Appendix to
Campbell, Serfaty-de Medeiros and Viceira,
"Global Currency Hedging"

1 Hedged portfolio return

Let $R_{c,t+1}$ denote the gross return in currency c from holding country c stocks from the beginning to the end of period $t+1$, and let $S_{c,t+1}$ denote the spot exchange rate in dollars per foreign currency c at the end of period $t+1$. By convention, we index the domestic country by $c=1$ and the n foreign countries by $c=2,...,n+1$. Of course, the domestic exchange rate is constant over time and equal to 1: $S_{1,t+1}=1$ for all t.

At time t, the investor exchanges a dollar for $1/S_{c,t}$ units of currency c in the spot market which she then invests in the stock market of country c. After one period, stocks from country c return $R_{c,t+1}$, which the US investor can exchange for $S_{c,t+1}$ dollars, to earn an unhedged gross return of $R_{c,t+1}S_{c,t+1}/S_{c,t}$. For an arbitrarily weighted portfolio, the unhedged gross portfolio return is given by

$$R^{uh}_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right),$$

where $\boldsymbol{\omega}_t = \text{diag}\left(\omega_{1,t}, \omega_{2,t}, ..., \omega_{n+1,t}\right)$ is the $(n+1 \times n+1)$ diagonal matrix of weights on domestic and foreign stocks at time t, $\mathbf{R}_{t+1}$ is the $(n+1\times 1)$ vector of gross nominal stock returns in local currencies, $\mathbf{S}_{t+1}$ is the $(n+1\times 1)$ vector of spot exchange rates, and $\div$ denotes the element-by-element ratio operator, so that the c-th element of $(\mathbf{S}_{t+1} \div \mathbf{S}_t)$ is $S_{c,t+1}/S_{c,t}$. The weights add up to 1 in each period t:

$$\sum_{c=1}^{n+1} \omega_{c,t} = 1 \qquad \forall t. \tag{1}$$

We next consider the hedged portfolio. Let $F_{c,t}$ denote the one-period forward exchange rate in dollars per foreign currency c,[1] and $\theta_{c,t}$ the dollar value of the

[1] That is, at the end of month t, the investor can enter into a forward contract to sell one unit of currency c at the end of month $t+1$ for a forward price of $F_{c,t}$ dollars.

amount of forward exchange rate contracts for currency c the investor enters into at time t per dollar invested in her stock portfolio. At the end of period $t+1$, the investor gets to exchange $\theta_{c,t}/S_{c,t}$ units of the foreign-currency denominated return $R_{c,t+1}\omega_{c,t}/S_{c,t}$ back into dollars at an exchange rate $F_{c,t}$. She then exchanges the rest, which amounts to $(R_{c,t+1}\omega_{c,t}/S_{c,t} - \theta_{c,t}/S_{c,t})$ units of foreign currency c, at the spot exchange rate $S_{c,t+1}$. Collecting returns for all countries leads to a hedged portfolio return $R^h_{p,t+1}$ of

$$R^h_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \boldsymbol{\Theta}'_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) + \boldsymbol{\Theta}'_t\left(\mathbf{F}_t \div \mathbf{S}_t\right), \tag{2}$$

where $\mathbf{F}_t$ is the $(n+1\times1)$ vector of forward exchange rates, and $\boldsymbol{\Theta}_t = (\theta_{1,t}, \theta_{2,t}, ..., \theta_{n,t}, \theta_{n+1,t})'$. Of course, since $S_{1t} = F_{1,t} = 1$ for all t, the choice of domestic hedge ratio $\theta_{1,t}$ is arbitrary. For convenience, we set it so that all hedge ratios add up to 1:

$$\theta_{1,t} = 1 - \sum_{c=2}^{n+1}\theta_{c,t}. \tag{3}$$

Under covered interest parity, the forward contract for currency c trades at $F_{c,t} = S_{c,t}(1 + I_{1,t})/(1 + I_{c,t})$, where $I_{1,t}$ denotes the domestic nominal short-term riskless interest rate available at the end of period t, and $I_{c,t}$ is the corresponding country c nominal short-term interest rate. Thus the hedged dollar portfolio return (2) can be written as

$$R^h_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \boldsymbol{\Theta}'_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) + \boldsymbol{\Theta}'_t\left[\left(\mathbf{1} + \mathbf{I}^d_t\right) \div \left(\mathbf{1} + \mathbf{I}_t\right)\right], \tag{4}$$

where $\mathbf{I}_t = (I_{1,t}, I_{2,t}..., I_{n+1,t})$ is the $(n+1\times1)$ vector of nominal short-term interest rates and $\mathbf{I}^d_t = I_{1,t}\mathbf{1}$.

Equation (4) shows that selling currency forward—i.e., setting $\theta_{c,t} > 0$—is analogous to a strategy of shorting foreign bonds and holding domestic bonds, i.e. borrowing in foreign currency and lending in domestic currency.[2]

To capture the fact that the investor can alter the currency exposure implicit in her foreign stock position using forward contracts or lending and borrowing, we now define a new variable $\psi_{c,t}$ as $\psi_{c,t} \equiv \omega_{c,t} - \theta_{c,t}$. A fully hedged portfolio, in which the investor does not hold any exposure to currency c, corresponds to $\psi_{c,t} = 0$. A

[2] Note, however, that the two strategies are not completely equivalent except in the continuous time limit. We show in a later section of the appendix that, in continuous time, the two strategies are exactly equivalent.

positive value of $\psi_{c,t}$ means that the investor wants to hold exposure to currency c, or equivalently that the investor does not want to fully hedge the currency exposure implicit in her stock position in country c. Of course, a completely unhedged portfolio corresponds to $\psi_{c,t} = \omega_{c,t}$. Thus $\psi_{c,t}$ is a measure of currency demand or currency exposure. Accordingly we refer to $\psi_{c,t}$ as currency demand or currency exposure indistinctly.

For convenience, we now rewrite equation (4) in terms of currency demands:

$$\begin{aligned} R^h_{p,t+1} &= \mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \mathbf{1}'\boldsymbol{\omega}_t\left[\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \left(\mathbf{1} + \mathbf{I}^d_t\right) \div \left(\mathbf{1} + \mathbf{I}_t\right)\right] \\ &\quad + \boldsymbol{\Psi}'_t\left[\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \left(\mathbf{1} + \mathbf{I}^d_t\right) \div \left(\mathbf{1} + \mathbf{I}_t\right)\right], \end{aligned} \tag{5}$$

where $\boldsymbol{\Psi}_t = \left(\psi_{1,t}, \psi_{2,t}, ..., \psi_{n+1,t}\right)'$.

Note that $\boldsymbol{\Psi}_t = \boldsymbol{\omega}_t\mathbf{1} - \boldsymbol{\Theta}_t$. Given the definition of $\psi_{c,t}$, equations (1) and (3) imply that

$$\psi_{1,t} = -\sum_{c=2}^{n+1} \psi_{c,t}. \tag{6}$$

or $\Psi'_t\mathbf{1} = \mathbf{0}$, so that $\psi_{1,t}$ indeed represents the domestic currency exposure. That currency demands must add to zero is intuitive. Since the investor is fully invested in stocks, she can achieve a long position in a particular currency c only by borrowing—or equivalently, by shorting bonds—in her own domestic currency, and investing the proceeds in bonds denominated in that currency. Thus the currency portfolio is a zero investment portfolio.

2 Log portfolio returns over short time intervals

Assuming log-normality of the hedge returns, the derivation of the optimal Ψ requires an expression for the log-return on the hedged portfolio, $r^{hedge}_{p,t+1}$. We compute this log hedged return as a discrete-time approximation to its continuous-time counterpart. In order to do this, we need to specify, in continuous time, the return processes for stocks $P_{c,t}$, for currencies $X_{c,t}$ and for interest rates $B_{c,t}$.We assume that they all follow a geometric brownian motions:

$$\frac{dP_{c,t}}{P_{c,t}} = \mu_{P_c} dt + (\sigma_{P_c})_t \, dW_t^{P_c}, \qquad c = 1...n+1 \tag{7}$$

$$\frac{dB_{c,t}}{B_{c,t}} = \mu_{B_c} dt, \qquad c = 1...n+1 \tag{8}$$

$$\frac{dX_{c,t}}{X_{c,t}} = \mu_{X_c} dt + (\sigma_{X_c})_t \, dW_t^{X_c}, \qquad c = 1...n+1, \tag{9}$$

where $W_t^{P_c}$, $W_t^{B_c}$ and $W_t^{X_c}$ are diffusion processes. $\frac{dP_{c,t}}{P_{c,t}}$ represents the stock return, $\frac{dB_{c,t}}{B_{c,t}}$ the nominal return to holding a riskless bond from country and $\frac{dX_{c,t}}{X_{c,t}}$ the return to holding foreign currency c.

For notational simplicity, in what follows, we are momentarily dropping time subscripts for the standard deviations.

Using Ito's lemma, the log returns on each asset are given by:

$$\begin{aligned} d\log P_{c,t} &= \frac{dP_{c,t}}{P_{c,t}} - \frac{1}{2}\sigma_{P_c}^2 dt \\ d\log B_{c,t} &= \frac{dB_{c,t}}{B_{c,t}} - \frac{1}{2}\sigma_{B_c}^2 dt \\ d\log X_{c,t} &= \frac{dX_{c,t}}{X_{c,t}} - \frac{1}{2}\sigma_{X_c}^2 dt. \end{aligned}$$

Note that, because country 1 is the domestic country, which has a fixed exchange rate of 1, we have $d\log X_{1,t} = 0$. This implies $\mu_{X_1} = \sigma_{X_1} = 0$.

The domestic currency return on foreign stock is then given by $\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}}$. To derive an expression for this return, we will note that the return dynamics above, by standard calculations, imply :

$$\begin{aligned} \log P_{c,t}X_{c,t} &= \log P_{c,0}X_{c,0} + \left(\mu_{P_c} + \mu_{X_c} - \frac{1}{2}\sigma_{P_c}^2 - \frac{1}{2}\sigma_{X_c}^2\right) t \\ &\quad + \sigma_{P_c}\left(W_t^{P_c} - W_0^{P_c}\right) + \sigma_{X_c}\left(W_t^{X_c} - W_0^{X_c}\right) \end{aligned}$$

Differentiating, and then applying Ito's lemma, yields :

$$\begin{aligned}\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}} &= \frac{dP_{c,t}}{P_{c,t}} + \frac{dX_{c,t}}{X_{c,t}} + \sigma_{P_c}\sigma_{X_c}\rho_{P_c,X_c}dt \\ \frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}} &= d\log P_{c,t} + d\log X_{c,t} + \frac{1}{2}\operatorname{Var}_t\left(p_{c,t} + x_{c,t}\right)dt, \qquad (10)\end{aligned}$$

where $x_{c,t} = d\log X_{c,t}$ and $p_{c,t} = dlogP_{c,t}$. Note that for c=1, the formula does yield the simple stock return as $\frac{dP_{1,t}X_{1,t}}{P_{1,t}X_{1,t}} = \frac{dP_{1,t}}{P_{1,t}} + \frac{dX_{1,t}}{X_{1,t}} + \sigma_{P_1}\sigma_{X_1}\rho_{P_1,X_1}dt = \frac{dP_{1,t}}{P_{1,t}}$.

A similar calculation yields the following dynamics for the return of the strategy consisting in holding the domestic bond and shorting the foreign one :

$$\frac{d\left(B_{1,t}/B_{c,t}\right)}{B_{1,t}/B_{c,t}} = d\log B_{1,t} - d\log B_{c,t} \qquad (11)$$

We note V_t the value of the portfolio.The log return on the portfolio, by Ito's lemma, is :

$$d\log V_t = \frac{dV_t}{V_t} - \frac{1}{2}\left(\frac{dV_t}{V_t}\right)^2.$$

We can now derive each of the right-hand side terms:

$$\frac{dV_t}{V_t} = \sum_{c=1}^{n+1}\omega_{c,t}\left(\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}}\right) + \sum_{c=1}^{n+1}\theta_c\omega_{c,t}\frac{d\left(B_{1,t}/B_{c,t}\right)}{B_{1,t}/B_{c,t}} - \sum_{c=1}^{n+1}\theta_c\omega_{c,t}\frac{dX_{c,t}}{X_{c,t}},$$

which follows from our convention regarding the domestic country.

Using expressions (9) , (10) , and (11) to substitute and moving to matrix notation, we get :

$$\begin{aligned}\frac{dV_t}{V_t} &= \mathbf{1}'\boldsymbol{\omega}\left(\mathbf{p}_{t+1} + \mathbf{x}_{t+1}\right) - \boldsymbol{\Theta}_t'\left(\mathbf{x}_{t+1} - \mathbf{b}_t^d + \mathbf{b}_t\right) \\ &\quad + \frac{1}{2}\left[\mathbf{1}'\boldsymbol{\omega}_t diag\left(\operatorname{Var}_t\left(\mathbf{p}_{t+1} + \mathbf{x}_{t+1}\right)\right) - \boldsymbol{\Theta}_t' diag\left(\operatorname{Var}_t \mathbf{x}_{t+1}\right)\right]dt \quad ,\end{aligned}$$

where $\mathbf{p}_{t+1} = \left(d\log P_{1,t},\ d\log P_{2,t}..., d\log P_{n+1,t}\right)'$, $\mathbf{x}_{t+1} = \left(d\log X_{1,t},\ d\log X_{2,t}..., d\log X_{n+1,t}\right)'$, $\mathbf{b}_t^d = \left(d\log B_{1,t}\right)\mathbf{1},\ \mathbf{b}_t = \left(d\log B_{1,t},\ d\log B_{2,t}..., d\log B_{n+1,t}\right)'$ and $diag\left(X\right)$ denotes, for a symmetric $(n \times n)$ matrix X, the $(n \times 1)$ vector of its diagonal terms.

Then,

$$\left(\frac{dV_t}{V_t}\right)^2 = \mathrm{Var}_t\left[\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right)\right]dt+o\left(dt\right)$$
$$= \left[\begin{array}{c}\mathbf{1}'\boldsymbol{\omega}_t\,\mathrm{Var}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)\boldsymbol{\omega}_t\iota \\ -2\mathbf{1}'\boldsymbol{\omega}_t\,\mathrm{cov}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1},\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right)\boldsymbol{\Theta}_t \\ +\boldsymbol{\Theta}'\boldsymbol{\omega}_t\,\mathrm{Var}_t\left(\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right)\boldsymbol{\Theta}_t\end{array}\right]dt+o\left(dt\right).$$

So, finally,

$$\begin{aligned} d\log V_t &= \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right) \\ &+\frac{1}{2}\left[\mathbf{1}'\boldsymbol{\omega}_t diag\left(\mathrm{Var}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)\right)-\boldsymbol{\Theta}_t' diag\left(\mathrm{Var}_t\,\mathbf{x}_{t+1}\right)\right]dt \\ &-\frac{1}{2}\mathrm{Var}_t\left[\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right)\right]dt+\mathrm{o}\left(dt\right) \quad . \end{aligned} \tag{12}$$

Now, we get the approximation for $r_{p,t+1}^h$ by computing the previous expression for $dt=1$, replacing $d\log X_{c,t}=\Delta s_{c,t+1}$, $dlogP_{c,t}=r_{c,t+1}$, and $dlogB_{c,t}=i_{c,t}$ and neglecting the higher order terms. Noting, for any variable, $\mathbf{z}_t$, the $(n+1\times 1)$ vector $\left(z_{1,t},z_{2,t}..z_{n+1,t}\right)$, this is equivalent to replacing in equation (12) $\mathbf{p}_{t+1}$ by $\mathbf{r}_{t+1}$, $\mathbf{x}_{t+1}$ by $\boldsymbol{\Delta}\mathbf{s}_{t+1}$, $\mathbf{b}_t^d$ by $\mathbf{i}_t^d$ and $\mathbf{b}_t$ by $\mathbf{i}_t$.

$$r_{p,t+1}^h \simeq \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)+\frac{1}{2}\Sigma_t^h$$

where Σ_{t+1}^h is equal to :

$$\begin{aligned}\Sigma_t^h &= \mathbf{1}'\boldsymbol{\omega}_t diag\left(\mathrm{Var}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)-\boldsymbol{\Theta}_t' diag\left(\mathrm{Var}_t\,\boldsymbol{\Delta}\mathbf{s}_{t+1}\right) \\ &\quad -\mathrm{Var}_t\left[\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right]\end{aligned}$$

where, for any variable z, $\mathbf{z}_t$ denotes the vector of country observations $\left(z_{1,t},z_{2,t}...z_{n+1,t}\right)'$ and small case letters denote logs in the following fashion : $r_{c,t+1}=\log\left(R_{c,t+1}\right)$, $s_{t+1}=\log\left(S_{t+1}\right)$, $i_t^d=\log\left(1+I_{1,t}\right)\mathbf{1}$ and $i_{c,t}=\log\left(1+I_{c,t}\right)$.

We can now rewrite the portfolio return as a function of $\boldsymbol{\Psi}_t$ by substituting for $\boldsymbol{\Theta}_t$. This yields :

$$\begin{aligned} r_{p,t+1}^h &= \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}+\mathbf{i}_t^d-\mathbf{i}_t\right)+\boldsymbol{\Psi}_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)+\frac{1}{2}\Sigma_t^h \\ &= i_{1,t}^d+\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\boldsymbol{\Psi}_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)+\frac{1}{2}\Sigma_t^h, \end{aligned}$$

where:

$$\Sigma_t^h = \mathbf{1}'\boldsymbol{\omega}_t \operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)-\left(-\boldsymbol{\Psi}_t+\boldsymbol{\omega}_t\mathbf{1}\right)'\operatorname{diag}\left(\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right) \quad (13)$$
$$-\operatorname{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}+\mathbf{i}_t^d-\mathbf{i}_t\right)+\boldsymbol{\Psi}_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right).$$

3 Equivalence between forward contracts and foreign currency borrowing and lending

With the same notations and assumptions as above, when the investor uses forward contracts to hedge currency risk, the portfolio return is:

$$R_{p,t+1}^h=\mathbf{R}_{t+1}'\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1}\div\mathbf{S}_t\right)-\boldsymbol{\Theta}_t'\left[\left(\mathbf{S}_{t+1}\div\mathbf{S}_t\right)-\left(\mathbf{1}+\mathbf{I}_t^d\right)\div\left(\mathbf{1}+\mathbf{I}_t\right)\right]$$

Another natural view is one in which the investor borrows in foreign currency and lends in domestic currency to hedge currency risk. Then, the portfolio return is:

$$R_{p,t+1}^{BL}=\mathbf{R}_{t+1}'\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1}\div\mathbf{S}_t\right)-\boldsymbol{\Theta}'\left(\mathbf{S}_{t+1}\div\mathbf{S}_t\right)\left(1+\mathbf{I}_t\right)+\boldsymbol{\Theta}'\left(1+\mathbf{I}_t^d\right)$$

Then, with V_t^{BL} the value of the portfolio with borrowing and lending, we have in continuous time:

$$\begin{aligned}
\frac{dV_t^{BL}}{V_t^{BL}} &= \sum_{c=1}^{n+1}\omega_{c,t}\left(\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}}\right)-\sum_{c=1}^{n+1}\Theta_{c,t}\frac{dX_{c,t}B_{c,t}}{X_{c,t}B_{c,t}}+\sum_{c=1}^{n+1}\Theta_{c,t}\frac{dB_{1,t}}{B_{1,t}}\\
&= \sum_{c=1}^{n+1}\omega_{c,t}\left(\log P_{c,t}+\log X_{c,t}+\frac{1}{2}\operatorname{Var}_t\left(p_{c,t}+x_{c,t}\right)dt\right)\\
&\quad -\sum_{c=1}^{n+1}\Theta_{c,t}\left(\log\left(X_{c,t}\right)+\log\left(B_{c,t}\right)+\frac{1}{2}\operatorname{Var}_t\left(x_{c,t}\right)dt\right)\\
&\quad +\sum_{c=2}^{n+1}\Theta_{c,t}\log\left(B_{1,t}\right)\\
&= \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\Theta'\left(\mathbf{x}_{t+1}+\mathbf{b}_t-\mathbf{b}_t^d\right)+\frac{1}{2}\mathbf{1}'\boldsymbol{\omega}_t\operatorname{diag}\operatorname{Var}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)dt\\
&\quad -\frac{1}{2}\Theta'\operatorname{diag}\operatorname{Var}_t\left(\mathbf{x}_{t+1}\right)dt
\end{aligned}$$

and

$$\left(\frac{dV_t^{BL}}{V_t^{BL}}\right)^2 = \mathrm{Var}_t\left(\boldsymbol{\omega}_t'\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\Theta'\left(\mathbf{x}_{t+1}+\mathbf{b}_t-\mathbf{b}_t^d\right)\right)dt+o\left(dt\right).$$

So

$$\begin{aligned} d\log V_t^{BL} &= \frac{dV_t^{BL}}{V_t^{BL}}-\frac{1}{2}\left(\frac{dV_t^{BL}}{V_t^{BL}}\right)^2 \\ &= \boldsymbol{\omega}_t'\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}'\left(\mathbf{x}_{t+1}+\mathbf{b}_t-\mathbf{b}_t^d\right)+\frac{1}{2}\boldsymbol{\omega}_t'\,\mathrm{diag}\,\mathrm{Var}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)dt \\ &\quad -\frac{1}{2}\boldsymbol{\Theta}'\,\mathrm{diag}\,\mathrm{Var}_t\left(\mathbf{x}_{t+1}\right)dt \\ &\quad -\frac{1}{2}\,\mathrm{Var}_t\left(\boldsymbol{\omega}_t'\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}'\left(\mathbf{x}_{t+1}+\mathbf{b}_t-\mathbf{b}_t^d\right)\right)dt+o\left(dt\right) \end{aligned}$$

We now go to the limit of $dt = 1$ and get :

$$\begin{aligned} r_{p,t+1}^{BL} &\simeq \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)-\boldsymbol{\Theta}'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}+\mathbf{i}_t-\mathbf{i}_t^d\right)+\frac{1}{2}\Sigma_t^h \\ &= r_{p,t+1}^h \end{aligned}$$

4 Mean-variance problem optimization

4.1 Unconstrained hedge ratio

In the general case, $r_{p,t+1}^h - i_{1,t}^d = \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)+\frac{1}{2}\Sigma_t^h$, and the Lagrangian is:

$$\begin{aligned} \pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\left(1-\lambda\right)\mathrm{Var}_t\left[\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right] \\ &\quad +\lambda\left[\mu_H-\mathrm{E}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)-\frac{1}{2}\Sigma_t^h\right] \end{aligned}$$

Substituting for Σ_t^h using equation (13), this expression is equivalent to :

$$\begin{aligned} \pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\mathrm{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right) \\ &\quad +\lambda\left[\mu_H-\mathrm{E}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)-\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)\right] \\ &\quad -\frac{\lambda}{2}\left[\mathbf{1}'\boldsymbol{\omega}_t\,\mathrm{diag}\left(\mathrm{Var}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)-\left(\boldsymbol{\omega}_t\mathbf{1}-\Psi_t\right)'\mathrm{diag}\left(\mathrm{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)\right] \end{aligned}$$

$$
\begin{aligned}
\pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\operatorname{Var}_t\left(\Psi_t'\left(\mathbf{\Delta s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)-\lambda\operatorname{E}_t\left(\Psi_t'\left(\mathbf{\Delta s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right) \\
&-\frac{\lambda}{2}\Psi_t'\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{\Delta s}_{t+1}\right)\right) \\
&+\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\Psi_t'\left(\mathbf{\Delta s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right) \\
&+\frac{1}{2}\operatorname{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)\right)-\lambda\operatorname{E}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)\right) \\
&+\frac{\lambda}{2}\mathbf{1}'\boldsymbol{\omega}_t\left[\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{\Delta s}_{t+1}\right)\right)-\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{r}_{t+1}+\mathbf{\Delta s}_{t+1}\right)\right)\right] \\
&+\lambda\mu_H
\end{aligned}
$$

$$
\begin{aligned}
\pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\Psi_t'\operatorname{Var}_t\left(\mathbf{\Delta s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\Psi_t-\lambda\Psi_t'\begin{bmatrix}\operatorname{E}_t\left(\mathbf{\Delta s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right) \\ +\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{\Delta s}_{t+1}\right)\right)\end{bmatrix} \\
&+\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\mathbf{\Delta s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)\Psi_t \\
&+K\left(\lambda\right)
\end{aligned}
$$

where

$$
\begin{aligned}
K\left(\lambda\right) &= \lambda\mu_H+\frac{1}{2}\operatorname{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)\right)-\lambda\operatorname{E}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)\right) \\
&+\frac{\lambda}{2}\mathbf{1}'\boldsymbol{\omega}_t\left[\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{\Delta s}_{t+1}\right)\right)-\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{r}_{t+1}+\mathbf{\Delta s}_{t+1}\right)\right)\right]
\end{aligned}
$$

$K\left(\lambda\right)$ is independent of $\widetilde{\Psi}_t$.

Now, we need to solve only for $\widetilde{\Psi}_t$ as Ψ_1, the demand for domestic currency, is given once the other currency demands are determined. We rewrite the Lagrangian in terms of $\widetilde{\Psi}_t$:

$$
\begin{aligned}
\pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\widetilde{\Psi}_t'\operatorname{Var}_t\left(\widetilde{\mathbf{\Delta s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\widetilde{\Psi}_t-\lambda\widetilde{\Psi}_t'\begin{bmatrix}\operatorname{E}_t\left(\widetilde{\mathbf{\Delta s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right) \\ +\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\widetilde{\mathbf{\Delta s}}_{t+1}\right)\right)\end{bmatrix} \\
&+\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\widetilde{\mathbf{\Delta s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\right)\widetilde{\Psi}_t \\
&+K\left(\lambda\right)
\end{aligned}
$$

The F.O.C. gives the following expression for the optimal $\widetilde{\Psi}_t$:

$$\begin{aligned} 0 &= \operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\right) \\ &\quad +\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\widetilde{\boldsymbol{\Psi}}_t^*-\lambda\left[\mathrm{E}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)+\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}\right)\right)\right] \end{aligned}$$

Finally, the optimal vector of currency demands is :

$$\begin{aligned} \widetilde{\boldsymbol{\Psi}}_t^*(\lambda) &= \lambda\operatorname{Var}_t^{-1}\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\left[E_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)+\frac{1}{2}diag\left(\operatorname{Var}_t\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}\right)\right] \\ &\quad -\operatorname{Var}_t^{-1}\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\left[\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\right)\right] \end{aligned}$$

4.2 Constrained hedge ratio

In the case where $\widetilde{\Psi}_t=\psi_t\widetilde{\mathbf{1}}$ (where $\widetilde{\mathbf{1}}$ denotes an $n\times 1$ vector of ones), we note ψ_t^* the optimal scalar constrained hedge ratio and we have :

$$\begin{aligned} \pounds\left(\psi_t\right) &= \frac{1}{2}\psi_t^2\widetilde{\mathbf{1}}'\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\widetilde{\mathbf{1}}-\lambda\psi_t\widetilde{\mathbf{1}}'\begin{bmatrix}\mathrm{E}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\\ +\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}\right)\right)\end{bmatrix} \\ &\quad +\psi_t\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\right)\widetilde{\mathbf{1}} \\ &\quad +K(\lambda) \end{aligned}$$

and

$$\begin{aligned} \psi_t^* &= \frac{\lambda\widetilde{\mathbf{1}}'\left[\mathrm{E}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)+\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}\right)\right)\right]}{\widetilde{\mathbf{1}}'\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\widetilde{\mathbf{1}}} \\ &\quad -\frac{\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\right)\widetilde{\mathbf{1}}}{\widetilde{\mathbf{1}}'\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\widetilde{\mathbf{1}}} \end{aligned}$$

In this case, ψ_t^* can equivalently be written in terms of the full matrices :

$$\psi_t^* = \frac{\lambda \mathbf{1}' \left[\mathrm{E}_t \left(\boldsymbol{\Delta} \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t \right) + \frac{1}{2} \operatorname{diag} \left(\operatorname{Var}_t \left(\boldsymbol{\Delta} \mathbf{s}_{t+1} \right) \right) \right]}{\mathbf{1}' \operatorname{Var}_t \left(\boldsymbol{\Delta} \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t \right) \mathbf{1}} - \frac{\mathbf{1}' \operatorname{cov}_t \left(\boldsymbol{\omega}_t \left(\mathbf{r}_{t+1} - \mathbf{i}_t \right), \left(\boldsymbol{\Delta} \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t \right) \right) \mathbf{1}}{\mathbf{1}' \operatorname{Var}_t \left(\boldsymbol{\Delta} \mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t \right) \mathbf{1}}$$

This case corresponds to a domestic investor hedging the same ratio of his foreign stock holdings for all foreign currencies.

5 Invariance of optimal currency demand with respect to base country

In the system of n^2 bilateral exchange rates, there are really only n free parameters as all exchange rates can be backed out of the n bilateral rates for one base domestic country. We use this fact to show that, for a portfolio of stocks from the $n+1$ countries in our model, the optimal hedge ratios on stocks from country c, Ψ_c^{j*} is the same for any base country j. Let us now use the subscript j to index the domestic country.

We assume for this derivation that weights on international stocks are the same for investors from all countries so that $\boldsymbol{\omega}_t^j = \boldsymbol{\omega}_t$. In terms of our empirical tests, this result will hence apply to the cases of an equally weighted or a value weighted world portfolios, in which weights do not vary with the base country. They do not hold for a home biased porftolio, in which weights by definition vary with base country.

Let us think of country 1 as our base country, and write the optimal vector of foreign currency demand assuming that $\lambda^j = 0$ for all values of j. We have :

$$\widetilde{\boldsymbol{\Psi}}_{RM}^{1*} = -\operatorname{Var}_t \left(\widetilde{\Delta s}_{t+1}^{1} - \widetilde{i}_t^{1,d} + \widetilde{i}_t^{1} \right)^{-1} \left[\operatorname{cov}_t \left(\mathbf{1}' \boldsymbol{\omega}_t \left(\mathbf{r}_{t+1} - \mathbf{i}_t \right), \widetilde{\Delta s}_{t+1}^{1} - \widetilde{i}_t^{1,d} + \widetilde{i}_t^{1} \right) \right] \quad (14)$$

$$= -\operatorname{Var}_t \left(\widetilde{x}_{t+1}^{1} \right)^{-1} \left[\operatorname{cov}_t \left(\mathbf{y}_{t+1}^{W}, \widetilde{x}_{t+1}^{1} \right) \right] \quad (15)$$

where $x_{t+1}^1 = \Delta s_{t+1}^1 - i_t^{1,d} + i_t^1$ and $\mathbf{y}_{t+1}^W = \mathbf{1}' \boldsymbol{\omega}_t \left(\mathbf{r}_{t+1} - \mathbf{i}_t \right)$.

Now, let us consider exchange rates from the perspective of country 2. By definition of the exchange rate between countries 1 and 2, it follows that $s^2_{t+1,1} = -s^1_{t+1,2}$.

Also, by definition of the exchange rates, $S^2_{t+1,3}$ units of currency 2 can be exchanged into one unit of currency 3. And one unit of currency 3 is equivalent to $S^1_{t+1,3}$ units of currency 1, which is equivalent to $S^1_{t+1,3}/S^1_{t+1,2}$ units of currency 2. So, the absence of arbitrage implies the equality: $S^2_{t+1,3} = S^1_{t+1,3}/S^1_{t+1,2}$. In logs, $s^2_{t+1,3} = s^1_{t+1,3} - s^1_{t+1,2}$. More generally, the following equality can be derived from the absence of arbitrage:

$$s^2_{t+1,c} = s^1_{t+1,c} - s^1_{t+1,2} \qquad c = 3...n+1$$

In matrix notation, this amounts to a linear relationship between $\widetilde{\mathbf{\Delta s}}^2_{t+1}$ and $\widetilde{\mathbf{\Delta s}}^1_{t+1}$:

$$\widetilde{\mathbf{\Delta s}}^2_{t+1} = A_2 \cdot \widetilde{\mathbf{\Delta s}}^1_{t+1}$$

where $A_2 = \begin{pmatrix} -1 & 0 & .. & .. & 0 \\ -1 & 1 & 0 & .. & .. \\ -1 & 0 & 1 & 0 & .. \\ -1 & 0 & 0 & .. & 0 \\ -1 & 0 & .. & 0 & 1 \end{pmatrix}$.

Given our notations :

$$\widetilde{\mathbf{i}}^{1,d}_t - \widetilde{\mathbf{i}}^1_t = (i_{t,2} - i_{t,1}, i_{t,3} - i_{t,1}, ..i_{t,n+1} - i_{t,1})'$$

and

$$\widetilde{\mathbf{i}}^{2,d}_t - \widetilde{\mathbf{i}}^2_t = (i_{t,1} - i_{t,2}, i_{t,3} - i_{t,2}, .., i_{t,n+1} - i_{t,2})'.$$

It follows that: $\widetilde{\mathbf{i}}^{2,d}_t - \widetilde{\mathbf{i}}^2_t = A\left(\widetilde{\mathbf{i}}^{1,d}_t - \widetilde{\mathbf{i}}^1_t\right)$.

Similarly, we have the following linear relationship between $\widetilde{\mathbf{x}}^2_{t+1}$ and $\widetilde{\mathbf{x}}^1_{t+1}$:

$$\widetilde{\mathbf{x}}^2_{t+1} = A\widetilde{\mathbf{x}}^1_{t+1} \qquad , \tag{16}$$

Let us substitute equation (16), the formula for $\widetilde{\mathbf{x}}^2_{t+1}$, into equation (**??**), the formula for the optimal hedge ratio. We use the properties of matrix second moments that

$\mathrm{Var}(AX) = A\,\mathrm{Var}(X)\,A'$, $\mathrm{cov}(AX,Y) = Acov\,(X,Y)$, and the property of inverse matrices that $(AB)^{-1} = B^{-1}A^{-1}$. Also, we note that $A_2 = (A_2)^{-1}$ and $(A_2')^{-1} = A_2'$. Substitution yields:

$$\begin{aligned}
\widetilde{\mathbf{\Psi}}^{2*}_{RM} &= -\,\mathrm{Var}_t\left(\widetilde{\mathbf{x}}^2_{t+1}\right)^{-1}\left[\mathrm{cov}_t\left(\mathbf{y}^W_{t+1}, \widetilde{\mathbf{x}}^2_{t+1}\right)\right] \\
&= -\,(A_2')^{-1}\,\mathrm{Var}_t\left(\widetilde{\mathbf{x}}^1_{t+1}\right)^{-1}(A_2)^{-1}\left[A_2\,\mathrm{cov}_t\left(\widetilde{\mathbf{x}}^1_{t+1}, \mathbf{y}^W_{t+1}\right)\right] \\
\widetilde{\mathbf{\Psi}}^{2*}_{RM}\left(\lambda^2\right) &= -\,(A_2')^{-1}\,\mathrm{Var}_t\left(\widetilde{\mathbf{x}}^1_{t+1}\right)^{-1}\mathrm{cov}_t\left(\widetilde{\mathbf{x}}^1_{t+1}, \mathbf{y}^W_{t+1}\right)
\end{aligned}$$

$$\widetilde{\mathbf{\Psi}}^{2*} = A_2'\widetilde{\mathbf{\Psi}}^{1*}$$

We write out the vector $\widetilde{\mathbf{\Psi}}^{2*}_{RM}$:

$$\widetilde{\mathbf{\Psi}}^{2*}_{RM} = \left(-\sum_{c=2}^{n+1}\Psi^{1*}_c, \Psi^{1*}_3, \Psi^{1*}_4, .., \Psi^{1*}_{n+1}\right)$$

Given the property that $\sum_{c=1}^{n+1}\Psi^{j*}_c = 1$ for $j = 1..n+1$, $\Psi^{1*}_1 = -\sum_{c=2}^{n+1}\Psi^{1*}_c$ so that $\widetilde{\mathbf{\Psi}}^{2*}_{RM} = \left(\Psi^{1*}_1, \Psi^{1*}_3, \Psi^{1*}_4, .., \Psi^{1*}_{n+1}\right)$. Applying this same property twice, $\Psi^{2*}_2 = -\sum_{c\neq 2}^{n+1}\Psi^{2*}_c = -\sum_{c\neq 2}^{n+1}\Psi^{1*}_c = \Psi^{1*}_2$, so that: $\Psi^{2*}_{RM} = \left(\Psi^{1*}_1, \Psi^{1*}_2, \Psi^{1*}_3, \Psi^{1*}_4, .., \Psi^{1*}_{n+1}\right) = \Psi^{1*}_{RM}$. Finally, the vector of optimal currency positions is the same for investors based in country 2 as that of country 1 investors.

Similar results hold for $j = 3...n+1$, where $A_3 = \begin{pmatrix} 1 & -1 & .. & .. & 0 \\ 0 & -1 & 0 & .. & .. \\ 0 & -1 & 1 & 0 & .. \\ 0 & .. & 0 & .. & 0 \\ 0 & -1 & .. & 0 & 1 \end{pmatrix}$, $A_4 = \begin{pmatrix} 1 & 0 & -1 & .. & 0 \\ 0 & 1 & .. & .. & .. \\ 0 & 0 & -1 & 0 & .. \\ 0 & .. & .. & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{pmatrix}$, etc...

This analysis justifies dropping the base-country subscript j and interpreting the $(n+1\times 1)$ vector $\mathbf{\Psi}^* = \left(-\sum_{c=2}^{n+1}\Psi^{1*}_c, \Psi^{1*}_2, \Psi^{1*}_3, .., \Psi^{1*}_{n+1}\right)'$ as a common vector of foreign currency demands that is independent of the country of origin.

A situation in which investors from all countries are hedged perfectly corresponds to $\mathbf{\Psi}^* = (0, 0, .., 0)'$.

A situation in which investors from country 1 are not hedged at all corresponds to $\mathbf{\Psi}^* = \left(-1, \omega_2^1, \omega_3^1 .., \omega_{n+1}^1\right)'$. That is, investors from country i undo the hedge of the fully hedged portfolio by taking long positions in each foreign currency proportional to the weight of each foreign country in their stock portfolio. (The perfectly hedged portfolio obtains by shorting each foreign currency by that same amount.) They need to borrow one unit of domestic currency to finance that.

Finally, note that this proof relies on the fact that all relevant exchange rates for an investor in a given base country are linear combinations of the relevant exchange rates for each other base country. In other words, the assumption is that all investors optimize over the same set of currencies.

6 Computation of Sharpe Ratios

Table 10 in the main text report in-sample Sharpe Ratios generated by the set of currency hedging strategies for global portfolios of stocks and bonds considered in the paper. The denominator of the Sharpe ratio is given by the standard deviations of log portfolio returns reported in Table 9.

The numerator of the Sharpe ratio is given by the log of the mean gros return on each of the portfolios. We compute a time series of gross returns for each strategy using equation (5), where $\mathbf{\Psi}_t$ is resplaced by the vector of fixed or time-varying currency demands that corresponds to each currency hedging strategy—for example, $\mathbf{\Psi}_t$ is a vector of zeroes for the "Full Hedge"strategy. Next we average the time series of gross returns, and take the natural log of the arithmetic mean.

Thus our Sharpe ratio is computed as

$$\frac{\log\left(\mathrm{E}\left[R_{p,t+1}^h\right]\right)}{\sqrt{\mathrm{Var}\left(r_{p,t+1}^h\right)}},$$

which at high frequency observation of returns or under lognormality is equivalent to

$$\frac{\mathrm{E}\left[r_{p,t+1}^{h}\right] + \frac{1}{2}\,\mathrm{Var}\left(r_{p,t+1}^{h}\right)}{\sqrt{\mathrm{Var}\left(r_{p,t+1}^{h}\right)}}.$$

Table A1

Currency return correlations

	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Base country: Euroland							
Euroland	.						
Australia	.	1.00					
Canada	.	0.70	1.00				
Japan	.	0.35	0.35	1.00			
Switzerland	.	-0.09	-0.11	0.20	1.00		
UK	.	0.31	0.34	0.24	-0.02	1.00	
US	.	0.63	0.87	0.40	-0.07	0.38	1.00
Base country: Australia							
Euroland	1.00						
Australia	.	.					
Canada	0.53	.	1.00				
Japan	0.67	.	0.46	1.00			
Switzerland	0.92	.	0.47	0.69	1.00		
UK	0.78	.	0.51	0.59	0.72	1.00	
US	0.59	.	0.85	0.55	0.54	0.58	1.00
Base country: Canada							
Euroland	1.00						
Australia	0.23	1.00					
Canada	.	.	.				
Japan	0.59	0.25	.	1.00			
Switzerland	0.91	0.20	.	0.62	1.00		
UK	0.71	0.24	.	0.50	0.65	1.00	
US	0.32	0.11	.	0.35	0.31	0.34	1.00
Base Country: Japan							
Euroland	1.00						
Australia	0.46	1.00					
Canada	0.55	0.74	1.00				
Japan	.	.	.	.			
Switzerland	0.87	0.33	0.39	.	1.00		
UK	0.72	0.49	0.56	.	0.60	1.00	
US	0.55	0.67	0.90	.	0.41	0.58	1.00
Base Country: Switzerland							
Euroland	1.00						
Australia	0.47	1.00					
Canada	0.52	0.77	1.00				
Japan	0.31	0.46	0.47	1.00			
Switzerland	.	.	.	.	.		
UK	0.56	0.49	0.53	0.37	.	1.00	
US	0.51	0.71	0.91	0.51	.	0.55	1.00
Base Country: UK							
Euroland	1.00						
Australia	0.35	1.00					
Canada	0.42	0.71	1.00				
Japan	0.50	0.42	0.44	1.00			
Switzerland	0.84	0.25	0.30	0.52	1.00		
UK	.	.	.	.	.	.	
US	0.42	0.64	0.88	0.48	0.32	.	1.00
Base Country: US							
Euroland	1.00						
Australia	0.26	1.00					
Canada	0.18	0.43	1.00				
Japan	0.55	0.25	0.10	1.00			
Switzerland	0.90	0.21	0.12	0.58	1.00		
UK	0.69	0.25	0.14	0.44	0.61	1.00	
US	.	.	.	.	.	.	.

Note. This table presents cross-country correlations of foreign currency log excess returns $s_{c,t}+i_{c,t}-i_{d,t}$, where d indexes the base country. Correlations are presented separately for investors from each base country. They are computed using monthly returns.

Table A2

Optimal currency exposure for an equally-weighted global equity portfolio: single-currency case

Base country	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Euroland		-0.37***	-0.45***	-0.25***	0.28*	-0.30***	-0.33***
		(0.09)	(0.10)	(0.07)	(0.15)	(0.09)	(0.11)
Australia	0.37***		0.02	0.14*	0.33***	0.21**	0.16**
	(0.09)		(0.08)	(0.07)	(0.07)	(0.09)	(0.08)
Canada	0.45***	-0.02		0.15*	0.38***	0.25**	0.55***
	(0.10)	(0.08)		(0.09)	(0.09)	(0.11)	(0.16)
Japan	0.25***	-0.14*	-0.15*		0.32***	0.05	-0.06
	(0.07)	(0.07)	(0.09)		(0.08)	(0.06)	(0.09)
Switzerland	-0.28*	-0.33***	-0.38***	-0.32***		-0.29***	-0.30***
	(0.15)	(0.07)	(0.09)	(0.08)		(0.07)	(0.09)
UK	0.30***	-0.21**	-0.25**	-0.05	0.29***		-0.13
	(0.09)	(0.09)	(0.11)	(0.06)	(0.07)		(0.11)
US	0.33***	-0.16**	-0.55***	0.06	0.30***	0.13	
	(0.11)	(0.08)	(0.16)	(0.09)	(0.09)	(0.11)	

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with equal weights, who chooses a position in one foreign currency at a time to minimize the variance of his portfolio. Rows indicate the base country of the investor, columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the excess return to the global equity portfolio onto the excess return of the column country currency to an investor based in the row country. All regressions include an intercept.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A3

Optimal currency exposure for an equally-weighted global equity portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
1 month	0.17	-0.16	-0.61*	-0.11	0.23	-0.11	0.60*
	(0.15)	(0.11)	(0.14)	(0.07)	(0.12)	(0.08)	(0.15)
2 months	0.29	-0.13	-0.63*	-0.19*	0.26	-0.11	0.51*
	(0.15)	(0.09)	(0.15)	(0.07)	(0.13)	(0.09)	(0.15)
3 months	0.32	-0.11	-0.61*	-0.17	0.27	-0.10	0.40*
	(0.17)	(0.09)	(0.16)	(0.09)	(0.15)	(0.11)	(0.18)
6 months	0.20	-0.05	-0.38	-0.25*	0.35	-0.06	0.19
	(0.26)	(0.14)	(0.25)	(0.12)	(0.20)	(0.16)	(0.28)
12 months	-0.20	0.21	-0.22	-0.41*	0.67*	-0.20	0.15
	(0.40)	(0.20)	(0.36)	(0.17)	(0.30)	(0.21)	(0.37)
Panel B : 5 country optimization							
1 month	0.37*	-0.29*		-0.08		-0.10	0.11
	(0.11)	(0.11)		(0.07)		(0.08)	(0.08)
2 months	0.50*	-0.27*		-0.15*		-0.09	0.01
	(0.11)	(0.09)		(0.07)		(0.09)	(0.11)
3 months	0.56*	-0.27*		-0.14		-0.09	-0.06
	(0.11)	(0.10)		(0.08)		(0.11)	(0.14)
6 months	0.53*	-0.21		-0.21*		-0.02	-0.09
	(0.14)	(0.13)		(0.10)		(0.15)	(0.18)
12 months	0.44*	0.05		-0.34*		-0.16	0.01
	(0.19)	(0.17)		(0.15)		(0.19)	(0.22)

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon T of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global equity portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping T-months returns, T varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A4

Subperiod analysis

Equally-weighted global equity portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
Subperiod I : 1975-1989							
1 month	0.15	-0.11	-0.73***	-0.06	0.08	-0.06	0.73***
	(0.20)	(0.16)	(0.23)	(0.12)	(0.13)	(0.11)	(0.24)
3 months	0.14	-0.05	-0.63**	-0.20	0.22	-0.09	0.62*
	(0.21)	(0.12)	(0.26)	(0.14)	(0.18)	(0.15)	(0.35)
12 months	-0.62	0.23	-0.15	-0.31	0.57*	-0.04	0.33
	(0.45)	(0.22)	(0.61)	(0.23)	(0.33)	(0.23)	(0.61)
Suberiod II : 1990-2005							
1 month	0.10	-0.25**	-0.49***	-0.15	0.51**	-0.20	0.48***
	(0.27)	(0.12)	(0.18)	(0.09)	(0.23)	(0.13)	(0.18)
3 months	0.44	-0.17	-0.65***	-0.08	0.37	-0.12	0.22
	(0.28)	(0.14)	(0.21)	(0.10)	(0.23)	(0.14)	(0.19)
12 months	0.56	-0.17	-0.31	-0.23	0.47	-0.22	-0.11
	(0.52)	(0.29)	(0.37)	(0.23)	(0.49)	(0.25)	(0.37)
Panel B : 5 country optimization							
Subperiod I : 1975-1989							
1 month	0.21	-0.22		-0.06		-0.06	0.13
	(0.19)	(0.16)		(0.12)		(0.11)	(0.10)
3 months	0.35**	-0.15		-0.15		-0.10	0.05
	(0.17)	(0.11)		(0.11)		(0.15)	(0.20)
12 months	-0.10	0.14		-0.20		-0.02	0.18
	(0.22)	(0.20)		(0.15)		(0.21)	(0.24)
Suberiod II : 1990-2005							
1 month	0.56***	-0.40***		-0.08		-0.20*	0.12
	(0.12)	(0.12)		(0.08)		(0.11)	(0.13)
3 months	0.79***	-0.47***		-0.06		-0.11	-0.15
	(0.13)	(0.12)		(0.11)		(0.13)	(0.17)
12 months	1.02***	-0.40*		-0.22		-0.20	-0.20
	(0.21)	(0.23)		(0.19)		(0.25)	(0.32)

Note. This table replicates Table 5 for two subperiods, respectively extending from 1975:7 to 1989:12 and from 1990:1 to 2005:12. Time horizons include 1, 3 and 12 months only.

Table A5

Optimal currency exposure for a value-weighted global equity portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
1 month	0.13	-0.09	-0.70***	-0.13*	0.22*	-0.09	0.66***
	(0.17)	(0.10)	(0.15)	(0.08)	(0.13)	(0.08)	(0.15)
2 months	0.22	-0.07	-0.73***	-0.22***	0.26*	-0.06	0.60***
	(0.16)	(0.09)	(0.15)	(0.08)	(0.13)	(0.09)	(0.16)
3 months	0.22	-0.04	-0.76***	-0.23**	0.30**	-0.03	0.55***
	(0.17)	(0.09)	(0.17)	(0.10)	(0.15)	(0.11)	(0.19)
6 months	0.11	0.01	-0.60***	-0.32**	0.39**	0.03	0.39
	(0.24)	(0.14)	(0.22)	(0.12)	(0.19)	(0.15)	(0.26)
12 months	-0.29	0.25	-0.49	-0.46**	0.72**	-0.09	0.36
	(0.39)	(0.22)	(0.36)	(0.18)	(0.30)	(0.21)	(0.37)
Panel B : 5 country optimization							
1 month	0.29***	-0.25***		-0.08		-0.08	0.12
	(0.11)	(0.09)		(0.07)		(0.08)	(0.09)
2 months	0.42***	-0.25**		-0.16**		-0.05	0.04
	(0.11)	(0.10)		(0.08)		(0.09)	(0.11)
3 months	0.46***	-0.24**		-0.17*		-0.03	-0.03
	(0.11)	(0.10)		(0.09)		(0.11)	(0.14)
6 months	0.45***	-0.20		-0.24**		0.06	-0.07
	(0.13)	(0.13)		(0.11)		(0.15)	(0.18)
12 months	0.39*	0.01		-0.32**		-0.08	0.00
	(0.20)	(0.20)		(0.16)		(0.21)	(0.23)

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with constant value weights (reflecting the end-of-period 2005:12 weights as reported in Table 7), who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon *T* of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global equity portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping *T*-months returns, *T* varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A6

Optimal currency exposure for a home-biased global equity portfolio: single and multiple currency cases

Base country	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
PANEL A : Single currency							
Euroland		-0.40***	-0.52***	-0.31***	0.34*	-0.32***	-0.45***
		(0.10)	(0.12)	(0.08)	(0.18)	(0.11)	(0.13)
Australia	0.37***		0.09	0.16*	0.31***	0.17	0.25**
	(0.10)		(0.11)	(0.09)	(0.09)	(0.12)	(0.10)
Canada	0.42***	-0.01		0.12	0.35***	0.17	0.88***
	(0.10)	(0.10)		(0.09)	(0.09)	(0.11)	(0.19)
Japan	0.31***	-0.09	-0.08		0.35***	0.15*	0.02
	(0.10)	(0.09)	(0.10)		(0.10)	(0.08)	(0.11)
Switzerland	-0.45***	-0.35***	-0.42***	-0.29***		-0.29***	-0.38***
	(0.15)	(0.08)	(0.09)	(0.09)		(0.08)	(0.10)
UK	0.25**	-0.24**	-0.30***	-0.10	0.25***		-0.21*
	(0.11)	(0.10)	(0.12)	(0.07)	(0.09)		(0.12)
US	0.23**	-0.14*	-0.71***	-0.01	0.22**	0.11	
	(0.11)	(0.08)	(0.16)	(0.09)	(0.09)	(0.11)	
Panel B : Multiple currencies at once							
Euroland	0.36	-0.08	-0.50**	-0.20*	0.33*	-0.08	0.17
	(0.23)	(0.11)	(0.21)	(0.11)	(0.18)	(0.13)	(0.22)
Australia	0.47**	-0.16	-0.68***	-0.14	0.15	-0.24*	0.60***
	(0.20)	(0.12)	(0.17)	(0.11)	(0.19)	(0.15)	(0.22)
Canada	0.30	-0.05	-0.94***	-0.22**	0.31	-0.23*	0.83***
	(0.20)	(0.10)	(0.21)	(0.10)	(0.20)	(0.13)	(0.21)
Japan	0.34*	-0.14	-0.63***	-0.25**	0.20	0.01	0.48**
	(0.17)	(0.13)	(0.21)	(0.12)	(0.16)	(0.12)	(0.21)
Switzerland	0.14	-0.12	-0.35*	-0.07	0.37**	-0.02	0.04
	(0.21)	(0.09)	(0.19)	(0.11)	(0.17)	(0.13)	(0.21)
UK	0.30	-0.11	-0.56***	-0.20**	0.28	-0.02	0.30
	(0.21)	(0.10)	(0.19)	(0.09)	(0.18)	(0.13)	(0.20)
US	0.15	0.00	-0.83***	-0.22**	0.30*	-0.03	0.62***
	(0.18)	(0.09)	(0.17)	(0.09)	(0.15)	(0.11)	(0.19)

Note. This table considers an investor holding a home-biased portfolio of global equity. The portfolio is constructed by assigning a 75% weight to the home country of the investor, and distributing the remaining 25% over the four other countries according to their value weights. The investor chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the base country of the investor, columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the excess return on the row country home biased global equity portfolio onto the excess return on the column country currency. Rows of Panel B (excluding diagonal terms) are obtained by regressing the excess return on the row country portfolio on the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in Panel B are obtained by computing the opposite of the sum of other terms and the corresponding standard deviation.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A7

Optimal currency exposure for an equally-weighted global bond portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
1 month	0.02	0.00	-0.12*	-0.06*	-0.04	-0.01	0.22*
	(0.05)	(0.02)	(0.05)	(0.03)	(0.04)	(0.03)	(0.05)
2 months	-0.01	0.03	-0.14*	-0.08*	-0.03	0.00	0.23*
	(0.07)	(0.03)	(0.06)	(0.03)	(0.05)	(0.04)	(0.05)
3 months	-0.03	0.04	-0.10	-0.07	-0.03	0.01	0.18*
	(0.07)	(0.04)	(0.08)	(0.04)	(0.07)	(0.05)	(0.06)
6 months	-0.08	0.13*	-0.05	-0.10	0.00	0.06	0.05
	(0.11)	(0.05)	(0.10)	(0.06)	(0.10)	(0.07)	(0.08)
12 months	-0.26	0.17	0.03	-0.11	0.11	0.14	-0.08
	(0.17)	(0.09)	(0.16)	(0.08)	(0.13)	(0.11)	(0.11)
Panel B : 5 country optimization							
1 month	-0.02	-0.03		-0.07*		-0.01	0.13*
	(0.03)	(0.02)		(0.03)		(0.03)	(0.03)
2 months	-0.04	-0.01		-0.09*		-0.01	0.15*
	(0.04)	(0.03)		(0.03)		(0.03)	(0.04)
3 months	-0.06	0.01		-0.08		0.01	0.11*
	(0.05)	(0.03)		(0.04)		(0.05)	(0.04)
6 months	-0.08	0.11*		-0.10		0.06	0.01
	(0.08)	(0.04)		(0.06)		(0.07)	(0.06)
12 months	-0.14	0.18*		-0.10		0.12	-0.06
	(0.12)	(0.06)		(0.07)		(0.11)	(0.07)

Note. This table considers an investor holding a portfolio composed of bonds from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon *T* of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global bond portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping *T*-months returns, *T* varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A8- Subperiod analysis

Optimal currency exposure for an equally-weighted global bond portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
Subperiod I: 1975-1989							
1 month	0.08	0.01	-0.24**	-0.11***	-0.06	0.00	0.33***
	(0.07)	(0.04)	(0.10)	(0.04)	(0.05)	(0.03)	(0.09)
3 months	-0.02	0.04	-0.30**	-0.16***	-0.01	0.02	0.43***
	(0.10)	(0.05)	(0.14)	(0.06)	(0.08)	(0.06)	(0.12)
12 months	-0.29*	0.24**	-0.14	-0.25***	0.12	0.20**	0.13
	(0.16)	(0.11)	(0.19)	(0.08)	(0.10)	(0.09)	(0.14)
Subperiod 2: 1990-2005							
1 month	-0.07	0.00	-0.05	-0.04	0.04	0.00	0.12**
	(0.09)	(0.03)	(0.06)	(0.03)	(0.07)	(0.04)	(0.06)
3 months	-0.17	0.13**	-0.08	-0.01	0.06	0.04	0.04
	(0.12)	(0.06)	(0.11)	(0.05)	(0.11)	(0.06)	(0.06)
12 months	-0.42**	0.24**	-0.14	0.04	0.25*	0.12	-0.10
	(0.21)	(0.12)	(0.22)	(0.09)	(0.15)	(0.11)	(0.10)
Panel B : 5 country optimization							
Subperiod I: 1975-1989							
1 month	0.02	-0.04		-0.11**		-0.01	0.15***
	(0.06)	(0.03)		(0.04)		(0.03)	(0.04)
3 months	-0.03	-0.04		-0.14**		0.01	0.20***
	(0.08)	(0.04)		(0.07)		(0.05)	(0.06)
12 months	-0.18	0.16*		-0.21***		0.20**	0.03
	(0.12)	(0.09)		(0.07)		(0.09)	(0.11)
Subperiod 2: 1990-2005							
1 month	-0.05	-0.01		-0.04		0.00	0.10**
	(0.04)	(0.03)		(0.03)		(0.04)	(0.04)
3 months	-0.13**	0.12***		0.00		0.03	-0.02
	(0.06)	(0.04)		(0.05)		(0.06)	(0.05)
12 months	-0.16	0.20***		0.04		0.08	-0.16*
	(0.13)	(0.05)		(0.08)		(0.11)	(0.10)

Note. This table considers an investor holding a portfolio composed of bonds from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon T of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global equity portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping T-months returns, T varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A9 - Subperiod I

Optimal conditional currency exposure for an equally-weighted global portfolio: single and multiple - currency case

Base Currency	Equity				Bonds			
	Single Currency		Multiple Currencies		Single Currency		Multiple Currencies	
	Slope	P-Value	Slope	P-Value	Slope	P-Value	Slope	P-Value
Euroland	-0.10	1.00	-7.57	0.11	0.10	0.99	1.92	0.00
	(0.94)		(7.55)		(0.34)		(3.62)	
Australia	-0.08	1.00	4.19	0.32	0.01	1.00	2.75***	0.61
	(0.51)		(2.58)		(0.28)		(0.84)	
Canada	-0.02	1.00	5.72	0.83	0.04	1.00	3.22	0.54
	(0.49)		(3.83)		(0.36)		(2.92)	
Japan	0.00	1.00	2.20	0.07	0.05	1.00	-0.07	0.03
	(0.07)		(4.81)		(0.44)		(2.07)	
Switz.	0.24	1.00	-8.22	0.04	0.11	1.00	1.74	0.03
	(0.87)		(5.53)		(0.35)		(2.92)	
UK	-0.05	1.00	2.77	0.11	0.12	1.00	-1.21	0.06
	(0.38)		(3.75)		(0.32)		(1.90)	
US	0.02	1.00	-1.88	0.21	0.07	1.00	-0.69	0.09
	(0.34)		(4.48)		(0.57)		(1.99)	

Note. This table reports optimal currency exposure conditional on interest rate. For each base country-currency pair, we now let the optimal currency position vary with the log interest rate differential (interest rate of the foreign country minus that of

The "Single Currency" columns consider the case of an investor using one currency at a time to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients from a SUR estimation are reported for each b

The "Multiple Currency" columns consider the case of an investor using all foreign currencies simultaenously to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients are reported for each base c

Table A9 - Subperiod II

Optimal conditional currency exposure for an equally-weighted global portfolio: single and multiple - currency case

Base Currency	Equity				Bonds			
	Single Currency		Multiple Currencies		Single Currency		Multiple Currencies	
	Slope	P-Value	Slope	P-Value	Slope	P-Value	Slope	P-Value
Euroland	-1.24	0.99	8.48**	0.23	0.04	1.00	-3.32*	0.58
	(2.90)		(3.62)		(0.39)		(1.95)	
Australia	-0.71	1.00	8.96**	0.91	0.00	1.00	0.42	0.41
	(1.79)		(3.81)		(0.30)		(1.50)	
Canada	-1.37	0.95	15.96**	0.42	0.04	1.00	-0.79	0.14
	(3.19)		(6.44)		(0.21)		(2.79)	
Japan	0.10	1.00	3.59	0.52	-0.01	1.00	-0.03	0.89
	(1.31)		(4.53)		(0.15)		(2.48)	
Switz.	-1.49	0.97	8.40***	0.11	0.00	1.00	-0.37	0.41
	(2.56)		(2.70)		(0.24)		(1.08)	
UK	-0.71	0.99	8.14	0.03	0.06	1.00	-0.89	0.16
	(2.34)		(7.16)		(0.20)		(3.27)	
US	-0.77	0.98	1.90	0.19	0.03	1.00	-1.84	0.42
	(1.78)		(4.46)		(0.20)		(1.74)	

Note. This table reports optimal currency exposure conditional on interest rate. For each base country-currency pair, we now let the optimal currency position vary with the log interest rate differential (interest rate of the foreign country minus that of

The "Single Currency" columns consider the case of an investor using one currency at a time to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients from a SUR estimation are reported for each b

The "Multiple Currency" columns consider the case of an investor using all foreign currencies simultaenously to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients are reported for each base c

Table A10

Optimal synthetic carry-trade currency exposure for equally-weighted global equity and bond portfolios

	Multiple Currencies								Single currency
	Euroland	Australia	Canada	Japan	Switz.	UK	US	Synthetic	Synthetic
Panel A: Stocks									
Full period	0.33**	-0.17*	-0.68***	-0.08	0.34**	-0.18	0.27*	0.27*	-0.23**
	(0.16)	(0.09)	(0.17)	(0.10)	(0.15)	(0.13)	(0.14)	(0.14)	(0.12)
Subperiod I	0.22	-0.25	-0.91***	0.00	0.48**	-0.23	0.69**	0.73*	-0.13
	(0.21)	(0.18)	(0.22)	(0.20)	(0.22)	(0.14)	(0.33)	(0.38)	(0.14)
Subperiod II	0.36	-0.14	-0.57***	-0.17*	0.38*	0.02	0.12	-0.26	-0.37*
	(0.30)	(0.14)	(0.21)	(0.10)	(0.23)	(0.20)	(0.21)	(0.17)	(0.20)
Panel B: Bonds									
Full period	-0.02	0.02	-0.14	-0.04	0.00	-0.02	0.20***	0.11	0.13***
	(0.07)	(0.04)	(0.09)	(0.05)	(0.07)	(0.06)	(0.07)	(0.08)	(0.05)
Subperiod I	0.00	-0.02	-0.39***	-0.10	0.08	-0.03	0.45***	0.24*	0.19***
	(0.09)	(0.07)	(0.15)	(0.07)	(0.09)	(0.07)	(0.12)	(0.14)	(0.07)
Subperiod II	-0.16	0.12**	-0.10	0.02	0.07	0.00	0.06	0.07	0.07
	(0.12)	(0.06)	(0.13)	(0.06)	(0.11)	(0.08)	(0.08)	(0.09)	(0.06)

Note: The first eight columns of this table consider an investor holding a global, equally weighted, stock (Panel A) or bond portfolio (Panel B) who chooses a vector of positions in available currencies to minimize the variance of his portfolio. Available currencies include all foreign currencies as well as a synthetic currency. At each point in time, the synthetic currency return is the average of the return of holding the currencies of the three highest interest rates countries and financing the position using the currencies of the three lowest interest rate countries. The time t return is based on currencies chosen using time t-1 interest rates.

The last column considers the same investor now choosing an optimal position in only one currency: the synthetic currency to minimize the variance of his portfolio.

Table A11

Optimal conditional currency exposure for an equally-weighted global portfolio: single and multiple - currency case using the real interest rate differential

Base Currency	Equity				Bonds			
	Single Currency		Multiple Currencies		Single Currency		Multiple Currencies	
	Slope	P-Value	Slope	P-Value	Slope	P-Value	Slope	P-Value
Euroland	-0.23	1.00	0.73	0.38	0.02	1.00	-0.24	0.09
	(0.62)		(2.22)		0.11		0.94	
Australia	0.00	1.00	0.41	0.10	0.00	1.00	-0.21	0.20
	(0.33)		(0.68)		0.07		0.28	
Canada	0.03	1.00	-2.60	0.06	-0.02	1.00	-1.55	0.19
	(0.37)		(1.98)		0.12		0.88	
Japan	-0.05	0.99	-0.64	0.06	-0.02	1.00	-0.27	0.00
	(0.17)		(1.45)		0.09		0.49	
Switz.	-0.09	1.00	0.48	0.37	0.00	1.00	-1.01	0.66
	(0.48)		(1.86)		0.10		0.73	
UK	0.01	1.00	2.59***	0.63	0.02	1.00	0.17	0.00
	(0.36)		(0.97)		0.07		0.43	
US	0.05	1.00	2.17	0.18	0.00	1.00	0.56	0.49
	(0.34)		(2.22)		0.11		0.98	

Note. This table reports optimal currency exposure conditional on real interest rate. For each base country-currency pair, we now let the optimal currency position vary with the log real interest rate differential (ex-post real interest rate of the foreign country minus that of the base country). Yet, we impose the constraints that the slopes of the optimal positions with respect to the interest rate differential be equal across foreign currencies.

The "Single Currency" columns consider the case of an investor using one currency at a time to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients from a SUR estimation are reported for each base country, followed by the P-value of a test of the constraint. A P-value of x% indicates that the constraint can be rejected at the x% level.

The "Multiple Currency" columns consider the case of an investor using all foreign currencies simultaenously to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients are reported for each base country, followed by the P-value of a test of the constraint. A P-value of x% indicates that the constraint can be rejected at the x% level.