COMPLEXITY, NETWORKS AND KNOWLEDGE FLOW*

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ABSTRACT

Because knowledge plays an important role in the creation of wealth, economic actors often wish to skew the flow of knowledge in their favor. Managers seek to spread knowledge widely within their organization but prevent its diffusion to rivals. Regional planners promote knowledge diffusion within a local economy but not beyond it. We ask, when will knowledge developed in one area of dense social connections – such as a firm, a geographic locale, or a technological community – tend to diffuse to the edge of that area but not beyond it? Marrying social network theory with a view of knowledge transfer as a search process, we argue that the degree of knowledge inequality across social boundaries depends crucially on the nature of the knowledge at hand. Simple knowledge diffuses readily across boundaries because outsiders with poor connections to the source of the knowledge can compensate for their limited access with unaided local search. Complex knowledge resists diffusion even within the social circles in which it originated. With knowledge of moderate complexity, however, high-fidelity transmission along social networks combined with local search allows insiders to reproduce and extend knowledge generated elsewhere within the social boundaries, while interdependencies stymie outsiders who rely more heavily on unaided search. Our core proposition, then, is that knowledge inequality across social boundaries reaches its maximum for knowledge of moderate complexity. To test this hypothesis, we examine patent data and compare citation rates across three types of social boundaries: within versus outside the firm, geographically near to versus far from the inventor, and internal versus external to the technological class. We find robust support for the proposition and discuss the important implications of our findings for managers and policy makers who aim to heighten or diminish knowledge inequality.
The flow of knowledge plays a central role in a wide variety of fields (for a review, see Rogers, 1995). Sociologists began investigating diffusion processes – and the importance of social structure to those processes – to understand the adoption patterns of agricultural and medical innovations (Ryan and Gross, 1943; Coleman, Katz and Menzel, 1957). To students of technology management, information transmission first arises as an important issue in the context of technology transfers within the firm (Allen, 1977; Teece, 1977), but questions of diffusion also arise when technology scholars ask whether incumbent firms or upstarts first develop and commercialize new inventions (Reinganum, 1981; Tushman and Anderson, 1986). Both students of organizational learning (for a review, see Argote, 1999) and industrial economists (Griliches, 1957; Zimmerman, 1982; Irwin and Klenow, 1994) study how knowledge moves through firms and how it spills over to other firms. In short, a diverse array of scholars shares an interest in diffusion processes.

The normative interpretation given to diffusion, however, differs dramatically across fields. Economists and sociologists tend to focus on the societal benefits of spillovers. The generation of new knowledge often requires substantial investment in research and development, but the repeated application of this knowledge, once produced, entails little if any incremental cost (Arrow, 1962). Knowledge diffusion, therefore, engenders scale economies and stimulates economic development by allowing several firms to benefit from the R&D activities undertaken by a single firm (Marshall, 1890; Scherer, 1984; Romer, 1987). Management scholars, by contrast, note that when knowledge escapes to competing firms the returns to innovation become fleeting at best. As rivals imitate new products and processes, the degree of differentiation or cost advantages accruing to the innovator erodes. The business literature thus urges managers to defend against spillovers (Lippman and Rumelt, 1982; Kogut and Zander, 1992).

Though their prescriptions differ, economists, sociologists, strategists, and students of technology management all seek a better understanding of why some knowledge disperses widely while other knowledge does not. In this quest, some scholars have focused on the attributes of the knowledge itself. For example, highly specific knowledge may flow slowly because few parties other than the initial innovator either have the baseline knowledge and skills necessary to absorb it (Cohen and Levinthal, 1990) or can benefit from its application (Henderson and Cockburn, 1996; McEvily and Chakravarthy, 2002). Other studies focus on how social networks structure the flow of knowledge (e.g., Coleman, Katz and Menzel, 1957; Davis and Greve, 1997; Uzzi and Lancaster, 2003), implicitly attributing the rate of diffusion to the locus of innovation in the network.
This paper seeks to augment our understanding of knowledge flow by examining the interplay between two features: the complexity of the underlying knowledge (measured by the degree of interdependence between technological components) and the density of social networks. Consider a network that contains patches of dense connections with sparse interstices between them. The dense patches might represent firms, for instance, or geographic regions or technical communities. When does knowledge diffuse within these dense patches but not beyond? In other words, when will knowledge diffuse within a firm but not to competitors or within a region but not to other locales? We argue that such inequality across social boundaries should peak when moderate complexity characterizes the underlying knowledge. Our expectation emerges from recognizing that the assimilation of knowledge frequently requires the recipient to engage in search to fill in gaps and correct transmission errors in the knowledge received—a difficult task when dealing with complex knowledge. Dense social networks diminish the need for search by facilitating high-fidelity transmission (i.e. complete information with negligible noise). However, when relatively sparse social networks—such as one might find spanning the borders of social groups—connect the would-be receiver and source, unaided search plays an increasingly important role in diffusion. Under such conditions, simple knowledge should flow universally—beyond the thick patch in which the knowledge originated across thin areas—because search can easily substitute for high-fidelity transmission. Highly interdependent knowledge meanwhile defies diffusion, regardless of whether one relies on search or a dense social network; it flows neither across thin areas nor within a thick patch. For knowledge of moderate complexity, however, a gap emerges between the ability of insiders, relative to that of outsiders, to acquire knowledge. High-fidelity transmission gives insiders sufficient insight that they can succeed in reproducing knowledge, even where outsiders, who rely more heavily on search, fail.

We analyze patent data to test this thesis empirically. Citation patterns across patents offer something like a fossil record of the flow of knowledge. To assess our argument, we estimate the impact of knowledge complexity on the rate of future citations across three salient social boundaries: (i) internal versus external to the firm, (ii) geographically proximate to, versus remote from, the inventor, and (iii) inside versus outside of a technological class. In all three cases, the disparity in knowledge diffusion across these borders peaks for knowledge of an intermediate level of interdependence.

This work makes several contributions to the literature. First, from the perspective of social networks, it identifies one condition under which social boundaries should prove especially important to information flow: for knowledge of intermediate complexity. Though researchers have usefully demonstrated that networks matter to the diffusion of information, relatively little research considers precisely when those networks should matter most (Strang and Soule, 1998; Baker, 2002). By synthesizing social network
theory with work on knowledge receipt as search, we identify scope conditions on the relevance of social
connections to the diffusion process, and hence to when these social structures reinforce inequality.
Second, with respect to evolutionary economics, our work highlights the social network as a mechanism
through which “insiders” gain superior access to sources of knowledge. Prior work asserts that insiders,
primarily those within a firm, have better access to an original success, which serves as a template in
efforts to reproduce and extend that knowledge (Nelson and Winter, 1982: 119; Rivkin, 2001). Yet this
work fails to establish the source of this preferential access. Does it come from incentives that reward
transfer, from the confidentiality agreements that employees sign, or from some other source? We identify
access to direct social connections as a critical factor differentiating insiders from outsiders. This
elaboration leads directly to a third contribution: recognition that the importance of knowledge
complexity to diffusion extends beyond the organizational boundary emphasized in prior research to other
dimensions along which social networks tend to cluster, specifically geographic and technological lines.
This recognition also yields a more nuanced understanding of the difficulties that a firm might face in
propagating knowledge within its own organizational boundaries but across regions. In addition, our
research proposes a new methodology for analyzing citations, and provides strong empirical support for
the conjecture that social networks matter most to the diffusion of knowledge of moderate complexity.

THE FLOW OF COMPLEX KNOWLEDGE

Our discussion begins with the most common finding of classic diffusion studies: the S-shaped
cumulative adoption curve (Ryan and Gross, 1943; Griliches, 1957; Rogers, 1995, provides an excellent
review). Researchers consistently find that the adoption of an innovation over time follows a common
pattern: growing slowly at first, then accelerating rapidly, and finally slowing to reach some asymptotic
saturation level. These dynamics resemble that of an epidemic spreading through a population; the
innovation first ‘infects’ those most at risk of exposure – actors closest to the original source
(Hägerstrand, 1953) – and those most susceptible to infection – those most prepared to accept the
uncertainty associated with an untested technology (Mansfield, 1968) or whose idiosyncratic features
make the innovation appear most attractive (Griliches, 1957). Over time, awareness of the innovation
spreads, uncertainty ebbs, and the economics of the invention become favorable to a larger share of the
population. As this happens, diffusion takes off. In this classic perspective, new knowledge resembles a
stone thrown into a calm pond, its ripples moving steadily across the entire surface.

Though this pattern accurately describes the diffusion of a wide variety of innovations and knowledge,
critics have faulted this focus on the S-curve for several reasons (cf. Mahajan et al., 1990; Hargadon,
1998). Two of these critiques have particular relevance here. First, the classic diffusion literature typically
depicts knowledge as moving unaltered as it passes from one actor to the next. Contrary to this depiction, in reality transmission rarely occurs without difficulty. Both gaps in the information sent and errors in its interpretation typically require the receiver to reconstruct portions of the knowledge. This process occurs so commonly that it even forms the basis of amusement in the children’s game of telephone.¹ Most knowledge, therefore, requires effort to acquire and transmutes to some extent in the process of diffusion and application; recipients assimilating new knowledge must actively process it by experimenting with its application to new problems and environmental contexts. Witness, for instance, the efforts of American automakers as they struggled to digest the knowledge embodied in Japanese lean production techniques (Womack et al., 1990), or the efforts of computer makers as they sought to imitate Dell’s direct distribution model (Porter and Rivkin, 1999). In both cases, the receipt of knowledge took years of trial, error, reflection, and adjustment and, arguably, remains incomplete. Even within the supportive infrastructure of an organization, transferring and incorporating new knowledge can prove difficult. Teece (1977), for example, reports that the transmission and assimilation of technical know-how accounted for 19% of project costs, on average—running as high as 59% in one case—in 26 international technology transfer projects. And Chew, Bresnahan, and Clark (1990) find the internal transfer of best practices so incomplete in multi-plant commercial food operations that, within a firm, the best plants produce twice as efficiently as the worst, even after controlling for differences in processing technology, location, and plant size (Szulanski, 1996, offers additional evidence). Hence, we regard the acquisition of knowledge not as the receipt of a complete, well-packaged gift, but rather as the beginning of a trial-and-error process.

Our second concern regarding the simple S-curve characterization of diffusion arises from its inattention to the crucial role that social networks play in diffusion. Several studies, largely out of sociology, demonstrate that knowledge spreads from its source not in concentric circles, but along conduits laid by social connections (Lazarsfeld, Berelson and Gaudet, 1944; Coleman, Katz and Menzel, 1966; Burt, 1987; see Marsden and Friedkin, 1993, for a review). Consider some of the relevant findings: Hedström (1994) discovered that network density and geographic proximity explained most of the spread of the idea of unionization in Sweden. In an analysis of adoption patterns for “poison pills” and “golden parachutes,” Davis and Greve (1997) offered strong evidence that information about these policies traveled through corporate board interlocks. And Hansen (1999) found that strong ties best conveyed complex knowledge across product development teams within a firm. A growing literature thus points to the importance of social networks as pathways that channel the flow of information across actors.

¹ In this game, one child whispers a message into the ear of another, who then whispers what she heard in the ear of a third child and so forth. At the end, the final person announces the message he heard and the first one reveals the message that she originally whispered; the two usually differ dramatically.
We synthesize these two perspectives—knowledge receipt as an active process of experimentation and search, and an appreciation for the role of social networks—into a model of knowledge flow. The model offers unique predictions regarding how knowledge complexity influences patterns in the diffusion and application of knowledge.

**Knowledge receipt as search**

Our perspective builds on the intellectual scaffolding of evolutionary economics (Nelson and Winter, 1982), conceptualizing a piece of knowledge as a recipe.² The list of potential ingredients encompasses both physical components and processes. The recipe further details how to combine these ingredients—in which proportions, in what order, under what circumstances—to achieve a desired end. For instance, a recipe for a McDonald’s outlet might read something like: “When a customer places a special order, the counter clerk keys the order into the register, which causes the order to show up on the computer screen in the kitchen, which induces the cook to put a raw hamburger on the grill…” Or “when opening a new outlet, a manager in the real estate department secures a site while the franchising office identifies a franchisee. Next, the franchisee contacts construction contractors while hiring shift managers…” Though these recipes may appear in writing, they more commonly reside in the form of behavioral routines, individual memory, or technology (March and Simon, 1958).

The conceptualization of knowledge as a recipe leads naturally to thinking of innovation as a process of searching for new recipes. Following a long tradition (Schumpeter, 1939; Gilfillan, 1935; Usher, 1954), Nelson and Winter (1982) explicitly treat innovation as a search process; inventors explore the space of possible combinations of ingredients, or recipes, for new and better alternatives. This exploration involves not just the search for the best combinations of ingredients but also the quest for the most effective methods of integration. Researchers who conceptualize innovation as search frequently allude to a landscape as a metaphor for the characteristics of the search space (Levinthal, 1997; Rivkin, 2000; Fleming and Sorenson, 2001). Innovators—depicted as myopic in their awareness of the surface—search these landscapes for peaks, which represent good recipes, useful inventions, and profitable strategies.

² This assumption limits the applicability of our theory to innovations that involve multiple components. This restriction should not severely constrain its scope, however; few innovations do not involve the combination of multiple physical components or processes. For example, even the synthesis of nylon, a polymer, involved the integration of several distinct processes (Smith and Hounshell, 1985).
Once a useful innovation has been located, transferring its recipe, even between cooperative actors, can fail for two reasons. First, the recipient rarely grasps the original recipe completely, due to imperfections in the transfer process. Gaps exist in the information conveyed by the sender—perhaps the chef forgets an ingredient or skips a step—and the receiver may misinterpret some of the transmitted data. And, unless the recipient understands perfectly the recipe that generated the success—an unlikely situation—she must engage in search to fill the gaps and correct the errors in her version of the recipe. Any attempt to apply a recipe to new settings will likewise require the recipient to rediscover the original combination, or some variant of it better suited to the new context. Second, the local ingredients and cooking experience of the receiving chef rarely match identically those of the sender. Research on absorptive capacity (Cohen and Levinthal, 1990) emphasizes that successful knowledge acquisition requires the receiver to possess a base of knowledge and skills to assimilate new information. Without this baseline, the transmission of new discoveries would often entail the communication of exorbitant amounts of data; imagine how long a recipe would become if one needed to detail every step of the process—how to chop vegetables, how to boil water, etc. These two factors imply that knowledge recipients rarely, if ever, act merely as passive beneficiaries; they actively search, recreate, and build upon the original recipes.

In this process, certain types of recipes prove particularly tricky to transfer because the sender finds it difficult to specify and communicate precisely where the original combination resides in the space of ingredients; on the figurative treasure map, it is hard to place the “X” that “marks the spot.” This communication difficulty could arise as a result of causal ambiguity (Lippman and Rumelt, 1982; Reed and DeFillippi, 1990): the innovator might not fully understand the connection between actions and outcomes so the roots of the original success remain unclear. It could also occur because the production process calls on tacit personal skills or connections among individuals that the involved parties themselves do not consciously understand (Polanyi, 1966; von Hippel, 1988), or that eludes codification (Zander and Kogut, 1995). These factors essentially increase the likelihood that the knowledge transmitted has gaps. The complexity of the recipe itself can also impair knowledge transfer by making it more difficult for the recipient to fill these gaps and correct transmission errors.

“Complexity” demands further explanation. Simon (1962) classifies a piece of knowledge as complex if it comprises many elements that interact richly (see also Kauffman, 1993). This definition corresponds closely to those used by other researchers. For example, one set of scholars defines the complexity of a piece of knowledge as the amount of information required to characterize it (Kolmogorov, 1965; Winter, 1987)—a direct function of the number of elements and their degree of interaction. Similarly, Zander and Kogut (1995), focusing on the interactions among components, consider a piece of knowledge complex if
it integrates many, distinct competencies within a firm. We adopt Simon’s definition, but like Zander and Kogut (1995) pay particular attention to the intensity of interdependence among the ingredients in the recipe. A high degree of interdependence indicates that many ingredients influence the effectiveness of others—like a continuous-flow production system without buffers. Low interdependence implies small cross-component effects, as one finds in cell manufacturing. Thus, a complex piece of knowledge, or recipe, marshals a large number of ingredients that depend delicately on one another.

Discovering, or rediscovering, a complex piece of knowledge poses a stiff challenge. Interdependence produces two effects that undermine the recipient’s attempts to regenerate the original. First, small errors in reproduction cause large problems when ingredients cross couple in a rich manner. In highly interdependent systems, implementers often realize no value from adopting a set of practices unless each-and-every component fits into place perfectly; a single error threatens the effectiveness of the entire system. An American automaker that attempts to adopt lean production techniques, for instance, may alter its human resource practices and inventory policies, yet see no benefit because it failed to invest appropriately in flexible production equipment. The fragility of such tightly coupled systems has been well documented (Weick, 1976; Perrow, 1984). Second, interdependence leads to a proliferation of “local peaks.” These internally consistent—though not necessarily optimal—ways of combining ingredients elude improvement through incremental change because changing any single element degrades the quality of the outcome (Kauffman, 1993). Such local peaks would pose no problem to omniscient actors that could assess the entire space of possibilities, but for individuals with finite cognitive abilities and a limited purview of the landscape such search proves difficult; in the face of high interdependence, searchers frequently find themselves trapped on local peaks. Moreover, these local peaks tend to correspond to poor recipes precisely when interdependence creates a thick web of potentially conflicting constraints.

**Complexity and access to a template**

Success in assimilating and using complex knowledge depends crucially on access to the original success, which serves as a *template* (Nelson and Winter, 1982: 119-120; Winter, 1995). For reasons explored below, individuals differ in their access to the template. Superior access facilitates the knowledge recipient’s search in at least two ways. First, the recipient begins searching in closer proximity to the ultimate target—as a result of either fewer errors in the interpretation of the transmission or smaller gaps in the information sent. Second, superior access allows the recipient to solicit advice when problems arise, making it possible to home in on the desired knowledge more efficiently.
Consider two actors both trying to apply a valuable piece of knowledge but who differ in their access to the template. The first has superior, though admittedly still imperfect, access to and understanding of the original, successful recipe. The second has far poorer access. To what degree does the first actor’s superior but imperfect access to the template have value, in the sense that it enables the actor to assimilate and build upon the original recipe more effectively? We contend that the value of this access depends on the complexity of the underlying knowledge in an inverted U-shaped relationship; that is, intermediate levels of interdependence make preferential access most valuable.

Suppose first that the ingredients making up the knowledge do not interact; getting one element in the recipe wrong diminishes that component’s contribution to the whole, but it does not undermine the other components. In this situation, the first actor’s access to the template does not educe a persistent advantage. Through routine, incremental search efforts, the second actor can reconstruct the recipe. Few local peaks threaten to trap the poorly informed recipient. As a result, both actors eventually fare equally well; search on the part of a recipient can easily substitute for high-fidelity transmission.

Next consider knowledge with an intermediate degree of interdependence. Local peaks now appear, but they remain relatively few in number. The well-informed actor begins its search near, but not precisely at, the original combination of ingredients. Through incremental search, and with recourse to the template, she can piece together the proper combination of ingredients. The second actor, who likely begins search farther from the target and receives less guidance about the direction in which to explore, more likely becomes ensnared on some local peak, away from and below the original success. Here superior access to the template gives the first actor an advantage that the second cannot recreate through search.

Finally, imagine a piece of maximally interdependent knowledge: ingredients depend on one another in an extremely delicate way, and none produces much benefit unless all align perfectly. Local peaks now pervade the landscape and neither actor’s incremental search will likely reproduce or build upon the original knowledge with any success. The first actor’s superior access to the template thus has little value beyond the second’s highly imperfect access.

Taken together, these arguments imply that the advantage of superior but imperfect access to the template reaches its peak at moderate levels of interdependence between knowledge components. With moderate interdependence, the smoothness of the landscape allows an actor who begins her search near the desired peak to rediscover it through local search. Yet the landscape also has sufficient ruggedness that an actor who begins search far from the target likely finds herself trapped on a lower peak. In contrast, the single-
peaked landscape that comes with independent components allows both parties to succeed in reproducing knowledge through local search. And the highly rugged landscape produced by extreme interdependence stymies both parties thoroughly. (For a more formal treatment, see the simulation in the Appendix.)

**Social networks and template access**

Access to the template depends crucially on the distribution of social relations, which provide the conduits through which valuable information travels (Hägerstrand, 1953). Strong, direct ties provide the most obvious and valuable links between inventors and those attempting to assimilate the knowledge because they permit bi-directional communication; thus, the recipient can interactively query the original source of the knowledge to correct errors or fill gaps in the original transmission. Indirect ties can also provide beneficial access to the template, as even second-hand knowledge provides important clues as to how to reconstruct and assimilate new knowledge. The value of this knowledge, however, undoubtedly declines rapidly as the number of actors between the innovator and the receiver increases; as in the game of telephone, each step in the path between the two parties offers an opportunity for errors and omissions to creep into the transmission. For those relying on indirect ties, redundancy—in the form of multiple indirect paths—may help the receiver identify and correct these errors (Sorenson and Stuart, 2001).

These social relations do not link actors at random. Rather, sociologists have consistently argued and demonstrated that networks concentrate within groups of actors that share one or more salient dimensions. Researchers attribute this pattern to two sources. First, an individual’s position in social and geographic space strongly influences one’s daily activities. Since one cannot easily form a relation with those whom one never meets, these behavioral patterns dramatically increase the likelihood that individuals encounter others with similar characteristics (Blau, 1977). For example, a college student meets other college students, through classes and social activities, rather than a sampling from the broader population that lives in the same city. Second, even when individuals do meet others from different backgrounds, they may prefer to interact, and hence form stronger ties, with those most like them (Lazerfeld and Merton, 1954; Rogers and Kincaid, 1981). Because existing social networks can also strongly influence whom one meets—for example, through meeting friends of friends—these processes reinforce one another.

Our study tests the salience of three dimensions that structure social networks—organizational membership, geographic location, and technological community membership—in influencing the flow of

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3 “Weak” ties, however, likely play a minor role in diffusing information in this context. Weak ties have long reach but low bandwidth; thus, they operate most prominently in the diffusion process when transferring only a small amount of information (Hansen, 1999)–for example, in Granovetter’s (1973) search for jobs.
knowledge. Consider organizational boundaries first. A firm attempting to replicate and build on its own prior success has better access to its knowledge than would an outside imitator, for several reasons. First, simple physical barriers impede the flow of information. The fence around the innovator’s factory prevents anyone but an employee of the firm from watching the knowledge in action and inspecting its physical embodiment. In many cases, though, physical barriers likely play a minor role. Many firms even host rivals on tours of their facilities, confident that competitors learn little by merely seeing their sites. Second, formal and informal agreements with employees allow companies to prevent valuable knowledge from escaping the firm (Gilson, 1999; Stuart and Sorenson, 2003). Employment contracts nearly always forbid workers from leaking valuable information to the competition, and sometimes non-compete covenants within these agreements even prevent them from working for rivals. Third, fellow members of an organization also share codes, specialized languages, and beliefs that make high-fidelity transmission easier (Durkheim, 1912; Arrow, 1974). Fourth, and perhaps most important, the strong interpersonal ties and dense social networks necessary to facilitate access to the template reside primarily within the firm. Organizations demark some of the most prominent social boundaries in commercial activity. On a daily basis, most fully employed individuals spend more waking hours at work than in any other single location, perhaps with the exception of the home. Employees regularly meet other employees at work to cooperate on projects, to confer on decisions, to transfer information, and to socialize. Thus, the far greater density of social connections within firm boundaries should not surprise us (Granovetter, 1985).

As argued above, the value of this access peaks for the transmission of knowledge of intermediate complexity. Thus we expect:

**Hypothesis 1:** The advantage in receiving and applying knowledge that members of the same firm have over members of different firms reaches its maximum for knowledge of intermediate interdependence.

That is, the insider’s advantage over the outsider has an inverted U-shaped relation to the interdependence of the knowledge.

Actors in close physical proximity to the innovator also have superior access to the template. Some of the earliest literature on social networks noted the dramatic decline in the likelihood of a social tie as two parties became increasingly distant (e.g., Park, 1926; Bossard, 1932). Social networks localize in space

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4 The relative salience of the firm in structuring relations may vary from firm to firm, or even from region to region. For example, Saxenian (1994) notes that the extraordinary mobility of employees among firms in Silicon Valley has engendered an unusually high frequency of inter-firm contacts, an outcome that may stem from California’s labor laws (Gilson, 1999; Stuart and Sorenson, 2003). Regardless, even in Silicon Valley, social ties occur more commonly within firms than across them.
for a variety of reasons: Individuals in close proximity more likely have the chance encounters that lead to lasting ties (Festinger, Schacter and Back, 1950; Hawley, 1971); local bonds form at neighborhood gatherings, coffee shops, softball leagues, PTA meetings, and the like. Owen-Smith and Powell (2004) further note that these local institutional networks can themselves facilitate the flow of information. As the distance between two actors grows, the likelihood of an intervening opportunity—an equally preferred but closer contact—increases as well (Stouffer, 1940). Finally, the cost of maintaining a relationship also increases with distance; thus, ties that span physical distances more likely erode over time (Zipf, 1949; Boalt and Janson, 1957).

Another reason for the geographic concentration of social relations concerns the local nature of labor markets. When individuals leave one employer and move to another, they most frequently enter an employment relation with another firm located in the same region. These local labor markets facilitate the dense formation of networks within, but not beyond the region. Almeida and Kogut (1999), for instance, demonstrate this convincingly in their study of inventors’ networks within the semiconductor industry, showing that employee mobility patterns can explain much of the variation in the inter-firm citation patterns of semiconductor patents; Breschi and Lissoni (2002) demonstrate a similar phenomenon across a broad range of industries using patent data from firms in Northern Italy. Though a variety of processes may account for the formation and maintenance of geographically localized social networks, these ties then become the pathways along which knowledge moves.

Not only do social networks localize in space, but also these regions often develop their own unique cultures. Indeed, the literature on culture originated with the study of the beliefs and activities of local populations of individuals (e.g., Benedict, 1934; Radcliffe-Brown, 1952). In the extreme, local populations develop distinct languages making the communication of complex concepts across borders all but impossible. But even between regions that share a common language, differences in assumptions and background can impede effective communication. Accordingly, we expect that actors physically close to an original source of knowledge have better access to it. Thus, we anticipate:

**Hypothesis 2:** A nearby knowledge recipient’s advantage in receiving and applying knowledge over a distant recipient peaks for knowledge of intermediate interdependence.

This expectation raises an interesting issue: When managers seek to diffuse knowledge within a firm, they often hope to extend the advantages accruing to their existing knowledge to new regions (Winter, 1995). For example, a retail chain might wish to open a store in a new market, a manufacturer may aim to build
factories in many locations, or a consulting firm might try to share best practices across offices. One school of thought suggests that the ownership rights embodied in firms should allow for the alignment of incentives necessary to facilitate this transfer (e.g., Nelson and Winter, 1982; Rivkin, 2001). The recognition that social networks provide the critical access to the template necessary to transfer knowledge of moderate complexity suggests otherwise. To the extent that networks localize geographically within the firm, organizations likely find it difficult to diffuse knowledge beyond its point of origin. Within a firm, then, we expect simple knowledge to spread broadly and highly complex knowledge to remain isolated within a single team or department. Knowledge of moderate complexity, however, should spread within a firm to the edges of a facility or locale, but not to distant installations:

**Hypothesis 3:** Within a firm, a nearby knowledge recipient’s advantage in receiving and applying knowledge over a distant recipient reaches its maximum for knowledge of intermediate interdependence.

An analogous argument applies to technological proximity. Actors who work in the same technological domain as an inventor likely have superior access to the template. Universities, trade associations, professional societies, industry consortia, and work experience foster dense social connections within such technological communities. These institutions provide common grounds both for inventors to meet in the first place, and thereby potentially establish a social relation, and for them to maintain their relations over time. Journals, and particularly conferences, also play a role in alerting members of technological communities to who else currently works on similar problems. Though inventors may already know these individuals, they may not always be aware of everything that their acquaintances have been doing. Knowing who knows what enables an inventor to tap her network to request access to the template. Prior interactions and overlapping social connections make it more likely that the holder of the template grants the request (Smith, 1763; Gouldner, 1960).

Technological communities also develop common knowledge that can facilitate knowledge transfer. Through training—both graduate programs and post-doctoral fellowships—universities help disseminate tacit knowledge regarding the basic techniques of the field (e.g., procedures at the lab bench). They also promote the development of communal languages—systematic methods for describing and communicating knowledge (Allen, 1977; Rosenkopf and Tushman, 1998). This baseline knowledge provides firms and inventors with the “absorptive capacity” necessary to integrate new knowledge (Cohen and Levinthal, 1990). Technological proximity thus engenders superior access to the template, which should have its greatest impact when the target knowledge displays moderate interdependence:
**Hypothesis 4:** The advantage in receiving and applying knowledge that a member of a technological community has over a non-member of the community reaches its maximum with knowledge of intermediate interdependence.

In brief, we view knowledge diffusion and appropriation as a search to rediscover and apply an effective recipe. Recipients within the same firm, within the same locale, or within the same technological community as the knowledge source have superior, though still imperfect, access to the original recipe. This advantage in access translates into higher fidelity reproduction that benefits the actor most significantly when the ingredients of the recipe display moderate interdependence.

**EMPIRICAL CORROBORATION**

To test these hypotheses, we analyzed prior art citations to all U.S. utility patents granted in May and June of 1990 (n = 17,264). The data came from the Micro Patent database and NBER public access data on patents (Hall, Jaffe and Trajtenberg, 2001). Following much previous research (e.g., Jaffe, Trajtenberg, and Henderson, 1993; Podolny and Stuart, 1995; Almeida and Kogut, 1999), we view a prior art citation as evidence of knowledge diffusion: the applicant has successfully assimilated the knowledge underlying the original patent to a new setting and built upon it. Patents and their citation patterns provide an attractive test bed for our hypotheses for several reasons. First, these citations have been carefully assigned. The U.S. Patent Office requires all applicants to demonstrate awareness of their invention’s precedents by citing similar “prior art” patents. Patent examiners in each technological domain review and supplement the prior art references to ensure accurate and comprehensive citations. Second, the inventions that patents describe have often been characterized as combinations of technological components, consistent with our earlier description of knowledge. Third, Fleming and Sorenson (2001; 2004) have developed a technique for measuring the interdependence among the components, which draws on information uniquely revealed in patents and potentially difficult to duplicate in other settings.

There remain, however, some potential concerns about this empirical setting. First, our hypotheses rest on the assumption that some potential knowledge recipients have better access to the template than others. If one believed that the prior patent fully revealed the inventor’s underlying knowledge of the invention, one would question this assumption. Inventor’s incentives, however, make this unlikely. Patent applicants prefer to disclose as little as possible to limit their competitors’ ability to benefit from their disclosure.

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5 We selected May as a starting month at random from 1990, a year that we selected to maximize the timeliness of the data while still allowing sufficient time to observe the citations of future patents to this focal set. Two months of data represents the maximum period we could examine given computational costs; calculating the K measure (described below) for this sample required two months of workstation CPU time.
(Lim, 2001). Indeed, conversations with the U.S. Patent Office indicate that applicants intentionally obfuscate their descriptions to diminish the value of the knowledge revealed (Stern, 2001). Second, one might question whether a citation represents the reproduction of old knowledge or simply the generation of new knowledge. We merely claim, however, that the citation signals replication of some underlying knowledge embodied in both the old and the new invention, not that the new invention merely replicates the old. Third, patents admittedly offer imperfect measures of invention. Inventors may limit their patent applications to a subset of their discoveries, and one must ask whether this selection process biases our results. Inventors most likely seek legal protection when a patent raises a meaningful barrier to imitation (e.g., when inventing around the patent proves difficult), when the invention will not quickly become obsolete, and when few alternative “natural” defenses protect the knowledge (Levin et al., 1987). Of these conditions, the last seems most germane to our study. It implies that our sample may under-represent inventions that involve highly tacit, causally ambiguous and complex knowledge. Empirical research, however, suggests that this selection bias may not exist: Cohen, Nelson, and Walsh (2000), for example, find that firms in industries with complex products disproportionately choose to patent.

**Case-control design**

Our statistical approach involves estimating the likelihood that a focal patent receives a citation from a future patent as a function of several factors: the interdependence of the knowledge underlying the focal patent, the status of the citing patent’s inventor as an insider or outsider with respect to the focal patent, the interaction of interdependence and insider / outsider status, and a set of control variables. The results of the estimation allow us to examine how the likelihood of insider citation compares to the likelihood of outsider citation and, crucially, whether the gap between the two probabilities peaks when the focal patent embodies moderately interdependent knowledge.

Our unit of analysis is a patent dyad, one patent issued in May or June of 1990 and one issued later that may or may not cite the first. Hence our approach conceptually follows that of other studies of the likelihood of tie formation—in this case, the likelihood that a future patent builds on the knowledge embodied in one of our focal patents. These studies have typically estimated tie formation on the entire matrix of possible relations (e.g., Podolny, 1994; Gulati, 1995). This approach has two disadvantages. With large numbers of nodes, in this case patents, it can generate enormous, sparse matrices, making estimation and the efficient construction of variables difficult. In our situation, this method would generate nearly 20 billion dyads but only around 60,000 realized citations. In addition, this approach raises questions regarding network autocorrelation and the non-independence of repeated observations on
the same patents across multiple observations in the error structure, requiring one to make assumptions about the nature of this autocorrelation to estimate efficient standard errors.

Instead, our analysis follows Sorenson and Stuart (2001) in adopting a case-control approach to analyzing the formation of ties. The case-control sampling procedure works as follows. We begin by including all cases of future patents, from July 1990 to June 1996, that cite any of our 17,268 focal patents: 60,999 in total. Since these citations occur, the dependent variable for these cases takes a value of “1” to denote a realized citation. In addition, we pair each of the 17,268 focal patents with four future patents that do not cite it (but that could have). We set the dependent variable for these control cases to zero. Though this generates a data set of 130,055 dyads, our analyses restrict the sample used for estimation to the 72,801 cases where both inventors reside in the U.S. To address the fact that focal patents enter the data more than once, we report robust standard errors estimated without the assumption of independence across repeated observations of the same focal patent.

The use of a matched sample introduces one new problem. Logistic regression can yield biased estimates when the proportion of positive outcomes (citations) in the sample does not match the proportion of citations in the population. In particular, uncorrected logistic regression using a matched sample tends to produce underestimates of the factors that predict a positive outcome (King and Zeng, 2001). Large samples do not necessarily alleviate this problem.

We adjust the coefficient estimates using the method proposed by King and Zeng (2001) for the logistic regression of rare events. The traditional logistic regression model considers the dichotomous outcome variable a Bernoulli probability function that takes a value 1 with the probability $\pi$:

6 We tracked future citations for six years as this period should capture the bulk of future citations for most patents; prior research suggests that most of the information available in patent citations appears in the first five years (Trajtenberg, 1990; Lanjouw and Schankerman, 1999).

7 We chose four patents for the “control” group so that the sample would have a roughly equal proportion of realized and unrealized dyads. Although some feel that conditioning on important factors improves the statistical power of a case-control sample (e.g., Jaffe, Trajtenberg and Henderson, 1993, implicitly make such an argument in drawing controls from the same classes as the citing patents), the ideal method of selecting controls remains an open debate. Matching controls to cases on one or more dimensions can lead to two problems in particular that concern us. First, correcting the logit for over-sampling on the dependent variable requires that one knows the sampling probabilities (King and Zeng, 2001); matching controls to cases precludes the possibility of estimating this information. Second, matching on an endogenously determined factor risks generating biased results (e.g., when investigating diffusion processes, one would not want to consider the geographic distribution of activity exogenous). Given these concerns, we sample future patents at random and control for heterogeneity in the estimation.

8 Including the foreign patents does not change the results qualitatively.
\[ \pi_i = \frac{1}{1 + e^{-X_i\beta}}, \]

where \( X \) represents a vector of covariates and \( \beta \) denotes a vector of parameters. Researchers typically use maximum likelihood methods to estimate \( \beta \). King and Zeng (2001) prove that the following weighted least squares expression estimates the bias in \( \beta \) generated by oversampling rare events:

\[
\text{bias} \left( \hat{\beta} \right) = (X'WX)^{-1}X'W\xi,
\]

where \( \xi = 0.5 \) \( Q \left[ (1 + w_1)\bar{p} - w_1 \right] \), the \( Q \) are the diagonal elements of \( Q = X(X'WX)^{-1}X' \), \( W = \text{diag} \left\{ \bar{p}(1 - \bar{p}) \right\} \), and \( w_1 \) represents the fraction of ones (events) in the sample relative to the fraction in the population. At an intuitive level, one regresses the independent variables on the residuals using \( W \) as the weighting factor. Tomz (1999) implements this method in the relogit Stata procedure.

This case-control approach offers two principal advantages over the count models employed in most patent research (e.g., Fleming and Sorenson, 2001). First, this method permits far more fine-grained controls for heterogeneity in citing patents. Count models preclude the possibility of controlling for detailed features of a citing patent. The ability to account for the attributes of the potential citing patents proves critical, however, to testing our hypotheses, which suggest that the ability of future inventors to build on the original knowledge depends on their position in social and geographic space. Second, analyzing citations at the level of the citing-patent/cited-patent dyad avoids the potential for aggregation bias inherent in count models.

**Interdependence**

Following Fleming and Sorenson (2001), we measure the complexity of the knowledge in a patent by observing the historical difficulty of recombining the elements that constitute it. Though it involves intensive calculation, the intuition behind the metric is straightforward: a technology whose components have, in the past, been mixed and matched readily with a wide variety of other components has exhibited few sensitive interdependencies. The measure takes the subclasses identified in a patent as proxies for the underlying components. Though in many cases subclasses correspond quite closely to physical components (such as in the example below), they do not always align so closely. Our measure, however, only requires that these subclasses define pieces of knowledge rather than identifiable physical components. Combining some pieces that interact sensitively to each other proves more difficult than connecting relatively independent chunks of knowledge.
We calculate the measure of interdependence, $k$, in two stages. Equation 1 details our measurement of the ease of recombination—the inverse of interdependence—for subclass $i$ used in patent $j$. We first identified every use of the subclass $i$ in previous patents from 1980 to 1990. The sum of the number of prior uses provided the denominator. For the numerator, we counted the number of different subclasses appearing with subclass $i$ on previous patents. Hence, our measure increases as a particular subclass combines with a wider variety of technologies, controlling for the total number of applications, capturing the ease of combining a particular technology. To create our measure of interdependence for an entire patent, we averaged the inverted ease of recombination scores for the subclasses to which it belongs (equation 2).

\[
\text{Ease of recombination of subclass } i = E_i = \frac{\text{Count of subclasses previously combined with subclass } i}{\text{Count of previous patents in subclass } i}
\]

\[
\text{Interdependence of patent } j = k_j = \frac{\text{Count of subclasses on patent } j}{\sum_{i \in j} E_i}
\]

As an example, consider a digital technology patent, #5,136,185, filed by the third author of this paper. Figure 1 outlines the calculation of $k$ for this patent and the mapping of the USPTO classification scheme to the components used. 326/16 identifies the “Test facilitate feature” subclass, which implements a testing mode within a semiconductor chip. Prior to its appearance here, this subclass had been recombined 116 times with 205 other components, implying an observed ease of recombination score of $205/116=1.77$. 326/56 indicates the “Tristate subclass,” and 326/82 points to “Current driving fan in/out.” 326/31 identifies the “Switching threshold stabilization” subclass (essentially a priority encoder). Figure 1 illustrates the location of these components on the circuit, the calculation of their ease of recombination scores, and the calculation of the patent’s interdependence, $k$ (≈ 0.61)—a level slightly above the mean $k$ for our sample.

The invention described above assists engineers in testing the logic gates on new chips—a difficult task when chips can contain hundreds of thousands or even millions of such gates. As one might expect from its moderate level of interdependence, this invention has resisted transmittal outside the firm in which it

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9 Some might worry about the stability of this measure over time. To test its robustness, we constructed a second $k$ measure using data from 1780 to 1990; that measure yielded a qualitatively identical set of results.
originated. Even though the patent appears to disclose much of the important information, it does not disclose the proprietary test generation algorithm, and how that algorithm manipulated the components (in particular, the “test facilitate feature”). Without access to, or an understanding of, that algorithm, rivals could see the components of the knowledge in the patent but not how the components worked together. As a result, competitors faced an uphill battle in taking full advantage of the knowledge. Even within the firm, effective transmission required the inventor to travel around the country to teach others how to work with the technology. Similarly, competitors found it difficult to reproduce IBM’s copper interconnect technology—another invention of intermediate complexity—until enough engineers defected to rivals to diffuse the relevant knowledge of how to fabricate the copper interconnect without contaminating the wafer’s other materials (Lim, 2001).

By comparison, inventions involving extremely high levels of interdependence defy diffusion even within a social boundary. Plasmid preparation, for example, a biological technique, involves an intricately intertwined sequence of actions involving various chemicals, reagents and manual operations. As Jordan and Lynch (1992: 84) note, “Although the plasma prep is far from controversial and is commonly referenced as a well established and indispensable technique, how exactly it is done is not effectively communicated, either by print, word of mouth, or demonstration.” On the other hand, inventions involving a low degree of interdependence diffuse rapidly. For instance, patent #4,927,016, one of the patents in the bottom quartile of the \(k\) range, involves the production of monoclonal antibodies. The industry associated with this technology has essentially become a commodity business since one can easily acquire all the necessary knowledge components by reading a textbook and piece them together without concern for sensitive interdependencies. Polymerase chain reaction, a technique for amplifying DNA sequences, has followed a similar route. Or, one might think of Sun’s workstation technology. The modular design of their system has allowed rivals to match the performance of their hardware quickly, making it difficult for the company to maintain an advantage in the hardware market. Though these examples give a sense of interdependence, Fleming and Sorenson (2004) also validated the measure by surveying inventors; they found a strong, significant correlation between inventors’ perceptions of coupling and the degree of interdependence calculated through this technique.

**Social networks**

The analyses investigate the effect of knowledge complexity on the diffusion of knowledge to insiders, whose dense social connections give them good access to the template, versus outsiders, with sparse connections and poor access. Three variables proxy for the insider/outsider status of the potential citing
inventor with respect to the holder of a focal patent. The variables reflect proximity in organizational, geographic, and technological space.

**Same assignee** – If the patents in a dyad share the same assignee (i.e., owning organization), then dense social networks more likely connect the two inventors, increasing the likelihood of a citation—particularly for inventions of moderate complexity. Same assignee is simply a dummy variable indicating whether the two patents share a common owner.

**Geographic proximity** – A continuous variable, the natural log of the distance in miles between the first inventors listed on the two patents multiplied by negative one (so that we expect larger values to increase the likelihood of citation), provides our measure of geographic proximity. All patents list the address of the inventor on the front page of the patent application. To locate each inventor, we match the inventor’s 3-digit zip code\(^{10}\) to the latitude and longitude of the center of the area in which the inventor resides based on information from the U.S. Postal Service. We then use spherical geometry to calculate the distance between the points. Geographic proximity should increase the likelihood of citation, particularly for inventions of moderate complexity.

**Same class** – A second dummy variable denotes whether the two patents both belong to the same primary technological class. Again, we expect that technological similarity—a proxy for shared membership in a community—should enhance the citation likelihood, particularly at intermediate levels of interdependence.

In all three cases, we test our hypotheses by interacting our measure of interdependence and its square with a measure of the density of social networks—whether due to firm boundaries, geographic proximity, or technological similarity (we mean-deviate the variables before creating the interaction terms to reduce multicollinearity). The benefits of social proximity should peak for inventions of moderate complexity. We also enter these variables without interacting them with \(k\) to control for any consistent effects of these factors on diffusion across knowledge of all levels of interdependence.

We believe it useful to elaborate our hypotheses in light of our particular empirical context, patent citations. The hypotheses describe the impact of interdependence on the *gap* between insider reproduction of knowledge and outsider reproduction. In developing the hypotheses, however, we also paint a picture

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\(^{10}\) The USPTO includes 5-digit zip information, but we chose to reduce the measurement error by using cleaned data. CHI, an information provider, has called every patent holder to verify the inventor’s location; however, it only records this information at the 3-digit level.
of the impact of interdependence on insider reproduction alone and on outsider reproduction by itself: we suggest that greater interdependence makes it more difficult for each type of party to receive and build upon prior knowledge. This argument concerns the success of an insider or an outsider in reproducing knowledge conditional on an attempt at reproduction being undertaken. Patent citation data reflect not only success conditional on an attempt being undertaken, but also the sheer number of attempts being undertaken. We have reason to believe, however, that the number of attempts by insiders and outsiders alike may rise with interdependence, simply because interdependence increases the fertile variety that may come from mixing and matching components (Fleming and Sorenson, 2001). As a result, patent citation rates, by insiders alone or by outsiders alone, may rise with interdependence, fall with interdependence, or have a non-monotonic relationship. The gap between insider and outsider citation rates, however, should continue to have an inverted-U relation to interdependence. The simulation of knowledge flow presented in the Appendix presents this argument in more detail.

**Controls**

The non-monotonic interactions between interdependence and proximity that we predict—if discovered in the data—lend themselves to few alternative interpretations. The models nevertheless include as controls several of the most important variables used in prior patent studies (e.g., Lanjouw and Schankerman, 1999). The controls increase the precision of our estimates and account for other types of heterogeneity:

*Subclass overlap* — In addition to geographic distance, one could also imagine that distance in the technological sense influences the likelihood of citation. To measure the technological distance between the two patents in each dyad, we examined the overlap in subclass assignments between the focal patent, \( i \), and the potentially citing patent, \( j \):

\[
o_{ij} = \frac{s_i \cdot s_j}{|s_i|},
\]

where \( s \) represents a vector of subclass assignments, with each cell being a binary indicator variable of membership in a subclass (i.e., 1 denoting assignment to the subclass). The measure ranges from 0 to 1, with larger values indicating more similar technologies.\textsuperscript{11}

\textsuperscript{11} One could think of this measure as also reflecting the similarity of the technological communities linking the two inventors and hence the likely density of the social networks connecting them. Indeed, substituting subclass overlap for same class membership to test the relevance of the boundaries of technological communities generates equivalent qualitative results: the gap between insiders and outsiders peaks for inventions of moderate complexity.
Activity control – The activity control accounts for the typical number of citations received by a patent in the same technological areas as the focal patent. In the first stage, we calculated the average number of citations that each patent in a particular USPTO class received from patents granted between January of 1985 and June of 1990 (equation 4).\textsuperscript{12} We weighted these parameters according to the patent’s class assignments (equation 5), where \( p_{ik} \) indicates the proportion of patent \( k \)’s sub-class memberships that fall in class \( i \).

\[
\text{Average citations in patent class } i = \mu_i = \frac{\sum_{j \in i} \text{Citations}_j \text{ (before 7/90)}}{\text{Count of patents } j \text{ in subclass } i}
\]  

Technology mean control patent \( k = M_k = p_{ik} \mu_i \)  

Recent technology – Inventions can also differ in the degree to which they build on recently developed technologies versus older, potentially better developed, fields. Focal patents in areas of technological ferment may receive citations at a higher rate. A control variable equal to the average of the patent numbers of the focal patent’s prior art (higher numbers indicating more recent technology) captures this potential influence on the likelihood of citation.\textsuperscript{13}

Backward citations – The models include counts of two types of backward patent citations, also known as prior art citations, which appear on the focal patents. First, they include a tally of the number of prior patents cited. Podolny and Stuart (1995) argued that this variable measures the degree of local search; it may also capture idiosyncratic differences in citation propensity that our activity control misses. Second, the models include a control for the number of non-patent prior art citations (e.g., references to published articles). As these published sources enjoy wider circulation than the patent database, patents that build on published knowledge likely diffuse more rapidly in social and geographic space.

\textsuperscript{12} We allow all patents issued between January 1985 and June 30, 1990 to enter the estimation of the technology control, meaning that the patents used to calculate it vary in the time during which they can receive citations. Alternatively, we could select a small set of patents from 1985 and base the measures on the subsequent five years of citations; however, this approach would ignore the patent activity just prior to our sample.

\textsuperscript{13} This variable made use of the fact that the USPTO assigns patent numbers sequentially. This assignment pattern generates a correlation between a patent number and the grant date of the patent of 0.98.
Classes – The models include a count of the number of major classes. Patents that belong to a larger number of classes may differ from other patents in their breadth of future applicability. These patents may therefore receive more citations.

Subclasses – The number of subclass references assigned to a patent provided a control for the number of components in the invention (Fleming and Sorenson, 2001). Descriptive statistics for all of the variables used in the models appear in Table 1.

RESULTS

The results of these models appear in Tables 2 and 3. Model 1 estimates the effects of the control variables alone. Interdependence, k, has the non-monotonic effect on the baseline rate of future citations that prior studies have reported (Fleming and Sorenson, 2001). Both geographic and technological proximity increase the likelihood of a future citation, as one would expect. Interestingly, however, while controlling for other types of heterogeneity, firms do not appear more likely to cite their own prior patents, on average. Of the other control variables, only the number of non-patent references cited by the focal patent has an effect on future citation likelihood: consistent with faster diffusion, these patents receive future citations at a higher rate.

Model 2 tests hypothesis 1 by interacting interdependence with same assignee. As expected, membership within the same firm produces the greatest diffusion advantage over outsiders for knowledge of intermediate complexity, as evidenced by the positive coefficient on $k \times \text{same assignee}$ and the negative coefficient on $k^2 \times \text{same assignee}$. The interactions between geographic distance and interdependence in model 3 test hypothesis 2, again showing strong support. Model 4 tests hypothesis 3 by re-estimating model 2, but only for dyads where both patents belong to the same firm. In essence it asks: Does geography still matter for knowledge diffusion within firms? Indeed, the results (in support of H3) reveal that even within firm boundaries, social networks influence the flow of knowledge, with the greatest disparity between local diffusion and distant diffusion arising for knowledge of moderate interdependence. Finally, model 5 tests the salience of technological communities. Once again, the estimates show strong support; the impact of technological community membership on citation probability peaks for intermediate interdependence.

Models 6 and 7 test the robustness of these results in two ways. First, model 6 estimates the effects of all of the interaction terms simultaneously. One might otherwise worry that our models simply reflect a
single dimension of social similarity as firm, geographic, and technological boundaries have a high degree of overlap. Model 6 demonstrates that we can reject this alternative as each dimension of social similarity has an independent and significant effect (in support of H1, H2 and H4) when estimated simultaneously. Second, model 7 checks for whether our functional form— a linear and quadratic term—produces the results as an artifact of another process. Though the coefficient estimates suggest that the inequality between insiders and outsiders peaks in the heart of the range of the observed data (see Table 4), one might worry that inventions of high complexity never actually realize a downturn but rather that the data simply exhibit decreasing marginal returns to increasing levels of interdependence. Model 7 tests for this possibility by using a log-quadratic specification for interdependence. In other words, for both the main effects and interaction terms, this model includes the natural log of $k$ and the square of $k$. Since this functional form allows for decreasing marginal effects without a significant effect on the quadratic interdependence term, it allows us to test this possibility. Even under this specification, the interactions between the squared interaction measure and the proximity indicators appear strongly significant, meaning that we can also reject this possibility.

INSERT TABLES 2, 3, & 4 ABOUT HERE

Figures 2, 3, and 4, based on model 6, depict how insider and outsider citation rates typically vary as a function of interdependence. Each figure graphs the estimated probability of a citation as a function of the interdependence of the knowledge in the focal patent and its proximity to the template in social or geographic space. Because of the interactions inherent in the logit, we set all other variables to their mean values for the purpose of creating these charts. Figure 2 shows the difference between inventors inside versus outside the firm. Figure 3 compares inventors residing in the same zip code as the focal patent to those located an average distance away (~665 miles). And Figure 4 illustrates within versus cross-class citations. As expected, the value of superior access to the template reaches a maximum for knowledge of moderate interdependence, regardless of whether the superior access comes from organizational membership, geographic proximity, or technological community. In addition to being significant, the effects have substantial economic import. For simple or highly complex knowledge, the insider has no greater likelihood than the outsider of attaining and building on the knowledge in a focal patent. For knowledge of moderate complexity at the gap-maximizing levels of $k$ shown in Table 4, a firm insider is 218% more likely than an outsider to transfer knowledge effectively; an inventor located in the same zip
code is 160% more likely to absorb the knowledge in a region than one 665 miles distant; and a technological insider is 238% as likely as a technological outsider to build on knowledge in the class.\textsuperscript{14}

\textbf{DISCUSSION}

The analysis of patent citation patterns supports our basic theoretical perspective on knowledge diffusion: search in the space of possible combinations of ingredients offers a useful lens for understanding the flow of knowledge. Recipients connected to the source of the knowledge by dense social networks have preferential access to the original success, which serves as a template during the assimilation effort. Our analyses considered the relationship between knowledge complexity, access to the original knowledge, and the likelihood of diffusion, using three proxies for social connection: within vs. outside a firm, nearby vs. distant in physical space, and within vs. outside a technological community. All recipients, near and far, compete on equal footing when assimilating simple knowledge; incremental search suffices to reproduce simple knowledge, so guidance from a prior success has little value. Highly complex knowledge, on the other hand, equally resists diffusion to both classes of would-be recipients. Hence, at both extremes of complexity, the close recipient has no lasting advantage over the distant. In contrast, for knowledge whose ingredients display a moderate degree of interdependence, superior but imperfect access to the template translates into better knowledge reproduction. The close recipient can complete its initially imperfect reproduction via local search, but local search alone cannot guide the distant recipient to an accurate replica. Thus in our patent data, the largest gap between the ability of a close recipient to build on prior knowledge relative to the ability of a distant recipient arises when the cited patent involves moderate interdependence.

\textsuperscript{14} In all three cases, we compare the magnitude of the gap for knowledge of minimal complexity to its size at its maximum point to assess the magnitude of the effects of knowledge interdependence.
Our findings have an array of practical and theoretical implications. At the most general level, the results speak to the question, when does inequality of knowledge arise across social borders? Our results suggest that the nature of the knowledge, specifically its degree of complexity, plays a critical role. One might initially suspect that highly complex knowledge, the most difficult to reproduce, would create the greatest inequality. But this intuition ignores the fact that inequality in its sharpest form requires some diffusion: to create the most inequity across social boundaries, knowledge must creep up to the edge of the thick patch of connections in which it originated but not beyond. This, we have argued, most likely occurs for moderately complex knowledge.

Accordingly, the results suggest a resolution to the replication / imitation dilemma that has puzzled evolutionary economists and strategy scholars. To achieve a competitive advantage from knowledge, a firm must typically leverage that knowledge across multiple applications, for example, across all its production facilities (Winter, 1995). Yet any would-be replicator with a valuable piece of knowledge faces a dilemma: The profits produced by its original knowledge attract the envious attention of imitators. Valuable knowledge provides a source of sustained advantage only to the extent that it lends itself to replication yet defies imitation. Unfortunately for the innovator, replication and imitation typically go hand-in-hand (Nelson and Winter, 1982). Our results suggest, however, that replication-without-imitation can occur when two conditions arise together: (1) the target knowledge entails moderate complexity; and, (2) the replicator has superior access to the original template. Future research might examine whether these micro-level processes manifest themselves in outcomes at the firm and/or industry level. One might expect that, ceteris paribus, industries based on moderately complex knowledge would display especially wide intra-industry dispersion in long-run financial returns.

The results also speak interestingly to the literature on industrial agglomeration. Researchers frequently cite knowledge spillovers as a prominent reason that firms within an industry cluster together (Marshall, 1890; Krugman, 1991) and congregate near universities (Zucker, Darby and Brewer, 1997). Our results certainly support this point of view: dense social networks, which tend to localize geographically, give firms and individuals close to the source of knowledge an important advantage in reproducing and building on the knowledge. Regardless, some industries cluster while others do not. Though research on economic geography points out that knowledge spillovers can contribute to agglomeration, it does not identify what type of knowledge most likely engenders these clusters. Our findings suggest that industries that rely on moderately complex knowledge more likely form industrial districts. Simple knowledge can
diffuse far and wide because incremental search efforts can substitute for high fidelity communication. As the complexity of knowledge increases, however, a gap emerges between local diffusion and distant diffusion; thus, the potential return to locating near to innovators rises.

More broadly speaking, the level of knowledge complexity in a particular industry or population may determine the regime governing its structure and evolution. At one extreme, much of neo-classical economics views information as flowing freely across firm boundaries influencing, among other things, both the expectations for externalities and growth (e.g., Romer, 1987) and firm-level incentives to innovate (Arrow, 1962). Although this worldview may fit industries marked by relatively simple knowledge, industries drawing on more complex knowledge appear to violate the assumptions of these models. At the opposite extreme, the resource-based view of the firm often construes knowledge as an advantage that remains embedded in its original birthplace, providing rents to those firms fortunate enough to possess it but not able to travel far from its point of origin (e.g., Lippman and Rumelt, 1982; Barney, 1991). Again, this perspective appears congruent with a world characterized by information involving a high level of interdependence, but not by one with simpler knowledge. In the middle—the realm of intermediate complexity—lays a region of path dependency: firms not only benefit from the initial discovery of valuable knowledge, but also leverage that knowledge to extend their competitive advantage.

In addition to influencing industry structure, the nature of the underlying knowledge used by the firm may also have implications for organizational design. Though our empirics depict the firm as a homogenous body (perhaps with geographic divisions internally), real firms have both formal and informal structures that influence the degree to which actors within the firm interact with each other. Managers can influence who likely interacts with whom through the assignment of individuals to facilities, the design of laboratories and factories, and the structure of reporting relationships (Allen, 1977). To distribute knowledge effectively, a firm might usefully expend resources to foment dense social connections between sources and intended recipients of complex knowledge, while letting networks remain sparse elsewhere. Indeed, leaders might fruitfully construe the task of knowledge management not as the construction of central databases of information (as sometimes presented today), but rather as an effort to build appropriate social networks. Effective organizational design, however, surely requires a deeper understanding of how social structure affects knowledge diffusion than considered here; networks have subtle features and nuances that doubtlessly influence their ability to convey knowledge, both simple and complex (Hansen, 1999).
To this point, our argument has largely assumed that the degree of interdependence between combinations of components remains immutable. In the long term, however, the effective interdependence of knowledge may change. Firms and inventors can make investments in R&D to specify interfaces and embed knowledge within physical components, thereby reducing the difficulty of combining a particular combination of components with other elements in the future (Baldwin and Clark, 2000). In structuring knowledge, managers must perform a delicate balancing act. Isolating interdependencies within substructures has important attractions, including the ability to perform a greater number of independent experiments (Baldwin and Clark, 2000) and the capacity to adjust more readily to environmental shifts (Levinthal, 1997). Engineering curricula support this preference with a strong emphasis on reliability, black box design techniques, and the re-use of previously combined components (e.g., Mead and Conway, 1980). Such modularization, however, also entails frequently overlooked costs. Designing and implementing an architecture that isolates interdependencies within substructures involves considerable engineering costs (O’Sullivan, 2001). But those direct costs likely pale in comparison to the indirect costs—the opportunities that the lack of complexity opens for new entrants (Rivkin, 2000), the reduction in variety from which developers can select (Christensen, Verlinden, and Westerman, 2002), and the constraints on potential performance (Fleming and Sorenson, 2001; 2004).

Despite these costs, a secular trend towards modularization may influence the evolution of industries, creating a distinctive pattern. Direct costs likely strike firms as more tangible than indirect costs as they make decisions about where to direct R&D effort. Thus, firms may over-invest in less complex technology as they seek to maximize efficiency. As this process reduces the effective interdependence of the knowledge being diffused, knowledge should flow more easily, generating two industry-level patterns. First, an industry that begins its life in a concentrated region should become less concentrated geographically as the advantage of preferential access to the template declines (see Audretsch and Feldman, 1996, for related ideas).15 Second, the move towards less complex knowledge likely reduces differentiation across firms’ products over time, leading to more intense price competition and efforts to control standard interfaces and key modules—a pattern identified in the product lifecycle literature.

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15 This pattern seems consistent with the evolution of the software industry, for instance. Early on, knowledge localized to an extreme: understanding of a new piece of code resided in the head of a single developer or a small group of developers in a university, government, or large corporate computing facility. Inventors developed local languages for specific hardware. Over time, programmers developed techniques for reducing the interdependencies in code. Higher-level languages such as Cobol and C allowed programmers to divorce code from specific hardware. Meanwhile, software production has dispersed geographically—beyond Silicon Valley, Route 128, and IBM’s Armonk home, to Seattle, Austin, and even Bangalore.
To reiterate, our results demonstrate that knowledge complexity importantly influences the dynamics of diffusion. More specifically, the degree of inequality across social boundaries reaches its climax when moderately complex knowledge lies beneath these differences. Moderately complex knowledge generates the greatest disparities across social borders because preferential access to information—available through the denser social networks within these boundaries—provides the largest advantage in absorbing and extending knowledge when that information involves an intermediate degree of interdependence. Our empirical analyses support this proposition by finding that knowledge complexity moderates the proportion of future citations: (i) within versus outside the firm, (ii) geographically proximate to, versus distant from, the inventor, and (iii) internal versus external to the technological class. In all three cases, intermediate levels of knowledge complexity drive a wedge between the spread of knowledge internally and its diffusion across boundaries. Though our empirical results come from patent data alone, the basic logic of our hypotheses applies to knowledge in general, not just the knowledge underlying inventions. Hence, future research might usefully examine these dynamics across a wide range of applications—including organizational learning, the diffusion of management practices, knowledge management, and the sustainability of knowledge-based competitive advantage.

**BIBLIOGRAPHY**


Durkheim, Emile. 1912. *The Elementary Forms of Religious Life*.


Scherer, Frederic M. 1984. Innovation and Growth: Schumpeterian Perspectives. Cambridge, MA:


Figure 1: Calculation of interdependence for patent #5,136,185

Subclass 326/16: Test facilitate feature
E = 205 other subclasses /116 patents = 1.77

Subclass 326/82: Current driving fan in/out
E = 101/50 = 2.02

Subclass 326/31: Switching threshold stabilization
E = 104/69 = 1.51

Interdependence k = 4 subclasses /(1.77+2.02+1.21+1.51) = 0.61

Table 1: Descriptive statistics and correlations

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<th>8</th>
<th>9</th>
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<td>0.49</td>
<td>0.30</td>
<td>.03</td>
<td>.00</td>
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<td>.02</td>
<td>.10</td>
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<td>-.03</td>
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<td>.05</td>
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<td>.02</td>
<td>-.04</td>
<td>-.02</td>
<td>-.03</td>
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<td>.05</td>
<td>.05</td>
<td>.01</td>
<td>.00</td>
<td>.00</td>
<td>.01</td>
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<td>.43</td>
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<td>-.01</td>
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<td>.06</td>
<td>.02</td>
<td>.06</td>
<td>.10</td>
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<td>6. Activity control</td>
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<td>.08</td>
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<td>.05</td>
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<td>9. Backward non-patent citations</td>
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<td>10. Number of classes</td>
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<td>.49</td>
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<td>11. Number of subclasses</td>
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<td></td>
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Table 2: Rare events logit models of the likelihood of a focal patent receiving a citation from a future patent

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4 Only same assignee</th>
<th>Model 5</th>
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<tbody>
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<td>k</td>
<td>1.362*** (0.298)</td>
<td>1.687*** (0.302)</td>
<td>1.526*** (0.307)</td>
<td>4.821*** (0.359)</td>
<td>1.444*** (0.321)</td>
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<tr>
<td>k²</td>
<td>-0.535*** (0.084)</td>
<td>-0.892*** (0.082)</td>
<td>-0.793*** (0.074)</td>
<td>-4.208*** (0.117)</td>
<td>-0.704*** (0.098)</td>
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<td>k x same assignee</td>
<td>2.969*** (0.515)</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>k² x same assignee</td>
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<td>6.047*** (0.213)</td>
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<td>k² x - ln (dist)</td>
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<td>-0.794*** (0.044)</td>
<td>-4.566*** (0.074)</td>
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<tr>
<td>k x same class</td>
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<td>3.019** (1.146)</td>
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<tr>
<td>k² x same class</td>
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<td>-1.396*** (0.363)</td>
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<td>Same assignee</td>
<td>.423 (0.278)</td>
<td>.343 (0.280)</td>
<td>.389 (0.276)</td>
<td>.432 (0.281)</td>
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<tr>
<td>- Ln (dist)</td>
<td>.516*** (0.031)</td>
<td>.499*** (0.031)</td>
<td>.428*** (0.031)</td>
<td>.500*** (0.066)</td>
<td>.499*** (0.030)</td>
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<tr>
<td>Same class</td>
<td>3.838*** (0.307)</td>
<td>3.800*** (0.306)</td>
<td>3.663*** (0.307)</td>
<td>1.878*** (0.394)</td>
<td>3.837*** (0.302)</td>
</tr>
<tr>
<td>Subclass overlap</td>
<td>4.255*** (0.317)</td>
<td>4.230*** (0.316)</td>
<td>4.190*** (0.317)</td>
<td>3.767*** (0.349)</td>
<td>4.114*** (0.316)</td>
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<tr>
<td>Activity control</td>
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<td>.393 (0.287)</td>
<td>.389 (0.287)</td>
<td>-.746** (0.248)</td>
<td>.477 (0.388)</td>
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<td>Recent technology</td>
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<td>.122 (0.171)</td>
<td>.195 (0.170)</td>
<td>.010 (0.309)</td>
<td>.096 (0.165)</td>
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<td>.002 (0.013)</td>
<td>.013 (0.013)</td>
<td>.025** (0.008)</td>
<td>-.001 (0.014)</td>
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<td>Backward non-patent citations</td>
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<td>.018** (0.006)</td>
<td>.014* (0.006)</td>
<td>-.126*** (0.037)</td>
<td>.019** (0.005)</td>
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<td>.070 (0.140)</td>
<td>.054 (0.140)</td>
<td>.456 (0.249)</td>
<td>.041 (0.138)</td>
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<td>-.016 (.045)</td>
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<td>.170*** (.048)</td>
<td>.001 (.044)</td>
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<tr>
<td>Constant</td>
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<td>-9.224*** (0.714)</td>
<td>-9.953*** (0.703)</td>
<td>-7.148*** (1.208)</td>
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<td>-22,262.4</td>
<td>-22,261.4</td>
<td>-2,294.1</td>
<td>-22,255.9</td>
</tr>
</tbody>
</table>

* 72,801 dyads (52% realized ties versus .0004% in population); • p < .05; •• p < .01; ••• p < .001
Table 3: Rare events logit models of the likelihood of a focal patent receiving a citation from a future patent

<table>
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<th>Model 7</th>
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<tr>
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<td>Log-quadratic</td>
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<tr>
<td>(k^1)</td>
<td>1.070••</td>
<td>.407</td>
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<tr>
<td></td>
<td>(.362)</td>
<td>(.253)</td>
</tr>
<tr>
<td>(k^2)</td>
<td>-.359••</td>
<td>-.279••</td>
</tr>
<tr>
<td></td>
<td>(.107)</td>
<td>(.091)</td>
</tr>
<tr>
<td>(k^1) x same assignee</td>
<td>6.231***</td>
<td>7.449***</td>
</tr>
<tr>
<td></td>
<td>(.555)</td>
<td>(.270)</td>
</tr>
<tr>
<td>(k^2) x same assignee</td>
<td>-9.851***</td>
<td>-12.347***</td>
</tr>
<tr>
<td></td>
<td>(.273)</td>
<td>(.118)</td>
</tr>
<tr>
<td>(k^1) x - \ln (dist)</td>
<td>.885***</td>
<td>.553***</td>
</tr>
<tr>
<td></td>
<td>(.139)</td>
<td>(.074)</td>
</tr>
<tr>
<td>(k^2) x - \ln (dist)</td>
<td>-1.025**</td>
<td>-.938***</td>
</tr>
<tr>
<td></td>
<td>(.066)</td>
<td>(.058)</td>
</tr>
<tr>
<td>(k^1) x same class</td>
<td>5.733***</td>
<td>4.722***</td>
</tr>
<tr>
<td></td>
<td>(1.032)</td>
<td>(.605)</td>
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<tr>
<td>(k^2) x same class</td>
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<td>-5.874***</td>
</tr>
<tr>
<td></td>
<td>(.330)</td>
<td>(.253)</td>
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<tr>
<td>Same assignee</td>
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<td>-.084</td>
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<td></td>
<td>(.278)</td>
<td>(.281)</td>
</tr>
<tr>
<td>- \ln (dist)</td>
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<td>.345***</td>
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<tr>
<td></td>
<td>(.029)</td>
<td>(.029)</td>
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<tr>
<td>Same class</td>
<td>3.448***</td>
<td>11.028***</td>
</tr>
<tr>
<td></td>
<td>(.299)</td>
<td>(1.013)</td>
</tr>
<tr>
<td>Subclass overlap</td>
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<td>3.984***</td>
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<tr>
<td></td>
<td>(.314)</td>
<td>(.400)</td>
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<td>.611***</td>
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<td>(.289)</td>
<td>(.294)</td>
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<td>(.014)</td>
<td>(.013)</td>
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<tr>
<td>Backward non-patent citations</td>
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<td>(.005)</td>
<td>(.005)</td>
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<td>(.045)</td>
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<td>-10.678***</td>
</tr>
<tr>
<td></td>
<td>(.675)</td>
<td>(.779)</td>
</tr>
</tbody>
</table>

Log-likelihood     | -22,251.1        | -22,248.8        |

† First order \(k\) terms linear in model 6 and logged in model 7

* 72,801 dyads (52% realized ties versus .0004% in population); • p < .05; •• p < .01; ••• p < .001
Table 4: Implied points of maximal insider advantage

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Model 5</th>
<th>Model 6</th>
</tr>
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<tbody>
<tr>
<td>Same vs. other assignee</td>
<td>k = .32</td>
<td>k = .55</td>
</tr>
<tr>
<td>Geographic proximity</td>
<td>k = .43</td>
<td>k = .54</td>
</tr>
<tr>
<td>Same vs. different tech</td>
<td>k = .53</td>
<td>k = .63</td>
</tr>
</tbody>
</table>

Figure 2: Citation probability as a function of interdependence within and outside firm boundaries
Figure 3: Citation probability as a function of interdependence near to and far from the inventor in geographic space

Figure 4: Citation probability as a function of interdependence within and across technological classes