Risk-spreading properties of common tax and contract instruments

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Many tax systems require payment by means of fixed fees, percentages of gross revenues (royalties or ad valorem taxes), or percentages of net income (profit shares or income taxes). Even when payments due under such instruments have the same expected value, their risk-spreading properties may differ. For equal expected levies, profit-sharing is often ranked as the most effective means of risk-spreading, followed by royalty payments, and finally by fixed fees. When revenues and costs are both uncertain, however, we demonstrate that this common risk-ranking is not generally valid and discuss reasons for its breakdown.

1. Introduction

In this article we analyze the relative risk-spreading properties of lump-sum taxes, royalties, and profit shares. For equal expected payments, conventional wisdom generally ranks profit-sharing as the most effective means of risk-spreading, followed in turn by royalties and lump-sum taxes. We demonstrate that while this ranking is often valid, it need not hold in nonpathological instances when both costs and revenues are stochastic.

It should be noted immediately that we focus only on how these three contractual instruments spread purely exogenous risks, and do not examine many of the diverse aspects of contractual relations that have interested economists. Specifically, we do not treat incentive questions that might arise from differences in risk attitudes, differential information and ability to take actions, or imperfect monitoring ability. Likewise, we address neither problems arising from imperfect auditing of revenues and costs, nor the issues raised by effects of a levy on production and investment decisions.

Even after restricting our focus along these lines, however, the risk-spreading aspects of many policy-relevant contracts remain within our purview. For instance, not only have various combinations of fixed fees, royalties, and profit taxes been the primary forms of levy in past Third World mineral contracts, but Law of the Sea Conference negotiators have turned to these instruments when working out financial arrangements for future deep seabed mining of manganese nodules. Similarly, oil leases on the U.S. Outer Continental Shelf have predominantly consisted of a fixed-fee bonus bid with a prescribed royalty. Other examples include: fixed advances combined with subsequent royalty payments in book publishing; management compensation schemes made up of bonuses, salaries, and profit-shares; and fixed wages, rents, or share contracts under sharecropping.

If markets for contingent claims were complete and perfect, of course, there would be no need to examine the risk-sharing properties of tax instruments whose levels depend

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on the variation of revenues and costs across states of nature. In actuality, however, there is little question about the importance of risk-spreading properties of contractual provisions. There is also little question about which instruments are held most risky, even when expected payments are constant. For example, in justifying their "resource rent tax" proposal, Garnaut and Ross (1977, p. 81) explain that from the standpoint of "... risk of loss for any given tax yield... a fixed annual or once-for-all license fee is the least satisfactory and a profits tax the most satisfactory of [the fixed license fee, the specific or ad valorem royalty, and the profits tax]."

Commenting on oil leasing arrangements, Leland (1978, p. 433) similarly notes that "profit sharing payments share risk more effectively [than royalties]," while Reece (1979, p. 668) goes on to say that "bonus bidding schemes, of course, do not share risk at all." In an agricultural context, fixed rent and wage contracts are said to place all the risk on tenant and landowner, respectively, while share tenancy (akin to royalty payments) "... may then be regarded as a device for risk sharing" (Cheung, 1969, p. 26).

To examine the validity of this intuitive risk ranking, we analyze the case of a risk averse party (the "firm") opposite a risk neutral party (the "government"). Because the government will be indifferent among all uncertain prospects with the same expected value, we can compare the firm's expected utility across alternative contractual instruments to determine which is "riskiest." A number of important problems, such as the effects of tax policies of a risk neutral government on private risk-taking (Arrow and Lind, 1970), fall directly within this framework, while others have the same formal structure.

For the firm we assume an increasing, concave, twice continuously differentiable utility function $U$ over a single monetary attribute. Revenues ($r$) and costs ($c$) are random variables with a joint density that defines profit ($\pi = r - c$). We assume agreement between the firm and government on probabilities. The means of $r$, $c$, and $\pi$ are denoted by $\bar{r}$, $\bar{c}$, and $\bar{\pi}$, and their (finite) variances by $\sigma^2_r$, $\sigma^2_c$, and $\sigma^2$. The covariance of $r$ and $c$ is given by $\sigma_{rc}$. Profits are shared at a constant rate $\rho \in (0, 1)$, with expected payments thus equal to $\rho(\bar{r} - \bar{c})$. Royalties are levied on $r$ at a constant rate $\beta \in (0, 1)$, and are set to yield the same expected payments as profit-sharing. Thus, $\beta \bar{r} = \rho(\bar{r} - \bar{c})$, or $\beta = \rho(1 - \bar{c}/\bar{r}) = \rho m$, where $m$ is the expected profit margin, which is assumed to be positive. Finally, fixed fees are set at the expected level of the other charges: $\rho(\bar{r} - \bar{c}) = \rho m \bar{r} = \beta \bar{r}$.

2. Profit-sharing rules and fixed fees compared

In this case, the firm either pays a random share of profits $\rho \pi$ or a sure fee $\rho \bar{\pi}$ with the government receiving the same expected payment in either case. Since the risk averse firm would prefer to receive a sure payment rather than an uncertain one (Jensen's inequality), intuition suggests that the firm would rather pay the uncertain income tax. Mean variance analysis shows the same mean, $(1 - \rho)\bar{\pi}$, for both alternatives, but a variance for the profit-sharing alternative, $(1 - \rho)^2 \sigma^2$, which is less than the variance of the postfee residual $\sigma^2$. In fact, this fee-profit-share comparison is the only case that we consider in which conventional wisdom is unconditionally vindicated. Interestingly, $E[U(1 - \rho)\pi] \geq E[U(\pi - \rho \bar{\pi})]$ does not immediately follow from Jensen's inequality or a first-order stochastic dominance argument.

This general result does obtain, however, by noting that if $x$ and $h$ lie in the domain of the firm's concave utility function $U$, then $U(x) \leq U(h) + U'(h)(x - h)$. Letting

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1 Clearly, the optimal mode of risk-sharing in such instances is for the firm to receive the mean value of the contract in all states, while the government bears all the risk. In light of the obvious incentive difficulties with such arrangements and our exclusion of such issues in the present context, we shall ignore questions of contractual optimality.
\[ x = \pi - \rho \bar{\pi} \text{ and } h = \pi - \rho \pi, \text{ monotonicity of integration over } \pi \text{ gives} \]
\[ EU(\pi - \rho \bar{\pi}) \leq EU(\pi - \rho \pi) + \rho E \{ U'(\pi - \rho \pi)(\pi - \bar{\pi}) \} \]
\[ \leq EU(\pi - \rho \pi) + \rho \text{ cov } \{ \pi, U'(\pi - \rho \pi) \}. \]

Since \( \text{cov } \{ \pi, U'(\pi - \rho \pi) \} < 0 \), the desired result follows.\(^2\) Recall that Rothschild and Stiglitz (1970) prove that every risk averter will prefer a random payoff \( X \) to a random payoff \( Y \) if and only if \( Y \) is a mean-preserving spread of \( X \).\(^3\) Thus, (1) says that the probability density of the postfee residual is a mean-preserving spread of the density of the postprofit-share residual.

As a simple but illuminating special case of this result, let the firm’s prelevy profits take on just two values, \( \pi_1 \) and \( \pi_2 \), with equal probability, where \( 0 < \pi_1 < \pi_2 \). The firm then receives either \( \pi^F_1 = \pi_1 - \rho \pi_1 \) or \( \pi^F_2 = \pi_2 - \rho \pi_2 \) with equal probability under profit-sharing. Under the fee the firm receives
\[ \pi^F_1 = \pi_1 - \frac{1}{2} \rho (\pi_1 + \pi_2) \text{ or } \pi^F_2 = \pi_2 - \frac{1}{2} \rho (\pi_1 + \pi_2) \]
with equal probability. An easy computation shows that the firm receives the same expected profits \( \bar{\pi} \) in each case. Furthermore, \( \pi^F_1 < \pi^F_1 \) and \( \pi^F_2 > \pi^F_2 \), since \( \pi_1 < \pi_2 \). This situation is shown in Figure 1, which illustrates the preference of a risk averse firm for a profit-sharing over a fee contract. Here, \( \pi^F_i \) and \( \pi^F_j \) refer to profits in state \( i \) under the fee and profit-share, respectively.

Figure 1 suggests that the effect of a fee is to move probability mass from the center of the profit-share density toward its tail while preserving its mean, and thus that the random postfee residual is a mean-preserving spread of the random postprofit-sharing residual. Equation (1), in conjunction with the Rothschild-Stiglitz result, shows this result to be generally valid. In the sequel we shall use this figure to illustrate nonpathological cases that reverse the intuitive risk-rankings of fees relative to royalties and royalties relative to profit shares.

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\(^2\) We thank Michael Ward for greatly simplifying our initial argument.

\(^3\) Intuitively, this means that \( Y \) can be obtained from \( X \) by taking some of the probability mass from the center of the probability density of \( X \) and moving it to the density’s tails in a way that leaves its mean unchanged. Prospect \( Y \) is then “riskier” than prospect \( X \).
3. Fixed fees and royalties compared

Consider first the choice between royalty and fee contracts in a mean-variance framework. Under the fee contract the firm keeps \( r - c - \beta \bar{F} \), with mean \((1 - \beta) \bar{F} - \bar{c}\) and variance \( \sigma_c^2 + \sigma_{rc}^2 - 2 \sigma_{rc} \). Likewise, under a royalty scheme it keeps \((1 - \beta)r - c\), with the same mean but variance \((1 - \beta)^2 \sigma_c^2 + \sigma_{rc}^2 - 2(1 - \beta)\sigma_{rc}\). A royalty will be preferred to a fee if and only if the variance of the postroyalty residual is less than the variance of the postfee residual, or

\[
\sigma_{rc} < \frac{(2 - \beta) \sigma_c^2}{2}.
\]

Note that this condition always holds when \( \sigma_{rc} < 0 \): In this case cost and revenue can be regarded as negatively correlated securities, the variance of whose combination is less than the sum of their individual variances. Moreover, (2) suggests that a royalty will also be preferred to a fee when \( \sigma_{rc} \) is positive but small relative to \( \sigma_c^2 \), but that for sufficiently large \( \sigma_{rc} \) a seemingly counterintuitive preference for fees might arise.

We can use this insight to construct a simple von Neumann-Morgenstern example of such a preference. Suppose that there are two equiprobable revenue-cost realizations, \((r_1, c_1)\) and \((r_2, c_2)\). Under a fee contract in state \( i \), the firm receives \( \pi_f^i = r_i - c_i - \beta \bar{F} \) and achieves expected utility \( \frac{1}{2} U(\pi_f^i) + \frac{1}{2} U(\pi_c^i) \). Assume that \( r_2 < r_1, c_2 < c_1, \) and \( r_i - c_i < \bar{F} - \bar{c} < r_2 - c_2 \), or, equivalently, that the covariance of revenues and costs is sufficiently positive. An easy computation then gives \( \pi_f^i < \pi_c^i < \pi_c^j \). Under a royalty contract, on the other hand, the firm receives \( \pi_c^i = r_i - c_i - \beta \bar{F} \) in state \( i \) and achieves expected utility \( \frac{1}{2} U(\pi_f^i) + \frac{1}{2} U(\pi_c^i) \). Since \( r_1 > \bar{F} \) and \( r_2 < \bar{F} \), however, we have immediately that \( \pi_f^i < \pi_f^j \) and \( \pi_c^i > \pi_c^j \), respectively. If we interpret \( A \) as a royalty and \( B \) as a fee in Figure 1, then the constructed relationships among the various postroyalty residuals illustrate that a fee can produce higher expected utility than a royalty for concave \( U \). Since the reverse can also be true, depending on the covariance structure of revenues and costs, we see that an unambiguous royalty-fee risk-ranking is impossible without prior knowledge of these structures.

To understand the intuition underlying this example, note that state \( 1 \) is characterized by high revenues and costs but by low preroyalty profits. In state 2, on the other hand, revenues and costs are both low, but preroyalty profits are high. Hence, in state 1 a royalty payment, which falls on revenue alone, makes the low-profit situation under a fee payment even worse by decreasing the revenues that the firm has available to meet its relatively high costs. This implies that \( \pi_f^i < \pi_f^j \). In state 2, in contrast, a royalty scheme further improves the high-profit situation resulting under fee payment: In effect, the government absorbs part of the firm’s revenue decline through decreased royalty collection relative to the level of fee collection. Hence, the government aids the firm in meeting its already low costs, implying that \( \pi_f^i > \pi_f^j \). Since the fee shields the postroyalty residual in the low-residual state, while leaving the mean of the residual unchanged, a risk averter would choose a fee over a royalty.

Finally, we derive conditions under which the density of \( \pi_f \) is a mean-preserving spread of the density of \( \pi_c \). Note that \( \pi_f = \pi_c + \beta (r - \bar{F}) \) and, since \( U \) is concave, that

\[
U(\pi_f) \leq U(\pi_c) + \beta (r - \bar{F}) U'(\pi_c).
\]

Taking the expectation of (3) over \( r \) and \( c \) then gives

\[
EU(\pi_f) \leq EU(\pi_c) + \beta E\{(r - \bar{F})U'(\pi_c)\} \leq EU(\pi_c) + \beta \operatorname{cov} \{r, E_{cr} U'(\pi_c)\}, \tag{4}
\]

where \( E_{cr} \) denotes the expectation operator over \( c \) conditioned on \( r \).

Two remarks about (4) are in order. First, whether the density of \( \pi_f \) is a mean-

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\textsuperscript{4} If \( E_x \) is the expectation operator over \( X \), then

\[
E_x\{(r - \bar{F})U'(\pi_c)\} = E_x\{rU'(\pi_c)\} - \bar{F} E_x U'(\pi_c) = E_x\{r E_{cr} U'(\pi_c)\} - \bar{F} E_x \{E_{cr} U'(\pi_c)\} = \operatorname{cov} \{r, E_{cr} U'(\pi_c)\}.
\]
preserving spread of the density of $\pi^f$ turns on the sign and magnitude of the covariance term. The intuition behind this result is analogous to that behind the two-state example described earlier. If the covariance of revenue and conditional expected marginal utility is negative, then (4) indicates that a royalty is preferred to a fee. This situation will hold when a rise in revenue causes a rise in the conditional expected postroyalty residual, and hence a fall in conditional expected marginal utility. Likewise, a fee can be preferred in the reverse instance, or when a surge in revenue entails a decline in the postroyalty residual because of increased costs.

Second, note that (4) and its associated interpretation can also be used to rank instruments involving mixtures of fees and royalties relative to each pure instrument. Suppose that the residual of a mixed instrument $\pi^M$ is given by $\alpha\pi^R + (1 - \alpha)\pi^F$, $\alpha \in (0, 1)$. Since $\pi^F = \pi^R + \beta(r - \bar{r})$, we can substitute in this expression to obtain $\pi^M = \pi^R + \alpha\beta(r - \bar{r})$, which can be used to derive analogs to (3) and (4) and to rank $\pi^M$ relative to $\pi^F$. Substitution for $\pi^R$ in the expression for $\pi^M$ can be used in the same way to rank $\pi^M$ relative to $\pi^F$.

4. Profit-sharing rules and royalties compared

Finally, consider the choice between profit-sharing and royalty contracts. In the mean-variance framework, a risk averse firm will prefer paying a share of profits $\rho(r - c)$ to a royalty $\rho mr = \rho(1 - \bar{c}/\bar{r})r$ if and only if var $\{(1 - \rho)(r - c]\} < \text{var } \{(1 - \rho m)r - c\}$, or

$$\sigma_{r_c} < F\sigma_r^2 + G\sigma_r^2,$$

(5)

where $F$ and $G$ are positive constants independent of the covariance structure of $r$ and $c$. Equation (5) thus suggests that a sufficiently positive covariance of revenue and cost can lead a risk averse firm to prefer a royalty contract.

In the von Neumann-Morgenstern context we can, in fact, construct an example in which revenue and cost covary positively, and the postprofit-sharing residual is a mean-preserving spread of the postroyalty residual. Under profit-sharing in the two-state example of Section 3, the firm receives $\pi^F_i = (1 - \rho)(r_i - c)$ in state $i$ and expected utility $\frac{1}{2} U(\pi^F_i) + \frac{1}{2} U(\pi^F_{\bar{r}})$. Similarly, a royalty with the same mean yields a residual $\pi^R_i = (1 - \rho m)r_i - c_i$ in state $i$ and expected utility $\frac{1}{2} U(\pi^R_i) + \frac{1}{2} U(\pi^R_{\bar{r}})$. In state 1 suppose that revenue and cost are both low relative to their respective means, but that revenue is high relative to cost. In state 2 the reverse holds. Specifically, let: (i) $r_1/c_1 > \bar{r}/\bar{c}$; (ii) $r_2/c_2 < \bar{r}/\bar{c}$, and (iii) $(1 - \rho m)(r_1 - c_1) < c_1 - c_2$. These conditions yield in succession: (i) $\pi^F_1 < \pi^F_2$; (ii) $\pi^R_2 > \pi^R_1$; and (iii) $\pi^F_1 < \pi^F_2$. This configuration of residuals is illustrated in Figure 1, if instrument $B$ is interpreted as a royalty and $A$ as a profit share. In this example, then, the postprofit-share residual is a mean-preserving spread of the postroyalty residual, leading a risk averse firm to prefer the royalty.

The intuition underlying this example is similar to that described in the preceding royalty-fee comparison. Note first that revenue under a royalty exceeds revenue under a profit share if both are required to have the same mean payment, since

$$(1 - \rho m)r > (1 - \rho)r.$$

Likewise, costs under a royalty exceed those under a profit share, since $c > (1 - \rho)c$. Hence, when the realization of revenues sufficiently exceeds the realization of costs, as in state 1, the first effect can dominate, with the royalty leading to a higher residual. Similarly, since the profit share deducts costs in arriving at its levy, while the royalty does not, the profit share can become the preferred instrument when costs are relatively high, as in state 2. Finally, since the royalty here shields profits in the low-profit state, and the profit share does not, firm risk aversion suggests preference for the royalty.

To derive general conditions under which the density of the postroyalty residual is a mean-preserving spread of postprofit-sharing residual, note that $\pi^R = \pi^F + \rho(\bar{c}/\bar{r})r - c]$. Concavity of $U$ then yields
\[ U(\pi^r) \leq U(\pi^p) + \rho U'(\pi^p)(\bar{c}/\bar{r})r - c, \] 

which, upon taking the expectation over \( r \) and \( c \), becomes

\[
EU(\pi^r) \leq EU(\pi^p) + \rho EU'(\pi^p)(\bar{c}/\bar{r})r - c
\]

\[
\leq EU(\pi^p) + \rho \{ (\bar{c}/\bar{r}) \text{cov}[r, U'(\pi^p)] - \text{cov}[c, U'(\pi^p)] \}. \tag{7}
\]

Like (4) in the case of the royalty-fee comparison, (7) allows us to determine when a royalty is "more risky" than a profit share in light of the covariance structure of revenue and cost. Interpretation of (7) generalizes the discussion of this section's two-state example. If a cost increase signals a decrease in the postlevy residual (an increase in marginal utility) and a revenue increase an increase in postlevy residual (a decrease in marginal utility), then \( \text{cov}(c, U') > 0 \) and \( \text{cov}(r, U') < 0 \). In this case cost and revenue are negatively correlated, (7) holds, and a risk averter always prefers the profit share to the royalty. When cost and revenue are independent or negatively correlated, the situation becomes somewhat more complex. If costs rise in the last case, the increase in revenue may exceed the cost increase, thus decreasing marginal revenue and yielding \( \text{cov}(c, U') < 0 \). The net effect, however, is determined by \( \text{cov}(r, U') < 0 \). If the absolute value of this term is small relative to \( \text{cov}(c, U') \), then it is possible for a risk lover to prefer the royalty, implying a reversal of the usual risk ranking.

Finally, we note in passing that the preceding analysis and its interpretation can be used to rank a mixed profit-share-royalty instrument \( \pi^M = \alpha \pi^p + (1 - \alpha)\pi^R \) relative to each of the pure instruments. The details are entirely analogous to the ranking of a fee-royalty mixed instrument described in the preceding section.

5. Conclusion

We began this article with a conventional risk-ranking that placed profit-sharing as least risky, followed in turn by royalties and fees. We saw that profit-sharing unambiguously dominates fee payment, but that comparisons among the other instruments depend on the underlying covariance structure of revenues and costs. Reversals of the usual risk-ranking can occur for sufficiently positive covariance between revenue and cost, which seemingly cannot be excluded in a variety of contexts. Intermediate goods industries in a competitive economy, especially those with constant returns, are likely to display strong positive correlation between input and output prices. For example, a firm processing oil into plastics or buying logs and selling wood chips may show parallel trends in revenues and costs that could result in preferences for fees over royalties or royalties over profit shares.

Hence, when costs and revenues are independent or move in opposite directions, the conventional ranking of these instruments holds well. When they move together, a closer examination is necessary to evaluate the risk-spreading properties of the system.

References