CONSTITUENCIES WITH FINITE LIVES AND THE VALUATION OF GOVERNMENT BONDS*

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Abstract

The equilibrium value of government debt is derived in a model in which the government cannot precommit to payments on its debt. The reason bondholders receive such payments is the government's concern for the welfare of the old in the context of an overlapping generations model in which capital accumulation is allowed as well. It is shown that under reasonable conditions this model can explain the increased value of the public debt during episodes of high government spending such as wars.

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I Introduction

This paper is concerned with the valuation of government bonds. The usual approach to this problem takes the payments the government will make to bondholders in the future as given. Then, the current value of the bonds is just the current value to individuals of these future income streams. The issue that is modelled here is the equilibrium value of those future payments. Unlike in the positive theories of government debt of Barro (1979) and Lucas and Stokey (1983) I do not permit the government to determine (or precommit) the payments to bondholders when the bonds are issued.[1] Instead governments must at each point decide how much they wish to pay the bondholders whose bonds are maturing.

In the presence of costless lump-sum taxation the cost to society from raising one dollar of taxes to pay out to bondholders is zero. In this case, which is considered by Kotlikoff (1986), the government is indifferent between paying and not paying bondholders. In fact it is also indifferent between these alternatives and paying bondholders one hundred times the amount they were originally promised. The value of government bonds in such an economy is thus essentially indeterminate. On the other hand, suppose that there are some costs, be they minuscule, of raising taxes. Then the question becomes why any money at all is paid to bondholders. By foregoing such payments the governments not only reduces distortions but even avoids all administrative costs. Thus the argument for not paying bondholders is if anything even stronger, although very related, to the argument for capital levies. If bonds are to have any value, the government must have some reason to pay bondholders over and above the fact that it has received money for these bonds in the past. Similarly if capital levies are not to be the most important source of revenue there must be a similar reluctance to exploit the existing capitalists.

There is one traditional reason which has been given for restraint in capital taxation

[1] That such precommittment is unlikely to be relevant is illustrated by the recurrence of nonpayments on international debts. While committment would have the same effect on foreign debt as on the domestic debt considered here, there is an important difference. The creditors in the case of foreign (or municipal) debt are different from the constituents of the government issuing the debt. This makes it more attractive to repudiate such debts but also means that others are more likely to punish a government who engages in such repudiation.

and payments on government bonds. That is that the government fears that if capital levies are imposed they will be expected to become prevalent in the future thus reducing capital accumulation. Similarly, unless the government maintains a reputation for paying on its bonds it will not be able to borrow in the future. Equilibria in which the government has this view and in which both capitalists and bondholders receive some resources can be constructed. These equilibria require that in response to the government paying "too little" to, say, bondholders the equilibrium switch to another equilibrium. In that other equilibrium government debt is valueless because it will never be honored. The difficulty with this argument is empirical. Many tax reforms are essentially equivalent to lump sum taxes on existing capitalists and bondholders.[2] Any increase in the tax rate on interest receipts has at least some effect of this type. Thus to believe the reputation based models are the sole reason the government abstains from both capital levies and repudiation of bonds one must believe that the actual path of tax rates has always remained just within the bounds of what maintains the government's trustworthiness.[3]

The equally traditional reason that I consider here is that instead (or in addition) the government cares directly for the welfare of bondholders and capitalists (see Keynes (1931) and Clark (1945)). I consider an overlapping generations model with a structure similar to Diamond's (1965) in which people work while young and consume their savings when they are old. Capital levies and expropriation of bondholders then have important distributional consequences. In the model the government is concerned with the welfare of both the young and the old so that it is not indifferent to these distributional effects of taxation.

The paper uses the model to ask under what conditions its qualitative features resemble

[2]Similarly, Chamley (1982) shows that it is possible to construct apparently innocuous tax reforms involving changes in the investment tax credit which are equivalent to capital levies.

[3]A more modern variant of this argument is presented by Kotlikoff, Persson and Svensson (1986). They argue that the reputation for payment can be embodied in a law which the young would purchase from the old. In their model these payments are explicit although they suggest that implicit payments are possible as well. The key ingredient of the model is that deviations from the prescribed strategies in the form of either excessive capital taxation or insufficient payment by the young for the law render the law worthless. Formally, both the level of capital taxation and the payments required from the young are constant. Yet, in practice, both vary. For the model to be correct, these variations must be those that are permitted by the law; they must remain just within the bounds that keep the value of the law intact.

those of actual data on the public debt. In particular I am concerned with explaining why the real value of government debt grows during wars.[4] I also make the model consistent with the observation that wars have had negligible effects on real interest rates at least in the US.[5] I show that as long as preferences are not time separable and, in particular, consumptions in the two periods of life are good substitutes it is possible to explain the increase in the value of public debts during wars.

The use of an overlapping generations model with a government concerned for its subjects is common to the models of optimal fiscal policy of Abel (1986) and Calvo and Obstfeld (1985). The major difference between those models and the model in this paper is that here the current government cares only about the individuals currently alive. In Abel (1986) and Calvo and Obstfeld (1985) the government cares instead about future generations as well. In fact the utility of the generation which will be born next is discounted relative to the utility of the currently young at the same rate as the utility of the young is discounted relative to that of the current old and so on for future generations. This particular form of preferences ensures that the government optimal policy is time consistent. While this is of great technical convenience it is difficult to see why the current government cares about unborn generations when the individuals who are its constituency do not. One would for instance have to believe that individuals care more about the average welfare of future generations than they care about that of their direct descendants.

The paper proceeds as follows. In Section II the model is presented and the path of consumption is derived when there is a government which has a welfare function which depends on the utility functions of the agents currently alive. In Section III I show how to support this equilibrium with government bonds. In Section IV and V I consider instead an economy in which there is explicit bargaining between the young and the old. Section IV studies the Nash bargaining solution while Section V studies the Rubinstein solution. I show that the results do not differ qualitatively from those obtained with the aid of a social

[4] See Barro (1984) pp. 370-71.

[5] See Barro (1984) pp.315-316. However, Benjamin and Kochin (1984) show that in the UK wars are associated with high nominal interest rates.

welfare function in Sections II and III.

II The model: Consumption over Time

The basic structure of the model is an overlapping generations model with constant population. For expository purposes I first make the horizon finite. In other words, there is a last generation born at T which, for simplicity, lives for one period only. Generations born before T live for two periods. This finite horizon model clarifies the issues. I subsequently extend the model to the more usual infinite horizon overlapping generation model.

Output per member of each generation in period t, Y_t is given by:

$$Y_t = e + fK_{t-1} \tag{1}$$

where e is an endowment, K_t is capital per member of each generation at t and f is a constant. The economy thus has available to it a technology with a constant rate of return. The output Y_t can either be used by the government, invested, or consumed. Thus

$$C_{1t} + C_{2t-1} + K_t + G_t = Y_t \tag{2}$$

where G_t is government consumption per member of each generation at t and C_{it} is consumption of individuals born at t during the i'th period of their life.

The utility function of a generation born at t (where t must be smaller than T) can be written as $U(C_{1t}, C_{2t})$. The last generation has a utility function given simply by $V(C_{1T})$.

Following Barro (1979) and Lucas and Stokey (1983) government expenditures are taken as exogenous. This is a weakness in a model in which at least some of the government is modelled explicitly. Yet the sorts of considerations of intergenerational equity that I take as central in the determination of the financing of government expenditures are probably far less central in the determination of major changes in the expenditures themselves. This is particularly true for the sorts of large changes in expenditures such as wars whose financing provides the motivation for this paper.

When choosing how to finance its expenditures the government cares about the two generation currently alive. For simplicity I assume that at each point in time the government seeks to maximize the sum of the utility of the "representative" currently old and times the utility of the "representative" currently young. If all the agents of a given age carry out the same actions (as they will in equilibrium) then the "representative" currently old person has the same preferences as each old person. Thus the objective function of the government at t can be written as:

$$U(C_{1t-1}, C_{2t-1}) \quad +\Phi U(C_{1t}, C_{2t}) \qquad\qquad t < T \tag{3}$$

$$U(C_{1t-1}, C_{2t-1}) \quad +\Phi V(C_{1t}) \qquad\qquad t = T \tag{4}$$

When the government maximizes a welfare function of this type the solution is obviously not invariant even to affine transformations of the agents utility functions. Thus it makes sense to assume in particular that the utility functions in (3) and (4) are concave. I start by letting the government pick the consumptions of the two types of agents at each point in time. I thus defer the discussion of how the government achieves this allocation. Consider first the government's problem at T where its objective function is given by (4). Given that this is the last period the optimal choice for K_T is zero. Thus the maximization of (4) subject to (2) yields:

$$U_{2T-1} - \Phi V_{1T} = 0 \tag{5}$$

where U_{it} represents the partial derivative of the utility of individuals born at t with respect to its i'th argument. The solution of (5) together with (1) and (2) is a set of functions which give consumption of both young and old at T as a function of the consumption of the currently old when they were young, of capital at $T-1$ and of government spending at T. In particular one can write the consumption of the old at T as:

$$C_{2T-1} = g(C_{1T-1}, K_{T-1}, G_T, T) \tag{6}$$

To characterize this function g it is sufficient to differentiate (5) using (1) and (2):

$$\begin{aligned} dC_{2T-1} &= g_{1T} dC_{1T-1} + g_{2T} dK_{T-1} + g_{3T} dG_t 7) \qquad (7) \\ g_{1T} &= -U_{12T-1}/[U_{22T-1} + \Phi V_{11T}] \end{aligned}$$

$$
\begin{aligned}
g2_T &= \Phi f V_{11T}/[U_{22T-1} + \Phi V_{11T}] \\
g_{3T} &= -g_{2T}/f
\end{aligned}
\tag{8}
$$

where U_{ijt} represents the second partial derivative with respect to the i'th and j'th argument of the utility of those born at t. Note that the concavity of the utility function implies that g_{2T} is positive and strictly between 0 and f. Thus the output due to an increase in the capital received from previous generations is distributed in part to members of both generations. Given that G_t is exogenous an increase in this variable simply represents a loss in resources available for consumption. Thus its effect on consumption is the same as that of the loss of some of the output produced by capital.

The sign of g_{1T} is ambiguous. If U_{12} is positive so that increases in consumption while young raise the marginal utility of consumption while old then g_{1T} is positive. If, instead U_{12} is negative so that penuries suffered when young raise the marginal utility of consuming when old, g_{1T} is negative. In this case a fall in consumption when young of the generation born at $T-1$ leads the government at T to value the consumption of those born at $T-1$ more highly. As a result those born at $T-1$ get more and those born at T get less.[6]

If the utility functions are quadratic g_{1T} and g_{2T} are independent of the allocation of consumption. This particular implication of quadratic utility functions is attractive because, otherwise, the variation of $g1$ with respect to C_{1T-1} which would play a role below depends on the relative importance of U_{112T-1} and U_{221T-1}. In what follows I thus restrict attention to quadratic utility functions. This means that I can drop the t subscript from the second partials of the utility function.

Now consider the problem of the government at $T-1$. It wants to maximize (3) for t equal to $T-1$ subject to (1) and (2) and subject to the fact that the consumption of the currently young when they become old will be chosen by the future government according to (6). Substituting for C_{2T-2} using (1) and (2) and for C_{2T} using (6) into (3) and differentiating

[6] It is worth pointing out that the sign of U_{12} is invariant with respect to affine transformations of the utility function even though it is not invariant to general monotone transformations.

with respect to C_{1T-1} and K_{T-1} one obtains the following first order conditions:

$$-U_{2T-2} \quad +\Phi(U_{1T-1}+g_{1T}U_{2T-1})=0 \tag{9}$$

$$-U_{2T-2} \quad +\Phi g_{2T}U_{2T-1}=0. \tag{10}$$

These can be combined to yield:

$$g_{2T}-g_{1T}=U_{1T-1}/U_{2T-1}. \tag{11}$$

By writing these first order conditions as equalities I am assuming interior solutions to the governments optimization problem. From (10) it is clear that such interiority requires that $g_2 - g_1$ be positive. This condition is always met when U_{12} is negative. For (9) to hold with equality g_2 must also be sufficiently big, which as we saw can be achieved with a high value of f. In particular, if one drops the requirement that there be a last generation, the existence of a steady state requires that g_2 equal $1/\Phi$.[7]

Substituting again for C_{2T-1} and C_{2T} into (8) and (9) and differentiating one obtains:

$$A \begin{matrix} dC_{1T-1} \\ dC_{1T-1} \end{matrix} = -U_{22}dG_{T-1}+fU_{22}dK_{T-2}+U_{12}dK_{T-1} \tag{12}$$

where:

$$A=\begin{pmatrix} U_{22}+\Phi[U_{11}+2g_{1T}U_{12}+g_{1T}^2U_{22}] & U_{22}+\Phi g_{2T}[U_{12}+g_{1T}U_{22}] \\ U_{22}+\Phi g_{2T}[U_{12}+g_{1T}U_{22}] & U_{22}+\Phi g_{2T}^2U_{22} \end{pmatrix}$$

For the government's problem to satisfy the second order conditions for a maximum the matrix A must be negative definite. This requires that the diagonal elements of A be negative which is ensured by the concavity of U. It also requires that the determinant of A, Δ be positive. This determinant can be written as:

$$\Delta_T=\Phi U_{22}[U_{11}+2(g_{1T}-g_{2T})U_{12}+(g_{1T}-g_{2T})^2U_{22}]+(\Phi g_{2T})^2[U_{11}U_{22}-U_{12}^2] \tag{13}$$

both of whose elements are positive if U is concave. Note that the second order conditions are thus met independently of the values of g_1 and g_2.

[7] This is the analogue of the familiar condition that in steady states of infinite horizon models the discount rate and the rate of return on capital coincide. Here what is relevant is the rate of return to the members of the currently young generation which is of course lower than the physical rate of return.

Now consider the effect of an increase in G_{T-1} on the sum of C_{1T-1} and K_{T-1}. Since the only reason the government invests in capital is to provide for the consumption of the currently young the sum of C_{1T-1} and K_T can be thought of as the total resources the government spends on the young. The changes in the sum are given by:

$$d(C_{1T-1}+K_{T-1})/dG_{T-1} = -\Phi U_{22}[U_{11}+2(g_{1T}-g_{2T})U_{12}+(g_{1T}-g_{2T})^2 U_{22}]/\Delta \qquad (14)$$

which is negative and whose absolute value is between zero and one. Thus when the government spends one more dollar, the currently young pay for some fraction of this dollar as is reasonable. The actual response of the consumption of the young is given by:

$$dC_{1T-1}/dG_{T-1} = -\Phi U_{22} g_{2T}^2 [U_{22}-U_{12}/f]/\Delta \qquad (15)$$

which is negative as long as either f is sufficiently big or U_{12} is sufficiently small. For f bigger than or equal to one this requires only that the cross derivative U_{12} be smaller than U_{22} which is plausible. The response of capital is given be the difference between expressions (13) and (14) and thus has an ambiguous sign. Yet for plausible parameter values, such as $U_{11}=1$, $U_{22}=.9$, $U_{12}=.5$, $f=1.4$, $\Phi=1/f$ capital also fall when G is big. Indeed for this numerical example as for the others I have considered the fall in capital is so severe that the consumption of the old at T falls as well in spite of the fact that the fall in C_{1T-1} tends to raise C_{2T}.

So far I have considered a model with a last generation. Now suppose instead that new two period lived generations continue to be born ad infinitum. There is then still a function $g(C_{1t,}K_t, G_{t+1,t+1})$ which gives the consumption of the old at $t+1$ in response to their consumption while young and the inherited capital stock. Given this function g, the governments problem at t is identical to the problem I solved for period $T-1$. In particular, equations (13) and (14) are valid, an increase in government spending at t leads to a fall in resources spent on the young at t where these resources are equal to $(C_{1t}+K_t)$.

For this case it is still necessary to derive the function g. This function must give the actual response of C_{2t+1}. In the case of quadratic preferences this response can be computed from the solution of (11). Equation (11) gives the response of C_{1T-1} and K_{T-1} to changes in C_{1T-2} and K_{T-2}. As I mentioned above these responses can be interpreted as the responses of C_{1t} and K_t to changes in C_{1t-1} and K_{t-1}. Using (1), (2) and (11):

$$\begin{aligned} g_{1t} &= dC_{2t-1}/dC_{1t-1} = -d(C_{1t}+K_t)/dC_{1t-1} \\ &= -\Phi U_{12}[U_{11} + 2(g_{1t+1} - g_{2t+1})U_{12} + (g_{1t+1} - g_{2t+1})^2 U_{22}]/\Delta_{t+1} \qquad (16) \\ g_{2t} &= dC_{2t-1}/dK_{t-1} = f - d(C_{1t}+K_t)/dK_{t-1} \\ &= f - f\Phi U_{22}[U_{11} + 2(g_{1t+1} - g_{2t+1})U_{12} + (g_{1t+1} - g_{2t+1})^2 U_{22}]/\Delta_{t+1} \\ &= f(\Phi g_{2t+1})^2[U_{11}U_{22} - U_{12}^2]/\Delta_{t+1} \qquad (17) \end{aligned}$$

which establishes that g_{1t} has the same sign as U_{12} while g_{2t} lies strictly between zero and f. For quadratic preferences, g_{1t} and g_{2t} depend only on calendar time and not on the path of either consumption or capital accumulation. Note that equations (6), (15) and (16) form a system of difference equations backwards in time. Equations (6) give the values of the g's at t, while (15) and (16) give the values at t conditional on the values at $t+1$. The analysis is considerably simplified when one focuses on "steady states" in which the g's do not vary over time. I now investigate whether the economy converges to such steady states when the horizon $(T-t)$ becomes arbitrarily long. To study this question I analyze whether, starting near a steady state, the economy tends to converge to the steady state when we move backwards in time. Before carrying out this computation, it is worth pointing out that, combining (15) and (16):

$$g_{1t}(U_{22}/U_{12}) + g_{2t}/f = 1$$

so that, if g_{2t} converges to the steady state g_2, g_{1t} converges to the steady state g_1. Using this expression for g_{1t} in (16), differentiating and evaluating at the steady state:

$$dg_{2t}/dg_{2t+1} = 21 - g_2/f[1 + (fU_{22} - U_{12})^2/\Phi(U_{11}U_{22} - U_{12}^2)] = 2\Phi[U_{11}U_{22} - U_{12}^2]/\Delta$$

Stability requires that this expression be smaller than one in absolute value. By the second equality the expression is positive. Thus, we can see from the first equality that, as long as g_2/f is larger than one half (so that the effective tax on capital is less than one half), the g's are stable around their steady state values. For instance, for U_{11} equal 1, U_{22} equal .8 and U_{12} equal .5, there exists a stable steady state with g_2/f equals one half for f equal to 1.39 and Φ equal to 2.44. For the same individual utility function stability is ensured even with g_2/f equal to .37 as long as f and Φ equal 1.9 and 1.48 respectively. It is worth noting that, for any quadratic utility function, there is a steady state satisfying (15) and (16) in which g_2 is zero. This steady state is never stable.

II Implementing the equilibrium

So far I have derived the consumptions the government deems optimal. In this section I consider the taxes and subsidies that are necessary to support this allocation. In particular I consider one way in which this allocation can be supported in which government bonds play a central role. I suppose the endowment accrues initially to the young. The young are subject to lump sum taxes. They can also purchase assets. Thus their first period budget constraint is:

$$e - L - C_{1t} = a_t \tag{18}$$

where L are lump sum taxes, while a_t is the value (in terms of consumption units) of individual holdings of bonds and capital at t. The after tax return on holding these assets is R_t. Thus the second period budget constraint is:

$$R_t a_t + S = C_{2t} \tag{19}$$

where S are lump sum transfers. The young at t thus maximize $U(C_{1t}, C_{2t})$ subject to (17) and (18). The first order condition for this problem is:

$$U_{1t}/U_{2t} = R_t$$

It can be seen from (10) that, to support the allocation from the previous section in the steady state the rate of return must obey:

$$R_t = g_2 - g_1 \tag{20}$$

so that the after tax real rate of return is constant. The first term g_2 represents the increased consumption by the old when one more unit of capital is brought forward while the term $-g_1$ corresponds to the increased consumption that results from the decline in the consumption of the young.[8] Obviously the constancy of the real rate is not a general result. It hinges on the constancy of the marginal product of capital, on the fact that the utility function is quadratic and on a horizon of sufficient length so that steady state values of the g's are appropriate.

Given (19), implementation of the allocation from the previous section requires only that the budget sets be such that individuals can afford their consumptions and that the amount of capital they choose to carry over is K_t. The first requirement can be met through appropriate lump sum transfers. In the case of the young these are lump sum taxes (or transfers) which we capture through L. In the case of the old there are several ways of distributing the C_2 units of consumption. The first approach is to simply carry out some lump sum transfers which can be thought to resemble social security payments.

The other approach is to let the old receive some earnings from their assets. In this latter case the government will choose earnings on assets so as to transfer the desired amount of consumption to those old who carry over the expected level of assets. But, how much will an old person receive if she carries over fewer assets ? Can she claim to have lost them and thereby receive a larger lump sum transfer? If this were so there would be a clear disincentive to the accumulation of assets. I assume that there is no credible way for individuals to convey to the government their own consumption and asset holdings. So the government chooses only the earnings per unit of asset. In equilibrium, all individuals carry over equal amounts of assets and receive equal transfer. Yet, if an individual deviates and saves less, she will also consume less in the second period.

As Kotlikoff (1986) points out, it is possible to simultaneously increase earnings on assets

[8] It is worth pointing out that these are "social returns" in the sense that it is only if all the young consume one unit less and invest it in capital that they will all receive $g_2 - g_1$ more units of consumption when they are old. If only one young person consumes less and buys more capital, then, the government would have essentially no incentive to increase the consumption of the old in the future.

and decrease the lump sum transfers to the old (or increase their taxes) thereby continuing to implement the allocation from the previous section. This multiplicity in methods for supporting the allocation can be circumvented to some extent by assuming that there is some cost to operating a program that taxes the old and some other costs associated with giving them earnings on their assets. For simplicity, and so as not to disturb the structure of the model these can be thought of as fixed costs of setting up any particular payments system. Then the government will always use the minimal set of payments systems.

What is unsatisfactory about this method of circumventing the problem is that, in the US economy, we observe simultaneously transfers in the form of Social Security and payments to bondholders. Similarly the government pays both bondholders and owners of capital even though it might well be administratively simpler to, for instance, let only capital gather a rate of return. Obviously a better solution to the problem is to realize that there is heterogeneity among the old. Some have more bargaining power than others and they consume more in their old age. The way the government gives more to some than to others is to have a variety of programs through which funds are disbursed to the old. This bargaining among the old is, however, beyond the scope of this paper. I thus assume simply that there is a certain invariant amount S, which can be thought of as zero, of government transfers to each member of the old generation. The rest of the consumption of the old is received in the form of payments on assets.Thus:

$$C_{2t} = v_{t+1}a_t + S \tag{21}$$

where v_t is the **ex post** ratio of earnings (in consumption units) to the value of individuals's portfolios.[9]

Now suppose that, as (19) requires, v_t is equal to $g_2 - g_1$. The question of whether the allocation from the previous section can be supported as a competitive equilibrium with this interest rate then reduces to whether individuals will choose to carry over the required amount of capital. In other words do the assets the young purchase at t whose value is, via

[9] It is worth noting that the requirement of (20) that all assets earn the same return **ex post** is very weak. In principle it is consistent both with very small payments on government debt (which makes their value small) and with much larger payments.

(20), $(C_{2t} - S)/(g_2 - g_1)$ "correspond" to K_t. There are at least two ways of ensuring that this is so.

The first involves no government debt. The government simply institutes either a tax or a subsidy on capital formation. Suppose this tax equals τ_t so that a unit of capital costs $(1 + \tau_t)$ units of consumption. Then a_t corresponds to K_t if:

$$1 + \tau_t = (C_{2t} - S)/[K_t(g_2 - g_1)]. \tag{22}$$

With this tax there exists a competitive equilibrium in which the return to assets is $g_2 - g_1$ and the allocation is given by that in the previous section. To see what is involved in supporting the allocation it is convenient to analyze Figure 1, the budget set of a young person at t. The allocation requires that the individual consume C_{1t} and C_{2t} where we know by (10) that these consumptions are on an indifference curve with slope $-(g_2 - g_1)$. We ask, suppose that interest rates are given by $g_2 - g_1$, is the allocation an equilibrium. For this to be true, the young's perceived first period endowment must be given by $e - L + S/(g_2 - g_1)$ which can be ensured by varying L. Moreover, a_t must correspond to K_t units of capital so that individuals indeed receive C_{2t} when they are old. If a_t happens to equal K_t no government intervention is needed. If it is less than K_t individuals must receive a subsidy for holding capital while a tax is needed if a_t is bigger than K_t. With this tax or subsidy we achieve the desired goal of making the rate of return equal to $(g_2 - g_1)$ if S is distributed in lump sum fashion while the rest of C_2 is distributed as return on assets.

It is worth noting that, at least in the case in which there is a finite horizon, this equilibrium is locally unique. In other words, if the government imposes a tax like (21) at $T - 1$, then there generally no equilibrium with an interest rate close to but not equal to $(g_{2T} - g_{1T})$. The reason for this is that such a small change in the interest rate would lead to a small change in capital. Yet, it is unlikely that the resulting change in C_{2T} would exactly match the different interest rate and level of assets.

The second method of making a_t correspond to K_t is for the government to issue debt. Naturally, if both capital and bonds are held in a perfect foresight equilibrium with homogeneous agents the payments per dollar invested must be the same for both assets.[10] The easiest way of ensuring this is to assume that the government at $t+1$ decides only on a return per dollar invested at t but does not discriminate between bonds and capital.[11] This means that, even though bonds issued at t need not, in principle, affect resource allocation at $t+1$, they do so because the government decides to treat all assets symmetrically. Then, to implement the equilibrium, the government at t must issue debt whose value per member of the young generation is B_t so that:

$$B_t + K_t = a_t = (C_{2t} - S)/(g_2 - g_1). \tag{23}$$

It is worth noting that the value of government debt will be positive if and only if τ would be positive in (21). It is necessary to issue debt only if the young's accumulation of assets is in excess of the desired capital stock. So, the allocation of the previous section can be supported as a competitive equilibrium if there is government debt whose value is given by (21). Yet, how does the government make debt have this value?

The government must be thought of as issuing pieces of paper. Individuals know that these pieces of paper have the same rate of return as physical assets so they are willing to pay a positive price for them. Moreover, suppose that the market price for these pieces of paper is q. Then individuals are strictly indifferent between an additional piece of paper and q units of current consumption or capital. This means that any allocation with a level of a_t different from that given by (22) can be disrupted by changes in the bonds sold by the government. Note again that individuals who deviate from the proposed equilibrium and accumulate fewer assets (or lose their bonds) actually receive lower consumption so that they will not chose to deviate in this fashion.

One immediate implication of (22) is that an increase in the lump sum transfers S reduces the value of government bonds B. After all bonds are only valued because they are the

[10]In a model with uncertainty it would be enough that the agents that hold the two assets receive the same expected utility per dollar invested in both. Such a model would be consistent with situations in which the relative bargaining power of owners of capital and bondholders is random.

[11]This corresponds to the fact that actual taxation of government bonds moves very much in tandem with taxation on private bonds.

mechanism through which the government distributes payments to the old. If this mechanism is replaced in part with lump sum transfers bonds lose some of their value. It is also clear from the analysis that if this change is just a change in the way money is distributed to the old it would have no effect on the allocation of resources and thus no effect on K.[12]

I now use (22) to analyze how the value of government debt would respond to an increase in G_t with government spending constant in all other periods.

$$\begin{aligned} dB_t/dG_t &= [(dC_{2t}/dC_{1t})(dC_{1t}/dG_t) + (dC_{2t}/dK_t)(d_{Kt}/dG_t)]/(g_2 - g_1) - dK_t/dG_t \\ &= g_1[d(C_{1t} + K_t)/dG_t]/(g_2 - g_1) \qquad (24) \end{aligned}$$

Equation (13) guarantees that the term in brackets is negative while $g_2 - g_1$ must be positive for the government's problem to be well defined. Thus the sign of dB_t/dG_t depends on the sign of U_{12}. If U_{12} is negative, consumption in the two periods are "substitutes" in the sense that consuming more in one reduces the marginal utility of consuming in the other and temporary increases in government spending raise the value of government debt.

When U_{12} is negative the government compensates those with low consumption in the past with more consumption when they are old. If this meant that those who are young during wars can look forward to relatively high consumption when they become old this could rationalize a high value for the assets of the young. Yet, consumption when old of those who are young during wars is not unambiguously high since capital accumulation tends to be low in wars as well. Yet, consumption when old will be high relative to the capital stock. It is for this reason that the value of bonds which represents the value of consumption not given by the value of the capital stock is high when G is high.

IV Nash Bargaining

Up to this point I have solved the model assuming that there is a government interested in the welfare of the two generations. What I have in mind is that the two generations elect, or otherwise sanction, the government which then pursues some combination of the two generations's interest. Perhaps a more convincing, although technically more taxing,

[12]Of course, if the change in S reflects an underlying change, such an increase in the bargaining power of the old then it will affect the distribution of resources.

way of formalizing this is to have the two generations explicitly bargain with each other. This idea is pursued in the next two sections.

I assume that unless this bargaining leads to agreement the two generations can only procure for themselves some subsistence level of consumption s. Thus the budget constraint (2) is only valid if agreement is reached and otherwise consumption is s for both generations. The preferences discussed below (2) remain valid however. I first adopt as the solution to this bargaining problem the axiomatic solution proposed by Nash (see Roth (1979)). This solution is the unique solution which treats players symmetrically, is invariant with respect to affine transformations of the utility function, is independent of irrelevant alternatives and leads to Pareto Optimal outcomes.[13] As shown by Harsanyi (1977) (see Roth (1979)) this solution can also be obtained by assuming that the two agents negotiate in two rounds. In the first round both players make offers. If the offers are compatible all is well and the negotiations end. If the offers are not compatible there is a second round in which one of the agents, (the one who is in a weaker bargaining position) is given the choice of accepting the other agent's offer or consuming s.[14]

This solution can be obtained by maximizing the product of the difference between the utilities of each agent and the utility that agent would obtain if he consumed s.

In period T, and once (2) is taken into account this implies the maximization of:

$$[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)][V(Y_T - G_T - C_{2T-1}) - V(s)] \tag{25}$$

with respect to C_{2T-1} where the C's represent consumptions if agreement is reached. Differentiating (24) and rearranging one obtains:

$$U_2/[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)] = V_1/[V(Y_T - G_T - C_{2T-1}) - V(s)] \tag{26}$$

[13]As Roth (1979) shows if one drops the Pareto Optimality requirement, the Nash solution remains one of only two solutions, the other being that no agreement is reached.

[14]An even more recent (and similar) interpretation is provided in Binmore, Rubinstein and Wolinsky (1986). They show that this solution is the unique equilibrium of a specific extensive form game. In this game each side makes alternating offers. After one side makes an offer, the other side can either accept it, at which point the game ends or make a counteroffer. However if the offer is not accepted and before the counteroffer can be uttered there is a probability that any possibility of agreement disappears and that the agents simply receive their subsistence levels of consumption.

U_2 is the value to an old agent of an additional unit of consumption while $[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)]$ is the loss in utility if the current agreement falls apart. Thus the LHS of (25) is the ratio of the benefits of a slightly better agreement for an old person to the value of the equilibrium agreement. The RHS can be given the same interpretation for the case of a young agent.

In the interpretation of Harsanyi (based on Zeuthen's (1930) model of negotiation) one imagines that if an agent insists on a higher payoff there is some probability that the agreement will fall apart, otherwise he will get his higher payoff. Then the ratio in the LHS of (25) is proportional to the probability of destroying the agreement that leaves an old agent indifferent between insisting on a slightly higher payoff and accepting the status quo. Now suppose that the LHS is bigger than the RHS. This means that the old are willing to accept a higher probability of failure when pressing for an improvement in their allocation. In the Zeuthen-Harsanyi model this means the young are in a weaker bargaining situation and therefore concede, C_{2T-1} rises and C_{1T} falls. This tends to reestablish the equality of the LHS and the RHS because an increase in C_{2T-1} lowers the numerator of the LHS while reducing the denominator of the LHS so the LHS falls. Similarly, a fall in C_{1T} raises the numerator of the RHS and reduces the denominator of the RHS so the RHS rises.

To see how the outcome of the bargaining process changes when C_{1T-1}, K_{T-1} or G_{T-1} change one differentiates (25) and obtains:

$$dC_{2T-1} = g_1 dC_{1T-1} + g_2 dK_{T-1} + g_3 G_{T-1} \tag{27}$$

$$\begin{aligned}
g_1 &= -\frac{U_{12}/U_2 - [U_1(C_{1T-1}, C_{2T-1}) - U_1(C_{1T-1}, s)]/[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)]}{[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)]D/U_2} \\
g_2 &= f\{V_{11}/[V(Y_T - G_T - C_{2T-1}) - V(s)] - V_1^2/[V(Y_T - G_T - C_{2T-1}) - V(s)]^2\}/D \\
g_3 &= -g_2/f \\
D &= \frac{U_{22}}{[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)]} - \frac{U_2^2}{[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)]^2} + \\
&\quad \frac{V_{11}}{[V(Y_T - G_T - C_{2T-1}) - V(s)]} - \frac{V_1^2}{[V(Y_T - G_T - C_{2T-1}) - V(s)]^2}
\end{aligned}$$

where the derivatives whose subscripts are omitted are evaluated at the Nash equilibrium.

Hence, as in (7), g_2 is strictly between zero and f while g_3 is strictly between zero and minus one. Thus any extra resources in the form of capital or lower government spending are shared by both generations.

Unlike in previous sections the sign of U_{12} is now not enough to determine the sign of g_1. A negative U_{12} means that falls in C_{1T-1} raise the old's marginal utility of consumption. This, by itself, would tend to make the old more bold in their demands and raise their consumption. Yet, as we saw before what is relevant when bargaining is the ratio of the marginal utility of consumption to the loss from scrapping the current agreement i.e. the change in $U_2/[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)]$. The proportional change in this expression is equal to the proportional change in the numerator (the first term in the numerator of the equation which gives g_1) minus the proportional change in the denominator (the second term in the same numerator). A negative U_{12} means that this first term is negative but, since it also implies that $[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)]$ rises the second term is positive. In other words a negative U_{12} raises the loss from moving to the disagreement outcome as well.

However, by the mean value theorem there is a level of consumption x between C_2 and s such that the change in $[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)]$ equals $U_{12}(x)$ times $(C_2 - s)$. Similarly, by the mean value theorem the ratio $(C_2 - s)/[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)]$ is equal to U_2 evaluated at a level of consumption y between C_2 and s. This means that the proportional change in $[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)]$ equals $U_{12}(x)/U_2(y)$. I noted above that the proportional change in U_2 equals $U_{12}(C_2)/U_2(C_2)$. The concavity of the utility function implies that $U_2(C_2)$ is smaller than $U_2(y)$. Thus as long as $U_{12}(C_2)$ is not much below $U_{12}(x)$, i.e. as long as U_{12} does not fall substantially when consumption in the second period rises, the expression $U_2/[U(C_{1T-1}, C_{2T-1}) - U(C_{1T-1}, s)]$ falls when C_{1T-1} rises. In particular, if the preferences are quadratic so that U_{12} is constant and negative, a rise in the consumption when young lowers the bargaining ability of the old and makes them receive a lower level of consumption.[15]

The main difficulty with the bargaining approach considered in this section is that I

[15] The same result now obtains when U_{12} is positive as long as U_{122} is sufficiently negative.

know of no utility functions for which g_1 and g_2 are globally independent of the levels of consumption of the two agents. Obviously, the g's can be locally constant, i.e. invariant to small perturbations in K and C_{1T-1}. For simplicity, such local constancy is simply assumed in what follows.[16]

I now consider the bargaining that takes place at $T-1$. At this point a quantity of resources Z_{T-1} is allocated to the young while the old receive C_{2T-2}. The amount Z_{T-1} in turn is divided into two parts. The first is a certain amount of consumption at $T-1$, C_{1T-1} while the rest is capital at $T-1$ whose sole purpose is to increase consumption of the old at T. I first consider the optimal division of Z_{T-1} into these two components and then turn to the determination of Z_{T-1}.

The old at $T-1$ do not care how Z_{T-1} is divided so that the Nash solution requires that this division maximize the utility of the young $U(C_{1T-1}, C_{2T-1})$ subject to (27). The first order condition for this maximization problem is identical to (10). In other words, $g_2 - g_1$ represents once again the social rate of return from foregoing one unit of consumption at $T-1$ and investing it in capital. If $g_2 - g_1$ is constant, the marginal rate of substitution between consumption in the two periods must be constant as well and so will be the rate of return on assets if one tries to support the allocation with government bonds as in Section III.

One further implication of this subsidiary maximization is worth noting. If the utility function of the young is strictly concave, as has been assumed throughout, the maximum (or indirect) utility the young can obtain as a function of Z_{T-1} is concave as well. Denote this indirect utility function by $V(Z_{T-1})$.

Now consider the determination of Z_{T-1}. This is the result of bargaining between the young and the old at T-1 so that it can be computed by maximizing the product of the differences between the utilities of the two types of agents and their respective reservation

[16] This property ceases to be local if one requires that it hold at every instant. Thus in the remainder of this section I consider g's only for period T.

levels of utility. Using (2) this requires the maximization of:

$$[U(C_{1T-2}, Y_{T-1} - G_{T-1} - Z_{T-1}) - U(C_{1T-2}, s)][V(Z_{T-1}) - V(s)]$$

with respect to Z_{T-1} where $V(s)$ represents the indirect utility of the young if no agreement is reached. An argument analogous to the one below (25) which shows that g_3 is strictly between zero implies that with concave utility functions an increase in G_{T-1} leads to a fall in Z_{T-1}.

Suppose now that we try to implement this bargaining equilibrium using government bonds as in Section III. Then (22) gives once again the formula for the value of these government bonds. Upon differentiation of (22) we again obtain that:

$$dB_{T-1}/dG_{T-1} = g_1[dZ_{T-1}/dG_{T-1}]/(g_2 - g_1)$$

which, given that dZ_{T-1}/dG_{T-1} is negative has the opposite sign as g_1. So, I have shown that the results of the previous section can be extended to an economy in which there is explicit bargaining between the two generations. There is a sense in which the results are weaker however, since it only applies locally where the g's are constant and I have needed to bound certain third derivatives to draw a relationship between the sign of U_{12} and that of g_1.[17]

V Rubinstein Bargaining

In the previous section I considered the axiomatic model of bargaining due to Nash. In this section I consider instead the bargaining procedure developped by Rubinstein (1981). This bargaining procedure describes an explicit game in extensive form whose perfect equilibrium is unique. The idea is that agents make alternating offers. After an agent has made an offer, the other agent has the choice of accepting it or countering with another offer. The key ingredient in the model is that if an offer is rejected and agreement is postponed the utility of the agreement becomes discounted by a factor δ. Thus consider period T. If agreement results from the first offer of C_{2T-1} then the utilities are given by $U(C_{1T-1}, C_{2T-1})$

[17]This bound was implicitly present in previous sections as well since only quadratic preferences were considered explicitly.

and $V(Y_T - G_T - C_{2T-1})$ for the old and young respectively. Instead if it results from the second offer the utilities are $\delta U(C_{1T-1}, C_{2T-1})$ and $\delta V(Y_T - G_T - C_{2T-1})$ respectively. Unfortunately, unlike the case of a labor market considered for instance by Shaked and Sutton (1984) this timing of negotiations is not particularly natural in the present context. Yet, it is instructive to note that it produces similar answers to the model considered in previous sections.

To save on notation I will now exclude government spending (which can be thought of as being subtracted from the income measure I use) and remove the time subscripts (with the understanding that bargaining is taking place at time T). To obtain the equilibrium one reasons as follows. Let the amount of consumption the old receive when the young are allowed to make an offer that is accepted be C_2^y. Then, if it is the old's turn to make an offer they would offer to consume an amount C_2^o which leaves the young indifferent between consuming $Y - C_2^o$ now or $Y - C_2^y$ in the next period. In other words:

$$V(Y - C_2^o) = \delta V(Y - C_2^y). \tag{28}$$

Similarly, given that C_2^o is the amount the old can obtain when it is their turn to make an offer, the young will, when it is their turn, offer them an amount that leaves them indifferent between C_2^y now and C_2^o later:

$$U(C_1, C_2^y) = \delta U(C_1, C_2^o). \tag{29}$$

These two equations determine both C_2^o and C_2^y. Without affecting the qualitative results I now assume that the young make the first offer so that C_2^y is the outcome of the bargaining process.[18] Totally differentiating both equations and solving one obtains:

$$dC_2^y = g_1 dC_1 + g_4 dY$$

where:

$$g_1 = V_1^o[-U_1^y + \delta U_1^o]/D$$

[18] As δ goes to one the two levels of consumption converge.

$$
\begin{aligned}
&= V_1(\delta - 1)U_1^o/D + V_1^o(U_1^o - U_1^y)/D \\
g_4 &= \delta U_2^o[V_1^o - \delta V_1^y]/D \\
D &= U_2^y V_1^o - \delta^2 U_2^o V_1^y
\end{aligned}
$$

where the superscripts denote the levels of consumption at which the derivatives are taken. Since C_2^o exceeds C_2^y, U_2^o is smaller than U_2^y while V_1^o is bigger than V_1^y. This means that D is positive and g_4 is strictly between zero and one. Once again any extra resources are shared by the two generations.

The effect of increases in C_1 on C_2 can be decomposed in two terms. The first term represent the fact that with more C_1 the old individual enjoys more utility. Yet, he can only collect on this utility when an agreement is actually struck. Since he has more to collect, he cares more about collecting soon and is therefore willing to give up more to the young. This term thus makes g_1 negative even when U_{12} is zero. The second term, which is proportional to the difference between U_1^o and U_1^y has the same sign as U_{12} as in the earlier analysis.

To value government debt in this case one proceeds just as before. One can show that at the bargain struck in period $T - 1$, an increase in government spending lowers resources for both generations so that Z falls. Then, if one assumes that the g's are constant (23) can be used to demonstrate that the effect of an increase in G on the value of government debt has the opposite sign as g_1. Thus, once again a negative value of U_{12} helps explain why government debt is valued more when government expenditures are high.

V Conclusions

This paper has presented a simple model in which there is a reason for debt to be valued. The government cares for bondholders who are old and distributes some of the payments to them in the form of a return on assets. The main strength of the model is that it rationalizes the fact that government debt has a high value during wars with the at least plausible assumption that those who consume little during wars have a high marginal utility of consumption when the war ends.

The main weakness of the model, and the natural topic for further research is that the model does not provide a complete explanation for the decision to distribute some of the payments to the old in the form of returns on assets while others are made in the form of lump sum transfers. For this a more complex model in which different members of the old generation have different bargaining power and in which somehow those with assets have more bargaining power than those without assets will have to be constructed.

REFERENCES

Abel, Andrew:"Optimal Monetary Growth", Mimeo, July 1985

Barro, Robert J:"On the Determination of the Public Debt", *Journal of Political Economy*, 87, (1979), 940-71

Barro, Robert J. :" *Macroeconomics*, Wiley, NY, 1984

Benjamin, Daniel K. and Levis A. Kochin: "Wars, Prices and Interest Rates, A Partial Solution to Gibson's Paradox" in M.D. Bordo ed. *A Retrospective on the Gold Standard*, Chicago: University of Chicago Press.

Binmore, Ken, Ariel Rubinstein and Asher Wolinsky:"The Nash Bargaining Solution in Economic Modelling" *Rand Journal of Economics*, 17, Summer 1986, 176-88

Calvo, Guillermo and Maurice Obstfeld: "Optimal, Time-Consistent Fiscal Policy with Uncertain Lifetimes" *NBER WORKING PAPER N 1593*, March 1985

Chamley, Christophe:"On Optimal Taxation in Stylized Models of Intertemporal General Equilibrium", Mimeo 1982

Clark, Colin: "Public Finance and Changes in the Value of Money" *Economic Journal*, Dec 1945, 371-89

Diamond, Peter:"National Debt in a Neoclassical Growth Model" *American Economic Review*, December 1965, 1126-50

Harsanyi, John: *Rational Behavior and Bargaining Equilibrium in Games and Social Situations*, Cambridge University Press, Cambridge, 1977

Keynes, John M:*Essays in Persuasion*, 1931

Kotlikoff, Laurence J:"Is Debt Neutral in the Life-Cycle Model", mimeo, September 1986

—, Torsten Persson and Lars E.O.Svensson: "Laws as Assets: A Solution to the Time Consistency Problem", mimeo September 1986

Lucas Robert E. Jr. and Nancy Stokey:"Optimal Monetary and Fiscal Policy in an Economy without Capital", *Journal of Monetary Economics*, 12, 1983, 55-93

Roth, Alvin E.:*Axiomatic Models in Bargaining*, Springer, Berlin ,1979

Rubinstein, Ariel:"Perfect Equilibrium in a Bargaining Model" *Econometrica*, 50, 1982, 97-109

Shaked, Avner and John Sutton:"Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model", *Econometrica*, 52, November 1984, 1351-64

Zeuthen, Frederik: *Problems of Monopoly and Economic Welfare*, Routledge, London, 1930

Figure 1

The Budget Set of the Young