &= \left[\begin{array}{c}
C^{\$}_{z,n-1}\phi_z^2 - \frac{1}{2}\sigma_m^2 + \frac{1}{2}g_{33} + 2g_{77}C^{\$2}_{z,n-1}\phi_z^2 + \frac{1}{2}g_{88}C^{\$2}_{z\psi,n-1}\phi_z^2 \\
-2g_{37}C^{\$}_{z,n-1}\phi_z - g_{38}C^{\$}_{z\psi,n-1}\phi_z + 2g_{78}C^{\$}_{z,n-1}C^{\$}_{z\psi,n-1}\phi_z^2
\end{array} \right] \\
C^{\$}_{\psi,n} &= \left[\begin{array}{c}
2g_{28}B^{\$}_{\lambda,n-1}C^{\$}_{\psi,n-1}\phi_\psi + g_{27}B^{\$}_{\lambda,n-1}C^{\$}_{z\psi,n-1}\phi_\psi \\
+g_{26}B^{\$}_{\lambda,n-1}B^{\$}_{\xi,n-1} - g_{24}B^{\$}_{\lambda,n-1} + \frac{1}{2}g_{22}B^{\$2}_{\lambda,n-1} \\
+C^{\$}_{\psi,n-1}\phi_\psi^2 - \frac{1}{2}\sigma_\pi^2 \\
+\frac{1}{2}g_{44} + \frac{1}{2}g_{66}B^{\$2}_{\xi,n-1} + \frac{1}{2}g_{77}C^{\$2}_{z\psi,n-1}\phi_\psi^2 + 2g_{88}C^{\$2}_{\psi,n-1}\phi_\psi^2 \\
-g_{46}B^{\$}_{\xi,n-1} - g_{47}C^{\$}_{z\psi,n-1}\phi_\psi \\
-2g_{48}C^{\$}_{\psi,n-1}\phi_\psi + g_{67}B^{\$}_{\xi,n-1}C^{\$}_{z\psi,n-1}\phi_\psi \\
+2g_{68}B^{\$}_{\xi,n-1}C^{\$}_{\psi,n-1}\phi_\psi + 2g_{78}C^{\$}_{\psi,n-1}C^{\$}_{z\psi,n-1}\phi_\psi^2
\end{array} \right] \\
C^{\$}_{z\psi,n} &= \left[\begin{array}{c}
g_{28}B^{\$}_{\lambda,n-1}C^{\$}_{z\psi,n-1}\phi_z + 2g_{27}B^{\$}_{\lambda,n-1}C^{\$}_{z\psi,n-1}\phi_z - g_{23}B^{\$}_{\lambda,n-1} \\
+C^{\$}_{z\psi,n-1}\phi_z\phi_\psi + g_{34} - g_{36}B^{\$}_{\xi,n-1} + 2g_{77}C^{\$}_{z,n-1}C^{\$}_{z\psi,n-1}\phi_z\phi_\psi \\
+2g_{88}C^{\$}_{\psi,n-1}C^{\$}_{z\psi,n-1}\phi_z\phi_\psi - g_{37}C^{\$}_{z\psi,n-1}\phi_\psi - 2g_{38}C^{\$}_{\psi,n-1}\phi_\psi \\
-2g_{47}C^{\$}_{z,n-1}\phi_z - g_{48}C^{\$}_{z\psi,n-1}\phi_z \\
+2g_{67}B^{\$}_{\xi,n-1}C^{\$}_{z,n-1}\phi_z + g_{68}B^{\$}_{\xi,n-1}C^{\$}_{z\psi,n-1}\phi_z \\
+g_{78}\left(4C^{\$}_{z,n-1}C^{\$}_{\psi,n-1} + C^{\$2}_{z\psi,n-1}\right)\phi_z\phi_\psi
\end{array} \right]
\end{aligned}
$$

where $B^{\$}_{x,1} = -1$, $B^{\$}_{\lambda,1} = -1$, $B^{\$}_{\xi,1} = -1$, $C^{\$}_{z\psi,1} = \sigma_{m\pi}$ and all other coefficients are zero at $n = 1$.

A.1.3 Expected Excess Returns

Real Bond Premia The log expected return on an n-period zero-coupon real bond is

$$\begin{aligned} E_t\left[r_{n,t+1} - r_{1,t+1}\right] &= E_t\left[\begin{array}{c} -\frac{1}{2}B^2_{x,n-1}\sigma^2_x - \frac{1}{2}B^2_{z,n-1}\sigma^2_z + B_{x,n-1}B_{z,n-1}\sigma_{xz} + \left(B_{x,n-1}\sigma_{xm} + B_{z,,n-1}\sigma_{zm}\right)z_t \\ +B_{x,n-1}\varepsilon_{x,t+1} + B_{z,n-1}\varepsilon_{z,t+1} \end{array}\right] \\ &= \left(B_{x,n-1}\sigma_{xm} + B_{z,,n-1}\sigma_{zm}\right)z_t \end{aligned}$$

since the shocks are conditionally jointly normal.

Nominal Bond Premia The log conditional expected real return on a 1-period zero-coupon nominal bond is

$$E_t\left[r^{\$}_{1,t+1} - \pi_{t+1}\right] = -\sigma_{m,\pi}z_t\psi_t$$

The log conditional expected gross excess return on an n-period zero-coupon nominal bond is

$$\begin{aligned} \log E_t\left[\frac{P^{\$}_{n-1,t+1}}{P^{\$}_{n,t}}\right] - E_t\left[r^{\$}_{1,t+1}\right] &= \log E_t\left[\exp\left\{p^{\$}_{n-1.t+1} - p^{\$}_{n,t}\right\}\right] - x_t - \lambda_t - \xi_t + \sigma_{m,\pi}z_t\psi_t \\ &= \left[\begin{array}{c} A^{\$}_{n-1} - A^{\$}_n + B^{\$}_{x,n-1}\mu_x\left(1-\phi_x\right) + B^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + B^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) \\ +C^{\$}_{z,n-1}\mu^2_z\left(1-\phi_z\right)^2 + C^{\$2}_{\psi,n-1}\mu^2_\psi\left(1-\phi_\psi\right)^2 + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\mu_\psi\left(1-\phi_\psi\right) \\ +\left(B^{\$}_{x,n-1}\phi_x - B^{\$}_{x,n} - 1\right)x_t + \left(B^{\$}_{\lambda,n-1} - B^{\$}_{\lambda,n} - 1\right)\lambda_t + \left(B^{\$}_{\xi,n-1}\phi_\xi - B^{\$}_{\xi,n} - 1\right)\xi_t \\ +\left(C^{\$}_{z,n-1}\phi^2_z - C^{\$}_{z,n}\right)z^2_t + \left(C^{\$}_{\psi,n-1}\phi^2_\psi - C^{\$}_{\psi,n}\right)\psi^2_t + \left(\sigma_{m,\pi} + C^{\$}_{z\psi,n-1}\phi_z\phi_\psi - C^{\$}_{z\psi,n}\right)z_t\psi_t \\ +\left(B^{\$}_{z,n-1}\phi_z - B^{\$}_{z,n} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right)\phi_z + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\phi_z\right)z_t \\ +\left(B^{\$}_{\psi,n-1}\phi_\psi - B^{\$}_{\psi,n} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\phi_\psi + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\phi_\psi\right)\psi_t \end{array}\right] \\ &\quad + \log E_t\left[\exp\left\{\begin{array}{c} B^{\$}_{x,n-1}\varepsilon_{x,t+1} + B^{\$}_{\lambda,n-1}\psi_t\varepsilon_{\lambda,t+1} + B^{\$}_{\lambda,n-1}\varepsilon_{\Lambda,t+1} + B^{\$}_{\xi,n-1}\psi_t\varepsilon_{\xi,t+1} \\ +C^{\$}_{z,n-1}\varepsilon^2_{z,t+1} + C^{\$}_{\psi,n-1}\varepsilon^2_{\psi,t+1} + C^{\$}_{z\psi,n-1}\varepsilon_{z,t+1}\varepsilon_{\psi,t+1} \\ +\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right)\right)\varepsilon_{z,t+1} \\ +\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right)\right)\varepsilon_{\psi,t+1} \end{array}\right\}\right] \end{aligned}$$

Note that the coefficient recursions imply that $B^{\$}_{x,n} = B^{\$}_{x,n-1}\phi_x - 1$, $B^{\$}_{\lambda,n} = B^{\$}_{\lambda,n-1} - 1$,and $B^{\$}_{\xi,n} = B^{\$}_{\xi,n-1}\phi_\xi - 1$, so that the terms involving x_t, λ_t, and ξ_t drop out. Following Campbell, Chan, and Viceira (2003), we calculate the expectation by completing the square. Let

$$\boldsymbol{\nu}' = (\varepsilon_{\Lambda,t+1}, \varepsilon_{x,t+1}, \varepsilon_{\lambda,t+1}, \varepsilon_{\xi,t+1}, \varepsilon_{z,t+1}, \varepsilon_{\psi,t+1}) \tilde{\ } N(0, \boldsymbol{\Sigma}_v),$$

$$\mathbf{f}_1 = \begin{pmatrix} B^{\$}_{\lambda,n-1} \\ B^{\$}_{x,n-1} \\ B^{\$}_{\lambda,n-1}\psi_t \\ B^{\$}_{\xi,n-1}\psi_t \\ \left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}(\mu_z(1-\phi_z)+\phi_z z_t) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right) \\ \left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}(\mu_z(1-\phi_z)+\phi_z z_t)\right) \end{pmatrix}$$

$$\mathbf{F}_2 = \begin{pmatrix} 0 & \cdots & & 0 \\ & \ddots & & \\ \vdots & & & \\ & & C^{\$}_{z,n-1} & \frac{1}{2}C^{\$}_{z\psi,n-1} \\ 0 & & \frac{1}{2}C^{\$}_{z\psi,n-1} & C^{\$}_{\psi,n-1} \end{pmatrix}$$

Then

$$\begin{aligned} E_t\left[\exp\{\mathbf{f}_1'\boldsymbol{\nu} + \boldsymbol{\nu}'\mathbf{F}_2\boldsymbol{\nu}\}\right] &= E_t\left[\exp\left\{\begin{array}{c} B^{\$}_{x,n-1}\varepsilon_{x,t+1} + B^{\$}_{\lambda,n-1}\psi_t\varepsilon_{\lambda,t+1} + B^{\$}_{\lambda,n-1}\varepsilon_{\Lambda,t+1} + B^{\$}_{\xi,n-1}\psi_t\varepsilon_{\xi,t+1} \\ +C^{\$}_{z,n-1}\varepsilon^2_{z,t+1} + C^{\$}_{\psi,n-1}\varepsilon^2_{\psi,t+1} + C^{\$}_{z\psi,n-1}\varepsilon_{z,t+1}\varepsilon_{\psi,t+1} \\ +\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}(\mu_z(1-\phi_z)+\phi_z z_t) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)\varepsilon_{z,t+1} \\ +\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}(\mu_z(1-\phi_z)+\phi_z z_t)\right)\varepsilon_{\psi,t+1} \end{array}\right\}\right] \\ &= \exp\left\{-\frac{1}{2}\log|\boldsymbol{\Sigma}_\nu| + \frac{1}{2}\log|\mathbf{H}| + \frac{1}{2}\mathbf{f}_1\mathbf{H}\mathbf{f}_1'\right\} \end{aligned}$$

where $\mathbf{H} = \left(\boldsymbol{\Sigma}_\nu^{-1} - 2\mathbf{F}_2\right)^{-1}$. Let h_{ij} be the ij-th element of $\mathbf{H}$. Then expanding and collecting terms gives

$$
\log E_t\left[\frac{P^{\$}_{n-1,t+1}}{P^{\$}_{n,t}}\right]-E_t\left[r^{\$}_{1,t+1}\right]=\left[\begin{array}{c}
A^{\$}_{n-1}-A^{\$}_{n}+B^{\$}_{x,n-1}\mu_x\left(1-\phi_x\right)+B^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right)+B^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\\
+C^{\$}_{z,n-1}\mu_z^2\left(1-\phi_z\right)^2+C^{\$2}_{\psi,n-1}\mu_\psi^2\left(1-\phi_\psi\right)^2+C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\mu_\psi\left(1-\phi_\psi\right)\\
+\left(C^{\$}_{z,n-1}\phi_z^2-C^{\$}_{z,n}\right)z_t^2+\left(C^{\$}_{\psi,n-1}\phi_\psi^2-C^{\$}_{\psi,n}\right)\psi_t^2+\left(\sigma_{m,\pi}+C^{\$}_{z\psi,n-1}\phi_z\phi_\psi-C^{\$}_{z\psi,n}\right)z_t\psi_t\\
+\left(B^{\$}_{z,n-1}\phi_z-B^{\$}_{z,n}+2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right)\phi_z+C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\phi_z\right)z_t\\
+\left(B^{\$}_{\psi,n-1}\phi_\psi-B^{\$}_{\psi,n}+2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\phi_\psi+C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\phi_\psi\right)\psi_t\\
-\frac{1}{2}\log\left|\mathbf{\Sigma}_\nu\right|+\frac{1}{2}\log\left|\mathbf{H}\right|+\frac{1}{2}h_{11}B^{\$2}_{\lambda,n-1}+\frac{1}{2}h_{22}B^{\$2}_{x,n-1}+\frac{1}{2}h_{33}B^{\$2}_{\lambda,n-1}\psi_t^2+\frac{1}{2}h_{44}B^{\$2}_{\xi,n-1}\psi_t^2\\
+\frac{1}{2}h_{55}\left(B^{\$}_{z,n-1}+2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)+C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)^2\\
+\frac{1}{2}h_{66}\left(B^{\$}_{\psi,n-1}+2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)+C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)^2\\
+h_{12}B^{\$}_{\lambda,n-1}B^{\$}_{x,n-1}+h_{13}B^{\$2}_{\lambda,n-1}\psi_t+h_{14}B^{\$}_{\lambda,n-1}B^{\$}_{\xi,n-1}\psi_t\\
+h_{15}B^{\$}_{\lambda,n-1}\left(B^{\$}_{z,n-1}+2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)+C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)\\
+h_{16}B^{\$}_{\lambda,n-1}\left(B^{\$}_{\psi,n-1}+2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)+C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)\\
+h_{23}B^{\$}_{x,n-1}B^{\$}_{\lambda,n-1}\psi_t+h_{24}B^{\$}_{x,n-1}B^{\$}_{\xi,n-1}\psi_t\\
+h_{25}B^{\$}_{x,n-1}\left(B^{\$}_{z,n-1}+2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)+C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)\\
+h_{26}B^{\$}_{x,n-1}\left(B^{\$}_{\psi,n-1}+2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)+C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)\\
+h_{34}B^{\$}_{\lambda,n-1}B^{\$}_{\xi,n-1}\psi_t^2\\
+h_{35}B^{\$}_{\lambda,n-1}\psi_t\left(B^{\$}_{z,n-1}+2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)+C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)\\
+h_{36}B^{\$}_{\lambda,n-1}\psi_t\left(B^{\$}_{\psi,n-1}+2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)+C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)\\
+h_{45}B^{\$}_{\xi,n-1}\psi_t\left(B^{\$}_{z,n-1}+2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)+C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)\\
+h_{46}B^{\$}_{\xi,n-1}\psi_t\left(B^{\$}_{\psi,n-1}+2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)+C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)\\
+h_{56}\left(B^{\$}_{z,n-1}+2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)+C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)\\
\times\left(B^{\$}_{\psi,n-1}+2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)+C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)
\end{array}\right]
$$

Thus, we can write

$$
\log E_t\left[\frac{P^{\$}_{n-1,t+1}}{P^{\$}_{n,t}}\right]-E_t\left[r^{\$}_{1,t+1}\right]=\kappa_n+\eta_{z,n}z_t+\eta_{\psi,n}\psi_t+\beta_{z,n}z_t^2+\beta_{\psi,n}\psi_t^2+\beta_{z\psi,n}z_t\psi_t
$$

where the coefficients are given by

$$
\kappa_n = \left[\begin{array}{c}
A^{\$}_{n-1} - A^{\$}_{n} + B^{\$}_{x,n-1}\mu_x\left(1-\phi_x\right) + B^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + B^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) \\
+C^{\$}_{z,n-1}\mu_z^2\left(1-\phi_z\right)^2 + C^{\$2}_{\psi,n-1}\mu_\psi^2\left(1-\phi_\psi\right)^2 + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\mu_\psi\left(1-\phi_\psi\right) \\
-\frac{1}{2}\log\left|\mathbf{\Sigma}_\nu\right| + \frac{1}{2}\log\left|\mathbf{H}\right| + \frac{1}{2}h_{11}B^{\$2}_{\lambda,n-1} + \frac{1}{2}h_{22}B^{\$2}_{x,n-1} \\
+\frac{1}{2}h_{55}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right)^2 \\
+\frac{1}{2}h_{66}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)^2 \\
+h_{12}B^{\$}_{\lambda,n-1}B^{\$}_{x,n-1} \\
+h_{15}B^{\$}_{\lambda,n-1}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right) \\
+h_{16}B^{\$}_{\lambda,n-1}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right) \\
+h_{25}B^{\$}_{x,n-1}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right) \\
+h_{26}B^{\$}_{x,n-1}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right) \\
+h_{56}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right)\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)
\end{array}\right]
$$

$$
\eta_{z,n} = \left[\begin{array}{c}
B^{\$}_{z,n-1}\phi_z - B^{\$}_{z,n} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right)\phi_z + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\phi_z \\
+2h_{55}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right)C^{\$}_{z,n-1}\phi_z \\
+h_{66}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)C^{\$}_{z\psi,n-1}\phi_z \\
2h_{15}B^{\$}_{\lambda,n-1}C^{\$}_{z,n-1}\phi_z + h_{16}B^{\$}_{\lambda,n-1}C^{\$}_{z\psi,n-1}\phi_z \\
+2h_{25}B^{\$}_{x,n-1}C^{\$}_{z,n-1}\phi_z + h_{26}B^{\$}_{x,n-1}C^{\$}_{z\psi,n-1}\phi_z \\
+h_{56}\left(\begin{array}{c}
\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right)C^{\$}_{z\psi,n-1}\phi_z \\
+2C^{\$}_{z,n-1}\phi_z\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)
\end{array}\right)
\end{array}\right]
$$

$$
\eta_{\psi,n} = \left[\begin{array}{c}
\left(B^{\$}_{\psi,n-1}\phi_\psi - B^{\$}_{\psi,n} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\phi_\psi + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\phi_\psi\right) \\
+h_{55}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right)C^{\$}_{z\psi,n-1}\phi_\psi \\
+2h_{66}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)C^{\$}_{\psi,n-1}\phi_\psi \\
+h_{13}B^{\$2}_{\lambda,n-1} + h_{14}B^{\$}_{\lambda,n-1}B^{\$}_{\xi,n-1} + h_{15}B^{\$}_{\lambda,n-1}C^{\$}_{z\psi,n-1}\phi_\psi + 2h_{16}B^{\$}_{\lambda,n-1}C^{\$}_{\psi,n-1}\phi_\psi \\
+h_{23}B^{\$}_{x,n-1}B^{\$}_{\lambda,n-1} + h_{24}B^{\$}_{x,n-1}B^{\$}_{\xi,n-1} + h_{25}B^{\$}_{x,n-1}C^{\$}_{z\psi,n-1}\phi_\psi + 2h_{26}B^{\$}_{x,n-1}C^{\$}_{\psi,n-1}\phi_\psi \\
+h_{35}B^{\$}_{\lambda,n-1}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right) \\
+h_{36}B^{\$}_{\lambda,n-1}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right) \\
+h_{45}B^{\$}_{\xi,n-1}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right) \\
+h_{46}B^{\$}_{\xi,n-1}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right) \\
+h_{56}\left[\begin{array}{c}
2C^{\$}_{\psi,n-1}\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right) \\
+C^{\$}_{z\psi,n-1}\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)
\end{array}\right]\phi_\psi
\end{array}\right]
$$

$$\beta_{z,n} = \left[C^{\$}_{z,n-1}\phi_z^2 - C^{\$}_{z,n} + 2h_{55}C^{\$2}_{z,n-1}\phi_z^2 + \frac{1}{2}h_{66}C^{\$2}_{z\psi,n-1}\phi_z^2 + 2h_{56}C^{\$}_{z,n-1}C^{\$}_{z\psi,n-1}\phi_z^2 \right]$$

$$\beta_{\psi,n} = \left[\begin{array}{c} C^{\$}_{\psi,n-1}\phi_\psi^2 - C^{\$}_{\psi,n} + \frac{1}{2}h_{33}B^{\$2}_{\lambda,n-1} + \frac{1}{2}h_{44}B^{\$2}_{\xi,n-1} + \frac{1}{2}h_{55}C^{\$2}_{z\psi,n-1}\phi_\psi^2 + 2h_{66}C^{\$2}_{\psi,n-1}\phi_\psi^2 \\ +h_{34}B^{\$}_{\xi,n-1}B^{\$}_{\lambda,n-1} + h_{35}B^{\$}_{\lambda,n-1}C^{\$}_{z\psi,n-1}\phi_\psi + 2h_{36}B^{\$}_{\lambda,n-1}C^{\$}_{\psi,n-1}\phi_\psi \\ +h_{45}B^{\$}_{\xi,n-1}C^{\$}_{z\psi,n-1}\phi_\psi + 2h_{46}B^{\$}_{\xi,n-1}C^{\$}_{\psi,n-1}\phi_\psi + 2h_{56}C^{\$}_{\psi,n-1}C^{\$}_{z\psi,n-1}\phi_\psi^2 \end{array} \right]$$

$$\beta_{z\psi,n} = \left[\begin{array}{c} \sigma_{m,\pi} + C^{\$}_{z\psi,n-1}\phi_z\phi_\psi - C^{\$}_{z\psi,n} + 2h_{55}C^{\$}_{z,n-1}C^{\$}_{z\psi,n-1}\phi_z\phi_\psi + 2h_{66}C^{\$}_{\psi,n-1}C^{\$}_{z\psi,n-1}\phi_z\phi_\psi + 2h_{24}B^{\$}_{\lambda,n-1}C^{\$}_{z,n-1}\phi_z \\ +2h_{35}B^{\$}_{\lambda,n-1}C^{\$}_{z,n-1}\phi_z + h_{36}B^{\$}_{\lambda,n-1}C_{z\psi,n-1}\phi_z + 2h_{45}B^{\$}_{\xi,n-1}C^{\$}_{z,n-1}\phi_z + h_{46}B^{\$}_{\xi,n-1}C^{\$}_{z\psi,n-1}\phi_z + h_{56}\left(4C^{\$}_{z,n-1}C^{\$}_{\psi,n-1} + C^{\$2}_{z\psi,n-1}\right)\phi_\psi\phi_z \end{array} \right]$$

A.1.4 Observation Equations

Stock Returns We model the unexpected stock return as

$$r_{e,t+1} - E_t r_{e,t+1} = \beta_{ex}\varepsilon_{x,t+1} + \beta_{em}\varepsilon_{m,t+1}$$

The standard pricing equation then implies that the expected equity return satisfies

$$\begin{aligned} 1 &= E_t\left[\exp\left(r_{e,t+1} + m_{t+1}\right)\right] \\ &= \exp\left(E_t r_{e,t+1} - x_t - \frac{1}{2}z_t^2\sigma_m^2\right)\exp\left(\begin{array}{c} \frac{1}{2}\beta_{ex}^2\sigma_x^2 + \frac{1}{2}\beta_{em}^2\sigma_m^2 + \frac{1}{2}z_t^2\sigma_m^2 \\ +\beta_{ex}\beta_{em}\sigma_{xm} - \beta_{ex}z_t\sigma_{xm} - \beta_{em}z_t\sigma_m^2 \end{array} \right) \end{aligned}$$

so that

$$r_{e,t+1} = -\frac{1}{2}\beta_{ex}^2\sigma_x^2 - \frac{1}{2}\beta_{em}^2\sigma_m^2 - \beta_{ex}\beta_{em}\sigma_{xm} + x_t + \left(\beta_{ex}\sigma_{xm} + \beta_{em}\sigma_m^2\right)z_t + \beta_{ex}\varepsilon_{x,t+1} + \beta_{em}\varepsilon_{m,t+1}$$

Stock-Bond Return Covariance The final observation equation uses the conditional covariance of log stock returns with log bond returns. As we saw above, the holding period return on an n-period bond is

$$
\begin{aligned}
r_{n,t+1} &= p_{n-1,t+1} - p_{n,t} \\
&= \left[\begin{array}{c}
A^{\$}_{n-1} - A^{\$}_{n} + B^{\$}_{x,n-1}\mu_x\left(1-\phi_x\right) + B^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + B^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) \\
+C^{\$}_{z,n-1}\mu_z^2\left(1-\phi_z\right)^2 + C^{\$2}_{\psi,n-1}\mu_\psi^2\left(1-\phi_\psi\right)^2 + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\mu_\psi\left(1-\phi_\psi\right) \\
+\left(B^{\$}_{x,n-1}\phi_x - B^{\$}_{x,n} - 1\right)x_t + \left(B^{\$}_{\xi,n-1}\phi_\xi - B^{\$}_{\xi,n} - 1\right)\xi_t \\
+\left(C^{\$}_{z,n-1}\phi_z^2 - C^{\$}_{z,n}\right)z_t^2 + \left(C^{\$}_{\psi,n-1}\phi_\psi^2 - C^{\$}_{\psi,n}\right)\psi_t^2 + \left(\sigma_{m,\pi} + C^{\$}_{z\psi,n-1}\phi_z\phi_\psi - C^{\$}_{z\psi,n}\right)z_t\psi_t \\
+\left(B^{\$}_{z,n-1}\phi_z - B^{\$}_{z,n} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right)\phi_z + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\phi_z\right)z_t \\
+\left(B^{\$}_{\psi,n-1}\phi_\psi - B^{\$}_{\psi,n} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\phi_\psi + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\phi_\psi\right)\psi_t
\end{array}\right] \\
&\quad + \left[\begin{array}{c}
B^{\$}_{x,n-1}\varepsilon_{x,t+1} + B^{\$}_{\lambda,n-1}\psi_t\varepsilon_{\lambda,t+1} + B^{\$}_{\lambda,n-1}\varepsilon_{\Lambda,t+1} + B^{\$}_{\xi,n-1}\psi_t\varepsilon_{\xi,t+1} \\
+C^{\$}_{z,n-1}\varepsilon^2_{z,t+1} + C^{\$}_{\psi,n-1}\varepsilon^2_{\psi,t+1} + C^{\$}_{z\psi,n-1}\varepsilon_{z,t+1}\varepsilon_{\psi,t+1} \\
+\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right)\right)\varepsilon_{z,t+1} \\
+\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right)\right)\varepsilon_{\psi,t+1}
\end{array}\right]
\end{aligned}
$$

We assume that the unexpected stock return is assumed to be

$$r_{e,t+1} - E_t r_{e,t+1} = \beta_{ex}\varepsilon_{x,t+1} + \beta_{em}\varepsilon_{m,t+1}$$

Thus, the conditional covariance with the real return on short term nominal bond is

$$Cov_t\left(r_{e,t+1}, r^{\$}_{1,t+1} - \pi_{t+1}\right) = Cov\left(\beta_{ex}\varepsilon_{x,t+1} + \beta_{em}\varepsilon_{m,t+1}, -\psi_t\varepsilon_{\pi,t+1}\right) = -\psi_t\left(\beta_{ex}\sigma_{x\pi} + \beta_{em}\sigma_{m\pi}\right)$$

And the conditional covariance with the return on a long bond is

$$Cov_t\left(r_{e,t+1}, r^{\$}_{n,t+1}\right) = Cov_t\left(\beta_{ex}\varepsilon_{x,t+1} + \beta_{em}\varepsilon_{m,t+1}, \left[\begin{array}{c} B^{\$}_{x,n-1}\varepsilon_{x,t+1} + B^{\$}_{\lambda,n-1}\psi_t\varepsilon_{\lambda,t+1} + B^{\$}_{\lambda,n-1}\varepsilon_{\Lambda,t+1} + B^{\$}_{\xi,n-1}\psi_t\varepsilon_{\xi,t+1} \\ +C^{\$}_{z,n-1}\varepsilon^2_{z,t+1} + C^{\$}_{\psi,n-1}\varepsilon^2_{\psi,t+1} + C^{\$}_{z\psi,n-1}\varepsilon_{z,t+1}\varepsilon_{\psi,t+1} \\ +\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right)\right)\varepsilon_{z,t+1} \\ +\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right)\right)\varepsilon_{\psi,t+1} \end{array}\right]\right)$$

$$= \beta_{ex}\left(\begin{array}{c} B^{\$}_{x,n-1}\sigma^2_x + B^{\$}_{\lambda,n-1}\sigma_{x,\lambda}\psi_t + B^{\$}_{\lambda,n-1}\sigma_{x,\Lambda} + B^{\$}_{\xi,n-1}\sigma_{x,\xi}\psi_t \\ +C^{\$}_{z,n-1}Cov_t\left(\varepsilon_{x,t+1}, \varepsilon^2_{z,t+1}\right) + C^{\$}_{\psi,n-1}Cov_t\left(\varepsilon_{x,t+1}, \varepsilon^2_{\psi,t+1}\right) + C^{\$}_{z\psi,n-1}Cov_t\left(\varepsilon_{x,t+1}, \varepsilon_{z,t+1}\varepsilon_{\psi,t+1}\right) \\ +\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right)\right)\sigma_{x.z} \\ +\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right)\right)\sigma_{x,\psi} \end{array}\right)$$

$$+\beta_{em}\left(\begin{array}{c} B^{\$}_{x,n-1}\sigma_{xm} + B^{\$}_{\lambda,n-1}\sigma_{m,\lambda}\psi_t + B^{\$}_{\lambda,n-1}\sigma_{m,\Lambda} + B^{\$}_{\xi,n-1}\sigma_{\xi,m}\psi_t \\ +C^{\$}_{z,n-1}Cov_t\left(\varepsilon_{m,t+1}, \varepsilon^2_{z,t+1}\right) + C^{\$}_{\psi,n-1}Cov_t\left(\varepsilon_{m,t+1}, \varepsilon^2_{\psi,t+1}\right) + C^{\$}_{z\psi,n-1}Cov_t\left(\varepsilon_{m,t+1}, \varepsilon_{z,t+1}\varepsilon_{\psi,t+1}\right) \\ +\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right)\right)\sigma_{z,m} \\ +\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right)\right)\sigma_{\psi,m} \end{array}\right)$$

Since the ε's are conditionally jointly normal and mean zero we have $Cov_t\left(\varepsilon_{a,t+1}, \varepsilon^2_{b,t+1}\right) = 0$ and $Cov_t\left(\varepsilon_{a,t+1}, \varepsilon_{b,t+1}\varepsilon_{c,t+1}\right) = 0$ for all a, b, c. Additionally, note that we impose $\sigma_{x,\Lambda} = \sigma_{x,\Xi} = \sigma_{m,\Lambda} = \sigma_{m,\Xi} = 0$ so that the real-nominal covariance is unaffected by the homoskedastic shocks to expected inflation. Thus, the expression simplifies to

$$Cov_t\left(r_{e,t+1}, r_{n,t+1}\right) = \beta_{ex}\left(\begin{array}{c} B^{\$}_{x,n-1}\sigma^2_x \\ +\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right)\sigma_{x.z} \\ +\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)\sigma_{x,\psi} \end{array}\right)$$

$$+\beta_{em}\left(\begin{array}{c} B^{\$}_{x,n-1}\sigma_{xm} \\ +\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right)\sigma_{z,m} \\ +\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)\sigma_{\psi,m} \end{array}\right)$$

$$+\left[\begin{array}{c} \beta_{ex}\left(2C^{\$}_{z,n-1}\sigma_{xz}\phi_z + C^{\$}_{z\psi,n-1}\sigma_{x\psi}\phi_z\right) \\ +\beta_{em}\left(2C^{\$}_{z,n-1}\sigma_{zm}\phi_z + C^{\$}_{z\psi,n-1}\sigma_{\psi m}\phi_z\right) \end{array}\right] z_t$$

$$+\left[\begin{array}{c} \beta_{ex}\left(B^{\$}_{\lambda,n-1}\sigma_{x,\lambda} + B^{\$}_{\xi,n-1}\sigma_{x,\xi} + C^{\$}_{z\psi,n-1}\sigma_{xz}\phi_\psi + 2C^{\$}_{\psi,n-1}\sigma_{x\psi}\phi_\psi\right) \\ +\beta_{em}\left(B^{\$}_{\lambda,n-1}\sigma_{m,\lambda} + B^{\$}_{\xi,n-1}\sigma_{m,\xi} + C^{\$}_{z\psi,n-1}\sigma_{zm}\phi_\psi + 2C^{\$}_{\psi,n-1}\sigma_{\psi m}\phi_\psi\right) \end{array}\right]\psi_t$$

Volatility of Bond Returns We have

$$r_{n,t+1} - E_t r_{n,t+1} = \begin{array}{c} B^{\$}_{x,n-1}\varepsilon_{x,t+1} + B^{\$}_{\lambda,n-1}\psi_t\varepsilon_{\lambda,t+1} + B^{\$}_{\lambda,n-1}\varepsilon_{\Lambda,t+1} + B^{\$}_{\xi,n-1}\psi_t\varepsilon_{\xi,t+1} \\ +C^{\$}_{z,n-1}\varepsilon^2_{z,t+1} + C^{\$}_{\psi,n-1}\varepsilon^2_{\psi,t+1} + C^{\$}_{z\psi,n-1}\varepsilon_{z,t+1}\varepsilon_{\psi,t+1} \\ +\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right)\right)\varepsilon_{z,t+1} \\ +\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right)\right)\varepsilon_{\psi,t+1} \end{array}$$

so that

$$\begin{aligned} Cov_t\left(r^{\$}_{1,t+1} - \pi_{t+1}, r^{\$}_{n,t+1} - E^{\$}_t r_{n,t+1}\right) &= Cov_t\left(-\psi_t\varepsilon_{\pi,t+1}, \begin{array}{c} B^{\$}_{x,n-1}\varepsilon_{x,t+1} + B^{\$}_{\lambda,n-1}\psi_t\varepsilon_{\lambda,t+1} + B^{\$}_{\lambda,n-1}\varepsilon_{\Lambda,t+1} + B^{\$}_{\xi,n-1}\psi_t\varepsilon_{\xi,t+1} \\ +C^{\$}_{z,n-1}\varepsilon^2_{z,t+1} + C^{\$}_{\psi,n-1}\varepsilon^2_{\psi,t+1} + C^{\$}_{z\psi,n-1}\varepsilon_{z,t+1}\varepsilon_{\psi,t+1} \\ +\left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right)\right)\varepsilon_{z,t+1} \\ +\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\left(\mu_\psi\left(1-\phi_\psi\right) + \phi_\psi\psi_t\right) + C^{\$}_{z\psi,n-1}\left(\mu_z\left(1-\phi_z\right) + \phi_z z_t\right)\right)\varepsilon_{\psi,t+1} \end{array}\right) \\ &= -\left(\begin{array}{c} B^{\$}_{x,n-1}\sigma_{x\pi} + B^{\$}_{\lambda,n-1}\sigma_{\Lambda\pi} + \left(B^{\$}_{z,n-1} + 2C^{\$}_{z,n-1}\mu_z\left(1-\phi_z\right) + C^{\$}_{z\psi,n-1}\mu_\psi\left(1-\phi_\psi\right)\right)\sigma_{z\pi} \\ +\left(B^{\$}_{\psi,n-1} + 2C^{\$}_{\psi,n-1}\mu_\psi\left(1-\phi_\psi\right) + C^{\$}_{z\psi,n-1}\mu_z\left(1-\phi_z\right)\right)\sigma_{\psi\pi} \end{array}\right)\psi_t \\ &\quad -\left(B^{\$}_{\lambda,n-1}\sigma_{\lambda\pi} + B^{\$}_{\xi,n-1}\sigma_{\xi\pi} + C^{\$}_{z\psi,n-1}\phi_\psi\sigma_{z\pi} + 2C^{\$}_{\psi,n-1}\phi_\psi\sigma_{\psi\pi}\right)\psi^2_t \\ &\quad -\left(2C^{\$}_{z,n-1}\phi_z\sigma_{z\pi} + C^{\$}_{z\psi,n-1}\phi_z\sigma_{\psi\pi}\right)z_t\psi_t \end{aligned}$$

and

$$
Var_t\left(r_{n,t+1}\right)=\begin{array}{c}
B_{x,n-1}^{\$2}\sigma_x^2+B_{\lambda,n-1}^{\$2}\psi_t^2\sigma_\lambda^2+B_{\lambda,n-1}^{\$2}\sigma_\Lambda^2+B_{\xi,n-1}^{\$2}\psi_t^2\sigma_\xi^2\\
+C_{z,n-1}^{\$2}Var_t\left(\varepsilon_{z,t+1}^2\right)+C_{\psi,n-1}^{\$2}Var_t\left(\varepsilon_{\psi,t+1}^2\right)+C_{z\psi,n-1}^{\$2}Var_t\left(\varepsilon_{z,t+1}\varepsilon_{\psi,t+1}\right)\\
+\left(B_{z,n-1}^{\$}+2C_{z,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)+C_{z\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)^2\sigma_z^2\\
+\left(B_{\psi,n-1}^{\$}+2C_{\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)+C_{z\psi,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)^2\sigma_\psi^2\\
+2B_{x,n-1}^{\$}B_{\lambda,n-1}^{\$}\sigma_{x,\lambda}\psi_t+2B_{x,n-1}^{\$}B_{\lambda,n-1}^{\$}\sigma_{x,\Lambda}+2B_{x,n-1}^{\$}B_{\xi,n-1}^{\$}\sigma_{x,\xi}\psi_t\\
+2\left(B_{z,n-1}^{\$}+2C_{z,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)+C_{z\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)B_{x,n-1}^{\$}\sigma_{xz}\\
+2\left(B_{\psi,n-1}^{\$}+2C_{\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)+C_{z\psi,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)B_{x,n-1}^{\$}\sigma_{x\psi}\\
+2B_{\lambda,n-1}^{\$2}\sigma_{\lambda,\Lambda}\psi_t+2B_{\lambda,n-1}^{\$}B_{\xi,n-1}^{\$}\sigma_{\lambda,\xi}\psi_t^2\\
+2\left(B_{z,n-1}^{\$}+2C_{z,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)+C_{z\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)B_{\lambda,n-1}^{\$}\sigma_{z,\lambda}\psi_t\\
+2\left(B_{\psi,n-1}^{\$}+2C_{\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)+C_{z\psi,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)B_{\lambda,n-1}^{\$}\sigma_{\psi,\lambda}\psi_t\\
+2B_{\lambda,n-1}^{\$}B_{\xi,n-1}^{\$}\sigma_{\Lambda,\xi}\psi_t\\
+2\left(B_{z,n-1}^{\$}+2C_{z,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)+C_{z\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)B_{\lambda,n-1}^{\$}\sigma_{z,\Lambda}\\
+2\left(B_{\psi,n-1}^{\$}+2C_{\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)+C_{z\psi,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)B_{\lambda,n-1}^{\$}\sigma_{\psi,\Lambda}\\
+2\left(B_{z,n-1}^{\$}+2C_{z,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)+C_{z\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)B_{\xi,n-1}^{\$}\sigma_{\xi,z}\psi_t\\
+2\left(B_{\psi,n-1}^{\$}+2C_{\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)+C_{z\psi,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)B_{\xi,n-1}^{\$}\sigma_{\xi,\psi}\psi_t\\
+2\left[\begin{array}{c}\left(B_{z,n-1}^{\$}+2C_{z,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)+C_{z\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)\right)\\
\times\left(B_{\psi,n-1}^{\$}+2C_{\psi,n-1}^{\$}\left(\mu_\psi\left(1-\phi_\psi\right)+\phi_\psi\psi_t\right)+C_{z\psi,n-1}^{\$}\left(\mu_z\left(1-\phi_z\right)+\phi_z z_t\right)\right)\end{array}\right]\sigma_{z,\psi}
\end{array}
$$

Rearranging gives

$$
\begin{aligned}
Var_t\left(r_{n,t+1}\right) \quad = \quad & \left[\begin{array}{c}
B_{x,n-1}^{\$2}\sigma_x^2 + B_{\lambda,n-1}^{\$2}\sigma_\Lambda^2 \\
+2C_{z,n-1}^{\$2}\sigma_z^4 + 2C_{\psi,n-1}^{\$2}\sigma_\psi^4 + C_{z\psi,n-1}^{\$2}\left(\sigma_z^2\sigma_\psi^2 + \sigma_{z\psi}^2\right) \\
+\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)^2\sigma_z^2 \\
+\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)^2\sigma_\psi^2 \\
+2\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)B_{x,n-1}^{\$}\sigma_{xz} \\
+2\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)B_{x,n-1}^{\$}\sigma_{x\psi} \\
+2\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)B_{\lambda,n-1}^{\$}\sigma_{\psi,\Lambda} \\
+2\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)\sigma_{z,\psi}
\end{array}\right] \\
& + \left[\begin{array}{c}
4\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)C_{z,n-1}^{\$}\sigma_z^2\phi_z \\
+2\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)C_{z\psi,n-1}^{\$}\sigma_\psi^2\phi_z \\
+4C_{z,n-1}^{\$}B_{x,n-1}^{\$}\sigma_{xz}\phi_z + 2C_{z\psi,n-1}^{\$}B_{x,n-1}^{\$}\sigma_{x\psi}\phi_z \\
+2C_{z\psi,n-1}^{\$}B_{\lambda,n-1}^{\$}\sigma_{\psi,\Lambda}\phi_z \\
+2\left[\begin{array}{c}
2C_{z,n-1}^{\$}\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right) \\
+C_{z\psi,n-1}^{\$}\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)
\end{array}\right]\sigma_{z,\psi}\phi_z
\end{array}\right] z_t \\
& + \left[\begin{array}{c}
2B_{x,n-1}^{\$}B_{\lambda,n-1}^{\$}\sigma_{x,\lambda} + 2B_{x,n-1}^{\$}B_{\xi,n-1}^{\$}\sigma_{x,\xi} \\
+2\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)C_{z\psi,n-1}^{\$}\sigma_z^2\phi_\psi \\
+4\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)C_{\psi,n-1}^{\$}\sigma_\psi^2\phi_\psi \\
+2C_{z\psi,n-1}^{\$}B_{x,n-1}^{\$}\sigma_{xz}\phi_\psi + 4C_{\psi,n-1}^{\$}B_{x,n-1}^{\$}\sigma_{x\psi}\phi_\psi \\
+2B_{\lambda,n-1}^{\$2}\sigma_{\lambda,\Lambda} \\
+2\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)B_{\lambda,n-1}^{\$}\sigma_{z,\lambda} \\
+2\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)B_{\lambda,n-1}^{\$}\sigma_{\psi,\lambda} \\
+2B_{\lambda,n-1}^{\$}B_{\xi,n-1}^{\$}\sigma_{\Lambda,\xi} + 4C_{\psi,n-1}^{\$}B_{\lambda,n-1}^{\$}\sigma_{\psi,\Lambda}\phi_\psi \\
+2\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right)B_{\xi,n-1}^{\$}\sigma_{\xi,z} \\
+2\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)B_{\xi,n-1}^{\$}\sigma_{\psi,\xi} \\
+2\left[\begin{array}{c}
2C_{\psi,n-1}^{\$}\left(B_{z,n-1}^{\$} + 2C_{z,n-1}^{\$}\mu_z\left(1-\phi_z\right) + C_{z\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right)\right) \\
+C_{z\psi,n-1}^{\$}\left(B_{\psi,n-1}^{\$} + 2C_{\psi,n-1}^{\$}\mu_\psi\left(1-\phi_\psi\right) + C_{z\psi,n-1}^{\$}\mu_z\left(1-\phi_z\right)\right)
\end{array}\right]\sigma_{z,\psi}\phi_\psi
\end{array}\right]\psi_t
\end{aligned}
$$

$$+\left[4C^{\$2}_{z,n-1}\phi_z^2\sigma_z^2 + C^{\$2}_{z\psi,n-1}\phi_z^2\sigma_\psi^2 + 4C^{\$}_{z,n-1}C^{\$}_{z\psi,n-1}\sigma_{z,\psi}\phi_z^2\right] z_t^2$$

$$+\left[\begin{array}{c} B^{\$2}_{\lambda,n-1}\sigma_\lambda^2 + B^{\$2}_{\xi,n-1}\sigma_\xi^2 + C^{\$2}_{z\psi,n-1}\phi_\psi^2\sigma_z^2 + 4C^{\$2}_{\psi,n-1}\phi_\psi^2\sigma_\psi^2 + 2B^{\$}_{\lambda,n-1}B^{\$}_{\xi,n-1}\sigma_{\xi\lambda} + 2C^{\$}_{z\psi,n-1}B^{\$}_{\lambda,n-1}\sigma_{z,\lambda}\phi_\psi \\ +4C^{\$}_{\psi,n-1}B^{\$}_{\lambda,n-1}\sigma_{\psi,\lambda}\phi_\psi + 2C^{\$}_{z\psi,n-1}B^{\$}_{\xi,n-1}\sigma_{\xi,z}\phi_\psi + 4C^{\$}_{\psi,n-1}B^{\$}_{\xi,n-1}\sigma_{\psi,\xi}\phi_\psi + 4C^{\$}_{\psi,n-1}C^{\$}_{z\psi,n-1}\sigma_{z,\psi}\phi_\psi^2 \end{array}\right] \psi_t^2$$

$$+\left[\begin{array}{c} 4C^{\$}_{z,n-1}C^{\$}_{z\psi,n-1}\sigma_z^2\phi_z\phi_\psi + 4C^{\$}_{\psi,n-1}C^{\$}_{z\psi,n-1}\sigma_\psi^2\phi_z\phi_\psi + 4C^{\$}_{z,n-1}B^{\$}_{\lambda,n-1}\sigma_{z,\lambda}\phi_z + 2C^{\$}_{z\psi,n-1}B^{\$}_{\lambda,n-1}\sigma_{\psi,\lambda}\phi_z \\ +4C^{\$}_{z,n-1}B^{\$}_{\xi,n-1}\sigma_{\xi,z}\phi_z + 2C^{\$}_{z\psi,n-1}B^{\$}_{\xi,n-1}\sigma_{\psi,\xi}\phi_z + 2\left(4C^{\$}_{z,n-1}C^{\$}_{\psi,n-1} + C^{\$2}_{z\psi,n-1}\right)\sigma_{z\psi}\phi_\psi\phi_z \end{array}\right] z_t\psi_t$$

A.2 Additional Results

This section presents results for alternative versions of the model. We examine variants that 1) drop the TIPS observation equation; 2) hold fixed the risk aversion variable z_t; 3) hold fixed the nominal-real covariance ψ_t; and 4) hold fixed both z_t and ψ_t.

Table 1. Parameter estimates.

Parameter Estimates					
Parameter	Full Model	w/o TIPS	Constant z	Constant ψ	Constant $z\psi$
$\mu_x \times 10^2$	1.02	0.69	1.01	1.00	1.01
	(0.02)	(0.05)	(0.02)	(0.02)	(0.02)
$\mu_z \times 10^1$	2.36	2.33	2.37	2.27	1.96
	(0.32)	(0.38)	(0.33)	(0.69)	(0.72)
$\mu_\psi \times 10^3$	3.97	2.80	4.45	1.58	1.03
	(1.24)	(0.70)	(1.43)	(0.09)	(0.40)
ϕ_x	0.95	0.94	0.95	0.93	0.93
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)
ϕ_ξ	0.86	0.79	0.86	0.89	0.89
	(0.02)	(0.04)	(0.02)	(0.02)	(0.02)
ϕ_z	0.96	0.97		0.96	
	(0.08)	(0.11)		(0.08)	
ϕ_ψ	0.88	0.77	0.89		
	(0.08)	(0.09)	(0.09)		
$\sigma_x \times 10^4$	9.17	11.42	9.52	5.57	5.87
	(0.73)	(1.02)	(0.77)	(0.34)	(0.38)
$\sigma_\lambda \times 10^4$	3.94	18.68	5.84	7.26	9.95
	(36.16)	(14.21)	(43.40)	(40.85)	(26.15)
$\sigma_\Lambda \times 10^4$	7.01	7.64	6.99	9.26	9.26
	(0.28)	(0.27)	(0.28)	(0.22)	(0.22)
σ_ξ	0.64	0.57	0.63	2.51	3.97
	(0.09)	(0.06)	(0.08)	(0.01)	(0.16)
$\sigma_z \times 10^3$	6.52	8.70		5.22	
	(3.77)	(8.70)		(2.24)	
$\sigma_\psi \times 10^3$	1.99	2.82	1.98		
	(0.32)	(0.42)	(0.28)		
β_{ex}	22.77	14.81	21.69	−56.70	−47.32
	(0.01)	(0.01)	(0.01)	(0.91)	(0.31)
$\beta_{em} \times 10^2$	8.42	8.33	8.40	7.31	7.48
	(0.05)	(0.05)	(0.05)	(0.44)	(0.45)
b_d	0.44	0.34		0.59	
	(0.01)	(0.01)		(0.07)	
$a_d \times 10^1$	−0.72	−0.46		−1.01	
	(0.14)	(0.13)		(0.44)	

Parameter Estimates					
Parameter	Full Model	w/o TIPS	Constant z	Constant ψ	Constant $z\psi$
$\rho_{x\xi}$	-0.00	-0.00	-0.00	0.01	-0.00
	(0.01)	(0.01)	(0.01)	(0.03)	(0.01)
ρ_{xm}	-0.03	-0.03	-0.03	-0.05	-0.07
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$\rho_{x\pi}$	-0.12	-0.11	-0.12	-0.13	-0.12
	(0.03)	(0.01)	(0.03)	(0.01)	(0.01)
$\rho_{\lambda m}$	0.01	-0.00	0.01	0.02	-0.00
	(1.36)	(0.36)	(1.09)	(0.31)	(0.07)
$\rho_{\xi m}$	-0.48	-0.56	-0.48	-0.08	-0.05
	(0.04)	(0.02)	(0.03)	(0.01)	(0.02)
$\rho_{\xi\pi}$	0.16	0.17	0.16	0.03	0.03
	(0.07)	(0.05)	(0.06)	(0.01)	(0.01)
ρ_{zm}	0.03	0.03		0.04	
	(0.37)	(0.07)		(0.27)	
$\rho_{\pi m}$	-0.03	-0.06	-0.03	-0.06	-0.06
	(0.04)	(0.01)	(0.07)	(0.02)	(0.04)

Log Likelihoods					
	Full Model	w/o TIPS	Constant z	Constant ψ	Constant $z\psi$
ΔLog-likelihood	0	N/A	-5	-303	-310
p-value		N/A	0.01	0.000	0.000

Table 2. Sample and implied moments for 3mo excess returns. Yield spreads (YS) are calculated over the 3mo yield. Realized excess returns (RXR) are calculated over a 3mo holding period, in excess of the 3mo yield. Units are annualized percentage points. Simulation columns report means across 1000 replications, each of which simulates a time-series of 250 quarters. The $\sigma\left(\widehat{CP}\right)$ row reports the standard deviation of the fitted values from a Cochrane-Piazzesi style regression of RXR on the 1-, 3-, and 5-yr forward rates at the beginning of the holding period. The $\sigma\left(\widehat{CS}\right)$ row reports the standard deviation of the fitted values from a Campbell-Shiller style regression of RXR on the same-maturity YS at the beginning of the holding period. Below each entry we report in brackets the fraction of simulation runs where the simulated value exceeds the data value. † Data moments for the 10yr return require 117mo yields. We interpolate the 117mo yield linearly between the 5yr and the 10yr ‡ TIPS entries refer to the 10yr spliced TIPS yield. We have this data 1/1985-12/2005.

Sample and Implied Moments						
Moment	Actual Data	Full model	w/o TIPS	Constant z	Constant ψ	Constant $z\psi$
3yr YS mean	.674	.355 [.201]	.219 [.043]	.400 [.239]	.159 [.047]	.116 [.034]
10yr YS mean	1.13	.478 [.154]	.232 [.016]	.557 [.185]	.233 [.036]	.247 [.028]
3yr YS stdev	.401	.714 [.996]	.615 [.997]	.748 [1.00]	.725 [1.00]	.743 [1.00]
10yr YS stdev	.642	1.16 [.998]	.910 [.990]	1.23 [1.00]	1.29 [1.00]	1.33 [1.00]
3yr RXR mean	1.06	.694 [.308]	.395 [.085]	.781 [.357]	.225 [.128]	.121 [.092]
10yr RXR mean	1.79	.910 [.238]	.455 [.082]	1.05 [.289]	.330 [.126]	.184 [.110]
3yr RXR stdev	4.01	4.85 [.928]	3.70 [.125]	5.12 [.969]	5.58 [1.00]	5.75 [1.00]
10yr RXR stdev	10.00	8.40 [.011]	7.48 [.000]	8.69 [.035]	10.13 [.631]	10.32 [.761]
10yr TIPS yield mean	3.37‡	4.07 [.999]	2.74 [.000]	4.05 [1.00]	4.04 [1.00]	4.04 [1.00]
10yr TIPS YS mean		.001	−.011	.000	.014	.019
10yr TIPS RXR mean		.039	.027	.041	.027	.035
10yr TIPS RXR stdev		3.27	3.36	3.35	1.43	1.49

Predictive Regressions						
Moment	Actual Data	Full model	w/o TIPS	Constant z	Constant ψ	Constant $z\psi$
3yr EXR stdev		.334	.284	.339	.007	.000
10yr EXR stdev		.408	.307	.420	.010	.000
10yr TIPS EXR stdev		.003	.001	.000	.001	.000
3yr RXR $\sigma\left(\widehat{CS}\right)$	.810	.300 [.037]	.213 [.005]	.314 [.056]	.285 [.023]	.293 [.032]
10yr RXR $\sigma\left(\widehat{CS}\right)$	$2.55^{\dagger}$	.488 [.000]	.395 [.000]	.493 [.000]	.510 [.000]	.520 [.000]
10yr TIPS RXR $\sigma\left(\widehat{CS}\right)$		.179	.180	.076	.184	.079
3yr RXR $\sigma\left(\widehat{CP}\right)$	1.00	.653 [.118]	.473 [.007]	.678 [.142]	.647 [.093]	.646 [.119]
10yr RXR $\sigma\left(\widehat{CP}\right)$	$2.23^{\dagger}$	1.10 [.013]	.969 [.000]	1.13 [.022]	1.27 [.033]	1.19 [.049]

Table 3. Sample and implied moments for 1yr excess returns. Yield spreads (YS) are calculated over the 1yr yield. Realized excess returns (RXR) are calculated over a 1yr holding period, in excess of the 1yr yield. Units are annualized percentage points. Simulation columns report means across 1000 replications, each of which simulates a time-series of 250 quarters. The $\sigma\left(\widehat{CP}\right)$ row reports the standard deviation of the fitted values from a Cochrane-Piazzesi style regression of RXR on the 1-, 3-, and 5-yr forward rates at the beginning of the holding period. The $\sigma\left(\widehat{CS}\right)$ row reports the standard deviation of the fitted values from a Campbell-Shiller style regression of RXR on the same-maturity YS at the beginning of the holding period. Below each entry we report in brackets the fraction of simulation runs where the simulated value exceeds the data value.† Data moments for the 10yr return require 9yr yields. We interpolate the 9yr yield linearly between the 5yr and the 10yr.‡ TIPS entries refer to the 10yr spliced TIPS yield 1/1985-12/2005.

Sample and Implied Moments						
Moment	Actual Data	Full model	w/o TIPS	Constant z	Constant ψ	Constant $z\psi$
3yr YS mean	.360	.210 [.233]	.115 [.045]	.234 [.294]	.123 [.079]	.118 [.067]
10yr YS mean	.821	.348 [.128]	.191 [.015]	.393 [.196]	.169 [.054]	.160 [.036]
3yr YS stdev	.486	.878 [.998]	.702 [.991]	.925 [.999]	.928 [1.00]	.953 [1.00]
10yr YS stdev	1.05	1.80 [.999]	1.32 [.937]	1.91 [.997]	2.06 [1.00]	2.13 [1.00]
3yr RXR mean	.647	.438 [.292]	.226 [.093]	.482 [.356]	.110 [.137]	.047 [.112]
10yr RXR mean	1.49†	.718 [.191]	.314 [.081]	.797 [.255]	.204 [.140]	.081 [.126]
3yr RXR stdev	2.86	3.40 [.876]	2.56 [.104]	3.59 [.922]	3.96 [1.00]	4.08 [1.00]
10yr RXR stdev	9.10†	7.44 [.010]	6.67 [.000]	7.68 [.038]	9.06 [.507]	9.23 [.570]
10yr TIPS yield mean	3.37‡	4.07 [.999]	2.74 [.000]	4.05 [1.00]	4.04 [1.00]	4.04 [1.00]
10yr TIPS YS mean		−.003	−.013	−.003	.011	.015
10yr TIPS RXR mean		.026	.014	.028	.022	.028
10yr TIPS RXR stdev		2.94	2.99	3.02	1.26	1.31

Predictive Regressions						
Moment	Actual Data	Full model	w/o TIPS	Constant z	Constant ψ	Constant $z\psi$
3yr EXR stdev		.359	.238	.367	.009	.000
10yr EXR stdev		.504	.282	.525	.015	.000
10yr TIPS EXR stdev		.003	.004	.000	.002	.000
3yr RXR $\sigma\left(\widehat{CS}\right)$	0.88	.363 [.051]	.240 [.002]	.386 [.069]	.280 [.072]	.292 [.082]
10yr RXR $\sigma\left(\widehat{CS}\right)$	3.44†	.717 [.010]	.602 [.000]	.745 [.000]	.722 [.002]	.739 [.003]
10yr TIPS RXR $\sigma\left(\widehat{CS}\right)$		.314	.307	.322	.123	.132
3yr RXR $\sigma\left(\widehat{CP}\right)$	1.23	.805 [.095]	.570 [.007]	.843 [.147]	.762 [.149]	.786 [.151]
10yr RXR $\sigma\left(\widehat{CP}\right)$	4.49†	1.79 [.000]	1.61 [.000]	1.83 [.000]	2.08 [.006]	2.12 [.005]

Table 4. Forecasting excess returns. The table below reports the R^2 for regressions in our data of actual data RXR on linear combinations of the actual data 1-, 3-, and 5-yr forward rates at the beginning of the holding period. The unconstrained column estimates the best combination in the data, and thus corresponds to the first stage of the Cochrane-Piazzesi procedure. In the other columns, the combination is restricted to be the one estimated in long-sample simulation regressions of simulated RXR on simulated forward rates. In the first panel, we allow this simulation-generated combination to be scaled up. In the second panel, we do not allow scaling. Realized excess returns (RXR) are calculated over 3mo and 1yr holding periods. † Data moments for the 10yr return require 9yr yields. These yields are in our dataset 8/1971-12/2005. For the earlier part of the sample we interpolate the 9yr yield linearly between the 5yr and the 10yr.

Forecasting Excess Returns

	Moment	Unconstrained	Full model	w/o TIPS	Constant z	Constant ψ	Constant $z\psi$
3-month	3yr RXR	.049	.030	.029	.030	.029	N/A
holding period	10yr RXR	.050	.031	.027	.031	.023	N/A
1-year	3yr RXR	.184	.146	.141	.146	.134	N/A
holding period	10yr RXR	.194†	.172	.150	.171	.129	N/A

Forecasting Excess Returns: No scaling

	Moment	Unconstrained	Full model	w/o TIPS	Constant z	Constant ψ	Constant $z\psi$
3-month	3yr RXR	.049	.029	.024	.029	.028	.000
holding period	10yr RXR	.050	.017	.017	.017	.017	.000
1-year	3yr RXR	.180	.124	.139	.124	.134	.000
holding period	10yr RXR	.214†	.056	.070	.055	.071	.000