Towards a compact, empirically-verified rational expectations model for monetary policy analysis

A comment

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In a series of prominent and influential papers of which the current paper is an outgrowth, Jeffrey Fuhrer has been proposing a radical rethinking of much of empirical macroeconomics. I very much like Fuhrer's basic approach to macroeconomic modeling which involves the specification and estimation of three behavioral equations. The first of these is the interest-rate rule which gives the behavior of the central bank as it conducts monetary policy. The second is the IS curve which gives the behavior of purchasers of final output as a function of the real interest rate. Finally, the third equation is the price-setting rule; it characterizes the behavior of the producers of output – be they workers or firms. Each of these three equations involves three variables, namely, the nominal interest rate, the rate of inflation, and the level of output. Given the centrality of these three variables in our field, this structure provides a wonderfully convenient vehicle for discussing many of the basic questions with which macroeconomics is concerned.

In the current paper, Fuhrer spends most of his time developing and estimating a set of equations describing consumption and investment which represent potential replacements for the IS curve of his original papers with George Moore. These newer equations are derived from optimizing behavior by at least some of the agents doing the consuming and the investing. These equations are then embedded into a complete model that also contains the interest-rate and price-setting rules from the earlier papers. The complete model is stochastically simulated and the correlations among the variables of the simulated model are then compared with the actual correlations among macroeconomic variables. As Fuhrer shows, many of the simulated correlations, particularly those relating real variables and interest rates, are grossly
counterfactual. Because the match between simulated and actual correlations was better in the earlier papers with George Moore, Fuhrer concludes that the earlier IS curve may have been better after all. This analysis is very similar in structure to that in Fuhrer and Moore (1995a). In that earlier paper, the authors compare the simulated correlations produced by a model with a standard Taylor rule for price setters to the simulated correlations for a behaviorally less comprehensible price-setting rule which I will call the Fuhrer-Moore price-setting rule. They conclude that the simulated correlations using the Fuhrer-Moore rule are closer to the actual dynamic correlations and that this establishes that the Taylor rule does not fit the facts.

I want to argue today that neither the earlier nor the current conclusion of Fuhrer’s work is warranted so that there is hope for optimizing models both of price-setting and of aggregate demand. The reason Fuhrer reaches these conclusions, it seems to me, is that his method of model evaluation is problematic. I want to start my discussion by describing this problem in general. Then I want to turn to the specific empirical failures that Fuhrer highlights for optimizing IS curves and Taylor rules and suggest how they can be circumvented while also making the models more appealing from a behavioral perspective.

Supposing for expository purposes that there is a single IS curve, the models in Fuhrer’s papers can be written as

\[ \Phi(L)z_t = \epsilon_t \]

where \( z_t = (y_t, i_t, \pi_t) \)

\( y_t, i_t \) and \( \pi_t \) represent detrended output, the interest rate and inflation at \( t \), respectively, and where the elements of \( \Phi \) can be derived from the parameter estimates. Even though the decision rules in Fuhrer’s models often involve expectations of future values of \( y, i, \) and \( \pi \), these can be made functions of past values of \( y, i, \) and \( \pi \) by using either the model itself or a vector autoregression to forecast these future values. One key feature of Fuhrer’s models is that, after the expected future values of \( y, i, \) and \( \pi \) are treated in this way, the equations do not fit perfectly so that there is a behavioral residual for each. A different point of view would be that these behavioral equations would fit exactly if only we observed agents’ expectations of the future correctly. This alternative seems to me less promising for empirical work because it seems unlikely that we will ever find deterministic relations among actual macroeconomic variables.

Assuming \( \Phi(L) \) is invertible, (1) can be written as

\[ z_t = M(L)\epsilon_t \quad M(L) = M_0 + M_1L + M_2L + \ldots \]
where the $M$ matrices represent the impulse responses of the endogenous variables to the $\epsilon$ shocks. Fuhrer assumes that the $\epsilon$'s are uncorrelated at all leads and lags. The vector autocovariance implied by the theoretical models is then

$$EZ_t Z'_{t-k} = \sum_i M_{i+k} E(\epsilon_i \epsilon'_i)M'_i$$

As Fuhrer shows, the implied vector autocorrelation is often different from the actual one. If, for instance, the impulse responses imply that $\epsilon$'s lagged $k$ periods have very small effect on the current endogenous variables, the elements of $M_{i+k}$ would be small and the corresponding elements of the vector autocorrelation of the $Z$'s would be small as well.

What is apparent from this discussion is that the model would be much less restrictive if the $\epsilon$'s were allowed to be serially correlated. The vector autocovariance would then be

$$EZ_t Z'_{t-k} = \sum_i \sum_j M_i E(\epsilon_{t-i} \epsilon'_{t-k-j})M'_j$$

The elements of this vector autocovariance could be large even if $M_{i+k}$ is small, as long as $\epsilon_t$ is highly correlated with $\epsilon_{t-k}$. Indeed, if one replaces $E(\epsilon_{t-i} \epsilon'_{t-k-j})$ in the above expression with the actual autocovariance matrix of the residuals in (1), one fits perfectly the elements of the vector autocovariance of $Z$. The answer to Fuhrer's question "Could the same structural model augmented to include serially correlated residuals, capture the dynamics inherent in the data?" is thus trivially yes and requires no calculation of the kind reported in the paper.

Thus, the key problem in fitting the autocorrelation function of $Z$ with appealing theoretical models is the assumption that the behavioral residuals are independently distributed over time. This assumption favors models which, when fitted to the data, have residuals which are indeed i.i.d. In particular, the above argument makes it clear that if the residuals from (1) happen to be i.i.d., the theoretical autocovariance of the $Z$'s would coincide with the empirical one. Thus, Fuhrer is led to favor models with white noise residuals even if these have limited theoretical appeal.

I now illustrate this general point by taking up the IS curve and the price-setting equation in turn. In both cases, I will argue that optimizing models with serially correlated residuals can help explain the patterns Fuhrer finds in the data in a way that is more appealing than the explanation provided by Fuhrer.

Ignoring constants, the original Fuhrer-Moore, IS curve was

$$y_t = 1.34y_{t-1} - .37y_{t-2} - .36\rho_t - .01 + \epsilon_t^{IS}$$

(3)
where $\rho_t$ is the ex ante real interest rate at $t$.\footnote{Fuhrer and Moore consider both specifications in which $\rho_t$ is a long-term real interest rate and specifications where it is a short-term rate.} Treated as a behavioral equation that is supposed to represent the behavior of consumers and purchasers of real assets, this specification is almost incomprehensible. Why should lags of output be so important? Why should lagged real interest rates but not current ones affect decisions at time $t$? Empirically, this specification has a great advantage, however. In particular, the residuals from an equation of this sort estimated by ordinary least squares are nearly serially uncorrelated.

The lack of theoretical appeal of this specification becomes even more apparent when one compares it to specifications that are actually consistent with intertemporal optimization by consumers. As is well-known, the logarithmic approximation to the first-order condition for an infinitely lived consumer maximizing a time separable utility function $U(C)$ is

$$E_t \beta \frac{U''(C)}{U'(C)} (c_{t+1} - c_t) + \rho_t + \epsilon^S_t = 0$$

where $c_t$ is the logarithm of consumption and $\epsilon^S_t$ is a behavioral residual which could be due to changes in the discount rate $\beta$.\footnote{The real interest rate in this equation is the real interest rate between $t$ and $t + 1$, whereas Fuhrer and Moore tend to use longer duration real rates. This difference is unlikely to be important empirically given that Fuhrer and Moore (1995b) show that their real interest rate is essentially identical to the nominal federal funds rate.} This says that high real interest rates at $t$ are associated with a high growth rate of consumption between $t$ and $t + 1$. If one ignores variations in the ratio of consumption over GNP, $c_{t+1} - c_t$ is the same as $y_{t+1} - y_t$ and this equation becomes the expectational IS curve of Kerr and King (1996) and McCallum and Nelson (1997). This IS curve can be rewritten as

$$y_{t+1} = y_t + \sigma \rho_t + \epsilon^s_t + \epsilon^{EIS}_{t+1} + \delta \epsilon^{S}_{t+1}$$

where $\sigma$ is positive because it equals the inverse of the coefficient of relative risk aversion and $\epsilon^{EIS}_{t+1}$ represents the revision at $t + 1$ of the expectation of $y_{t+1}$. If one treats this as a regression equation of $y$ on lagged $y$ and the lagged real rate, there are two differences between this expectational IS curve and the Fuhrer and Moore IS curve. The first is that the coefficient on lagged income is one in the former, whereas it is a bit larger in the latter. The more important difference is the sign of the coefficient on the expected real rate. This is positive when the IS curve is derived from consumer optimization and negative in the Fuhrer-Moore specification.

Since real interest rates are positively serially correlated, (4) must have positively correlated residuals and so cannot do well when it is evaluated using Fuhrer's methodology. To see whether this equation would do better
when it is assumed to have serially correlated residuals, I analyze whether a simulated model that includes a forward-looking IS curve with serially correlated residuals can lead to a positive correlation between current interest rates and lagged output. I focus on this particular correlation because Fuhrer calls attention to the failure of forward-looking models in this respect.

I thus simulate a model identical to the one in Fuhrer and Moore (1995b) except that I substitute (4) with $\sigma$ equal to one for their IS curve. Figure 1 shows the impulse response of output and interest rates for a one-percent increase in $c_s^s$ under two assumptions about the serial correlation of $c_t^s$. In the first, $c_t^s$ is i.i.d. while in the second it follows a first-order autoregressive process with parameter $\lambda$ equal to .99. The latter leads to more drawn-out responses of output. The result is that, assuming this is the only shock, the correlation between the interest rate at $t$ and lagged output is -.33 when $\lambda$ is zero while it is .09 when it is .99. Similarly, the correlations between the current interest rate and output lagged twice are -.24 and .05 respectively while those of the current interest rate with output lagged three times are -.19 and .01 respectively. Thus, it is certainly possible to obtain positive correlations between current interest rates and lagged output, which Fuhrer regards as an important feature that optimizing models have trouble reproducing. A similar result is reported in Fuhrer’s paper: making the residuals in the IS curves serially correlated ensures that the correlation between interest rates and lagged investment becomes positive (though there appears to be no effect on the correlation of interest rates with lagged consumption).

While this example is not necessarily ideal, it shows that changes in the serial correlation properties of the residuals that affect the forward-looking IS curve have significant effects on the resulting vector autocorrelation functions. And, it seems more reasonable to assume that such residuals are serially correlated than to assume that they are white noise. As far as the overall IS curve is concerned, it is shifted not only by changes in discount rates but also by changes in the profitability of new investment and by changes in government spending. Both of these are likely to be serially correlated as well.

I now turn my attention to price-setting rules. In Fuhrer and Moore (1995a), the Taylor rule is criticized because, once combined with the other ingredients of their model, it leads to insufficient positive serial correlation in inflation rates (which they label inflation persistence). I start with a theoretical discussion of how the Fuhrer-Moore price-setting rule differs from the

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3 In the figure, the magnitude of the time $t$ shock is reduced when $\lambda$ is equal to .99 to ensure that the initial responses of output are comparable. This adjustment obviously has no effect on the vector autocorrelations.

4 Fuhrer and Moore (1995b) shows that these correlations are positive, though they are modest in size.
Figure 1. Response to $\varepsilon^s$
Taylor rule and how it rationalizes inflation persistence. Then, I consider simulations with a Taylor rule subject to serially correlated disturbances.

For the theoretical discussion, I consider a setting where half the firms set prices in each period and where these prices remain in effect for two periods. The logarithm of the price chosen by a firm which changes its price at \( t \) is labelled \( x_t \) so that the logarithm of the price level at \( t, p_t \) is

\[
p_t = \frac{1}{2} (x_t + x_{t-1})
\]  

If one follows Taylor, price-setters at \( t \) set their price according to

\[
x_t = \frac{1}{2} (p_t + E_t p_{t+1}) + \gamma y_t + \epsilon_t^P
\]

where \( \gamma_y \) equals \((y_t + E_y y_{t+1})\). This equation can be rationalized in a number of ways. My own preference is for thinking of equations like this as arising from the optimization problem of imperfectly competitive firms that must keep their prices constant for a period of time, but there are other derivations that emphasize labor contracts. The basic implication is that price-setters want to set their price equal to the average price level that will prevail during the life of their fixed price, adjusted for increases in \( y \) which increase either the marginal cost of production, or the disutility from labor or both. Using (5), this equation can be written in two other revealing ways. These are

\[
x_t = \frac{1}{2} (x_{t-1} + E_t x_{t+1}) + 2\gamma y_t + 2\epsilon_t^P
\]

Equation (7) says that the price chosen by price-setters at \( t \) is an average of the prices set by other price-setters during the period for which prices will be fixed, again adjusted for \( y \). Equation (8) says exactly the same thing but makes explicit that price-setters are setting their relative price so that it is an average of the other relative prices that will prevail during the time that prices are fixed, again corrected for changes in \( y \).

Fuhrer and Moore suppose instead that price-setters behave according to

\[
x_t - p_t = \frac{1}{2} (x_{t-1} - p_{t-1}) + \frac{1}{2} (E_t x_{t+1} - p_{t+1}) + 2\gamma y_t + 2\epsilon_t^P
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\]
In other words, using \((x_t - p_t)\) on the left-hand side of (8) still leads to a dynamic equation in which current inflation depends only on the current level of output and the level of inflation expected in one period and where both of these variables have a positive effect on current inflation. The more important difference is that price-setters at \(t\) set their prices as a function of the relative price that people who set their prices at \(t - 1\) had at \(t - 1\) instead of setting it as a function of their relative price at \(t\). This seems hard to defend. Why should anyone at \(t\) care about relative prices that are no longer in existence?

While it is hard to rationalize from a behavioral point of view, this assumption is central in creating inflation persistence. This can be seen by imagining a shock to \(\varepsilon_t^P\). In either model, this leads to an increase in \(x_t\) which requires that inflation rise at \(t\). Also in either model, this increase in \(x_t\) tends to raise \(x_{t+1}\) which raises prices further at \(t + 1\). However, in the Fuhrer-Moore specification this second effect is larger, so there is more inflation at \(t + 1\). The reason is that, from the perspective of \(t + 1\), the lagged price \(p_t\) is lower than the current price \(p_{t+1}\) and this makes \(x_{t+1}\) larger when it is set according to (9) than when it is set according to (8). In economic terms, an increase in \(\varepsilon_t^P\) raises the relative price charged at \(t\) by those who set their prices at \(t\). In the Fuhrer-Moore model this leads to an increased desired relative price by those who set their price at \(t + 1\), whereas this desire is absent in the Taylor model. It seems more natural to suppose instead that whatever force led price-setters at \(t\) to raise their relative price might still be operative at \(t + 1\). Then, this force would also lead price-setters at \(t + 1\) to want to raise their relative prices. In other words, it seems more natural to appeal to serially correlated disturbances.

In Figure 2, I report three sets of impulse responses for inflation after a shock to \(\varepsilon_t^P\). The first is computed for the model of Fuhrer and Moore (1995b) (which involves contracts longer than two periods) under the assumption that \(\varepsilon^P\) is serially uncorrelated. This shows that, as they note, even a serially uncorrelated shock of this sort leads to persistent inflation in their model. The other two responses are constructed by replacing the Fuhrer-Moore pricing specification by the analogous pricing equation based on the Taylor model that is used in Fuhrer and Moore (1995a). This specification keeps the contracting parameters, including the parameter \(\gamma\), the same as in the model with the Fuhrer-Moore price-setting rule. The first response from this model is constructed assuming a serially uncorrelated \(\varepsilon^P\). As shown in Fuhrer and Moore (1995a), this leads to very little inflation persistence. Assuming this is the only source of shocks, the correlation of current inflation with lagged inflation is .42 while the correlation with inflation-lagged 4 periods is -.08. However, when \(\varepsilon^P\) is assumed to follow a first-order autoregressive process with an autoregressive parameter \(\lambda\) equal to .9, inflation proves to be per-
sistent in this model as well.\footnote{5} Assuming once again that this is the only source of shocks, the correlation of inflation with lagged inflation is .96 while that with inflation-lagged 4 periods is .70. This is very close to the numbers generated using the Fuhrer-Moore pricing rule, which are .91 and .70 respectively.

Among the shocks that can lead to changes in $e^P$ are technology shocks, shocks to the desired markup of price over marginal cost, and shocks to union power. It is hard to understand why any of these shocks would have to be serially uncorrelated over time so that the specification with correlated shocks seems \textit{a priori} more attractive.\footnote{6}

In conclusion, Jeffrey Fuhrer’s work poses an important challenge. His findings suggest that we either have to expunge the optimizing models we have already developed from our general equilibrium macro models or that we have to allow for serially correlated behavioral disturbances. To me the choice seems easy because I see no reason why the disturbances to the forward-looking IS curve and to the pricing equation should be serially uncorrelated.

\footnote{5}The impulse response that is plotted for this case involves a shock of smaller absolute magnitude to ensure that the initial response is comparable to the responses in the case of uncorrelated shocks.

\footnote{6}The existence of such serially correlated shocks also makes it difficult to interpret Phillips curve regressions relating the current inflation rate to the current unemployment rate and the lagged inflation rate. Depending on the sample, such regressions can lead to a coefficient estimate near one on the lagged inflation rate. This is then given an “accelerationist” interpretation in which a low unemployment rate is regarded as leading to an acceleration of inflation. Estimation of equations of this sort by OLS obviously leads to coefficient estimates that are contaminated by the serial correlation in the shocks to the price-setting equation and are thus a fragile source of inferences of this type. The evidence on the properties of price-setting rules presented in Roberts (1995) is less subject to this criticism and thus seems more compelling.
Figure 2. Response to $e^p$
References


