A behavioral model of the popularity and regulation of demandable liabilities

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Abstract

Overoptimism regarding one’s ability to arrive early in a queue is shown to rationalize deposit contracts in which people can withdraw their funds on demand even if consumption takes place later. Capitalized institutions serving overoptimistic depositors emerge in equilibrium even if depositors and bank owners have identical preferences and investment opportunities. Consistent with the evidence, runs can lead people to move their deposits from one intermediary to another. Regulatory policies, including deposit insurance, minimum capital requirements and restrictions on the assets held by depository institutions can increase the ex ante welfare of depositors.

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Many financial intermediaries have liabilities that are payable on demand. These liabilities satisfy two properties. First, individuals holding them know in advance the amount that they are entitled to withdraw, i.e., to receive at any particular moment in exchange for redeeming their claims. Second, creditors are paid in the order in which they ask to redeem their claims.

As emphasized by Diamond and Dybvig (1983), this contractual form induces equilibria with runs. Fearing that other individuals will seek to withdraw their funds, individuals have an incentive to attempt to arrive early enough that they receive their promised funds before the institution runs out. Run equilibria are generally inefficient, in part because the order of arrival becomes a source of risk. As noted by Wallace (1988) and Green and Lin (2000), an essential reason for this inefficiency is the “sequential service constraint,” the requirement that people be paid pre-specified amount in the order in which they arrive. As they show, efficiency is restored if the financial intermediary accumulates orders until the end of the period and then reduces everyone’s payments symmetrically if it has insufficient funds.\(^1\)

This raises the question of why contracts with sequential service are so popular. In their classic paper, Diamond and Dybvig (1983) say that this feature “represents some services that a bank provides and which we do not explicitly model.” These services are easy to picture if one imagines an individual withdrawing cash to make an immediate purchase.\(^2\) The benefits of paying individuals immediately rather than delaying payments while accumulating orders are less obvious when withdrawals are not used for immediate consumption but rather for acquiring alternate financial assets. Such withdrawals often take the form of electronic transfers between accounts whose ownership is the same. The result is that, during runs, intermediaries must sell their assets to those institutions in which people re-deposit their funds.

\(^1\)An important literature started by Calomiris and Kahn (1991) shows that allowing depositors to withdraw funds before the assets of a bank mature can help control moral hazard by banks. This does not appear to either require or benefit from sequential service so that treating all early withdrawers symmetrically may also be advantageous in their setting.

\(^2\)Banks provide much of this convenience through ATM networks, which generally limit the amount that can be withdrawn in a day.
With this as background, it would seem useful to have a model where contracts with sequential service arise even if people’s utility from consumption does not depend on receiving their funds before the period of sequential service is over. The model in this paper has this property. It rationalizes contracts with sequential service as resulting from people’s overconfidence in their ability to withdraw faster than others in a run. The result is that they prefer a contract that pays more to early arrivers than a contract whose actual payment is independent of the order of arrival. Sequential service is thus rationalized not by the transactions services, or “convenience” that it provides, but by the view that demandability adds to “safety,” which is valid for those that arrive early in a run. There is no reason, of course, why the convenience and safety motives cannot both be relevant at the same time. Still, to demonstrate the power of overconfidence, I abstract from transactions services.

One attractive aspect of overconfidence as a rationale for demandable deposits, is that it does not apply exclusively to assets that are held for short term use. In particular, it can explain why people and organizations might acquire demandable claims with funds that they intend to spend in the far future. In 2000, consumers held $3.5 trillion in “transactions balances” while their annual consumption was $6.8 trillion. It is possible, of course, that all these funds were held just in case they were needed immediately. These transactions balances are quite unevenly distributed in the population, however, so that those individuals who hold disproportionate amounts are unlikely to draw them suddenly down to zero. The level of demandable funds held by firms is quite remarkable as well. In 2008, nonfinancial corporations had $580 billion of demand deposits and $780 billion of money market mutual funds. One reason to give some credence to the possibility that these funds exceed those that firms might need to spend in an emergency is their considerable growth in a relatively short period. In 1995, with GDP being over one half of its 2008 level, this sector’s demand deposits equaled $205 billion while its money market mutual funds equaled $60 billion. While this

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3 At a conceptual level, this willingness to delay consumption until the end of the period implies that government provided deposit insurance that pays off only at the end can still provide considerable benefits. As emphasized in Wallace (1988), deposit insurance that pays off at the end of the period has limited benefits in his version of the Diamond-Dybvig (1983) model.
paper does not provide a reason for the overall increase in corporate saving over this period, it may help explain why so much of this additional saving was held in these extremely liquid forms.

An additional empirical regularity concerning demandable deposits is that the government is heavily involved in their operation. Even before the advent of deposit insurance, minimal levels of capital and restrictions on the assets held by depository institutions were common. Along the same lines, U.S. money market funds are restricted in the assets they may acquire if they wish to be allowed to maintain a “stable NAV” and thereby fulfill a key condition for demandability.

Here I show that various government interventions including deposit insurance, binding minimum capital requirements and restrictions in the assets that intermediaries can hold can increase the ex ante welfare of depositors when this is computed using the actual probabilities that they will arrive before funds run out. The reason they do so is that these policies can reduce the position-in-line risk that depositors incur. They can do so either by increasing the likelihood that banks pay out when their assets turn out to have low returns or by forcing banks to hold assets that are less likely to have low returns.

An essential component of the model is that depositors run on institutions that issue demand deposits when they hear bad news regarding the assets held by these institutions. The model is thus consistent with the empirical literature showing that banking panics took place when bank assets were impaired. Gorton (1988) shows this to be true for the U.S. in the National Banking Era, while Schumacher (2000) shows that the banks that experienced runs in Argentina after the 1994 Mexico crisis tended to have relatively weak balance sheets. Lastly, the run on the “Reserve Primary Fund” Money Market Mutual Fund took place after its Lehman assets had dropped in value.

One oft-cited finding in psychology is that survey responses display an “optimistic bias.”

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4 One interesting aspect of this run is the number of sophisticated institutions including Bank of America Securities LLC that complained because, having made their requests too late in the morning of Lehman’s bankruptcy, they received less than those who were faster. See Exhibit 3 at http://www.sec.gov/spotlight/reserve_primary_fund_investors.htm

5 In October 2010, Google Scholar lists about 5,000 works using the term.
Weinstein (1980) was one of the first to document this phenomenon. He showed that survey respondents tend to say that the probability that they will obtain desirable outcomes (like having a mentally gifted child) is higher than the probability that their peers will obtain these outcomes. By contrast, respondents reported that their own probability of a bad outcome such as a heart attack or a drinking problem is lower than that of their peers. These findings suggest that people are unlikely to believe that they will be left behind in a bank run.

Weinstein (1980) also showed that people believe themselves to be better than average at a variety of activities. This finding has been confirmed in a number of studies, but there is an important qualification. Moore and Small (2007) show that people expect to be worse than average at difficult tasks such as juggling. Faced with a difficult test, people predict (and bet) that they will do worse than others, while the opposite is true when they face an easier test (Hoelzl and Rustichini 2005, Moore and Small 2007). One reason to think that people may be optimistic when it comes to withdrawing funds during a run is that Weinstein (1980) and subsequent studies report that people are more biased towards overoptimism for events that they judge to be controllable.\(^6\)

Actual influence does not appear to be essential for people to perceive they have control. As Langer (1975) shows, numerous manipulations can give people an “illusion of control.” One of the most striking pieces of evidence for this can be found in Strickland, Lewicki, and Katz (1966) and Rothbart and Snyder (1970). These papers show that people express more confidence that they know the outcomes from rolls of dice, and are willing to wager more of their own funds on these outcomes, when they place their bets before they roll the dice rather than afterwards. In another intervention involving actual payoffs, Park and Santos-Pinto (2010) asked participants in poker and chess tournaments to forecast their rank and offered them bets whose payoff was highest if they forecast correctly. In both cases, significantly more than 50% of the participants bet that they would do better than the median.

People who entrust a debtor with funds often have a strong desire to receive it back. This

\(^6\)See Thompson, Armstrong, and Thomas (1998) for a valuable discussion.
may contribute to having a sense of control. As Thompson, Armstrong, and Thomas (1998, p. 151) put it, “When people have a strong need for an outcome or a strong commitment to getting the outcome, they know that their intention to obtain the outcome is strong, and this may influence their judgment of control.”

Related Literature. This paper is related to two behavioral literatures in economics. The first studies the equilibrium contracts that arise when some agents are overconfident. Yildiz (2003) analyzes the effect of overconfidence in bargaining games. Sandroni and Squintani (2004) propose a model in which overconfidence affects insurance markets while DellaVigna and Malmendier (2004) study contracts with people who are overconfident about their self-control. Manove and Padilla (1999) and Landier and Thesmar (2009) develop models of equilibrium contracts between rational lenders and overoptimistic entrepreneurs.

This paper is also related to a literature that studies the effect of cognitive limitations on the behavior of financial market participants. This includes, notably, Daniel, Hirshleifer, and Subrahmanyam (1998) and Odean (1998), who show that various puzzles in the behavior of stock prices can be rationalized by the existence of investors who are overconfident about their ability to predict the future. Grinblatt and Keloharju (2009) show that Finnish investors whose survey responses suggest that they are more overconfident trade stocks more frequently.

Incorrect predictions about the future play a role in several papers concerned with asset prices, including Barberis, Shleifer and Vishny (1998) and Hong, Stein, and Yu (2007). Gennaioli, Shleifer, and Vishny (2012) show that incorrect beliefs can also lead agents to hold new asset classes that appear safe to them because they have not yet become aware of some of their incipient risks. One distinction between this literature and the current paper is that I focus on beliefs that people have about their own future actions rather than beliefs.

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7See Spiegler (2011) for a broader discussion of how individual biases lead to equilibrium exchanges that can have deleterious effects on people’s “true” utility.

8These papers rely on miscalibration, i.e., people's tendency to overstate the precision of their knowledge. Ben-David, Graham, and Harvey (2010) distinguish between miscalibration, which they see as a form of overconfidence, and “optimism” regarding the mean exogenous random variables. Following Weinstein (1980), I use the term “optimism” to describe a different kind of overconfidence, namely people’s excessive confidence in their ability to affect their own outcomes favorably.
they have about external events. Indeed, mistaken beliefs about external events do not appear to provide a solid rationale for demandable deposits. The belief that an asset has a high expected return should lead people to hold this asset directly. It does not give people a reason to prefer to hold a demandable claim on an institution that holds this asset.⁹

The paper proceeds as follows. The next section introduces my formulation of overconfidence into a one period model of resource allocation. People’s belief that they will be among the first $x$ percent to arrive is measured by a one-parameter function which equals the rational value of $x$ for a particular parameter value and otherwise overstates this probability. The result is that, even with a certain degree of risk aversion, people can prefer to have $Y$ distributed to the first $Z/Y$ people who arrive in line rather than receiving $Z < Y$ for sure. This preference is not absolute, however, and there is both an ideal $Z/Y$ and a level of $Z/Y$ that is low enough that people prefer to receive $Z$ for sure.

Section 2 shows that such overconfident individuals can prefer to “deposit” a risky asset at a financial intermediary that promises to pay a state-independent sum until it runs out rather than holding the risky asset directly. In Section 3, I let the owners of these institutions capitalize their banks in order to attract depositors and show that they quite generally do it in equilibrium even though the preferences and endowments of the depositors and bank owners are the same. Section 4 studies policy interventions that raise the ex ante welfare of depositors. Section 5 expands the model to include three periods. Even though depositors intend to consume in the final period, their overconfidence can lead equilibrium contracts to give them the right to withdraw in the middle one. During runs, people who make withdrawals re-deposit the proceeds. This fits with the observation that, for example in the crisis of 1907, the assets of some intermediaries grew even while those subject to runs declined. These asset transfers may be an important cost of bank runs. Finally, Section 6

⁹A behavioral reason for people to hold demandable deposits that has been suggested to me is that people overestimate the probability that they will need funds in the short term. This is a less attractive assumption about beliefs than the one pursued in this paper for two reasons. First, learning one’s funding requirements should be easier than learning about how one will fare in a run, since the latter is relatively uncommon. Second, expecting to fare well in a run is consistent with the sort of optimism for which there is a great deal of psychological evidence, whereas the expectation that one will need a great many funds in a hurry is not particularly consistent with optimism.
concludes.

1 Queuing Optimism

Let there be $N$ individuals whose payoff depends on the order in which they arrive at a particular location and let $p_i$ be individual $i$’s position, which takes values between 1 and $N$. If individuals are symmetric, the probability that $i$ draws any particular $p_i$ equals $1/N$. Let $x_i$ denote $p_i/N$, so that $(1 - x_i)$ is the fraction of individuals who arrive after $i$. For values of $x_i$ that are multiples of $1/N$, the probability that a person has an $x_i$ smaller than or equal to $x$ equals $x$. Extending this to the case where $x_i$ can take all values in the unit interval, $x_i$ has a standard uniform distribution. This is not surprising; it is simply a restatement of the well known fact that the probability density function of any smooth cumulative density function is uniform.

Individuals are optimistic about being among the first $100x$ percent to be in line if their subjective probability that $x_i$ is smaller than $x$ exceeds $x$ itself. There are, of course, many distribution functions with this property. I focus on a particularly simple one, which has the advantage that the degree of excessive optimism is captured by a single parameter $\lambda$. Suppose, in particular, that each person believes that the probability that their own $x_i$ is less than or equal to $x$ is given by

$$G(x, \lambda) = \frac{1 - e^{-\lambda x}}{1 - e^{-\lambda}}.$$  \hfill (1)

In other words, they each believe that their own $x_i$ is drawn from a truncated exponential distribution. The limit of $G(x)$ when $\lambda$ goes to zero is $x$, so this limit corresponds to individuals who have correct beliefs. At the other extreme, an individual with a $\lambda$ equal to infinity is essentially certain that he will be first in line. More generally, the Appendix shows that $G(x)$ is strictly increasing in $\lambda$ for $0 < x < 1$, so that the parameter $\lambda$ is a measure of optimism. The function $G(x)$ is displayed for a few values of $\lambda$ in Figure 1.

The first and second derivatives of $G(x)$ with respect to $x$ are given by

$$G'(x) = \frac{\lambda e^{-\lambda x}}{1 - e^{-\lambda}}$$
and
$$G''(x) = -\frac{\lambda^2 e^{-\lambda x}}{1 - e^{-\lambda}},$$ \hfill (2)
so that \( G'(x) \) decreases as \( x \) increases. For low values of \( x \), the slope of people's subjective probability of being among the first \( x \) rises more rapidly than the actual probability. This can be seen by noting that \( G'(0) = \lambda/(1 - e^{-\lambda}) \), which is greater than one for \( \lambda > 0 \). For high values of \( x \), on the other hand, the subjective probability of being among the first \( x \) rises more slowly than the actual probability because, as shown below, the Einstein function \( \lambda e^{-\lambda}/(1-e^{-\lambda}) \) is smaller than one. There thus exists an interior value of \( x \) at which \( G'(x) = 1 \) and the overoptimistic bias \( G(x) - x \) is the largest. This value of \( x \) plays a role in what follows.

While queueing optimism affects people’s subjective probability of receiving funds, it need not affect how people value the funds they receive. It is thus consistent with individuals having a standard expected utility function. For convenience, suppose that this utility function takes the CRRA form with relative risk aversion between 0 and 1 so that \( U(0) = 0 \).\(^{10}\)

\(^{10}\)This degree of risk aversion may seem modest. It is important to stress, however, that people who hold demandable deposits typically also have other sources of funds for future consumption. Thus, variations in the return on demand deposits have a smaller effect on consumption than that assumed in the model. Using a low value of the risk aversion parameter provides is a mechanism for incorporating the relative low importance of this return variability.
Individuals thus expect their welfare to be

\[ U_c = E(C^\gamma) \quad 0 < \gamma \leq 1, \]  

(3)

where \( E \) takes expectations using their perceptions and \( C \) denotes the funds they receive.

To understand the implications of queueing optimism on financial contracts, it is worth starting with the simple case where an entity must distribute resources equal to \( Z \) among a continuum of individuals of mass one.\(^{11}\) One obvious solution is to give \( Z \) to each individual, since this would be the natural distribution if the claimants were indistinguishable from one another. This symmetric distribution would obtain, for example, if every claimant were entitled to the same amount greater than \( Z \) and payments were made at the end, with the available funds allocated to individuals in proportion to their claims. As stressed in the introduction, I am interested in situations where such an ex post symmetric distribution of funds is feasible.

Now suppose that it is also possible to rank individuals by the order in which they arrive to make a claim. The question, then, is whether it would be desirable to take advantage of this sequential service opportunity. If, as in Peck and Shell (2003), it is possible to implement mechanisms that depend arbitrarily on a person’s position in line, the sequential service opportunity allows one to assign consumption \( C(x) \) to the individual that arrives right after a fraction \( x \) have arrived. Using (2), the expected utility of individuals is then

\[ \int_0^1 \frac{\lambda e^{-\lambda x}}{1 - e^{-\lambda}} C(x)^\gamma dx, \]  

(4)

while feasibility then requires that

\[ \int_0^1 C(x)dx = Z. \]  

(5)

The maximization of (4) for each \( C(x) \) subject to (5) implies that\(^{12}\)

\[ C(x) = \kappa e^{\frac{\lambda x}{1-\gamma}} \quad \text{where} \quad \kappa = \frac{Z\lambda}{1 - \gamma} \left[ 1 - e^{\frac{-\lambda}{1-\gamma}} \right], \]  

(6)

\(^{11}\)Overoptimism about one’s position in line may also help explain why people often favor non-price rationing. For example, Frey and Pommerehne (1993) find that many respondents prefer to distribute a limited supply of water on a first-come first-serve rather than using high prices to discourage demand. It may also help explain why retailers sometimes offer limited quantities at lower prices “while supplies last.”

\(^{12}\)I am grateful to a referee for suggesting I consider this solution.
so that consumption falls smoothly with an individual’s position in line if \( \lambda > 0 \) whereas it is constant otherwise. For rational individuals, risk aversion is the only force determining \( C(x) \) and this calls for \( C(x) \) to be constant.

Payment schemes that make \( C \) depend finely on \( x \) seems difficult to enforce because, particularly in states of nature in which it does not run out of funds, the institution making the payment has a strong incentive to misrepresent \( x \). By pretending that people’s level of \( x \) entitles them to a low \( C(x) \), the institution gets to keep more for itself. It would seem that, in the case where people are paid as soon as they arrive, paying \( C(x) \) as in (6) requires that the number of people who arrived before, or at least the exact level of remaining funds at the institution, be verifiable in real time. Now consider instead the simpler solution in which people receive an amount stipulated in advance that is independent of \( x \) as soon as they arrive until the institution exhausts its funds, at which point they receive zero. In this case, there is no need to verify the number of people that have been paid. What remains necessary is that the institution demonstrate that it no longer has any assets, but this verification needs to be carried out only once and is then sufficient to cancel the payments of all individuals who arrive after the first who is unpaid.

In this simpler allocation scheme, the only decision that needs to be taken is the promised payment \( Y \). When \( Y \) exceeds \( Z \), \( C(x) \) equals \( Y \) for the first \( Z/Y \) to arrive and is zero for the rest. An individual’s expected utility when \( Y \) is greater than or equal to \( Z \) equals

\[
U_c = \frac{1 - e^{-\lambda Z/Y}}{1 - e^{-\lambda}} Y^\gamma, \tag{7}
\]

so that the gain from raising \( Y \) is

\[
\frac{dU_c}{dY} = \frac{Y^{\gamma-1}}{1 - e^{-\lambda}} \left[ -\frac{\lambda Z}{Y} e^{-\lambda Z/Y} + \gamma (1 - e^{-\lambda Z/Y}) \right]. \tag{8}
\]

This is positive at \( Y = Z \) if

\[
\gamma > \frac{\lambda}{e^\lambda - 1} \equiv n(\lambda). \tag{9}
\]

The function \( n(\lambda) \) is an Einstein function and l’Hôpital’s rule implies that it tends to one when \( \lambda \) tends to one. The Appendix shows that it is monotonically declining in \( \lambda \) for
\( \lambda > 0 \). This means that, for every \( \lambda > 0 \), there exists a critical \( \gamma < 1 \) such that individuals prefer sequential service with \( Y > Z \) for any \( \gamma \) larger than this critical value. Otherwise, they prefer to set \( Y \) equal to \( Z \), so that risk aversion dominates and people prefer not to have their outcome be determined by their random position in line. Notice that, because this contract is more restrictive than the one that leads to (6), having \( \lambda > 0 \) is no longer sufficient for individuals to wish to take advantage of sequential service.

When the degree of risk aversion is below \( n(\lambda) \), the optimal value of \( Y \), \( Y_o \) leads \( dU_c/dY \) in (8) to equal zero so that\(^{13}\)

\[
\frac{\lambda Z/Y_o}{e^{\lambda Z/Y_o} - 1} = n \left( \frac{\lambda Z}{Y_o} \right) = \gamma.
\]

Therefore

\[
Y_o(Z) = \frac{Z}{r_o(\gamma, \lambda)} \quad \text{where} \quad r_o(\gamma, \lambda) = \frac{n^{-1}(\gamma)}{\lambda}. \quad (10)
\]

The optimum thus requires that the ratio of \( Z \) to \( Y_o \), which is the probability that depositors are paid, be equal to a constant \( r_o \) that depends only on \( \gamma \) and \( \lambda \). Since the \( n \) function is decreasing, \( r_o \) falls when either \( \gamma \) or \( \lambda \) rises.

While (9) implies that individuals prefer a payment of \( Y_o \) to one of \( Z \), higher values of \( Y \) may be less desirable than \( Z \). This follows from (7), which implies that the limit of \( U_c \) when \( Y \) goes to infinity equals zero whenever \( \gamma \) is strictly less than one. With any risk aversion at all, there exist values of \( Y \) large enough that individuals prefer to receive \( Z \) with certainty to a lottery that pays \( Y \) to the first \( Z/Y \) to arrive. So, even when (9) is satisfied, there is an intermediate value of \( Y \) such that people are indifferent between having \( Z \) for sure and being promised the notional amount \( Y \). Using (7), this intermediate value of \( Y \), which I denote by \( Y_m(Z) \), satisfies

\[
\frac{1 - e^{-\lambda Z/Y_m}}{1 - e^{-\lambda}} (Y_m)^\gamma = Z^\gamma, \quad (11)
\]

or

\[
Y_m(Z) = \frac{Z}{r_m(\gamma, \lambda)} \quad \text{where} \quad r_m(\gamma, \lambda) \text{ solves } \frac{1 - e^{-\lambda r_m}}{1 - e^{-\lambda}} r_m^{-\gamma} = 1. \quad (12)
\]

\(^{13}\)While the function \( U_c \) need not be globally concave in \( Y \), a straightforward calculation establishes that its second derivative with respect to \( Y \) is negative at the point where this equation is satisfied.
The variable \( r_m(\gamma, \lambda) \) is the minimum value of \( Z/Y \) such that receiving \( Y \) with probability \( Z/Y \) is better than receiving \( Z \) for sure. For rational risk averse individuals, this minimum equals one, whereas it is lower if (9) is satisfied. Moreover, condition (9) implies that

\[
r_m < r_o, \tag{13}
\]

because a promised payment of \( Z/r_o \) gives strictly more utility than one of \( Z \) while \( r_m \) leads to indifference.

Lastly, it is worth noting that differentiation of the second equation in (12) implies that

\[
\left[ n(\lambda r_m) - \gamma \right] \frac{dr_m}{r_m} + \left[ n(\lambda r_m) - n(\lambda) \right] \frac{d\lambda}{\lambda} - \log(r_m) d\gamma = 0
\]

Given that \( r_m < r_o \) when (9) is satisfied, \( n(\lambda r_m) > n(\lambda r_o) = \gamma > n(\lambda) \). Since \( \log(r_m) < 0 \), this equation implies that both increases in \( \lambda \) and \( \gamma \) lower the smallest tolerable ratio \( r_m \).

As a result, individuals prefer arbitrarily low values of \( Z/Y \) rather than receiving \( Z \) for sure as long as \( \lambda \) and \( \gamma \) are large enough.

2 Deposit contracts with no bank equity

Let there be two periods labeled 0 and 2 (an intermediate period, period 1, will be added below). Each individual now has an endowment of one unit of \( A \) in period 0. There is a single real investment opportunity available that allows consumption to be deferred to period 2. For each unit of \( A \) that is invested in this opportunity, \( Z_H \) units of consumption are available in the high state, which occurs with probability \( \pi \). In the low state, the opportunity pays off \( Z_L = r Z_H \) with \( r < 1 \). The utility of the people who are alive at time 0 depends only on their consumption at the end of period 2. Therefore, assuming their preferences are given by (3), there exist a number of mechanisms that ensure that individuals consume \( Z_i \) in state \( i \) so that their expected utility is

\[
U_a = \pi Z_H^i + (1 - \pi) Z_L^i. \tag{14}
\]

If individuals can obtain the \( Z_i \) returns on their own, autarky brings about this level of utility. If these returns are only available to institutions, this utility can still be achieved
in several ways. Following Green and Lin (2000), institutions who receive a unit of \( A \) could promise a payment \( Y \) equal to \( Z_H \) and then only make payments after all individuals express their desire to withdraw. Utility level (14) is then achieved by divvying up the funds available among individuals in proportion to the amount that they are notionally entitled to. This should serve to clarify that there is no sequential service constraint in this setting and that the results of “equity” contracts can be obtained with a ”bankruptcy” solution even if promised payments are not allowed to vary as a function of the state of nature.

There is a simple condition which ensures that overoptimistic individuals shun “autarkic” contracts in which they receive the return to the asset in every state of nature in favor sequential service contracts that promise a payment \( Y \) equal to \( Z_H \). These two contracts have the same return in the high state. In the low state, the utility of an individual that puts a fraction \( \alpha \) of his endowment into the autarkic asset and a fraction \( 1 - \alpha \) in the sequential service contract has a subjective expected utility of

\[
\alpha Z_L^\gamma + (1 - \alpha) \left[ \frac{1 - e^{-\lambda r}}{1 - e^{-\lambda}} \right] Z_H^\gamma.
\]

Therefore, individuals optimally set \( \alpha \) to the minimum value of zero if

\[
\frac{1 - e^{-\lambda r}}{1 - e^{-\lambda}} r^{-\gamma} > 1 \quad \Rightarrow \quad r > r_m(\gamma, \lambda).
\] (15)

People invest their entire endowment in the sequential service contract if they prefer a payment of \( Z_L/r \) that is obtained by a fraction \( r \) of depositors to receiving \( Z_L \) for sure.

Now consider more general arrangements that pay \( Y \) to the first \( Z_i/Y \) individuals in state \( i \), with the rest receiving nothing. Notice that all these arrangements can be implemented by an institution with no capital of its own in which individuals deposit their endowment and receive a demandable claim of \( Y \) for period 2. Moreover, competition between institutions offering contracts of this sort would ensure that the favorite among these would be the one obtained by individuals in equilibrium. I study this equilibrium now.

Without bank capital, a sequential service contract in which \( Y \) exceeds \( Z_H \) does not have sufficient resources to pay \( Y \) to all depositors even in the high state. Therefore, the expected
utility from such a contract when $Y \geq Z_H$ is

$$U_e^+ = \left[ \pi \frac{1 - e^{-\lambda Z_H/Y}}{1 - e^{-\lambda}} + (1 - \pi) \frac{1 - e^{-\lambda Z_L/Y}}{1 - e^{-\lambda}} \right] Y^\gamma. \quad (16)$$

By contrast, even without capital, all depositors do receive $Y$ in the high state if $Y \leq Z_H$. As long as $Y > Z_L$, the expected utility of individuals under these sequential service contracts is

$$U_e^- = \left[ \pi + (1 - \pi) \frac{1 - e^{-\lambda Z_L/Y}}{1 - e^{-\lambda}} \right] Y^\gamma. \quad (17)$$

These two utility functions are different functions of $Y$, $Z_L$ and $Z_H$ because a reduction in the required payment below $Z_H$ reduces the expected value of payments while an increase in the required payment above $Z_H$ leaves this expected value unchanged. There is thus a range of parameters for which the feasible sequential service contract that depositors like the best has $Y = Z_H$. This is shown in the following proposition

**Proposition 1.** If $\gamma > n(\lambda)$, the set of parameters where

$$r_m(\gamma, \lambda) < r < r_o(\gamma, \lambda) \quad (18)$$

and

$$\frac{\psi}{\psi + \gamma} < \pi < \frac{\psi}{\psi + \gamma - n(\lambda)} \quad \text{where} \quad \psi = \frac{1 - e^{-\lambda r}}{1 - e^{-\lambda}} \left( n(\lambda r) - \gamma \right), \quad (19)$$

implies that the optimal contract has a notional payment of $Z_H$, and this gives more expected utility than consuming $Z_i$ in state $i$.

If (18) holds and, instead, $\pi < \frac{\psi}{\psi + \gamma}$, the optimal $Y$ is strictly smaller than $Z_H$.

Condition (9) implies that individuals prefer a payment larger than $Z_H$ in the high state. If $Z_H$ were lower than $Z_L/r_o$, they would prefer this in the low state as well, and the result would be that $Z_H$ would not be the optimal payment level. Thus, for $Z_H$ to be the optimal contract, it must be “too large” in the low state, which requires that $r < r_o$. Because individuals prefer a $Y$ above $Z_H$ in the high state, the probability of the high state $\pi$ must also be bounded above to keep the optimal $Y$ equal to $Z_H$. 

14
If \( r < r_o \), depositors actually prefer a \( Y \) below \( Z_H \) if the low state is common enough. That is why the optimality of \( Z_H \) requires that \( \pi \) be bounded below in (19). When \( \pi \) is lower, depositors prefer to receive less than \( Z_H \) in both states in exchange for having more depositors collect \( Y \) in the low state. This implies that banks make profits, and the dissipation of these profits is treated in the next section.

Figure 2 depicts the properties of optimal contracts implied by Proposition 1 for \( \lambda = 1.8 \) and \( \gamma = .5 \) when \( Z_L/Z_H \) is between \( r_m \) and \( r_o \). For a range of intermediate values of \( \pi \), the optimal contract has a \( Y \) exactly equal to \( Z_H \). Outside this intermediate range, the equilibrium promised payment \( Y \) is either higher or lower.

For simplicity, the rest of the paper focuses on situations in which \( \gamma > n(\lambda) \), while both (18) and the second inequality of (19) hold so that the optimal contract with no equity features \( Y \leq Z_H \). This seems like the empirically more relevant case, though this is partly because regulations, including regulations governing money market mutual funds, often forbid setting \( Y \) above \( Z_H \). In the face of such regulations, contracts with \( Y \leq Z_H \)
would also be observed for a broader range of parameters than in Proposition 1. Most of the results below carry over to the case where $Y$ is prevented from exceeding $Z_H$ by regulation.

3 Bank capital in the two period model

In this section, one group of individuals are potential creditors while the rest are potential bank owners. There are $\omega$ of the former for each of the latter. The difference between the two types is that potential bank owners have $\lambda = 0$ so that they are not overoptimistic and do not wish to invest their funds in a contract of the kind described in the previous section. Their utility functions are the same as those of potential creditors so that they equal

$$U_b = E(C_b^{\gamma}) \quad 0 < \gamma \leq 1,$$

where $C_b$ is their period 2 consumption and $E$ takes expectations given their beliefs. Because the two types have the same utility function and the same endowment, they have no standard motivation for trading with one another. The main result of this section is that, nonetheless, bank owners often provide insurance to depositors in equilibrium in the sense that depositors consume more than their endowment in the low state and less than their endowment in the high state. When it takes the form of capitalizing banks, this insurance reduces the risk of arriving too late to receive anything from a bank. This capitalization arises in equilibrium whenever, as in the cases discussed in the previous section, depositors like sequential service contracts because $\gamma > n(\lambda)$ while, at the same time, $\gamma < n(\lambda Z_L/Z_H)$ so that $Z_H$ is larger than the payment they would like to be promised when the funds available equal $Z_L$. When banks are capitalized, the equilibrium payment is lower than $Z_H$ but is received more often, and this makes depositors better off while leaving bank owners with their original level of utility.

Suppose, then, that sets of potential bank owners can get together and form banks. They then offer contracts to creditors that stipulate the amount $Y$ that the bank will distribute in period 2 on a first-come first-served basis to any creditor who has “deposited” their unit of $A$ in period 0. I consider only contracts where the contractual payment $Y$ is independent
of the state of nature. This assumption can be justified by supposing that the state is not verifiable at the time that individuals start making withdrawals.

Bank owners have the option of pledging some of their endowment to capitalize their banks. Because depositors have first priority, an increase in capital per depositor allows a higher fraction of depositors to withdraw \( Y \) in the low state if \( Y > Z_L \). If different banks offered the same \( Y \) but had different levels of capital per depositor, depositors would deposit their endowment at those whose capital per depositor was higher. I thus focus on situations where the amount of endowment held as capital per depositor is equalized at all institutions, and I denote its value by \( k \).

I consider a symmetric allocation where all potential bank owners provide the same amount of their endowment as capital to a bank. Since there are \( \omega \) creditors per owner, let each owner provide \( \omega k \), where \( k \geq 0 \). The owners of a given bank pool their resources and divide among themselves any amount that is left over after paying off the creditors. Since all owners and creditors are treated equally, these symmetric allocations do not depend on the number of banks. It is thus convenient to focus on a bank that has a unit mass of depositors (and \( 1/\omega \) owners). This bank’s total payouts to withdrawers in state \( i \) equal the minimum of \( Y \) and \( Z_i(1 + k) \). If \( Z_i(1 + k) < Y \), bank owners receive nothing from the bank in state \( i \), while they each receive \( \omega(Z_i(1 + k) - Y) \) otherwise. If \( (1 + k)Z_L = Y \), depositors are fully insured against the state since they each get \( Y \) in both states. I restrict attention to contracts with \( (1 + k)Z_L \leq Y \). This is without loss of generality because increasing \( k \) beyond this point does not have any effect on the consumption of depositors (who receive \( Y \)) or bank owners (who receive the returns from the additional units of \( A \)).

When \( (1 + k)Z_L \leq Y \), owners receive a distribution from their bank only in the high state. Denoting the consumption of bank owners in state \( i \) by \( C_{bi} \), we have

\[
C_{bL} = Z_L(1 - \omega k) \quad \text{and} \quad C_{bH} = Z_H(1 - \omega k) + \omega(Z_H(1 + k) - Y). \tag{21}
\]

Both increases in \( k \) and in \( Y \) thus lower the consumption of bank owners, though the former does it only in the low state and the latter only in the high one. Substituting the expressions
in (21) into (20) and differentiating, the effects on owner utility of small increases in $k$ and $Y$ are

$$U_{bk} = -\omega\gamma(1-\pi)Z_L^\gamma(1-\omega k)^{\gamma-1} \quad U_{bY} = -\omega\gamma\pi((1+\omega)Z_H - \omega Y)^{\gamma-1}$$  \hspace{1cm} (22)

The negative signs of $U_{bk}$ and $U_{bY}$ have three implications. First, the equilibrium contract keeps the expected utility of bank owners equal to $U_a$. If it were any higher, banks would deviate and attract additional deposits by raising $k$ or $Y$. Second, if $k$ is positive, $Y$ must be lower than $Z_H$, for otherwise bank owners would be better off in autarky than offering the equilibrium contract. As a result of this, runs only take place in the low state when $k > 0$, since the bank then has more funds than it needs to pay depositors in the high one.

While sure of receiving $Y$ in the high state when this is less than $Z_H$, depositors only receive $Y$ in the low state with probability $(1+k)Z_L/Y$. Their expected utility is thus

$$U_c = \left(\pi + (1-\pi)\frac{1-e^{-\lambda(1+k)Z_L/Y}}{1-e^{-\lambda}}\right)Y^{\gamma}.$$  

Differentiating this with respect to $k$ and $Y$ yields

$$U_{ck} = \left[\frac{(1-\pi)\lambda Z_L e^{-\hat{\lambda}}}{1-e^{-\lambda}}\right]Y^{\gamma-1} \quad \hat{\lambda} \equiv \lambda(1+k)Z_L/Y$$  \hspace{1cm} (23)

$$U_{cY} = \left[\gamma \left(\pi + (1-\pi)\frac{1-e^{-\hat{\lambda}}}{1-e^{-\lambda}}\right) - \frac{(1-\pi)\hat{\lambda}e^{-\hat{\lambda}}}{1-e^{-\lambda}}\right]Y^{\gamma-1}.$$  \hspace{1cm} (24)

It follows that

**Proposition 2.** The equilibrium contract features positive equity, $k > 0$ and a promised payment $Y$ below $Z_H$.

This proposition applies even to cases where $Y$ would equal $Z_H$ and banks would not make any profits if $k$ were equal to zero. The reason (rational) bank owners nonetheless capitalize their banks is that bank owners and depositors differ in their marginal rate of substitution of $k$ for $Y$ when this is evaluated at $k = 0$ and $Y = Z_H$. At this point, every
depositor and bank owner has the same consumption in the high state and small reductions in $Y$ reduce the consumption of every depositor by the same amount, so the utility cost to depositors of these reductions equals the utility benefit to owners times $\omega$. In the low state, by contrast, reductions in $Y$ do not affect owners at all but make depositors better off: Because $r$ is below $r_o$, they would prefer to receive a smaller $Y$ with higher probability. Since increases in $k$ also raise the probability of receiving $Y$ in the low state, they increase the utility of depositors by more than $\omega$ times the reduction they induce in owner utility. As a result of all this, one can find a combination of an increase in $k$ and a reduction in $Y$ that makes depositors better off while leaving owners indifferent. Even though depositors like to gamble with respect to the order with which they request their funds, the difference between $Z_L$ and $Z_H$ is high enough that depositors are willing to compensate bank owners for some insurance in the form of a higher probability of being paid in the low state.

In the case where the contract without capital yields a $Y$ below $Z_H$, and only in this case, there is an additional reason to capitalize banks. This capitalization allows owners to dissipate the profits that they receive otherwise. In a model in which consumption in period zero also gives utility, these rents could be dissipated instead by accepting a smaller deposit (or giving out a “free toaster”) while keeping period 2 payouts unchanged. Notice, however, that the willingness of depositors to pay for insurance provided by bank owners, and thus the incentive to capitalize banks, would remain. When $Y < Z_H$ for non-capitalized banks, depositors are giving up resources in the high state in exchange for redistributing the resources from early to late arrivers in the low state. Increases in $k$ are thus even more desirable to them because they directly increase the amount available to late arrivers. While this shows that other forms of dissipation would not lead to $k = 0$, it leaves open the question of whether the availability of other forms of dissipation could lower $k$ in this case. This is left for further research.

At least in numerical simulations, a key determinant of the equilibrium value of the amount of capital per depositor $k$ is the number of depositors per bank owner $\omega$. When $\omega$ is large, (21) implies that owners must give up more consumption in the low state per unit
of $k$. Therefore, their marginal utility of consumption is more sensitive to $k$, and $k$ is lower. For $\lambda$, $\gamma$, $r$ and $\pi$ equal to 1.8, 0.5, 0.5 and 0.8 respectively, the equilibrium value of $k$ is 0.139 when $\omega = 1$ and only 0.004 when $\omega = 100$.

4 The regulation of depository institutions

In this section I consider three policy interventions that are extremely common in the case of demandable deposits. These are capital requirements for depository institutions, deposit insurance financed by general taxation, and regulations concerning the assets that depository institutions may hold. What I demonstrate is that these policies can raise the expected welfare of depositors when this welfare is computed using actual probabilities that depositors will collect their funds in a run.

The analysis in this section can be thought of as a normative exercise that computes the policies a benevolent (and paternalistic) social planner would desire. Insofar as these preferences also represent those of voters, the analysis may also explain why the regulations considered here are so ubiquitous. There are two reasons voters might have these preferences. First, people might be rational at the moment of voting while realizing that they will be overoptimistic when they consider depositing their endowment. One difficulty with this interpretation is that most people are already involved in the sort of contracts I discuss at the moment that they vote so this requires them to be rational and overoptimistic almost at the same time. A second interpretation lets people have more consistent beliefs while relying on altruism for others. Overconfidence in oneself is essentially synonymous with relatively low confidence in others. Precisely because overconfident individuals expect that they themselves will do well in a run, they expect the overconfidence of others to cause widespread disappointment when the run actually occurs. If they are altruistic towards others, they would favor regulations that they see as making others better off even if this requires sacrificing opportunities for themselves.

Overoptimism about one’s own capacity to arrive early in line should actually lead one to be biased towards pessimism in assessing others’ prospects. In a large population, this effect
is likely to be small, however. I thus suppose that the individuals use objective probabilities to weigh the possible outcomes of other depositors. I thus analyze policies that raise

\[ \tilde{U}_c = \left( \pi + (1 - \pi) \frac{(1 + k)Z_L}{Y} \right) Y^\gamma, \]  

which I label bystander utility for short (where this bystander can be interpreted as the depositor in an earlier, rational, state).

### 4.1 Capital Requirements

This section shows that bystander utility rises if banks are required to hold more capital than they would in a laissez-faire equilibrium whenever an increase in \( k \) raises the actual probability that a depositor receives \( Y \) by more than it raises the depositor’s expected probability. The concavity of the subjective probability \( G \) discussed in (2) implies that this occurs when the actual probability that the depositor is paid is already relatively high, as is the case when \( Z_L \) is not too far below \( Z_H \).

Differentiating (25), the effect of changes in \( k \) and \( Y \) on bystander utility are

\[ \tilde{U}_{ck} = (1 - \pi)Z_L Y^{\gamma - 1}, \quad \tilde{U}_{cY} = \left[ \gamma \pi - (1 - \gamma)(1 - \pi) \frac{(1 + k)Z_L}{Y} \right] Y^{\gamma - 1}, \]  

It follows that:

**Proposition 3.** *If the free market equilibrium is interior, so that \( \omega k < 1 \) and the subjective probability of being among the first \( (1 + k)Z_L/Y \) depositors, \( G([1 + k]Z_L/Y) \), satisfies

\[ G'([1 + k]Z_L/Y) < 1, \]  

the welfare of depositors is increased by imposing binding bank equity requirements.*

Since \( G' \) is a decreasing function and Proposition 2 shows that an equilibrium with capitalized banks has \( k > 0 \) and \( Y < Z_H \), condition (27) is satisfied if \( G'([Z_L/Z_H]) \leq 1 \). This somewhat stronger condition requires that the subjective probability of receiving \( Z_H \) when \( Z_L \) per depositor is available rises more slowly with \( Z_L \) than the actual probability. Under
these conditions, overoptimistic depositors do not properly realize how much more insurance against being late they obtain by having banks that are better capitalized.

It turns out that this stronger conditions is not very demanding because the minimum value of $Z_L/Z_H$ such that $G'(Z_L/Z_H) \leq 1$ is quite low. Figure 3 shows this by displaying this minimum value for various values of $\lambda$. This approaches .5 for extremely low $\lambda$ and then falls steadily as $\lambda$ rises. For high values of $\lambda$, the subjective probability of being among the first $x$ is close to one even if $x$ is small, so that $G'(x)$ becomes smaller than one for quite small values of $x$.

Overoptimistic depositors think they will arrive early. It follows that their subjective probability of being among the first $x$ to arrive must often rise more slowly with $x$ than $x$ itself. This implies that capital requirements, which ensure that a higher fraction of individuals get paid in low states, are often desirable from the point of view of bystander utility.
4.2 Deposit insurance

The runs in this model occur in the low state, when the promises of banks to their depositors exceed their assets. If the government were to insure deposits, it would have to raise funds to make up the banks’ shortfall. The welfare consequences of such a scheme would then depend on how the government obtains these resources. This section shows that, in a simple extreme case, deposit insurance reaches the first best allocation for depositors because it reproduces the allocation under autarky. To obtain this result, I suppose that the government raises the necessary taxes in lump sum fashion from potential creditors.

In the low state, total payments by bank owners equal $Z_L(1 + k)$ per depositor. To make every depositor collect the promised amount $Y$, the government must make payments whose average value per depositor equals $Y - Z_L(1 + k)$. Since creditors take taxes as given, deposit insurance raises the attractiveness of deposits for a given $Y$ even though actual consumption of depositors in the low state remains equal to $Z_L(1 + k)$.

With this form of deposit insurance, the utility of depositors from a contract with $\{k, Y\}$ is then

$$U_d = \pi Y^\gamma + (1 - \pi)((1 + k)Z_L)^\gamma,$$

so the changes in subjective depositor utility as $k$ an $Y$ change satisfy

$$U_{dk} = \gamma(1 - \pi)Z_L^\gamma(1 + k)^{\gamma - 1}, \quad U_{dY} = \gamma\pi Y^{\gamma - 1}.$$  

When evaluated at $k = 0$ and $Y = Z_H$, these derivatives are proportional to those of bank owners in (22). Thus, the equilibrium contract without capital requirements has $k = 0$, and the allocation is the same as under autarky. Because deposit insurance equalizes consumption across depositors, and because there are no transfers from bank owners to depositors, all individuals have the same state-specific consumption when $k = 0$. There is thus no reason for bank owners to provide additional insurance to depositors, and any insurance they do provide is the result of government mandates.

When the taxes required to finance this deposit insurance are costly, or when deposit insurance affects the incentives of bank owners, bystanders would presumably choose to
combine deposit insurance with regulation concerning the assets that banks can hold. I do not pursue this analysis here. Rather, I show that bystanders will favor certain restrictions on assets even in the absence of deposit insurance.

4.3 Regulations on the asset-side of banks’ balance sheets

Given that bystander utility tends to rise when the probability that depositors do not get paid in the low state is reduced, it should not be surprising if bystander utility can be increased by requiring banks to hold safer assets. This section shows that bystanders can also be attracted to assets with some unusual characteristics. What leads to this result is that there is a difficult real-world issue for which there is no clear answer when there is no deposit insurance. The issue is the extent to which the focus of bank regulation should be to increase the probability that all depositors get paid in full and to what extent it should be to increase the fraction of depositors that can be paid in full in those states of nature in which bank resources are not sufficient to cover the claims of all depositors. The analysis below suggests that the former may be a more burning concern. To demonstrate this, I study an illustrative special case in which $k = 0$.

Since the economy considered so far has a single productive asset, at least one additional asset must be introduced to analyze formally the effect of regulations that force banks to change their asset composition. In this subsection, I consider what may be the simplest approach to having a second asset. An alternate investment opportunity is introduced that is similar to the original one in the sense that it has a payoff of $Z_H$ with probability $\pi + d\pi$ and a payoff $Z_L + dZ_L$ with probability $1 - \pi - d\pi$ where and $dZ_L$ and $d\pi$ are small. In Rotemberg (2010), the payoff in the high state is also varied by $dZ_H$. That allowed for a demonstration that a safer asset with a lower $Z_H$ and a higher value of $\pi$ could raise bystander utility. Here I focus on the case where $dZ_H = 0$ to isolate the possibility that increases in $\pi$ can be so valuable for bystander utility that this can increase with $d\pi > 0$ even if this is offset by a $dZ_L < 0$ large enough to make it less desirable to hold the asset on its own. This should not be taken to mean that reductions in $Z_L$ are always desirable, of course.
The endowment $A$ can be converted into this alternative asset. To keep the analysis tractable I impose some strong additional restrictions. First, individuals who manage their own assets can only convert either their entire endowment or none of it. Second, banks can convert their units of $A$ into the alternative asset but can only do it if they convert all the assets in their possession. Moreover, I require depositors to either hand over their entire endowment to a bank or to keep it all themselves.

These rather strong assumptions ensure that no one can hold a diversified portfolio. As a result, one can analyze the desirability of the alternative investment opportunity both to depositors and to bystanders by differentiating their equilibrium utility levels with respect to $\pi$ and $Z_L$.\footnote{In a richer model, several assets, including government bonds, would always be available. The issue would then be whether regulation should interfere with the proportion of the different kinds of assets that banks would hold on their own. Since changes in these proportions are likely to affect both the probability that banks can pay their obligations to all customers and the fraction of customers that are paid when they cannot, one should be able to obtain analogous results in this more complicated case.}

For bystanders, one must differentiate (25). Suppose that (19) holds and banks do not have equity, so that $Y = Z_H$ and $k = 0$. Then,

$$d\tilde{U}_c = \left( (1 - r) d\pi + (1 - \pi) r \frac{dZ_L}{Z_L} \right) Z_H^\gamma.$$  

(29)

For investors who hold the asset themselves, differentiation of the autarkic utility (14) yields

$$dU_{ca} = \left( (1 - r^\gamma) d\pi + (1 - \pi) r^\gamma \frac{dZ_L}{Z_L} \right) Z_H^\gamma.$$  

(30)

Lastly, differentiating (16) gives the change in subjective depositor utility. It is

$$dU_c = \left( \frac{e^{-\lambda r} - e^{-\lambda}}{1 - e^{-\lambda}} d\pi + (1 - \pi) \frac{\lambda r e^{-\lambda r} dZ_L}{1 - e^{-\lambda}} \right) Z_H^\gamma.$$  

(31)

Since $r^\gamma > r$, the effect of $d\pi$ on bystander utility in (29) is larger than its effect on autarkic utility in (30). Bystanders value payoffs in the high state more highly because the allocation in the low state is unnecessarily risky. The depositors subjective probability of arriving after a fraction $r$ of depositors has withdrawn, which is in square brackets in (31), is smaller than $(1 - r)$ and so they also gain less from an increase in $\pi$ than bystanders. Because all three utility functions above increase with $Z_L$, however, a calculation is needed.
to show that increases in $\pi$ combined with reductions in $Z_L$ can increase bystander utility $\tilde{U}_c$ while lowering both $U_{ca}$ and $U_c$.

**Proposition 4.** Suppose that $k = 0$ and (19) holds so that the equilibrium contract pays $Z_H$ to depositors until the bank runs out of funds. There then exist alternate assets with $dZ_L < 0$ and $d\pi > 0$ such that

$$\frac{r}{1 - r} \left| \frac{dZ_L}{Z_L} \right| < \frac{d\pi}{1 - \pi} < \min \left( \frac{r \gamma}{1 - r^\gamma}, \frac{\lambda r e^{-\lambda r}}{e^{-\lambda} - e^{-r}} \right) \left| \frac{dZ_L}{Z_L} \right|. \quad (32)$$

No one would hold these alternate assets under laissez-faire but bystanders believe that overoptimistic depositors are better off if banks are forced to hold them.

What this proposition suggests is that the regulation of bank assets when there are overoptimistic depositors may not be straightforward. The reason for bank regulation in the model is not, for example, that banks are refusing to hold assets that are preferred by rational risk averse investors. Moreover, the asset that bystanders would like banks to hold is “riskier” than the assets that they would hold under laissez-faire, at least in the sense that it features a wider disparity of potential payoffs. In this particular example, the increase in bystander utility is associated with a reduction in the probability that the bank will be unable to meet its contractual obligations. This is related, though not identical, to the requirement that banks be solvent under stringent stress tests, an idea that has become popular in practice.

5 A three period model with the potential for costly asset transfers

So far, the only negative effect of sequential service contracts is that bystanders dislike their effect on the distribution of resources. This section demonstrates that the logic of the model

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15 The variance of the original asset’s return is $\pi(1 - \pi)(Z_H - Z_L)^2$. Whether the variance of the returns on the alternate asset is higher than the variance of the returns on the original one thus depends also on the value of $\pi$. For $\pi = .5$ changes in $\pi$ have no first order effect on this variance so that the variance of the alternate asset is unambiguously higher. Interestingly, the first inequality of (32) implies that the mean return of the alternate asset is higher, so that this asset leads to a lower $U_{ca}$ solely as a result of risk aversion.
can also lead to the dissipation of tangible resources. The dissipation I focus on provides a simple explanation for a phenomenon that is widely observed during bank runs. This is that, as assumed for example in Stein (2012), banks who experience runs are forced to sell assets at a discount. Moreover, Moen and Tallman (1992, p. 617) show that “trust companies” sold nearly 40% of the loans they held as assets in a 4 month period during the Panic of 1907. These loans were sold to banks, whose total assets increased in this period. In this section, such transfers of assets take place in period 1.

The intermediaries who buy these assets in period 1 are assumed to be able to extract \( \rho \) times as much value from them as the banks that held them originally. Under the reasonable supposition that these new intermediaries have a comparative disadvantage in managing these assets, \( \rho < 1 \), and the fraction \( 1 - \rho \) of bank assets is dissipated if individuals make withdrawals in period 1 and use the proceeds to make deposits at a different bank. This section shows that, even with \( \rho < 1 \) there are two reasons to let individuals make these period 1 withdrawals instead of being forced to wait until they consume in period 2. Both of these reasons rely on the assumption that uncertainty about \( Z_i \) is resolved in period 1. The first involves depositors that are more overoptimistic about their response to information (in period 1) than they are about their chances to withdraw early in the period in which they consume. The second applies when, as is true quite generally, customers would prefer banks to promise different payments in different states. As in Allen and Gale (1998), the promised payment in period 1 can then be tailored to the low state.

Since individuals do not derive utility from consuming until period 2, depositors who withdraw \( Y_1 \) in period 1 redeposit these funds immediately at a new intermediary. Consistent with the Moen and Tallman (1992) evidence, the new intermediary uses these funds to purchase the assets held by the original one, though he obtains them at a discount. The new intermediaries are owned by a new class of agents, called arbitrageurs, who live only in periods 1 and 2.

Arbitrageurs are born with a substantial endowment \( B \) of a good that they can consume in period 1. This good is called good 1, with good 2 representing the good that is produced.
by investing $A$ in the original investment opportunity. In period 1, arbitrageurs are assumed to maximize the expected value of their utility

$$U_a = C_{1a} + C_{2a}, \quad (33)$$

where $C_{ia}$ represents their consumption of good $i$ in period $i$. Arbitrageurs are thus willing to buy risky assets in period 1 for a price that equals the yield they expect to receive in period 2. When controlled by arbitrageurs, the yield of one unit of the original asset is $\rho Z_i$ in state $i$.

Arbitrageurs are able to credibly pledge to pay out the proceeds from the assets they acquire in period 1. The existence of arbitrageurs then makes it possible for banks to offer payments of good 1 to depositors, since banks can obtain these goods by selling units of $A$ to the arbitrageurs. These depositors can then turn to the arbitrageurs, and offer them the units of good 1 they obtained from their bank in exchange for promises to receive good 2. Competition among arbitrageurs ensures that the price in terms of good 1 for a unit of good 2 is one. The result is that, if banks offer deposit contracts with a promised payment of $Y_1$ units of good 1 per unit of $A$ in period 1, those depositors who successfully withdraw their funds in period 1 obtain $Y_1$ units of consumption in period 2.

Consider now the case where depositors have more queue optimism when news are revealed in period 1 than in the period in which they intend to consume. The idea is that depositors see themselves as reacting more promptly than others to public information. The example of the 2007 run on Northern Rock, a UK bank dedicated to mortgages, may be useful as an illustration. Northern Rock financed many of its mortgages in public securities markets, and the cost of this funding rose through 2007 with the result that its stock price halved between January and September 1, 2007. Lines of depositors wishing to withdraw funds did not form until September 15, however. This was the day after the Bank of England first pledged that it would support Northern Rock with its own funds. This suggests that, while depositors have no reason to be optimistic about their capacity to know as much as stock markets, they may be optimistic about the rapidity with which they respond to public
To capture the importance of overoptimism at the moment information is released, let depositors’s subjective probability that they will be among the first $x$ to withdraw in the low state equals $G(x)$ in period 1 while it equals $x$ in period 2. Lack of overoptimism in period 2 ensures that, of all the contracts that pay off only when people need resources for consumption (period 2), autarky is the best. Overoptimism in period 1 can still justify the existence of sequential service contracts, however. The next proposition shows this to be true as long as the cost from transferring assets $(1 - \rho)$ is not too large.

**Proposition 5.** Suppose people are overoptimistic in period 1 and not in period 2 while

$$
(1 - e^{-\lambda \rho}) > r^\gamma (1 - e^{-\lambda}).
$$

*In this case, depositors prefer a sequential service contract that promises $Z_H$ in period 2 and a sum $Y_1$ in period 1 such that $Z_H - Y_1$ is an arbitrarily small positive number to autarky, which is itself preferable to a contract that only allows withdrawals of $Z_H$ in period 2.*

Recall that the optimality of offering a payment of $Z_H$ requires that $r$ be between $r_m$ and $r_o$. If $r$ equals $r_m$, which is given by (12), this proposition implies that $\rho$ cannot be smaller than 1.0. By contrast, a larger value of $r$ allows $\rho$ to be smaller. With $\lambda = 1.8$ and $\gamma = .5$, $r_o$ is .698 and $\rho$ can reach almost .95 if $r$ equals $r_o$. If $\gamma$ remains equal to .5 but $\lambda = 2.5$, $r_o$ is slightly above .5, and $r = r_o$ allows $\rho$ to be somewhat smaller than .84.

If $Y_1$ were set equal to $Z_H$, depositors who are able to withdraw “early” would be indifferent between making withdrawals in period 1 and waiting even if the state were high. To avoid these withdrawals in the high state, the proposition assumes that $Y_1$ is slightly smaller than $Z_H$. This value of $Y_1$ is not optimal, however, because it does not take advantage of the existence of two separate periods to tailor payments to the two states, as in Allen and Gale (1998). This gives a separate reason to offer contracts that allow withdrawals in period 1. If the firm offers payments in period 1 that will only be collected in the low state, equation (10) implies that the optimum payment in that period is $\rho Z_L/r_o$. This customization of this
payment can make it desirable to offer contracts that allow sequential withdrawals in period 1 even if people are just as optimistic about arriving early in period 2 as they are in period 1 and even if one restricts oneself to contracts that pay $Z_H$ in period 2. For a $(1 - \rho)$ large enough, this is shown in the next proposition.

**Proposition 6.** Suppose that (18) and (19) hold, and that $k = 0$ so that the optimal contract would promise $Z_H$ if only payments in period 2 were possible. Then, as long as

$$\frac{(1 - e^{-\lambda Z_L/Z_H})Z_H^\gamma}{(1 - e^{-n^{-1}Z_H})(\lambda Z_L/n^{-1}(\gamma))^\gamma} < \rho^\gamma < 1,$$

the equilibrium contract that promises $Z_H$ in period 2 also gives depositors the option of withdrawing $\rho Z_L/r_o$ in period 1. Depositors try to take advantage of this option in the low state, with those arriving late getting nothing.

6 Conclusion

The ubiquitousness of demandable liabilities suggests that people find them particularly valuable, and this paper has shown that overconfidence about arriving early enough to withdraw in the event that the institution has insufficient funds can help explain this phenomenon. This overconfidence creates a reason to pay creditors the amount they have been promised in the sequence in which they make their withdrawal requests even if none intend to consume the resulting funds before the period of sequential withdrawals is over. It thus rationalizes sequential service without the need for a sequential service constraint.

The demandable deposit contracts that cater to this overoptimism lead to inequality between people who arrive early and those who arrive late when there is a run. Even if agents are optimistic that they themselves will arrive early, their empathy for others should lead them to try to reduce this inequality, and this may explain why there are so many regulations seeking to reduce the risk of demandable deposits. An obvious question then is why these empathetic individuals don’t put even more roadblocks in front of strict sequential service contracts. One way of doing so is by applying to institutions offering demandable deposits the “preference” rules that apply to other institutions in bankruptcy.
In the United States, the ban against “preferences” in bankruptcy allows bankruptcy trustees to “claw back” funds, i.e., force creditors who have been paid in the past 90 days to return these funds and become regular claimants in bankruptcy. In principle, this eliminates the effects of sequential service. Depository institutions are presumably exempted from these rules because the transactions services that they offer, and which my analysis has abstracted from to focus on the role of overconfidence, would be impaired by these rules. Transaction benefits justify the existence of demandable deposits, and overconfidence then allows these deposits to grow well beyond actual transactions needs.

While the model has been motivated by the popularity of demandable claims, the form of overconfidence it introduces may help motivate other financial transactions. For example, “hot money” flows into developing countries often face the risk of a balance of payments crisis with an attendant dramatic change in the exchange rate. Nonetheless, investors might join such flows because they are convinced they will get out in time.

Overconfidence about arriving early in a bank run is quite closely related to overconfidence about learning early about an institution’s financial difficulties, and a similar modeling device might be applicable to both. Individuals with this related overconfidence might be willing to lend for a fixed term if they think that, even after they themselves become aware of an issuer’s difficulties, others will continue to roll over the issuer’s obligations. An increase in the likelihood that an issuer will be unable to pay might then lead to a shortening of the equilibrium maturity of debts, as this could allow overconfident investors to continue to believe that they will beat others to the exit.

One question that is left open by this analysis is whether overoptimism about exiting before others can be so important in financial markets given that private entrepreneurs do not seem to offer very many lotteries whose payoff depends on the order of one’s arrival after a triggering event. Specifying triggering events that do not appear to give informational advantages to insiders might be one source of difficulty for such lotteries. Relatedly, such lotteries might be subject to legal uncertainties that would prompt litigation by the “losers.”

REFERENCES

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### A Mathematical Appendix

**Proof that $G(x)$ is strictly increasing in $\lambda$ for $0 < x < 1$ and $\lambda > 0$:** Using (7), the derivative of $G(x)$ with respect to $\lambda$ is

$$\frac{dG(x, \lambda)}{d\lambda} = \frac{xe^{-\lambda}(1 - e^{-\lambda}) - e^{-\lambda}(1 - e^{-x\lambda})}{(1 - e^{-\lambda})^2}.$$  

This is zero at both $x = 0$ and $x = 1$. To demonstrate that it is positive for $0 < x < \lambda$, I first prove that, for $\lambda > 0$, $dG/d\lambda$ is increasing in $x$ at $x = 0$ and decreasing in $x$ at $x = 1$. I then prove that the $dG/d\lambda$ has no other zeros between 0 and 1 by showing that $dG/d\lambda$ is concave in $x$ at $x = 0$ and either remains concave or becomes convex in $x$ as $x$ rises. Since it never turns back from being convex to being concave, $dG/d\lambda$ remains above zero for $0 < x < 1$.

The derivatives of $dG/d\lambda$ with respect to $x$ satisfy

$$(1 - e^{-\lambda})^2 \frac{d^2G}{d\lambda dx} = e^{-x\lambda}(1 - e^{-\lambda})(1 - x\lambda) - \lambda e^{-\lambda(1+x)}$$

$$(1 - e^{-\lambda})^2 \frac{d^2G}{d\lambda dx^2} = \lambda e^{-x\lambda}[-(1 - e^{-\lambda})(1 - x\lambda) - (1 - e^{-\lambda}) + \lambda^2 e^{-\lambda}].$$
At $x = 0$, $d^2G/d\lambda dx$ has the same sign as $1 - e^{-\lambda} - \lambda e^{-\lambda}$ when $\lambda$ is positive. This is positive because $\lambda/(e^{\lambda} - 1)$ is an Einstein function that is smaller than one for $\lambda > 0$. At $x = 1$, $d^2G/d\lambda dx$ has the same sign as $e^{-\lambda}(1 - \lambda - e^{-\lambda})$. This is negative because the function $e^{-\lambda}$ is tangent to the function $1 - \lambda$ at $\lambda = 0$ and is above $1 - \lambda$ for higher values of $\lambda$.

The function $d^3G/d\lambda dx^2$ is increasing in $x$ for $\lambda > 0$. At $x = 0$, it has the same sign as $(2 + \lambda)e^{-\lambda} - 2$. This expression equals zero for $\lambda = 0$ and has a negative derivative with respect to $\lambda$. It follows that $d^3G/d\lambda dx^2 < 0$ at $x = 0$ for $\lambda > 0$ so that the function $dG/d\lambda$ is concave in $x$ at $x = 0$. As $x$ is increased, $d^3G/d\lambda dx^2$ rises. If it becomes positive, $dG/d\lambda$ becomes convex in $x$. It cannot, however, become concave once again for higher values of $x$. 

**Proof that $n(\lambda)$ is decreasing in $\lambda$ for $\lambda > 0$:**

The derivative of $n(\lambda)$ with respect to $\lambda$ is

$$
\frac{(1 - \lambda)e^\lambda - 1}{(e^\lambda - 1)^2}.
$$

For $\lambda > 0$, the denominator is positive. To see that the numerator is negative in this case, note first that the numerator is zero when $\lambda = 0$ and second that the derivative of the numerator with respect to $\lambda$ is $-\lambda e^\lambda$, which is negative.

**Proof of Proposition 1:** We already established that $\gamma > n(\lambda)$ implies that $r_m < r_o$ so that (18) gives a nontrivial region. Given that $\gamma > n(\lambda)$, (19) is a nontrivial region as well if and only if $\psi$ is positive. By definition, $\gamma = n(\lambda r_o)$. Since $n$ is a decreasing function and $r < r_o$, it follows that $n(\lambda r) > \gamma$ so that $\psi$ is indeed positive.

Now consider the desirability of setting $Y$ above $Z_H$. Differentiating (16),

$$
dU_+^+ = \gamma \left( \pi (1 - e^{-\lambda Z_H/Y}) + (1 - \pi)(1 - e^{-\lambda Z_L/Y}) \right) - \frac{\lambda}{Y} \left( \pi Z_H e^{-\lambda Z_H/Y} + (1 - \pi)Z_L e^{-\lambda Z_L/Y} \right) \frac{Y^{\gamma - 1}}{1 - e^{-\lambda}}.
$$

Evaluated at $Y = Z_H$, this derivative is

$$
\frac{dU_+^+}{dY} = \left[ \pi (\gamma - n(\lambda)) + (1 - \pi) \frac{(1 - e^{-\lambda r})}{(1 - e^{-\lambda})} (\gamma - n(\lambda r)) \right] Z_H^{\gamma - 1}.
$$

For individuals not to wish to be paid more than $Z_H$, this has to be smaller than or equal to zero. Given that $\gamma > n(\lambda)$ and $\gamma < n(\lambda r)$, this is satisfied when $\pi/(1 - \pi)$ is smaller than $\psi/\gamma(\gamma - n(\lambda))$, which is ensured by the second inequality of (19).

Now turn to the desirability of setting $Y$ below $Z_H$. Differentiating (17)

$$
dU_-^- = \gamma \left( \pi + (1 - \pi) \frac{1 - e^{-\lambda Z_L/Y}}{1 - e^{-\lambda}} \right) - (1 - \pi) \frac{\lambda Z_L e^{-\lambda Z_L/Y}}{Y} \frac{Y^{\gamma - 1}}{1 - e^{-\lambda}}.
$$

Evaluated at $Y = Z_H$, this derivative is

$$
\frac{dU_-^-}{dY} = \left[ \pi \gamma + (1 - \pi) \frac{(1 - e^{-\lambda r})}{(1 - e^{-\lambda})} (\gamma - n(\lambda r)) \right] Z_L^{\gamma - 1}.
$$

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If \( \pi/(1 - \pi) \) exceeds \( \psi/\gamma \), which is guaranteed by the first inequality in (19), this is positive so that individuals not to wish to be paid less than \( Z_H \).

**Proof of Proposition 2:** If the first inequality in (19) is violated and \( k = 0 \), \( Y < Z_H \). Bank owners then obtain more utility than \( U_{ck} \), so \( k \) must be positive. If the first inequality in (19) holds and \( k = 0 \), \( Y = Z_H \). To demonstrate that the equilibrium still features \( k > 0 \), it is enough to consider a small increase in \( k \) (starting at 0) that keeps owners indifferent and to demonstrate that this raises the utility of depositors. The first-order change in the utility of depositors is

\[
dU_c = U_{ck} dk + U_{cy} dY.
\]

Meanwhile, for owners to be indifferent \( dk \) must equal \(-(U_{by}/U_{bb})dY\). Substituting this for \( dk \) in the above expression, using (22), (23), and (24) for the partial derivatives of \( U_b \) and \( U_c \) and evaluating at \( Y = Z_H \) and \( k = 0 \) while remembering that \( Z_L/Z_H = r \), we have

\[
\frac{dU_c}{dY} = \left[ \gamma \pi + \frac{1 - \pi}{1 - e^{-\lambda}} \left( \gamma (1 - e^{-\lambda r}) - \lambda r e^{-\lambda r} \right) - \frac{(1 - \pi) \lambda e^{-\lambda r}}{1 - e^{-\lambda}} \frac{\pi}{1 - \pi} r^{1 - \gamma} \right] Y^{\gamma - 1}
\]

\[
= \left[ \frac{1 - \pi}{1 - e^{-\lambda}} \left( \gamma (1 - e^{-\lambda r}) - \lambda r e^{-\lambda r} \right) + \pi \left( \frac{\lambda r e^{-\lambda r}}{1 - e^{-\lambda}} - r^{-\gamma} \right) \right] Y^{\gamma - 1}
\]

\[
< \left[ \frac{1 - \pi}{1 - e^{-\lambda}} \left( \gamma (1 - e^{-\lambda r}) - \lambda r e^{-\lambda r} \right) + \pi \left( \frac{\lambda r e^{-\lambda r}}{1 - e^{-\lambda}} \right) \right] Y^{\gamma - 1} < 0,
\]

where the first inequality follows from \( r > r_m \) and the second from \( r < r_o \).

**Proof of Proposition 3:** Because \( \omega k < 1 \), the equilibrium is interior so that increasing bank capital above the equilibrium level is feasible. At an equilibrium, small increases in \( k \) accompanied by reductions in \( Y \) that keep bank owners indifferent also have no effect on depositor’s expected utility. Therefore

\[
U_{cy} - U_{ck} U_{by}/U_{bb} = 0,
\]

where \( U_{cy} \) and \( U_{ck} \) are positive, while \( U_{by} \) and \( U_{bb} \) are negative. Bystander utility would increase by raising \( k \) and keeping bank owners indifferent if \( \bar{U}_{cy} - \bar{U}_{ck} U_{by}/U_{bb} \) were negative. Comparing (24) and the first equation of (26), the first term of \( \bar{U}_{cy} \) is smaller than the first term of \( U_{cy} \) because \((1 - e^{-\lambda(1+k)Z_L/Y})/(1 - e^{-\lambda})\) is larger than \((1 + k)Z_L/Y\). The other terms in \( \bar{U}_{cy} - \bar{U}_{ck} U_{by}/U_{bb} \) are negative and equal \( \nu \) times the corresponding terms in \( U_{cy} - U_{ck} U_{by}/U_{bb} \) where

\[
\nu = \frac{\lambda e^{-\lambda(1+k)Z_L/Y}}{1 - e^{-\lambda}}.
\]

Thus, if \( \nu \) is less than one, \( \bar{U}_{cy} - \bar{U}_{ck} U_{by}/U_{bb} \) is negative in equilibrium. The equations in (2) show that \( \nu \) is less than one as long as \((1 + k)Z_L/Y\) exceeds the level that makes \( G'((1 + k)Z_L/Y) = 1 \).

**Proof of Proposition 4:** The first inequality in (32) implies that (29) is positive. The inequality with the first term of the min operator ensures that (30) is negative while the one with the second term leads \( dU_c \) to be negative. I first demonstrate that the left hand side of (32) is always smaller than the first term of the min operator. The former is the same as the latter with \( \gamma \) set to equal one. For the left hand side of (32) to be smaller, the derivative of \( \gamma/(r^{-\gamma} - 1) \) with respect to \( \gamma \) must be negative. To see that this is the case, notice first that this expression declines globally in the sense that its limit when \( \gamma \) tends to
zero, $1/\log(1/r)$ is larger than its value when $\gamma = 1$, which is $1/(1/r - 1)$. Moreover, the derivative of $\gamma/(r^{-\gamma} - 1)$ is

$$
\frac{r^{-\gamma} - 1 - \gamma \log(r)r^{-\gamma}}{(r^{-\gamma} - 1)^2}.
$$

The numerator of this expression is zero at $\gamma = 0$. For the derivative to change sign between $\gamma = 0$ and $\gamma = 1$, this numerator would have to have another zero in this range. But, this is impossible because the derivative of the numerator with respect to $\gamma$ equals $-\gamma^2(\log(r))^2r^{-\gamma}$, which is negative for $r > 0$. Since the derivative does not change sign and the expression is globally declining, its derivative must be negative, as claimed.

I now prove that the left hand side of (32) is smaller than the second term of the min operator. Since the denominators of both expressions are positive, this inequality is true if

$$
e^{-\lambda r} - e^{-\lambda} < \lambda(1-r)e^{-\lambda r} \quad \text{or} \quad 1 - e^{\lambda(r-1)} < \lambda(1-r).
$$

Since $\lambda(1-r)$ is an arbitrary positive number, this inequality requires that $x/(1 - e^{-x})$ be greater than one for all positive $x$. The limit of this expression when $x = 0$ is 1, while it equals infinity for $x$ infinite. The derivative of this expression with respect to $x$ is

$$
\frac{1 - e^{-x} - xe^{-x}}{(1 - e^{-x})^2}.
$$

The numerator of this is zero at $x = 0$ and the derivative of this numerator with respect to $x$ is $x^2e^{-x}$ which is always positive. Thus, there is no other value of $x$ for which the numerator equals zero. As a result, $x/(1 - e^{-x})$ is monotone increasing from $x = 0$ onward.

**Proof of Proposition 5:** Because the payments offered in period 1 is smaller than the payment $Z_H$ promised for period 2, and because the intermediary’s resources are sufficient to repay $Z_H$ in period 2, depositors wait to collect $Z_H$ in period 2 when they learn that the state is high (this is what the depositor who last has the right to withdraw would do if none of the others had withdrawn, and this would lead those who had the right to withdraw earlier to do the same). Therefore, the proposed sequential service contract has the same payoff as autarky in the high state. In the low state, a sequential service contract only has resources of $\rho Z_L$ per depositor in period 1. Because the promised payment for period 1 is only slightly smaller than that for period 2, depositors do not wait until period 2 to withdraw their funds when they learn the state is low. Even if all depositors did so, a depositor who deviated would ensure himself nearly $Z_H^\gamma$ by deviating and withdrawing in period 1 while he could only expect $Z_LZ_H^{\gamma-1}$ if he waited, which is less. If other depositors withdraw in period 1, the costs of waiting are even larger. The contract thus has a subjective expected payoff of $(1 - e^{\lambda Z_L/Z_H})Z_H^\gamma/(1 - e^{-\lambda})$ in the low state. Given (34), this exceeds the autarkic payoff of $Z_L^\gamma$ in the low state.

A contract that only allows withdrawals of $Z_H$ in period 2 has the same payoff as autarky in the high state. While its expected payoffs are also the same in the low state, it is riskier and thus less desirable.

**Proof of Proposition 6:** The expression in the left of (35) gives the ratio of $U_c$ in the low state when resources equal $Z_L$ and individual payments equal $Z_H$ to the level of $U_c$ when resources equal $Z_L$ and individual payments equal $Z_L/r_o$. This ratio is therefore smaller than one unless $Z_L/r_o = Z_H$. Condition (18), however, implies that $\gamma < n(\lambda Z_L/Z_H)$ so that $Z_L/r_o < Z_H$. Therefore, values of $\rho < 1$ that satisfy (35) can be found.

For any value of $\rho$ satisfying (35), the expected utility of depositors from withdrawing in period 1 when all others try to withdraw, resources per depositor equal $\rho Z_L$ and the payment
is set at $\rho Z_L/r_o > \rho Z_L$ equals

$$1 - e^{-\lambda \rho Z_L/(n-1(\gamma))} \left( \frac{\lambda \rho Z_L}{(n-1(\gamma))} \right)^\gamma = \frac{1 - e^{-n^{-1}(\gamma)}}{1 - e^{-\lambda}} \left( \frac{\lambda \rho Z_L}{n^{-1}(\gamma)} \right)^\gamma.$$  

Condition (35) then implies that this exceeds the utility they obtain if they all wait until period 2 where resources are $Z_L$ and the promised payment is $Z_H$. The contract that pays $\rho Z_L/r_o$ in period 1 yields this utility in the low state because depositors run in period 1. Even though they do not all succeed and those that arrive late get nothing, depositors who try to withdraw in period 1 when the state is low have higher expected utility than those who wait. This is true whether others do or do not withdraw in period 1.

Because the period 1 payment $\rho Z_L/r_o$ is less than $\rho Z_H$, depositors obtain $Z_H$ in period 2 when the state is high whether others withdraw in period 1 or not. Since withdrawing in period 1 nets depositors less, none of them do so when the state is high. Relative to a contract that only allows $Z_H$ to be withdrawn in period 2, the proposed contract therefore gives the same utility in the high state and higher utility in the low state.