Appendix: Monetary Policy Drivers of Bond and Equity Risks

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This document is a supplemental appendix to Campbell, Pflueger, and Viceira (2014). The contents of this appendix are as follows:

1. Section A provides additional details for the model solution. It discusses equilibrium selection criteria and provides the details of the solution of bond and stock returns. Section A.6 shows that the unconditional second moments of bond and stock returns are equal to the conditional second moments, evaluated at $x_t = 0$.

2. Section B provides details for the moment fitting procedure, including the objective function and optimization procedure.

3. Section C provides additional results for the calibrated model, including robustness results. In particular, Section C.1 shows that the convexity (or Jensen’s inequality) effect in bond returns has a small level effect on bond betas for the maturities that we focus on, but that it is not a driver of changes in betas. Section C also analyzes the effect of the monetary policy persistence parameter by comparing impulse responses for different values of $\rho^i$ when all other parameters are held fixed at their period 3 values.

4. Section D considers model extensions. Section D.3 shows the model can be reinterpreted as a model with shocks to the output gap, if we are willing to reinterpret the Phillips curve shocks as a combination of cost-push shocks and shocks to potential output. Section D.4 shows that adding a cointegrated random walk component to consumption and dividends leaves the model solution almost unchanged and that numerical effects from adding such a component are plausibly small.

5. Section E provides additional empirical results. It shows Taylor rule estimates before and after the financial crisis, which we date at 2008.Q3. Section E also shows that the bond-stock correlation has the opposite sign from the inflation-output gap correlation in all three subperiods.

### A Model Solution

Let $\pi^*_t$ denote the central bank’s inflation target at time $t$. We solve the model in terms of the output gap $x_t$ and inflation and nominal interest rate gaps:

\begin{align*}
\hat{\pi}_t &= \pi_t - \pi^*_t, \\
\hat{i}_t &= i_t - \pi^*_t.
\end{align*}

Denote the vector of state variables by:

\[\hat{Y}_t = [x_t, \hat{\pi}_t, \hat{i}_t]'\]
We can re-write the model dynamics in terms of the state variables as:

\[
x_t = \rho^x x_{t-1} + \rho^x E_{t-1} x_{t+1} - \psi \left( E_{t-1} \hat{i}_t - E_{t-1} \hat{\pi}_{t+1} \right) + u_t^{IS},
\]

\[
\hat{\pi}_t = \rho^\pi \hat{\pi}_{t-1} + (1 - \rho^\pi) E_{t-1} \hat{\pi}_{t+1} + \lambda x_t - \rho^\pi u_t^* + u_t^{PC},
\]

\[
\hat{i}_t = \rho^i \hat{i}_{t-1} + (1 - \rho^i) \left[ \gamma^x x_t + \gamma^\pi \hat{\pi}_t \right] + u_t^{MP},
\]

\[
\pi_t^* - \pi_{t-1}^* = u_t^*.
\]

Using \( E_{t-1} \hat{i}_t = \hat{i}_t - u_t^{MP} \), we can write the model as:

\[
0 = F E_{t-1} \hat{Y}_{t+1} + G \hat{Y}_t + H \hat{Y}_{t-1} + M u_t.
\]

where

\[
F = \begin{bmatrix}
\rho^x & \psi & 0 \\
0 & (1 - \rho^\pi) & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
G = \begin{bmatrix}
-1 & 0 & -\psi \\
\lambda & -1 & 0 \\
(1 - \rho^i) \gamma^x & (1 - \rho^i) \gamma^\pi & -1
\end{bmatrix},
\]

\[
H = \begin{bmatrix}
\rho^x & 0 & 0 \\
0 & \rho^\pi & 0 \\
0 & 0 & \rho^i
\end{bmatrix},
\]

\[
M = \begin{bmatrix}
1 & 0 & \psi & 0 \\
0 & 1 & 0 & -\rho^\pi \\
0 & 0 & 1 & 0
\end{bmatrix}.
\]

We focus on solutions of the form:

\[
\hat{Y}_t = P \hat{Y}_{t-1} + Q u_t.
\]

Additional solutions, such as solutions depending on two lags of state variables, may exist, see e.g. Evans and McGough (2005). \( P \) has to satisfy:

\[
FP^2 + GP + H = 0.
\]

Following Uhlig (1999), we first solve for the generalized eigenvectors and eigenvalues of \( \Xi \) with respect to \( \Delta \), where:

\[
\Xi = \begin{bmatrix}
-G & -H \\
I_3 & 0_3
\end{bmatrix},
\]

\[
\Delta = \begin{bmatrix}
F & 0_3 \\
0_3 & I_3
\end{bmatrix}.
\]

For three generalized eigenvalues \( \lambda_1, \lambda_2, \lambda_3 \) with generalized eigenvectors \( [\lambda_1 z_1', z_2']', [\lambda_2 z_2', z_2']', [\lambda_3 z_3', z_3']' \), a solution is given by

\[
P = \Omega \Lambda \Omega^{-1},
\]
where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \lambda_3)$ and $\Omega = [z_1, z_2, z_3]$. Generalized eigenvalues are stable if their absolute value is $< 1$.

Let $e_k$ denotes the row vector with a 1 in position $k$ and zeros otherwise. $Q$ has to satisfy

\begin{align*}
Q e_k' &= -[FP + G]^{-1} Me_k' \quad k = 1, 2, 4 \tag{18} \\
Q e_3' &= -G^{-1} Me_3' \tag{19}
\end{align*}

Provided that $G$ is nonsingular, $G \times Q \times e_3' = -Me_3' = -[\psi, 0, 1]'$ implies that $Q \times e_3' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, i.e. the monetary policy shock has no contemporaneous effect on $x_t$ or $\hat{\pi}_t$.

As long as we focus on solutions of the form (13) and the matrix of lagged terms $H$ is non-singular, the solution cannot contain arbitrary random variables, or ‘sunspots’. If we were to allow for more complicated solution forms, where $\hat{Y}_t$ can depend on two lags of itself as well as current and lagged shocks, sunspot solutions may be possible (Evans and McGough, 2005).

To see that solutions of the form (13) do not allow for sunspots, suppose the contrary. Assume that for some vector of random variables $\epsilon_t$ uncorrelated with $\hat{Y}_{t-1}$ and $u_t$:

$$\hat{Y}_t = P\hat{Y}_{t-1} + Qu_t + \epsilon_t.$$ \tag{20}

The expression (20) corresponds to the definition of sunspot equilibria, see e.g. Cho and Moreno (2011). Then substituting (20) into (8) gives the same conditions for $P$ and $M$ as before and:

$$(FP + G)\epsilon_t \equiv 0. \tag{21}$$

But from (14), $(FP + G) \times P = -H$ is non-singular. Therefore, $FP + G$ is non-singular and $\epsilon_t \equiv 0$. This completes the proof that there are no sunspot solutions.

### A.1 Equilibrium Selection and Properties

We are essentially solving a quadratic matrix equation, so picking a solution amounts to picking three out of six generalized eigenvalues. We only consider dynamically stable solutions with all eigenvalues less than 1 in absolute value, yielding non-explosive solutions for the output gap, inflation gap and interest rate gap. When there are only three generalized eigenvalues with absolute values less than 1, there exists a unique dynamically stable solution. For the period 1 calibration, we have $\gamma_\pi < 1$ and there exist multiple real-valued, dynamically stable solutions. The period 2 and 3 calibrations have unique dynamically stable solutions.

We only consider solutions that are real-valued, and have finite entries for $Q$. We also require the diagonal entries of $Q$ to be positive. This requirement means that
the immediate impact of a positive IS shock on the output gap is positive rather than negative.

We apply multiple equilibrium selection criteria, which have been proposed in the literature, to rule out “bubble” or unreasonable solutions. These different equilibrium refinements are not identical, but coincide in many cases. Therefore, there exists a unique solution satisfying all criteria for a large part of our parameter space.

McCallum (1983) proposes to pick the minimum state variable solution. This solution has a minimum set of state variables and satisfies a continuity criterion. Unfortunately, Uhlig (1999) points out that implementing this criterion directly can be computationally demanding. We therefore follow Uhlig (1999) in picking the solution with the minimum absolute eigenvalues, which under certain conditions coincides with the minimum state variable solution (McCallum 2004).

We also require that our solution is locally E-stable (Evans 1985, 1986, Evans and Honkapohja 1994) as a plausible necessary, but not sufficient, condition. Local E-stability intuitively requires that the solution is learnable. If agents expectations deviate slightly from equilibrium dynamics, the system will return to an E-stable equilibrium under a simple revision rule.

Finally, we ensure uniqueness of our solution by requiring that it equals the forward solution of Cho and Moreno (2011). The forward solution is obtained by imposing a zero terminal condition. Expectations about shocks far in the future do not affect the current equilibrium. Viewed differently, if we assume that all state variables are constant from time t+T onwards, we can solve for the time t output gap, inflation gap, and interest rate gap recursively. The forward solution obtains by letting T go to infinity.

Let vec denote vectorization. Applying Proposition 1.3 of Fudenberg and Levine (1998, p.25) the E-stability condition translates into the requirement that the eigenvalues of the derivative

\[ \frac{\partial \text{vec}([FP + G]^{-1}H)}{\partial \text{vec}(P)} \]

have eigenvalues with absolute values less than 1.

We implement the Cho and Moreno (2011) criterion by requiring that the following sequence \( P_n, n = 0, 1, \ldots \) converges to \( P \)

\[ P_0 = 0_{3 \times 3} \]

\[ P_{n+1} = -[FP_n + G]^{-1} \times H \] (24)

This sequence \( P_n \) has at most one limit and therefore this selection criterion yields a unique solution.
A.2 Solving for SDF and Model Dynamics Simultaneously

Figures 3 and 4 and Tables 8 and 9 conduct counterfactual exercises. We hold the heteroskedasticity parameter \( b \) constant and allow the degree of precautionary savings to vary as we vary \( \gamma^x, \gamma^\pi, \) and \( \rho^i \). The Euler equation coefficients \( \rho^{-}, \rho^{+}, \) and \( \psi \) depend on the volatility of the SDF. The volatility of the SDF in turn depends on the macroeconomic dynamics, which are determined by the Euler equation coefficients. This requires solving a fixed-point problem for the volatility of the SDF \( \bar{\sigma} \).

For any combination of model parameters, we solve for the Euler equation slope coefficients \( \rho^{-}, \rho^{+}, \) and \( \psi \) in terms of the preference parameters and volatilities:

\[
\begin{align*}
\rho^- &= \frac{\theta}{1 + \theta^*}, \\
\rho^+ &= \frac{1}{1 + \theta^*}, \\
\psi &= \frac{1}{\alpha(1 + \theta^*)}, \\
\theta^* &= \theta - \alpha b \bar{\sigma}^2/2.
\end{align*}
\]

Given \( \rho^+, \rho^- \) and \( \psi \), we solve for the matrices \( P \) and \( Q \). The volatility of the SDF therefore has to solve the fixed-point expression:

\[
\bar{\sigma}^2 = Q^M \Sigma_u Q^{Mt},
\]

\[
Q^M = e_1 Q - (1 + \theta^*)e_1.
\]

We solve the fixed-point problem (29) for every new combination of parameters.

A.3 Bond Returns

We solve for nominal and real bond log return surprises in terms of the fundamental vector of shocks \( u_t \). We use the loglinear framework of Campbell and Ammer (1993) and do not impose the Expectations Hypothesis. We maintain our previous simplifying approximation that risk premia on one period nominal bonds equal zero. Risk premia on longer-term bonds are allowed to vary.

We write \( r_{n-1,t+1} \) for the real one-period return on a real n-period bond from time \( t \) to time \( t+1 \) and \( xrr_{n-1,t+1} \) for the corresponding return in excess of \( r_t \). \( r^3_{n-1,t+1} \) denotes the nominal one-period return on a similar nominal bond and \( xrr^3_{n-1,t+1} \) the corresponding excess return over \( i_t \). We use the identities:
\[ r_{n-1,t+1}^s - E_t r_{n-1,t+1}^s = -(E_{t+1} - E_t) \sum_{j=1}^{n-1} \left( \hat{i}_{t+j} + \pi_{t+j}^* \right) \] (31)

\[ -(E_{t+1} - E_t) \sum_{j=1}^{n-1} x r_{n-j-1,t+1+j}^s \] (32)

\[ r_{n-1,t+1} - E_t r_{n-1,t+1} = -(E_{t+1} - E_t) \sum_{j=1}^{n-1} r_{t+j} \] (33)

\[ -(E_{t+1} - E_t) \sum_{j=1}^{n-1} x r_{n-j-1,t+1+j} \] (34)

We now derive recursive expressions for unexpected nominal and real bond returns. We guess the functional forms:

\[ E_t x r_{n-1,t+1}^s = (1 - bx_t) b^{s,n} \] (35)

\[ E_t x r_{n-1,t+1} = (1 - bx_t) b^n \] (36)

The functional forms (35) and (36) hold for \( n = 1 \) with \( b^{s,1} = b^1 = 0 \). Assuming (35) and (36) for maturities less than \( n \), we can express (32) and (34) as:

\[ -(E_{t+1} - E_t) \sum_{j=1}^{n-1} x r_{n-j-1,t+1+j}^s = b \sum_{j=1}^{n-1} b^{s,n-j} e_1 P^{j-1} Q u_{t+1} \] (37)

\[ -(E_{t+1} - E_t) \sum_{j=1}^{n-1} x r_{n-j-1,t+1+j} = b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q u_{t+1} \] (38)

We can express (31) and (33) as:

\[ -(E_{t+1} - E_t) \sum_{j=1}^{n-1} \left( \hat{i}_{t+j} + \pi_{t+j}^* \right) = -e_3 [I - P]^{-1} [I - P^{n-1}] Q u_{t+1} \]

\[ -(n - 1) u_{t+1}^* \] (39)

\[ -(E_{t+1} - E_t) \sum_{j=1}^{n-1} r_{t+j} = -(e_3 - e_2 P) [I - P]^{-1} [I - P^{n-1}] Q u_{t+1} \] (40)

Denoting

\[ S^{s,n} = -(n - 1) e_4 - e_3 [I - P]^{-1} [I - P^{n-1}] Q, \] (41)

\[ S^n = -(e_3 - e_2 P) [I - P]^{-1} [I - P^{n-1}] Q, \] (42)
we obtain:

\[ r^\$_{n-1,t+1} - E_{t}r^\$_{n-1,t+1} = \left[ S^\$_{n} + b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q \right] u_{t+1}, \]

\[ r_{n-1,t+1} - E_{t}r_{n-1,t+1} = \left[ S^{n} + b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q \right] u_{t+1}. \]

The conditional expected return adjusted for Jensen’s inequality equals the conditional covariance between bond excess returns and marginal utility. It hence follows that:

\[ b^{\$n} = \alpha \left[ S^{\$n} + b \sum_{j=1}^{n-1} b^{\$n-j} e_1 P^{j-1} Q \right] \Sigma_u Q^{M^t} \]

\[ \frac{1}{2} \left[ S^{\$n} + b \sum_{j=1}^{n-1} b^{\$n-j} e_1 P^{j-1} Q \right] \Sigma_u \left[ S^{\$n} + b \sum_{j=1}^{n-1} b^{\$n-j} e_1 P^{j-1} Q \right]'. \]

Similarly, we obtain the recursive expression:

\[ b^{n} = \alpha \left[ S^{n} + b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q \right] \Sigma_u Q^{M^t} \]

\[ \frac{1}{2} \left[ S^{n} + b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q \right] \Sigma_u \left[ S^{n} + b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q \right]'. \]

Up to a constant, log yields of nominal and real zero coupon bonds then equal:

\[ y^{\$n}_{n,t} = \frac{1}{n} E_{t} \sum_{j=0}^{n-1} r^{\$}_{n-j-1,t+1+j} \]

\[ = \frac{1}{n} E_{t} \sum_{j=0}^{n-1} i_{t+j} - \frac{1}{n} E_{t} \sum_{j=0}^{n-1} b^{\$n-j} x_{t+j} \]

\[ \pi^*_{t} + \left[ \frac{1}{n} e_3 [I - P]^{-1} [I - P^n] - \frac{b}{n} \sum_{j=0}^{n-1} b^{\$n-j} e_1 P^{j} \right] \hat{Y}_{t} \]

\[ y^{n}_{n,t} = \left[ \frac{1}{n} (e_3 - e_2 P) [I - P]^{-1} [I - P^n] - \frac{b}{n} \sum_{j=0}^{n-1} b^{n-j} e_1 P^{j} \right] \hat{Y}_{t} \]

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We can then calculate the conditional slope of the term structure as follows:

\[
y^s_{n,t} - i_t = \frac{1}{n} E_t \sum_{j=0}^{n-1} \hat{i}_{t+j} - \hat{i}_t + \frac{1}{n} E_t \sum_{j=0}^{n-1} b^s,n-j (1 - bx_{t+j}) \tag{53}
\]

\[
= \frac{1}{n} E_t \sum_{j=0}^{n-1} \hat{i}_{t+j} - \hat{i}_t + \frac{1}{n} E_t \sum_{j=0}^{n-1} b^s,n-j (1 - bx_{t+j}) \tag{54}
\]

\[
= (\Gamma^s,n - e_3) \hat{y}_t + \frac{1}{n} \sum_{j=0}^{n-1} b^s,n-j \tag{55}
\]

With \( \hat{y}_t \) mean zero, the average slope of the term structure and the average conditional expected bond excess return are:

\[
E (y^s_{n,t} - i_t) = \frac{1}{n} \sum_{j=0}^{n-1} b^s,n-j \tag{56}
\]

\[
E \left( E_t x_{n-1,t+1}^s + \frac{1}{2} \text{Var}_t(x_{n-1,t+1}) \right) = \alpha A^{s,n} \Sigma_u Q^{b_t} \tag{57}
\]

### A.4 Stock Returns

Modeling stocks as a levered claim on the output gap \( x_t \), we assume that dividends are given by:

\[
d_t = \delta x_t. \tag{58}
\]

We interpret \( \delta \) as capturing a broad concept of leverage, including operational leverage.

We write \( r^e_{t+1} \) for the log stock return and \( x r^e_{t+1} \) for the log stock return in excess of \( r_t \). Following Campbell (1991) we decompose stock returns into dividend news, news about real interest rates, and news about future excess stock returns ignoring constants:

\[
r^e_{t+1} - E_t r^e_{t+1} = \delta (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta x_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j x r^e_{t+1+j} \tag{59}
\]

\( \rho \) is a loglinearization constant close to 1. Now guess the functional form:

\[
E_t x r^e_{t+1} = (1 - bx_t) b^r. \tag{60}
\]

Then:

\[
r^e_{t+1} - E_t r^e_{t+1} = (\kappa A^x + A^r) u_{t+1}, \tag{61}
\]
where

\[ A^x = e_1 [I - \rho P]^{-1} Q, \]  
\[ A^r = -\rho (e_3 - e_2 P) [I - \rho P]^{-1} Q, \]  
\[ \kappa = \delta (1 - \rho) + \rho \times b \times b^e. \]  

We also write:

\[ A^e = (\kappa A^x + A^r). \] (65)

\( \kappa A^x \) captures the stock returns’ exposure to long-term news about the output gap. \( A^r \) captures the exposure of stock returns to real interest rate news.

The conditional equity premium adjusted for Jensen’s inequality equals the conditional covariance of excess stock returns and marginal utility:

\[ E_t x_{t+1} + \frac{1}{2} Var_t (x_{t+1}) = \alpha Cov_t (r_{t+1}^e, s_{t+1} + c_{t+1}) = \alpha A^e \Sigma_u Q^{M'} (1 - bx_t) \] (67)

The average conditional equity premium is then given by:

\[ E \left( E_t x_{t+1} + \frac{1}{2} Var_t (x_{t+1}) \right) = \alpha A^e \Sigma_u Q^{M'} \] (68)

It then follows that expected stock returns indeed take the hypothesized form, where \( \kappa \) is the positive root of the quadratic equation:

\[ 0 = \kappa^2 + \kappa \times 2 (\rho b)^{-1} - \alpha A^x \Sigma_u Q^{M'} + A^x \Sigma A^r + \frac{2 \delta (1 - \rho) (\rho b)^{-1} + A^x \Sigma_u A^r - 2 \alpha A^e \Sigma_u Q^{M'}}{A^x \Sigma_u A^r} \] (69)

Applying the Campbell and Shiller (1988) approximate loglinear present value model to equity prices (ignoring constants), we obtain log dividend price ratio:

\[ d_t - p_t = -\delta E_t \sum_{j=0}^{\infty} \rho^j \Delta x_{t+j} + E_t \sum_{j=0}^{\infty} \rho^j (r_{t+j}^e - r_{t+j}) + E_t \sum_{j=0}^{\infty} \rho^j r_{t+j} \] (70)

\[ = \delta e_1 (I - P) - (b \times b^e) e_1 + e_3 - e_2 P \] [I - \rho P]^{-1} \tilde{Y}_t. (71)

The model has implications for the relation between the log dividend price ratio and expected long-term excess stock returns. Denoting the k-period log equity return in excess of short-term real T-bills by \( x_{t \rightarrow t+k} \):

\[ E_t x_{t \rightarrow t+k} = -(b \times b^e) e_1 [I - P]^{-1} [I - P^k] \tilde{Y}_t. \] (72)
A.4.1 Bond-Stock Covariances

The conditional nominal and real bond-stock return covariances equal:

\[
\text{Cov}_t(r^{e}_{t+1}, r^{S}_{n-1,t+1}) = A^n \Sigma_u A^{et}(1 - bx_t) \quad (73)
\]

\[
\text{Cov}_t(r^{e}_{t+1}, r^{n-1}_{n-1,t+1}) = A^n \Sigma_u A^{et}(1 - bx_t) \quad (74)
\]

The nominal bond return loadings \(A^{n,n}\), as defined in (43), contain a term \(-(n - 1) \times [0, 0, 0, 1]\) increasing linearly in bond duration and for long-term nominal bonds this is the dominant term. If a positive shock to the inflation target increases stock returns, this term contributes negatively to the nominal bond-stock covariance. If a positive shock to the inflation target decreases stock returns, this term contributes positively.

The variances of equity excess returns, nominal and real bond excess returns are:

\[
\text{Var}_t(r^{e}_{t+1}) = A^{e} \Sigma_u A^{et}(1 - bx_t), \quad (75)
\]

\[
\text{Var}_t(r^{S}_{n-1,t+1}) = A^{S,n} \Sigma_u A^{S,nt}(1 - bx_t), \quad (76)
\]

\[
\text{Var}_t(r^{n-1}_{n-1,t+1}) = A^n \Sigma_u A^{nt}(1 - bx_t). \quad (77)
\]

The conditional stock market betas of nominal and real bonds are independent of \(x_t\) and given by:

\[
\beta_t(r^{S}_{n-1,t+1}) = \frac{A^{S,n} \Sigma_u A^{et}}{A^{e} \Sigma_u A^{et}}, \quad (78)
\]

\[
\beta_t(r^{n-1}_{n-1,t+1}) = \frac{A^n \Sigma_u A^{et}}{A^{e} \Sigma_u A^{et}}. \quad (79)
\]

A.5 Estimable VAR(1) in Output, Inflation, and Nominal Yields

While standard empirical measures are available for the output gap, we do not observe the interest rate and inflation gaps. We therefore cannot directly estimate the recursive law of motion (13). However, for a long-term bond maturity \(n\), we can estimate a VAR(1) in the vector:

\[
Y_t = \begin{bmatrix}
x_t \\
\pi_t \\
i_t \\
y^{S}_{n,t}
\end{bmatrix} = A \left[ Y_t, \pi_t^* \right]' \quad (80)
\]
The model implies that:

\[ Y_{t+1} = P^Y Y_t + Q^Y u^Y_{t+1}. \]  

Here:

\[
A = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 \\
\Gamma^s & 0 & 0 & 1 \\
\end{bmatrix},
\]

\[
P^Y = A \begin{bmatrix}
P & 0 \\
0 & 1 \\
\end{bmatrix} A^{-1},
\]

\[
Q^Y = A \begin{bmatrix}
0 & 0 & 0 & 1 \\
\end{bmatrix},
\]

\[
u^Y_t = u_t
\]

provided that the inverse of \(A\) exists.

### A.6 Unconditional Second Moments

Expressions (177) through (183) allow us to calculate conditional covariances, variances, and betas, conditional on the output gap being at zero. This section shows that unconditional second moments of bond and stock returns are equal to the conditional second moments, evaluated at \(x_t = 0\). The law of total variance says that for any random variables \(X_1\) and \(X_2\):

\[
Var(X_1) = E(Var(X_1|X_2)) + Var(E(X_1|X_2)).
\]  

We now apply the law of total variance to the unexpected equity return \(r^e_{t+1} - E_t r^e_{t+1}\) and the output gap \(x_t\). The unconditional variance of \(r^e_{t+1} - E_t r^e_{t+1}\) is given by:

\[
Var(r^e_{t+1} - E_t r^e_{t+1}) = E [Var (r^e_{t+1} - E_t r^e_{t+1} | x_t)] + Var (E (r^e_{t+1} - E_t r^e_{t+1} | x_t))
\]

But \(E (r^e_{t+1} - E_t r^e_{t+1} | x_t) = 0\) for any value of \(x_t\) and therefore

\[
Var(r^e_{t+1} - E_t r^e_{t+1}) = E [Var (r^e_{t+1} - E_t r^e_{t+1} | x_t)] + Var (E (r^e_{t+1} - E_t r^e_{t+1} | x_t))
\]

\[
= E [A^e \Sigma_u A^{e'} (1 - bx_t)]
\]

\[
= A^e \Sigma_u A^{e'}.
\]

The expression (93) shows that the unconditional variance of equity returns equals the conditional variance of equity returns, evaluated at \(x_t = 0\). It similarly follows
that:

\[
\begin{align*}
V\text{ar}(r^{s}_{n-1,t+1}) &= A^{s,n}\Sigma u A^{s,n} \\
V\text{ar}(r^{c}_{n-1,t+1}) &= A^{n}\Sigma u A^{n} \\
C\text{ov}(r^{c}_{t+1}, r^{s}_{n-1,t+1}) &= A^{s,n}\Sigma u A^{e'} \\
C\text{ov}(r^{c}_{t+1}, r^{c}_{n-1,t+1}) &= A^{n}\Sigma u A^{e'}
\end{align*}
\]  

(94)  

(95)  

(96)  

(97)  

(98)

The unconditional betas of nominal and real bonds are therefore also equal to the conditional betas (182) and (183).

It is useful to be able to simulate unconditional second moments of model real interest rates, dividend-price ratios etc. We show that we can simulate those moments by simulating a conditionally homoskedastic VAR(1) with matrix of slope coefficients \( P \) and a conditionally homoskedastic, independently and identically distributed vector of innovations.

First, we show that the unconditional second moments of the state variables \( \hat{Y}_t \) are the same as those for a conditionally homoskedastic VAR(1) with matrix of slope coefficients \( P \) and conditionally homoskedastic, independently and identically distributed vector of innovations \( \epsilon_t \sim \mathcal{N}(0, \Sigma_u Q') \). We denote such a conditionally homoskedastic VAR(1) process by \( \tilde{Y}_t \). The fundamental errors of \( \hat{Y}_t \) are given by:

\[
\hat{Y}_t - E(\hat{Y}_t|\hat{Y}_{t-1}, \hat{Y}_{t-2}, \ldots) = Qu_t. 
\]

(99)

The vector of fundamental errors is uncorrelated across time and it therefore is vector white noise (Chapter 10, Hamilton 1994). Applying again the law of total variance, we obtain the unconditional variance-covariance matrix of the fundamental errors:

\[
\begin{align*}
V\text{ar}(Qu_t) &= E \left( V\text{ar} \left( Qu_t|\hat{Y}_{t-1} \right) \right) + V\text{ar} \left( E \left( Qu_t|\hat{Y}_{t-1} \right) \right) \\
&= Q\Sigma_u Q'.
\end{align*}
\]

(100)  

(101)

We can then apply Wold’s theorem for vector processes (Chapter 10, Hamilton 1994) and write \( \hat{Y}_t \) as a vector MA(\( \infty \)) representation:

\[
\hat{Y}_t = [I - PL]^{-1} Qu_t, 
\]

(102)

where \( L \) denotes the lag operator. By Hamilton (1994) Proposition 10.2:

\[
C\text{ov}(\hat{Y}_t, \hat{Y}_{t-k}) = \sum_{j=0}^{\infty} P^{j+k} Q\Sigma_u Q' P^{j'}.
\]

(103)

The process \( \tilde{Y}_t \) has the same variance-covariance matrix of fundamental errors as \( \hat{Y}_t \). The unconditional variances and covariances of \( \tilde{Y}_t \) are hence also given by (103).
Second, we can simulate unconditional second moments of the log dividend price ratio, expected stock returns, and the real short-term interest rate by first simulating \( \tilde{Y} \) and then computing the short-term real interest rate, the dividend-price ratio, and expected equity excess returns according to (71), (72), and (52) with \( \tilde{Y}_t \) replacing \( \hat{Y}_t \). This follows from the observation that if \( \hat{Y}_t \) and \( \tilde{Y}_t \) have identical variances and covariances at all leads and lags, then so do any linear combinations of \( \hat{Y}_t \) and \( \tilde{Y}_t \). The second moments of other quantities that are linear combinations of the state variables can be simulated similarly.

**A.7 A Note on Units**

Our empirical yields and returns are in annualized percent units. Log real dividends and the log output gap are in natural percent units. Our empirical units are analogous to those used by CGG. Our empirical coefficients in Table 4 in the main paper can therefore be compared directly to those in CGG.

However, the Campbell and Shiller (1988) loglinearizations, the expression for the equity premium (169) and expected bond returns (46) are expressed in natural units. We therefore solve the model in natural units and subsequently report scaled parameters and model moments reflecting our choice of empirical units. Let a superscript \( c \) denote natural units used for solving the calibrated model. Values with no superscript denote the parameters and variables corresponding to empirical units.

Our quantities in empirical units are related to quantities in calibration units according to:

- \( x_t = 100x^c_t \), \( i_t = 400i^c_t \), \( \pi_t = 400\pi^c_t \), and \( y^s,n_t = 400y^s,n^c_t \) and \( \pi^*_t = 400\pi^*_t \).

We can therefore write the model as:

\[
\begin{align*}
x_t &= \rho^{x,c}x_{t-1} + \rho^{x+c}E_{t-1}x_{t+1} - \frac{\psi^c}{4} (E_{t-1}i_t - E_{t-1}E_{t+1}) + 100 \times u^IS,c_t \\
\pi_t &= \rho^{\pi,c}\pi_{t-1} + (1 - \rho^{\pi,c})E_{t-1}\pi_{t+1} + 4\lambda^cx_t + 400 \times u^PC,c_t \\
i_t &= \rho^{i,c}(i_{t-1} - \pi^*_t) + (1 - \rho^{i,c})[4\gamma^{x,c}x_t + \gamma^{\pi,c}(\pi_t - \pi^*_t)] + \pi^*_t + 400u^MP,c_t \\
\pi^*_t &= \pi^*_{t-1} + 400u^*_t
\end{align*}
\]

Equations (104) through (107) imply relations between the empirical and calibration parameters:

\[
\begin{align*}
\rho^{x-} &= \rho^{x-c}, \rho^{x+} = \rho^{x+c}, \psi = \frac{\psi^c}{4} \\
\rho^{\pi} &= \rho^{\pi,c}, \lambda = 4\lambda^c \\
\rho^{i} &= \rho^{i,c}, \gamma^{x} = 4\gamma^{x,c}, \gamma^{\pi} = \gamma^{\pi,c} \\
\bar{\sigma}^IS &= 100\bar{\sigma}^IS,c, \bar{\sigma}^PC = 400\bar{\sigma}^PC,c, \bar{\sigma}^MP = 400\bar{\sigma}^MP,c, \bar{\sigma}^* = 400\bar{\sigma}^*
\end{align*}
\]
Fuhrer (1997) estimates a Phillips curve with both backward-looking and forward-looking components. Using inflation in annualized percent, and the log output gap in natural units, he find a backward-looking component of 0.8, a forward-looking component of 0.2, and a weight on the output gap of 0.12. We can therefore compare the parameter $\lambda$ in empirical units directly to the magnitudes in CGG, Fuhrer (1997), and Roberts (1995).

Yogo (2004) scales interest rates and inflation to quarterly units. Our calibrated values for $\psi^c$ in natural units can therefore be compared directly to the estimated values in Yogo (2004). We therefore report the value $\psi^c$ corresponding to natural units rather than $\psi$ corresponding to empirical units throughout the paper.

We choose the leverage parameter $\delta$ to match the relative volatilities of log real dividend growth and log output gap growth. We use four quarter growth rates to smooth out some of the more seasonal fluctuations. We consider four quarter log output growth $\Delta x_t = x_t - x_{t-4}$. The standard deviation of this growth rate over the period 1960.Q1-2011.Q4 is 2.20%. Let $d_t$ denote the sum of log S&P 500 real dividends. Monthly real S&P 500 dividends are from Robert Shiller’s web site. These real dividends are deflated by the not seasonally adjusted CPI-U with a basis of 1982-84=100. We obtain quarterly dividends by summing the level real dividends within the quarter. The standard deviation 1960.Q1-2011.Q4 of the four quarter log dividend growth rate $\Delta d_t = d_t - d_{t-4}$ equals 5.35%. Our model specifies dividends as a levered claim on the output gap with $d_t = \delta x_t$. We therefore set the leverage parameter $\delta$ to match the relative standard deviations of output and dividend growth. This gives $\delta = 2.43$.

Due to our choice of empirical units, we use a slightly different transformation from the transition matrix $P^c$ of the state variables in natural unit to the transition matrix $P^Y$ of the estimable VAR(1). We have the relation:

$$Y_t = \begin{bmatrix} x_t \\ \pi_t \\ \iota_t \\ y^n_{n,t} \end{bmatrix} = A^c \begin{bmatrix} \hat{x}_t^c, \hat{\pi}_t^c, \hat{\iota}_t^c, \hat{\pi}_t^{c,*} \end{bmatrix}', \quad (112)$$

$$A^c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\gamma & 1 & 0 & 0 \\ 0 & -\gamma & 1 & 0 \\ 0 & 0 & -\gamma & 1 \end{bmatrix}, \quad (113)$$
where:

\[
A^c = \text{diag}(100, 400, 400, 400) \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ \Gamma^{s,n} & 1 \end{bmatrix},
\]

(114)

\[
P^Y = A^c \begin{bmatrix} P & 0 \\ 0 & 1 \end{bmatrix} A^{c-1},
\]

(115)

\[
Q^Y = A^c \begin{bmatrix} Q \\ 0 & 0 & 0 & 1 \end{bmatrix},
\]

(116)

\[
u^c_{t,Y} = \begin{bmatrix} u^{c,IS}_t, u^{c,PC}_t, u^{c,MP}_t, u^{c,*}_t \end{bmatrix}',
\]

(117)

\[
Y_t = P^Y Y_{t-1} + Q^Y \nu^c_{t+1}.
\]

(118)

We report annualized percent standard deviations of equity and bond returns at the average output gap \(x_t = 0\). We calculate annualized standard deviations of equity and bond returns in percent at \(x_t = 0\):

\[
\text{Std}_t(r^e_{t+1}) = 200 \sqrt{A^e \Sigma_u A'^e},
\]

(119)

\[
\text{Std}_t(r^s_{n-1,t+1}) = 200 \sqrt{A^{s,n} \Sigma_u A^{s,n'}},
\]

(120)

\[
\text{Std}_t(r^s_{n-1,t+1}) = 200 \sqrt{A^{n} \Sigma_u A^{n'}}.
\]

(121)

We back out empirical shocks for each sub period separately. From the empirical series \(Y^\text{emp}_t = [x^\text{emp}_t, \pi^\text{emp}_t, i^\text{emp}_t, y^\text{emp,s,n}_t]\), we back out fundamental shocks in empirical units:

\[
\begin{bmatrix} u^{c,IS}_t, u^{c,PC}_t, u^{c,MP}_t, u^{c,*}_t \end{bmatrix}' = 100u^{c,IS}_t, 400u^{c,PC}_t, 400u^{c,MP}_t, 400u^{c,*}_t \end{bmatrix}',
\]

(122)

\[
= \text{diag}(100, 400, 400, 400) \times Q^{Y-1}(Y^\text{emp}_t - P^Y Y^\text{emp}_{t-1}).
\]

(123)

We transform the parameter \(b\) into empirical units according to \(b = b^c/100\). Then \((1 - b^c x^c_t) = (1 - b x_t)\). We calculate the standard deviation of the volatility factor \((1 - bx_t)\) at \(x_t = 0\) according to \(b \sqrt{e_{1}Q \Sigma_u Q'e_{1}'}\).
### A.8 Partial Derivatives

We compute the partial derivative of the nominal bond beta with respect to $\ln(\bar{\sigma}_u^k)$ as follows:

\[
\frac{\partial \beta^s}{\partial \ln \bar{n}_{u}^{k}} = \frac{1}{A^e} \Sigma_{u} A^t 2 A^s_{n} \epsilon_k A^e \bar{\sigma}_{u}^{k^2}
\]

(124)

\[
\frac{\partial \text{Std}(r^e_{t+1})}{\partial \ln \bar{n}_{u}^{k}} = \frac{200 A^e \epsilon_k A^e \bar{\sigma}_{u}^{k^2}}{\sqrt{A^e} \Sigma_{u} A^e}
\]

(125)

\[
\frac{\partial \text{Std}(r^s_{n-1,t+1})}{\partial \ln \bar{n}_{u}^{k}} = \frac{200 A^s_{n} \epsilon_k A^s_{n} \bar{\sigma}_{u}^{k^2}}{\sqrt{A^s_{n}} \Sigma_{u} A^s_{n}}
\]

(126)

The partial derivatives for the nominal bond beta sum to two times the calibrated nominal bond beta for each sub period. The partial derivatives for the standard deviations of asset returns sum to the calibrated standard deviation of asset returns for each sub period.

### B Details of Moment Fitting Procedure

We minimize the distance between model and empirical moments summed over all three sub-periods. We use a superscript $p$ to denote period $p$ moments and a hat to denote empirically estimated moments. Our objective function is:

\[
\text{Obj} = \sum_{p=1}^{3} \left[ \left\| P^Y_p - \hat{P}^Y_p \right\|^2 + \left\| \text{diag}(Q^Y_{xu}) - \text{diag}(\hat{Q}^Y_{xu}) \right\|^2 ight]
\]

(127)

\[
+ \left( \frac{1}{10} (\text{Std}^p(r^s_{n-1,t+1}) - \text{Std}^p(\hat{r}^s_{n-1,t+1}))^2 ight)
\]

(128)

\[
+ \left( \frac{1}{10} (\text{Std}^p(r^e_{t+1}) - \text{Std}^p(\hat{r}^e_{n+1}))^2 ight)
\]

(129)

\[
+ (10 \times (\beta^p(r^s_{n-1,t+1}) - \beta^p(\hat{r}^s_{n-1,t+1}))^2)
\]

(130)

We optimize $\text{Obj}$ over the following parameters: $\alpha, b, \theta, \rho, \lambda, \sigma^{IS,p}, \sigma^{PC,p}, \sigma^{MP,p}$, and $\sigma^{*p}, p = 1, 2, 3$. We hold all other calibration parameters constant at the values shown in Table 5 in the main paper.

In order to reduce the dimensionality of the minimization problem, we minimize $\text{Obj}$ iteratively. First, we minimize with respect to all parameters except for $\alpha$ and $b$. Second, we minimize with respect to $\alpha$ and $b$.

Step 1: Starting from an initial guess of $\alpha = 20$ and $b = 0.7$, we minimize with respect to $\theta, \rho, \lambda, \sigma^{IS,p}, \sigma^{PC,p}, \sigma^{MP,p}$, and $\sigma^{*p}, p = 1, 2, 3$. 

16
We use a simple and robust minimization procedure. We randomly draw 100000 parameter vectors from independent multivariate uniform distributions. We first generate a random draw for the volatility of the SDF $\bar{\sigma}$ from a uniform distribution. For $p = 1, 2, 3$, we then draw a normalized vector of shock volatilities $[\tilde{\sigma}_{IS,p}, \tilde{\sigma}_{PC,p}, \tilde{\sigma}_{MP,p}, \tilde{\sigma}^{*+p}]$ from independent uniform distributions, where the volatility of the IS shock is fixed at 0.44 ($\tilde{\sigma}_{IS,p} = 0.44$). The volatilities of shocks are then given by $[\sigma_{IS,p}, \sigma_{PC,p}, \sigma_{MP,p}, \sigma^{*+p}] = [\tilde{\sigma}_{IS,p}, \tilde{\sigma}_{PC,p}, \tilde{\sigma}_{MP,p}, \tilde{\sigma}^{*+p}] / \sqrt{a}$, where the scaling factor $a$ is determined to be consistent with the volatility of the SDF $\bar{\sigma}_p$.

We then solve for the Euler equation coefficients $\rho^+, \rho^-$ and $\psi$ as functions of $\alpha, b$, and $\bar{\sigma}$, which allows us to solve for $P$ and $Q$. We then determine $a$ such that

$$a\bar{\sigma}^2 = (e_1 Q - (1 + \theta^*)e_1)\text{diag}((\tilde{\sigma}_{IS,p})^2, (\tilde{\sigma}_{PC,p})^2, (\tilde{\sigma}_{MP,p})^2, (\tilde{\sigma}^{*+p})^2)(e_1 Q - (1 + \theta^*)e_1).$$

We then rescale the volatilities of shocks to be consistent with the volatility of the SDF:

$$[\sigma_{IS,p}, \sigma_{PC,p}, \sigma_{MP,p}, \sigma^{*+p}] = [\tilde{\sigma}_{IS,p}, \tilde{\sigma}_{PC,p}, \tilde{\sigma}_{MP,p}, \tilde{\sigma}^{*+p}] / \sqrt{a}. \quad (132)$$

Figure A.1 shows the objective function $Obj$ against random draws for $\theta, \lambda, \rho^+, \bar{\sigma}^p$, and $[\tilde{\sigma}_{IS,p}, \tilde{\sigma}_{PC,p}, \tilde{\sigma}_{MP,p}, \tilde{\sigma}^{*+p}]$, where $p = 1, 2, 3$. Figure A.1 shows that the optimizing parameter values are in the middle of the ranges considered.

Step 2: In the second step, we minimize with respect to $\alpha$ and $b$ while all other parameters constant at their optimal values from Step 1.

Figure A.2 shows the objective function against the parameters fitted in Step 2. Figure A.2 shows that all parameters are in the interior of the intervals that we are optimizing over. This finding is reassuring in that it suggests that we are considering sufficiently large ranges.

The optimal parameters are shown in Table 5 in the main paper. These optimal parameters are close to the initial guesses, indicating convergence of our algorithm.

**C Additional Calibration Features and Robustness**

Table A.1 shows the calibration parameters with four significant digits.

Table A.2 shows the matrix of slope coefficients for the quarterly VAR(1) in the log output gap, inflation, the Federal Funds rate, and the 5 year nominal yield both in the model and in the data. Table A.2 shows that the calibrated model can generate substantial persistence in the output gap, inflation, Fed Funds rate, and the long-term nominal yield.

Figure A.3 shows a time series of smoothed shocks backed out from our sub period calibrations. For each sub period, we back out the empirical VAR(1) shocks according
to
\[ \hat{e}_t^Y = \hat{Y}_t - \hat{P}_t^Y \hat{Y}_{t-1}, \]  
(133)
and then compute model fundamental shocks according to:
\[ \hat{u}_t^Y = (Q^Y)^{-1} \hat{e}_t^Y. \]  
(134)

Table A.3 shows additional moments from the calibration. The average equity premium is 4.71% on an annualized basis.

C.1 Role of Jensen’s Inequality

As noted in the main text, bond returns in our model exhibit a Jensen’s inequality or convexity effect. This effect will increase bond prices when volatility is high, which tends to lower bond betas in our model. Since we have a shock with a unit root, this convexity effect increases sharply with maturity.

Tables A.4 and A.5 numerically investigate the magnitude of the convexity effect for the results reported in the main paper. Table A.4 shows that if we ignore the convexity effect in bonds, we obtain nominal bond betas that are slightly higher than those reported in Table 6 in the main paper. The differences are small, driving home the point that the effect of convexity on bond betas is mostly a level effect and small at the maturities considered. Without convexity, we still obtain a slightly positive bond beta (0.09) for the first period, a strongly positive bond beta for the second period (0.24), and a negative bond beta for the third period (-0.16). The lower panels of Table A.4 show similarly small effects on the standard deviation of nominal bond returns. Both total and partial derivatives look similar to the results reported in Table 8 in the main paper, illustrating that convexity is not a major determinant of the sensitivity of bond risks to individual model parameters.

Table A.5 decomposes model changes in bond risks without the convexity effect. It is analogous to Table 9 in the main paper, except that it uses the total derivatives reported in Table A.4. The numerical values in Table A.5 are almost identical to those in Table 9, illustrating that our conclusions are not driven by convexity effects in bond returns.

Figure A.4 shows the term structure of bond betas for each of the three subperiod calibrations. Nominal bond betas for periods 1 and 2 are positive for all maturities up to 40 quarters while nominal bond betas for period 3 are negative for all maturities up to 40 quarters. The convexity effect causes nominal bond betas to decrease sharply for longer maturities. However, the fact that nominal bond betas are close to linear in bond maturity for bond maturities up to 25 quarters indicates that the convexity effect does not dominate bond betas at 20 quarters, which is the maturity that we focus on.
C.2 Robustness to Phillips Curve Parameter $\rho^\pi$

In our calibration, we use a strongly backward-looking Phillips curve with a backward looking component of $\rho^\pi = 0.96$. In this section we verify that our conclusions are not dependent on the specific Phillips curve parameter. Table A.6 replicates Table 9 in the main text with a lower backward-looking component of $\rho^\pi = 0.94$ and shows that the numerical values and hence our conclusions are essentially unchanged. We cannot choose a significantly lower value for $\rho^\pi$, because then we obtain solutions that no longer satisfy the E-stability or learnability criterion.

C.3 Robustness to Different Leverage Parameter

Our baseline model assumes that the output gap and dividends are perfectly correlated, while in the data the correlation is much lower. We could follow a similar route as in Campbell and Cochrane (1999) and address this issue by modeling dividends as proportional to the output gap plus an idiosyncratic shock. In order to maintain a realistic ratio of the dividend growth volatility and output gap growth volatility, we would need to reduce the sensitivity of dividends to the output gap.

Campbell and Cochrane (1999) remark that their habit formation model has very similar implications no matter whether they model dividend growth as perfectly or imperfectly correlated with consumption growth.

We study the sensitivity of our model implications with respect to the leverage parameter $\delta$ in order to understand how important the assumption of perfectly correlated dividends and output gap is for our findings. Adding an idiosyncratic, unpriced, shock to dividends is unlikely to change substantially any asset pricing dynamics. Modeling dividends as imperfectly correlated with the output gap should therefore yield implications that are very similar to reducing the leverage parameter $\delta$.

Table A.7 is equivalent to Table 9 in the main text, but it sets $\delta = 1$, corresponding to an extremely low firm leverage ratio of 0%. We can see that the implications for nominal bond betas are qualitatively and quantitatively extremely similar to the baseline calibration.

C.4 The Role of Monetary Policy Persistence

Figures A.5 through A.7 have a closer look at how monetary policy persistence changes the impulse response functions, showing that the differential reaction of bond yields to PC shocks is an important reason why persistence matters for bond betas.

Figure A.5 shows three impulse responses. All three impulse responses correspond to the period 3 calibration, but they have different monetary policy persistence pa-
rameters. Bond yields and log dividend-price ratios commove more negatively (or less positively) when monetary policy is more persistent for the period 3 calibration. Figure A.6 decomposes the contributions of expected inflation, expected real rates, and the risk premium amplification mechanism for bond yields. This figure shows the Expectations Hypothesis component of the yield response (EH Infl+Real), the Expectations Hypothesis component due to expected inflation (EH Infl) and the Expectations Hypothesis component due to expected real short rates (EH Real). The difference between the bond yield and the Expectations Hypothesis component is the risk premium component (Yield RP). Figure A.7 similarly decomposes the impulse responses of the dividend yield into one component capturing current and expected dividends, one component capturing expected real short rates and a third component for expected excess returns (equation (71) in the appendix).

These plots tell us that the differential real short rate path after a PC shock is important for understanding why monetary policy persistence matters for the bond beta. A PC shock drives down output and drives up inflation. When monetary policy is persistent, the nominal Fed Funds rate increases much more slowly in response to a PC shock. The real rate decreases in the short-run and this means that long-term nominal yields respond less to a PC shock even in the absence of risk premia. This explains how more persistent monetary policy can dampen the effect of PC shocks on bond yields. Since PC shocks tend to generate positive bond betas and inflation target shocks tend to generate negative bond betas, more persistence means that inflation target shocks dominate and the bond beta turns negative. There is also an important risk premium channel, which further dampens the response of nominal bond yields after a PC shock and strengthens the response of bond yields after an inflation target shock when monetary policy is persistent.

Monetary policy persistence also affects the yield response to inflation target shocks. When monetary policy is less persistent, the central bank lowers real rates in response to inflation being below target, which will dampen the response of long-term nominal yields to inflation target shocks. When monetary policy is persistent, the central bank decreases real rates only with a lag, which means that nominal bond yields respond more positively to an inflation target shock even in the absence of risk premia. Risk premia further increase the response of nominal bond yields to an inflation target shock.

D Model Extensions

D.1 Shocks to Potential Output

Our model does not explicitly consider shocks to potential output. However, it is straightforward to reinterpret the model in terms of a model with shocks to the output gap. We may have to reinterpret the notion of IS or PC shocks, depending on
whether potential output enters into utility and/or dividends.

Consider a model with shocks to log potential output $x_t^*$, so that log total output $y_t$ is given by $y_t = x_t + x_t^*$. The simplest case is if potential output helps determine habit and as such does not enter into utility directly. Then the macro side of our model is unchanged. If potential output enters into dividends, it acts like an unpriced shock to dividends. Such an unpriced shock does not plausibly affect the volatility of equity returns, especially since our results are not sensitive to the leverage parameter $\lambda$ (previous section of the appendix).

### D.2 Utility with Total Output, Dividends with Output Gap

A more interesting case is when potential output enters into utility. If potential output enters into utility, but not into dividends, then shocks to potential output show up in the Euler equation, and can be reinterpreted as IS shocks.

In this subsection, we therefore assume that agents have utility over log total output. We continue to assume that dividends are proportional to the output gap $d_t = \delta x_t$. Such a specification may be reasonable if we think that long-term productivity gains accrue to labor or bond holders, but shorter term gains accrue to equity shareholders.

We assume that log marginal utility is given by:

$$\ln U_t' = -\alpha (y_t - \theta y_{t-1} - v_t).$$

We then again obtain the Euler equation as in the main paper, except that we have to substitute total output for the output gap:

$$y_t = \rho y_{t-1} + \rho^x E_t y_{t+1} - \psi (i_t - E_t \pi_{t+1}) + v_t / (1 + \theta^x).$$

Now assume that $x_t^*$ follows a deterministic trend plus an AR(1) process with autoregressive parameter $a$. We omit the deterministic trend for simplicity, since all it does is to introduce constants. Assume that $x_t^* = ax_{t-1}^* + v_t^*$. Then:

$$\rho^x x_{t-1}^* + \rho^x E_t x_{t+1}^* - x_t^* = \rho^x x_{t-1}^* - (1 - \rho^x a)x_t^*.$$  

Now, consider the special case where $a$ is such that $(\rho^x^2 + a^2 \rho^x - a) = 0$. We do not expect this to hold in general, but this illustrative case can help illuminate the
effects of fluctuations in potential output. Importantly, for any $\rho^\times \neq 0$ this implies that $a \neq \frac{1}{\rho^\times}$. We can then re-write the Euler equation as:

$$x_t = \rho^\times x_{t-1} + \rho^\times E_t x_{t+1} - \psi(i_t - E_t \pi_{t+1}) - (1 - a \rho^\times) v^*_t + v_t / (1 + \theta^*)$$ \quad (144)

The Euler equation (144) is identical to the one in the main paper, except now the shock to the Euler equation is a combination of preference shocks and shocks to potential output $u_{t}^{\text{IS}} = -v^*_t (1 - a \rho^\times) + v_t / (1 + \theta^*)$. The asset pricing equations are unchanged from the model presented in the main paper. It therefore follows that the impulse response for a negative shock to potential output is identical to that for a preference shock, appropriately scaled. Reinterpreting the standard deviation of $IS$ shocks as the standard deviation of the combined shock $-v^*_t (1 - a \rho^\times) + v_t / (1 + \theta^*)$, all results in the paper therefore continue to hold.

**D.3 Utility and Dividends in Terms of Total Output**

A second case is if both marginal utility and dividends depend on total output instead of the output gap. In this case, shocks to potential output, or total factor productivity shocks, have the potential to drive equity return volatility, as in a real business cycle model. For simplicity, we also assume that the central bank reacts to log output instead of the log output gap, which is plausible if potential output is hard to observe.

We now assume that dividends are proportional to total log output $d_t = \delta y_t$ and that agents have utility over log total output. The algebra in this section shows that we can obtain a model of exactly the same form as the one presented in the main paper. The only difference is that this extended model suggests that we should use output deviations from a deterministic trend instead of the output gap. The potential output measure from the CBO is well described by a quadratic function and the CBO uses extensive cycle-trend decompositions to construct potential output. It is therefore plausible to interpret the difference between log real GDP and log potential GDP from the CBO as deviations from a deterministic trend. The Phillips Curve shock needs to be reinterpreted as a combination of cost-push shocks and shocks to potential output. In this extended model, the Phillips Curve shock therefore captures all supply shocks, which is intuitive. We have also seen in Table 8 in the main paper, that these combined supply shocks are important determinants of equity return volatility.

We obtain the Euler equation:

$$y_t = \rho^\times y_{t-1} + \rho^\times E_t y_{t+1} - \psi(i_t - E_t \pi_{t+1}) + u^t_{IS}$$ \quad (145)

In this section, we assume that $x^*_t$ follows a deterministic trend plus iid noise for tractability. We omit the deterministic (quadratic) trend for simplicity, since all it
does is to introduce constants. Assume that $x_t^* = v_t^*$. The Philips Curve can be written in terms of $y_t$ as:

$$\pi_t = \rho^\pi \pi_{t-1} + (1 - \rho^\pi) E_{t-1} \pi_{t+1} + \lambda y_t - \lambda v_t^* + u_t^{PC} \quad (147)$$

We complete the model with a Taylor rule in terms of log output and deviations of inflation from target:

$$i_t = \rho^i (i_{t-1} - \pi_t^*) + (1 - \rho^i) \left[ \gamma^s y_t + \gamma^\pi (\pi_t - \pi_t^*) \right] + \pi_t^* + u_t^{MP}. \quad (148)$$

With dividends of the form

$$d_t = \delta y_t, \quad (149)$$

the macroeconomic dynamics and solution for equity returns is identical to the ones obtained for model obtained in the main paper. If we re-interpret the difference between log output and the CBO log potential output measure as deviations from a deterministic trend, and PC shocks as a combination of cost push shocks and shocks to potential output, all results in the paper continue to hold.

### D.4 Extended Model with Unit Root in Consumption and Dividends

In Campbell and Cochrane (1999), $s_t + c_t$ is not stationary. In contrast, our specification of marginal utility in equation (8) in the main text specifies a stationary process for marginal utility. This section shows that all the key features of the model are preserved when we add an independent random walk component to marginal utility. Consider the modified model:

$$s_t + c_t = x_t - \theta x_{t-1} - v_t - w_t, \quad (150)$$

$$\sigma_t^2 = \bar{\sigma} (1 - bx_t), \quad (151)$$

$$w_t = w_{t-1} + u_t^w, \quad (152)$$

$$Var(u_t^w) = (\sigma^w)^2 \quad (153)$$

Substituting (151) into the Euler equation (equation (7) in the main text), we get:

$$-\alpha(x_t - \theta x_{t-1} - v_t - w_t) = (i_t - E_t \pi_{t+1}) - \alpha E_t (x_{t+1} - \theta x_t - v_{t+1} - w_{t+1}) + \frac{\alpha^2}{2} \sigma_t^2 \quad (154)$$

With $w_t = E_t w_{t+1}$, the random walk term drops out of the Euler equation and we obtain the same linearized Euler equation as before:

$$x_t = \rho^x x_{t-1} + \rho^x E_t x_{t+1} - \psi (i_t - E_t \pi_{t+1}) + u_t^{IS}, \quad (155)$$

$$u_t^{IS} = \frac{v_t}{1 + \theta^x}, \quad (156)$$
D.4.1 SDF

We can solve for the conditional volatility of the SDF as before.

\[ s_{t+1} + c_{t+1} - E_t(s_{t+1} + c_{t+1}) = Q^M u_{t+1} - u^w_{t+1}, \quad (157) \]
\[ Q^M = e_1 Q - (1 + \theta^*) e_1. \quad (158) \]

From our conditional heteroskedasticity assumption for \( u_{t+1} \) it then follows that:

\[ \sigma_t^2 = Q^M \sum_u Q^M (1 - bx_t) + (\sigma^w)^2. \quad (159) \]

Hence, up to the constant term \((\sigma^w)^2\), we get the same expression as before. Importantly, shocks to trend marginal utility, or trend consumption, do not affect inflation.

D.4.2 Bond and Stock Returns

Since \( u^w_t \) shocks do not enter into real or nominal interest rates, the solution for bond returns is exactly the same as before.

The solution for stock returns depends on whether we allow \( w_t \) to enter into dividends or not. If we continue to assume that dividends, or de-trended dividends, are proportional to the output gap, the solution is unchanged from the basic model.

The more interesting case is when dividends are cointegrated with consumption, which is what we assume from now on in this section:

\[ d_t = \delta x_t - \delta w_t. \quad (160) \]

We write \( r^e_{t+1} \) for the log stock return and \( xr^e_{t+1} \) for the log stock return in excess of \( r_t \). Following Campbell (1991) we decompose stock returns into dividend news, news about real interest rates, and news about future excess stock returns ignoring constants:

\[ r^e_{t+1} - E_t r^e_{t+1} = \left( E_{t+1} - E_t \right) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+j} \]
\[ - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j x_{r^e_{t+1+j}} \quad (161) \]
\[ = -\delta u^w_{t+1} + \delta (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta x_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+j} \]
\[ - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j x_{r^e_{t+1+j}} \quad (162) \]
\( \rho \) is a loglinearization constant close to 1. Now guess the functional form:

\[
E_t x r_{t+1}^e = (1 - b x_t) b^e + c^e.
\]  

(163)

Then:

\[
\nu^e_{t+1} - E_t \nu^e_{t+1} = (\kappa A^x + A^r) u_{t+1} - \delta u_{t+1}^w .
\]  

(164)

where

\[
A^x = e_1 [I - \rho P]^{-1} Q ,
\]  

(165)

\[
A^r = -\rho (e_3 - e_2 P) [I - \rho P]^{-1} Q ,
\]  

(166)

\[
\kappa = \delta (1 - \rho) + \rho \times b \times b^e .
\]  

(167)

We also write:

\[
A^e = (\kappa A^x + A^r) .
\]  

(168)

\( \kappa A^e \) captures the stock returns’ exposure to long-term news about the output gap. \( A^r \) captures the exposure of stock returns to real interest rate news.

The conditional equity premium adjusted for Jensen’s inequality equals the conditional covariance of excess stock returns and marginal utility:

\[
E_t x r_{t+1}^e + \frac{1}{2} V a r_t (x r_{t+1}^e) = \alpha C o v_t (r_{t+1}^e, s_{t+1} + c_{t+1}) = \alpha A^e \Sigma_u Q^M (1 - b x_t) + \alpha \delta (\sigma^w)^2
\]  

(169)

Therefore

\[
E_t x r_{t+1}^e = \alpha A^e \Sigma_u Q^M (1 - b x_t) + \left( \alpha - \frac{1}{2} \right) (\sigma^w)^2 - \frac{1}{2} A^e \Sigma_u A^e (1 - b x_t)
\]  

(170)

\[
b^e = \alpha A^e \Sigma_u Q^M - \frac{1}{2} A^e \Sigma_u A^e
\]  

(172)

\[
= \alpha A^r \Sigma_u Q^M + \alpha \kappa A^x \Sigma_u Q^M - \frac{1}{2} A^r \Sigma_u A^r
\]  

(173)

\[
- \kappa A^x \Sigma_u A^r - \frac{1}{2} \kappa^2 A^x \Sigma_u A^x
\]  

(174)

The average conditional equity premium is then given by:

\[
E \left( E_t x r_{t+1}^e + \frac{1}{2} V a r_t (x r_{t+1}^e) \right) = \alpha A^e \Sigma_u Q^M + \alpha \delta (\sigma^w)^2 .
\]  

(175)

It then follows that expected stock returns indeed take the hypothesized form, where \( \kappa \) is the positive root of the quadratic equation:

\[
0 = \kappa^2 + \kappa \times 2 \left( \rho b \right)^{-1} - \alpha A^x \Sigma_u Q^M + A^x \Sigma A^r
\]  

\[
+ \frac{-2 \delta (1 - \rho) (\rho b)^{-1} + A^r \Sigma_u A^r - 2 \alpha A^r \Sigma_u Q^M}{A^x \Sigma_u A^x}
\]  

(176)
D.4.3 Bond-Stock Covariances

The conditional nominal and real bond-stock return covariances is exactly the same as before:

\[
\text{Cov}(r_{t+1}^e, r_{n-1,t+1}^s) = A^s u A^e (1 - bx_t) \quad (177)
\]

\[
\text{Cov}(r_{t+1}^e, r_{n-1,t+1}) = A^n u A^e (1 - bx_t) \quad (178)
\]

The variances of equity excess returns, nominal and real bond excess returns are:

\[
\text{Var}(r_{t+1}^e) = A^e u A^e (1 - bx_t) + \delta^2 (\sigma^w)^2, \quad (179)
\]

\[
\text{Var}(r_{n-1,t+1}^s) = A^s u A^s (1 - bx_t), \quad (180)
\]

\[
\text{Var}(r_{n-1,t+1}) = A^n u A^n (1 - bx_t). \quad (181)
\]

The conditional stock market betas of nominal and real bonds are given by:

\[
\beta_t(r_{n-1,t+1}^s) = \frac{A^s u A^s (1 - bx_t)}{A^e u A^e (1 - bx_t) + \delta^2 (\sigma^w)^2}, \quad (182)
\]

\[
\beta_t(r_{n-1,t+1}) = \frac{A^n u A^n (1 - bx_t)}{A^e u A^e (1 - bx_t) + \delta^2 (\sigma^w)^2}. \quad (183)
\]

D.4.4 Numerical Significance of Persistent Component

In order to understand whether shocks to the random walk component \(w_t\) are likely to be numerically significant, we consider an annualized volatility of 1%, corresponding to a quarterly standard deviation of 0.50%. Given that the standard deviation of annualized consumption growth in U.S. data is 1.1% (Campbell 2003), this numerical value provides a plausible upper bound for the volatility of permanent shocks to consumption.

For a calibration with \(\alpha = 19.06\) and \(\delta = 2.43\) and \(\sigma^w = 1\%\) in annualized units (\(\sigma^w = 0.005\) in natural quarterly units), this would increase the equity premium by 0.46% in annualized percent units. With this additional term, this would raise the equity premium for the new calibration to a plausible value of 5.15% in annualized units.

The additional term in the volatility of equity returns (179) with \(\delta = 2.43\) and \(\sigma^w = 1\%\) annualized increases the volatility of equity returns. However, at plausible values for the volatility of equity returns, the effect is small. For instance, adding a term \(\delta^2 (\sigma^w)^2\) would increase the volatility of equity returns from 20% to 20.14%.
E Additional Empirical Results

Table A.8 reports Taylor rule estimates for superiod 3 (1997.Q-2011.Q4) and splits it into a pre-Lehman subsample (1997.Q1-2008.Q2) and a post-Lehman subsample (2008.Q3-2011.Q4). Interestingly, the estimates for the pre-Lehman subsample are virtually identical to the estimates for the full subperiod 3. The estimates reported in Table 4 in the main paper therefore do not appear to be mainly driven by the financial crisis, just as the negative nominal bond beta does not appear mainly driven by the financial crisis. For the post-Lehman subsample, the estimates for all monetary policy coefficients ($\gamma^x$, $\gamma^\pi$, and $\rho^i$) are all close to zero with small standard errors. This is intuitive, since the most salient feature of monetary policy during the crisis was perhaps that the Federal Funds rate has been stuck at the zero lower bound. The empirical results in Table A.8 suggest that we can model post-Lehman monetary policy by setting all monetary policy parameters to zero.

Table A.9 tests for statistical significance of the changes in monetary policy parameters. It shows that the major changes (increase in $\gamma^\pi$ from period 1 to period 2; increase in $\rho^i$ from period 2 to period 3) are indeed statistically significant.

If changes in bond risks are driven by macroeconomic factors, then changes in bond risks should be reflected in changing macroeconomic correlations. Lower than expected inflation raises nominal bond prices, all else equal, so the inflation-output correlation should typically take the opposite sign from the bond-stock correlation.

Table A.10 compares sub-sample correlations of asset prices and macroeconomic variables. The empirical output gap is highly persistent and it is therefore unsurprising that three year equity excess returns are more strongly correlated with the output gap than highly volatile quarterly stock returns. We therefore use quarterly overlapping three year bond and stock excess returns for our comparison of asset return correlations and macroeconomic correlations. Table A.9 confirms our intuition that bond excess returns should at least partly reflect news about inflation and that equity excess returns should reflect the business cycle. In each sub period, empirical bond excess returns are negatively correlation with inflation and equity excess returns are positively correlated with the output gap.

Table A.10 confirms that the changes in the bond-stock comovement documented in Figure 1 and in Table 6 are robust to using three year returns instead of daily or quarterly returns. The correlation between three year stock returns and three year bond returns was positive and significant in the first sub-period, increased in the second sub period, and became negative and significant in the last sub period.

The bond-output gap, inflation-stock, and inflation-output gap correlations confirm our intuition that changing bond risks are related to the prevalence of inflationary recessions versus deflationary recessions during different regimes. The bond-output gap correlation typically has the same sign as the bond-stock correlation, while the
inflation-stock return correlation and the inflation-output correlation has the opposite sign. The only exception to this pattern is the first sub period bond-output gap correlation, which takes a negative, but small and insignificant, value.
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Tables and Figures

Table A.1: Parameter Choices with Four Significant Digits

Panel A: Calibration Parameters

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<th>Time-Invariant Parameters</th>
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<td>$\rho$</td>
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<td>$\delta$</td>
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<td>$\alpha$</td>
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<td>$b$</td>
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<td>$\theta$</td>
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<td>$\rho^\pi$</td>
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<td>$\lambda$</td>
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<tr>
<td>$\gamma^x$</td>
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Calibrated Std. Shocks

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Panel B: Implied Parameters

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<td>Coeff. on Lagged Variables</td>
<td>Coeff. on Lagged Variables</td>
<td>Coeff. on Lagged Variables</td>
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<tr>
<td>Output Gap</td>
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<td>Inflation Gap</td>
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<td>Fed Funds Rate</td>
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<td>Log Nom. Yield</td>
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<td>0.00</td>
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*P* is the matrix of slope coefficients of a quarterly VAR(1) in the log output gap, inflation, Fed Funds rate, and 5 Year Nominal Yield.
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<thead>
<tr>
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<td>79.Q3</td>
<td>97.Q1</td>
<td>Sub-Period Avg.</td>
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<td>4.16</td>
<td>4.34</td>
<td>4.71</td>
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<td>8.12</td>
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<td>5.36</td>
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<td>0.01</td>
<td>2.31</td>
<td>2.97</td>
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<td>0.80</td>
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<td>Corr(x, y5)</td>
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<td>-0.32</td>
<td>0.19</td>
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<td>-0.34</td>
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<td>-0.07</td>
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<td>Corr(x,i)</td>
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<td>-0.38</td>
<td>-0.09</td>
<td>-0.20</td>
<td>0.22</td>
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<td>Slope xروم on y5-i</td>
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<td>Slope xروم on dp</td>
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</table>

The equity premium and the average nominal bond excess return show average returns in excess of a short-term bond adjusted for Jensen’s inequality. The last three rows show regression coefficients of log 5 year bond excess returns (Annualized, %) onto the slope of the yield curve (Annualized, %), the output gap (%), and the log dividend price ratio (%), respectively. The last three rows show * when the coefficient is significant at the 5% level with Newey-West standard errors with two lags.
<table>
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<th>Total Derivative</th>
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<td>MP Coefficient Output</td>
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</tr>
<tr>
<td>MP Coefficient Inflation</td>
<td>$\gamma^\pi$</td>
<td>0.96</td>
<td>0.58</td>
<td>0.53</td>
<td>$\rho^i$</td>
<td>-0.06</td>
</tr>
<tr>
<td>MP Persistence</td>
<td>$\rho^i$</td>
<td>-0.06</td>
<td>-0.15</td>
<td>-2.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IS Shock Std.</td>
<td>$\sigma^{IS}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>$\sigma^{IS}$</td>
<td>0.00</td>
</tr>
<tr>
<td>PC Shock Std.</td>
<td>$\sigma^{PC}$</td>
<td>0.92</td>
<td>0.62</td>
<td>0.73</td>
<td>$\sigma^{PC}$</td>
<td>0.23</td>
</tr>
<tr>
<td>MP Shock Std.</td>
<td>$\sigma^{MP}$</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>$\sigma^{MP}$</td>
<td>0.01</td>
</tr>
<tr>
<td>Infl. Target Shock Std.</td>
<td>$\sigma^*$</td>
<td>-0.13</td>
<td>0.25</td>
<td>0.26</td>
<td>$\sigma^*$</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

| Model Std. Nominal Bond Ret. | 3.62            | 5.66                    | 5.49                     | 3.76              | 6.03                    | 5.25                     |
| Std. Nominal Bond Ret. w/o Convexity | 3.76          | 6.03                    | 5.25                     |                   |                         |                          |
| MP Coefficient Output    | $\gamma^x$      | -10.82                  | -7.92                    | 10.17             | $\gamma^x$              | 9.48                    | 5.79                    | -7.12                   |
| MP Coefficient Inflation | $\gamma^\pi$    | 9.48                    | 5.79                     | -7.12             | $\rho^i$                | 0.51                    | -1.62                   | 34.71                   |
| MP Persistence           | $\rho^i$         | 0.51                    | -1.62                    | 34.71             |                         |                         |                         |                         |
| IS Shock Std.            | $\sigma^{IS}$   | 0.00                    | 0.00                     | 0.01              | $\sigma^{IS}$           | 0.00                    | 0.00                    | 0.01                    |
| PC Shock Std.            | $\sigma^{PC}$   | 8.78                    | 7.16                     | -7.74             | $\sigma^{PC}$           | 1.72                    | 5.15                    | 0.44                    |
| MP Shock Std.            | $\sigma^{MP}$   | 0.63                    | 0.43                     | 0.25              | $\sigma^{MP}$           | 0.63                    | 0.43                    | 0.25                    |
| Infl. Target Shock Std.  | $\sigma^*$       | 0.73                    | -2.54                    | -5.13             | $\sigma^*$              | 1.42                    | 0.44                    | 4.55                    |

This table shows model bond betas and standard deviation of bond returns for each subperiod calibration computed without the Jensen’s inequality (or convexity) effect. Total and partial derivatives are computed analogously to Table 8 in the main text, but again exclude the convexity effect.
Table A.5: Decomposing Changes in Bond Risks Excluding Convexity Effect

<table>
<thead>
<tr>
<th>Change Date</th>
<th>Nominal Bond Beta</th>
<th>Std. Nom. Bond Returns</th>
<th>Std. Equity Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>79.Q3</td>
<td>97.Q1</td>
<td>79.Q3</td>
</tr>
<tr>
<td>Empirical Change</td>
<td>0.13</td>
<td>-0.36</td>
<td>4.26</td>
</tr>
<tr>
<td>Model Change</td>
<td>0.15</td>
<td>-0.40</td>
<td>2.26</td>
</tr>
</tbody>
</table>

Total Derivative x Parameter Change

|                          | $\gamma^x$ | 0.47                  | -0.41               | 4.61                | 0.22                | -6.50               | 9.35                |
|                          | $\gamma^\pi$ | 0.59                  | 0.27                | 5.83                | -0.08               | -7.05               | -5.63               |
|                          | $\rho^i$     | 0.01                  | -0.62               | 0.07                | 7.05                | 0.02                | 2.74                |
|                          | $\sigma^{MP}$ | 0.00                  | -0.02               | 0.17                | -0.61               | -0.05               | 0.11                |
|                          | $\sigma^*$   | 0.04                  | 0.02                | -0.62               | -0.34               | -13.42              | -3.15               |
|                          | $\sigma^{IS}$ | 0.00                  | 0.00                | 0.00                | 0.00                | 0.00                | 0.00                |
|                          | $\sigma^{PC}$ | -0.67                 | 0.06                | -6.90               | 0.03                | 14.56               | -1.59               |

Combined Effects

|                          | MP Sub-Total   | 1.11                  | -0.76               | 10.05               | 6.24                | -27.00              | 3.41                |
|                          | Shocks Sub-Total | -0.67                 | 0.06                | -6.90               | 0.03                | 14.56               | -1.59               |
|                          | Total Linear Changes | 0.44                 | -0.70               | 3.16                | 6.27                | -12.44              | 1.81                |
|                          | Nonlinearity Effect | -0.29                | 0.29                | -0.89               | -7.04               | 10.20               | -1.57               |

This table is constructed analogously to Table 9 in the main text, but it computes bond betas and standard deviations of bond returns without the Jensen’s inequality (or convexity) effect. This table uses total derivatives reported in Table A.4.
## Table A.6: Decomposing Changes in Bond Risks - Alternative Phillips Curve Parameter $\rho^\pi = 0.94$

<table>
<thead>
<tr>
<th>Change Date</th>
<th>Nominal Bond Beta</th>
<th>Std. Nom. Bond Returns</th>
<th>Std. Equity Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>79.Q3</td>
<td>97.Q1</td>
<td>79.Q3</td>
</tr>
<tr>
<td>Empirical Change</td>
<td>0.13</td>
<td>-0.36</td>
<td>4.26</td>
</tr>
<tr>
<td>Model Change</td>
<td>0.12</td>
<td>-0.41</td>
<td>2.06</td>
</tr>
</tbody>
</table>

**Total Derivative x Parameter Change**

<table>
<thead>
<tr>
<th></th>
<th>$\gamma^z$</th>
<th>$\gamma^\pi$</th>
<th>$\rho^i$</th>
<th>$\sigma^{MP}$</th>
<th>$\sigma^*$</th>
<th>$\sigma^{IS}$</th>
<th>$\sigma^{PC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP Coefficient Output</td>
<td>0.44</td>
<td>-0.39</td>
<td>4.50</td>
<td>0.92</td>
<td>-5.71</td>
<td>7.89</td>
<td></td>
</tr>
<tr>
<td>MP Coefficient Inflation</td>
<td>0.54</td>
<td>0.24</td>
<td>5.50</td>
<td>-0.55</td>
<td>-5.95</td>
<td>-4.45</td>
<td></td>
</tr>
<tr>
<td>MP Persistence</td>
<td>0.02</td>
<td>-0.64</td>
<td>0.08</td>
<td>9.25</td>
<td>0.02</td>
<td>2.05</td>
<td></td>
</tr>
<tr>
<td>MP Shock Std.</td>
<td>0.00</td>
<td>0.00</td>
<td>0.13</td>
<td>-0.49</td>
<td>-0.04</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Infl. Target Shock Std.</td>
<td>0.00</td>
<td>0.02</td>
<td>-0.32</td>
<td>-0.37</td>
<td>-12.12</td>
<td>-2.87</td>
<td></td>
</tr>
<tr>
<td>IS Shock Std.</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>PC Shock Std.</td>
<td>-0.63</td>
<td>0.06</td>
<td>-6.68</td>
<td>-0.15</td>
<td>14.27</td>
<td>-1.52</td>
<td></td>
</tr>
</tbody>
</table>

**Combined Effects**

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>MP Sub-Total</td>
<td>1.00</td>
<td>-0.78</td>
<td>9.88</td>
<td>8.77</td>
<td>-23.79</td>
<td>2.74</td>
</tr>
<tr>
<td>Shocks Sub-Total</td>
<td>-0.63</td>
<td>0.06</td>
<td>-6.68</td>
<td>-0.14</td>
<td>14.27</td>
<td>-1.52</td>
</tr>
<tr>
<td>Total Linear Changes</td>
<td>0.37</td>
<td>-0.72</td>
<td>3.20</td>
<td>8.62</td>
<td>-9.52</td>
<td>1.23</td>
</tr>
<tr>
<td>Nonlinearity Effect</td>
<td>-0.25</td>
<td>0.31</td>
<td>-1.14</td>
<td>-8.30</td>
<td>8.51</td>
<td>-1.22</td>
</tr>
</tbody>
</table>

This table is constructed analogously to Table 9 in the main text, but it uses a smaller backward-looking component in the Phillips curve of $\rho^\pi = 0.94$. All other parameters are as in Table 5 in the main text.
Table A.7: Decomposing Changes in Bond Risks - Alternative Leverage Parameter $\delta = 1$

<table>
<thead>
<tr>
<th>Change Date</th>
<th>Nominal Bond Beta</th>
<th>Std. Nom. Bond Returns</th>
<th>Std. Equity Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>79.Q3</td>
<td>97.Q1</td>
<td>79.Q3</td>
</tr>
<tr>
<td>Empirical Change</td>
<td>0.13</td>
<td>-0.36</td>
<td>4.26</td>
</tr>
<tr>
<td>Model Change</td>
<td>0.15</td>
<td>-0.41</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Total Derivative x Parameter Change

<table>
<thead>
<tr>
<th></th>
<th>$\gamma^x$</th>
<th>$\gamma^r$</th>
<th>$\rho^i$</th>
<th>$\sigma^{MP}$</th>
<th>$\sigma^*$</th>
<th>$\sigma^{IS}$</th>
<th>$\sigma^{PC}$</th>
<th>MP Sub-Total</th>
<th>Shocks Sub-Total</th>
<th>Total Linear Changes</th>
<th>Nonlinearity Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP Coefficient Output</td>
<td>0.48</td>
<td>-0.43</td>
<td>3.98</td>
<td>0.78</td>
<td>-6.39</td>
<td>9.27</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MP Coefficient Inflation</td>
<td>0.60</td>
<td>0.28</td>
<td>5.00</td>
<td>-0.45</td>
<td>-6.92</td>
<td>-5.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MP Persistence</td>
<td>0.02</td>
<td>-0.70</td>
<td>0.06</td>
<td>8.21</td>
<td>0.03</td>
<td>2.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MP Shock Std.</td>
<td>0.00</td>
<td>-0.01</td>
<td>0.13</td>
<td>-0.52</td>
<td>-0.06</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Infl. Target Shock Std.</td>
<td>0.05</td>
<td>0.03</td>
<td>-0.13</td>
<td>-0.33</td>
<td>-13.65</td>
<td>-3.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IS Shock Std.</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PC Shock Std.</td>
<td>-0.69</td>
<td>0.07</td>
<td>-6.05</td>
<td>-0.06</td>
<td>16.09</td>
<td>-1.61</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Combined Effects

|                      | 1.14             | -0.83           | 9.04       | 7.68          | -26.99     | 3.24          |               |              |                   |                     |                     |

This table is constructed analogously to Table 9 in the main text, but it uses a significantly smaller leverage parameter $\delta = 1$, corresponding to 0% firm leverage. All other parameters are as in Table 5 in the main text.
This table estimates the monetary policy rule before and after the Lehman brothers bankruptcy in 2008.Q3. All variables and test specifications are described in Table 4 in the main text.
Table A.9: Estimating Changes in the Monetary Policy Rule

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fed Funds</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_t$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy Period $\mathcal{T}$</td>
<td>79.Q3-96.Q4</td>
<td>97.Q1-11.Q4</td>
<td>08.Q3-11.Q4</td>
</tr>
<tr>
<td><strong>Output Gap</strong> $x_t$</td>
<td>0.18**</td>
<td>-0.04</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.13)</td>
<td>(0.13)</td>
</tr>
<tr>
<td><strong>Inflation</strong> $\pi_t$</td>
<td>0.30**</td>
<td>0.83**</td>
<td>0.27**</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.21)</td>
<td>(0.09)</td>
</tr>
<tr>
<td><strong>Lagged Fed Funds</strong> $i_{t-1}$</td>
<td>0.56**</td>
<td>0.43*</td>
<td>0.88**</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.17)</td>
<td>(0.09)</td>
</tr>
<tr>
<td><strong>Output Gap</strong> $x_t$ $I_{t \in \mathcal{T}}$</td>
<td>-0.22</td>
<td>0.09</td>
<td>-0.19</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.14)</td>
<td>(0.13)</td>
</tr>
<tr>
<td><strong>Inflation</strong> $\pi_t$ $I_{t \in \mathcal{T}}$</td>
<td>0.53*</td>
<td>-0.62**</td>
<td>-0.28**</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.22)</td>
<td>(0.09)</td>
</tr>
<tr>
<td><strong>Lagged Fed Funds</strong> $i_{t-1}$ $I_{t \in \mathcal{T}}$</td>
<td>-0.14</td>
<td>0.47*</td>
<td>-0.85**</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.18)</td>
<td>(0.10)</td>
</tr>
<tr>
<td><strong>Dummy</strong> $I_{t \in \mathcal{T}}$</td>
<td>0.84</td>
<td>-1.87</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>(1.00)</td>
<td>(0.96)</td>
<td>(0.40)</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>0.91*</td>
<td>1.75</td>
<td>-0.27</td>
</tr>
<tr>
<td></td>
<td>(0.38)</td>
<td>(0.92)</td>
<td>(0.27)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.77</td>
<td>0.85</td>
<td>0.93</td>
</tr>
<tr>
<td><strong>Implied</strong> $\Delta \hat{\gamma}^x$</td>
<td>-0.49</td>
<td>0.52</td>
<td>-1.45</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.30)</td>
<td>(0.41)</td>
</tr>
<tr>
<td><strong>Implied</strong> $\Delta \hat{\gamma}^\pi$</td>
<td>0.75*</td>
<td>0.47</td>
<td>-2.30</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(1.28)</td>
<td>(1.59)</td>
</tr>
<tr>
<td><strong>Implied</strong> $\Delta \hat{p}^i$</td>
<td>-0.14</td>
<td>0.47*</td>
<td>-0.85</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.18)</td>
<td>(0.10)</td>
</tr>
</tbody>
</table>

Variables and tests are described in Table 4 in the main text. We estimate $i_t = \alpha_0 + \alpha^x x_t + \alpha^\pi \pi_t + \alpha^i i_{t-1} + d^0 I_{t \in \mathcal{T}} + d^x x_t I_{t \in \mathcal{T}} + d^\pi \pi_t I_{t \in \mathcal{T}} + d^i i_{t-1} I_{t \in \mathcal{T}} + \epsilon_t$. Changes in monetary policy parameters, such as $\Delta \hat{\gamma}^x$, show estimated changes from the pre-$\mathcal{T}$ sub-sample to the $\mathcal{T}$ sub-sample. Standard errors for $\Delta \hat{\gamma}^x$ and $\Delta \hat{\gamma}^\pi$ are calculated by the delta method. Significance levels for changes in monetary policy parameters are based on a likelihood ratio test.
Table A.10: Estimating the Monetary Policy Function - Alternative Federal Funds Rate

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Gap $x_t$</td>
<td>0.07*</td>
<td>0.18**</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.05)</td>
<td>(0.10)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Inflation $\pi_t$</td>
<td>0.17</td>
<td>0.33**</td>
<td>0.61**</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.16)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>Lagged Fed Funds $i_{t-1}$</td>
<td>0.85**</td>
<td>0.58**</td>
<td>0.56**</td>
<td>0.89**</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.11)</td>
<td>(0.15)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.31</td>
<td>0.79*</td>
<td>1.39</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.31)</td>
<td>(0.74)</td>
<td>(0.32)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.86</td>
<td>0.84</td>
<td>0.77</td>
<td>0.93</td>
</tr>
<tr>
<td>Implied $\hat{\gamma}^x$</td>
<td>0.50*</td>
<td>0.42**</td>
<td>0.02</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.11)</td>
<td>(0.23)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>Implied $\hat{\gamma}^\pi$</td>
<td>1.13**</td>
<td>0.78**</td>
<td>1.39**</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>(0.39)</td>
<td>(0.12)</td>
<td>(0.20)</td>
<td>(1.13)</td>
</tr>
<tr>
<td>Implied $\hat{\rho}^i$</td>
<td>0.85**</td>
<td>0.58**</td>
<td>0.56**</td>
<td>0.89**</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.11)</td>
<td>(0.15)</td>
<td>(0.06)</td>
</tr>
</tbody>
</table>

This table is identical to Table 4 in the main text, but it uses a different measure of the Federal Funds rate, which is computed as the end-of-quarter Federal Funds rate excluding the last three days of the year. In contrast, the Federal Funds rate in Table 4 is the end-of-quarter Federal Funds rate including the last three days of the year.
Table A.11: Sub-Period Correlations of Bond Returns, Stock Returns, Output Gap, and Inflation

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Bond Excess Returns</th>
<th>Stock Excess Returns</th>
<th>Output Gap</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.Q1-79.Q2</td>
<td></td>
<td></td>
<td></td>
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Quarterly overlapping 3 year log equity returns in excess of log three month T-bill, 3 year log excess return on 5 year nominal bond in excess of three month log T-bill. Quarterly inflation and output as in Table 1. We report correlations of log excess returns from time $t - 12$ to $t$ and macroeconomic variables as of quarter $t$. * and ** denote significance at the 5% and 1% level. Significance levels not adjusted for time series dependence.
We minimize the objection function with respect to $\theta$, $\lambda$, $\rho^\pi$ and the volatilities of shocks while holding constant the preference parameter $\alpha = 20$ and the heteroskedasticity parameter $b = 0.7$. This figure shows the objective function $Obj$ against random draws for $\theta$, $\lambda$, $\rho^\pi$, and $[\hat{\sigma}^{IS,p}, \hat{\sigma}^{PC,p}, \hat{\sigma}^{MP,p}, \hat{\sigma}^{*p}]$, where $p = 1, 2, 3$. The volatilities of shocks are then given by $[\sigma^{IS,p}, \sigma^{PC,p}, \sigma^{MP,p}, \sigma^{*p}] = [\hat{\sigma}^{IS,p}, \hat{\sigma}^{PC,p}, \hat{\sigma}^{MP,p}, \hat{\sigma}^{*p}] / \sqrt{a}$, where the scaling factor $a$ is determined to be consistent with the volatility of the SDF $\hat{\sigma}^p$. The minimizing parameter values are indicated by circles.
This panel shows the objective function as a function of the standard deviation of $s_t + c_t$, $\sigma^p$, $p = 1, 2, 3$. 
This panel shows the objective function as a function of random draws of $[\tilde{\sigma}_{IS,p}, \tilde{\sigma}_{PC,p}, \tilde{\sigma}_{MP,p}, \tilde{\sigma}^{*p}]$, where $p = 1$. 
This panel shows the objective function as a function of random draws of $[\tilde{\sigma}^{IS,p}, \tilde{\sigma}^{PC,p}, \tilde{\sigma}^{MP,p}, \tilde{\sigma}^{*p}]$, where $p = 2$. 
This panel shows the objective function as a function of random draws of \([\tilde{\sigma}^{IS,p}, \tilde{\sigma}^{PC,p}, \tilde{\sigma}^{MP,p}, \tilde{\sigma}^{*,p}]\), where \(p = 3\).
We minimize the objection function with respect to $\alpha$ and $b$ while holding all other parameters at their optimal values from the minimization step 1. We draw parameter values randomly and independently according to uniform distributions.
Figure A.3: Time Series of Model Shocks

This figure plots the time series of smoothed IS, PC, MP and inflation target ($\pi^*$) shocks. IS shocks are in natural percent units, while PC, MP and inflation target shocks are in annualized percent units. The shocks are smoothed with a trailing exponentially-weighted moving average. The decay parameter equals 0.08 per quarter corresponding to a half life of 24 quarters.
Figure A.4: Term Structure of Model Bond Betas

Model Nominal Bond Beta and Bond Maturity

- Beta vs. Bond Maturity (Quarters)

Legend:
Figure A.5: Impulse Response Functions - Blue=Low Persistence, Green=Medium Persistence, Red=High Persistence

This figure shows impulses for the output gap, inflation, the nominal and real Federal Funds rates, the 5 year nominal yield, and the log dividend price ratio following one standard deviation shocks. We show impulse responses for the subperiod 1997.Q1-2011.Q4 calibration. We vary the monetary policy persistence parameter $\rho_i$ from 0.69 (blue line) to 0.79 (green dash line) to 0.89 (red dash-dot line, calibration 3 value). This figure shows impulse responses to the same size shocks for all three subperiods. The shock size equals the average subsample standard deviation, where the average is weighted by sample length. The output gap and the dividend price ratios are in percent deviations from the steady state. All other variables are in annualized percent units.
This figure shows impulses for five year nominal yields following one standard deviation shocks. This figure shows the five year nominal yield, the yield component due to future expected inflation (EH infl), the yield component due to future expected real rates (EH Real), the yield response due to both expected inflation and real rates (EH Infl+Real), and the difference between the five year yield and EH Infl+Real (Yield RP). We show impulse responses for the subperiod 1997.Q1-2011.Q4 calibration. We vary the monetary policy persistence parameter $\rho$ from 0.69 (blue line) to 0.79 (green dash line) to 0.89 (red dash-dot line, calibration 3 value). This figure shows impulse responses to the same size shocks for all three subperiods. The shock size equals the average subsample standard deviation, where the average is weighted by sample length. The output gap and the dividend price ratios are in percent deviations from the steady state. All other variables are in annualized percent units.
This figure shows impulses for log dividend yields following one standard deviation shocks. This figure shows the log dividend yield, as in Appendix equation (72). It separately shows the components due to expected dividends (first term in (72)), expected real short rates (third term in (72)) and risk premia (second term in (72)). We show impulse responses for the subperiod 1997.Q1-2011.Q4 calibration. We vary the monetary policy persistence parameter \( \rho^i \) from 0.69 (blue line) to 0.79 (green dash line) to 0.89 (red dash-dot line, calibration 3 value). This figure shows impulse responses to the same size shocks for all three subperiods. The shock size equals the average subsample standard deviation, where the average is weighted by sample length. The output gap and the dividend price ratios are in percent deviations from the steady state. All other variables are in annualized percent units.