

## **CEO Incentives and Firm Size**

George P. Baker,  
Harvard University and NBER

and

Brian J. Hall,  
Harvard University and NBER

June 11, 2002

George Baker  
Harvard Business School  
Baker Library 283  
Boston, MA 02163  
gbaker@hbs.edu

Brian Hall  
Harvard Business School  
Baker Library 185  
Boston, MA 02163  
bhall@hbs.edu

We thank Carliss Baldwin, Rob Gertner, Robert Gibbons, Thomas Knox, Ed Lazear, Ashish Nanda, Nancy Rose and an anonymous referee for helpful comments. We also thank Jia Liu, Ali Ahsan, Humayun Khalid and especially Sandra Nudelman for excellent research assistance.

### Abstract

What determines CEO incentives? A confusion exists among both academics and practitioners about how to measure the strength of CEO incentives, and how to reconcile the enormous differences in pay sensitivities between executives in large and small firms. We show that while one measure of CEO incentives (the dollar change in CEO wealth per dollar change in firm value) falls by a factor of ten between firms in the smallest and largest deciles in our sample, another measure of CEO incentives (the value of CEO equity stakes) *increases* by roughly the same magnitude. We resolve the confusion about which of these measures better reflects CEO incentives by developing and solving a model that allows CEO productivity to differ for firms of different sizes. The crucial parameter is shown to be the elasticity of CEO productivity with respect to firm size. Our empirical results suggest that CEO marginal products rise significantly, and overall CEO incentives are roughly constant or decline slightly with firm size. We thus confirm Rosen's (1992) conjecture that the actions of executives in larger firms have a "chain-letter like" effect on firm performance. We also show that the appropriate measure of incentives depends on the type of CEO activity being considered. For activities whose dollar impact is the same for large and small firms (such as the purchase of a corporate jet), the dollars-on-dollars measure is appropriate, and large firms suffer significant agency problems due to their weak incentives. For activities whose *percentage* impact is similar across firms of different sizes (such as a corporate reorganization), the equity stake measure is better, and the incentive problem faced by large firms is not as severe. Finally, using a multi-task model, we discuss the implications of our findings for the design of control systems.

## I. Introduction

How do the incentives for large-firm CEOs compare to those of small-firm CEOs? This seemingly simple question is fraught with ambiguity. A confusing debate rages among academics and practitioners about what determines CEO incentives.<sup>1</sup> This confusion manifests itself in a number of ways: in the range of empirical specifications for pay-to-performance regressions in the literature; in the wide discrepancy in estimates of pay sensitivities; in popular controversy over the appropriate level of executive holdings of stock and stock options.

Schaefer (1998) showed that the pay sensitivity (measured as the dollar change in CEO wealth per dollar change in firm value) falls with the square root of firm size. If CEO incentives are determined by pay sensitivity measured in this way, then Schaefer's estimates imply that CEO incentives are ten times lower for a \$10B firm than for a \$100M firm. But, as noted by Rosen (1992) and Holmstrom (1992), the key question is how to compare the incentives faced by the CEOs of companies of dramatically different sizes. In particular, the notion that CEOs of large companies have trivial incentives relative to CEOs of small companies seems odd given the very large wealth swings induced by the enormous stock and option holdings of large-company CEOs (Hall and Liebman, 1998).

Consider that the CEO of a \$10B firm, who owns only 0.5 percent of the firm's stock, still holds an equity stake worth \$50M. Given these large stakes, it seems implausible to many that the incentives of large-company CEOs are so much weaker than those of small-company CEOs. The large equity stake of the large-company CEO would seem to give strong incentives to raise the share price, since a small percentage change in the stock price changes the CEO's wealth by millions of dollars. Indeed, it seems plausible that the incentives of this large-company CEO are stronger, rather than weaker, than the incentives facing the small-company CEO. This raises the obvious question, which is central to this paper: in terms of the strength of CEO incentives, what matters? Is it only the "percent owned," or do "dollars at stake" also matter?

This paper attempts to answer several questions about the appropriate measure of incentives for top managers. We develop a model of pay-to-performance sensitivity that bridges the

---

<sup>1</sup> See, for example, Jensen and Murphy (1990), Garen (1994), Haubrich (1994), Joskow and Rose (1994), Hall and Liebman (1998), Murphy (1999) and Aggarwal and Samwick (1999).

gap between these two “measures” of incentives and enables us to analyze how the strength of incentives relates to the scale of the firm. We use this model to re-examine the data on CEO pay-to-performance relationships, deriving new insights on how variation in firm size affects CEO incentives, firm structure and control systems.

Our model predicts how the optimal pay-to-performance sensitivity of a CEO varies with firm size. As with the standard model, the optimal sensitivity depends on the variability of firm returns, CEO risk aversion and the marginal product of the CEO’s actions on firm value. The key difference between our model and the standard one is that we do not assume that this latter parameter—, the marginal product of CEO actions on firm value—is invariant to firm size. Indeed, we argue that the assumption that the marginal product of effort is invariant is one of two polar cases, the other polar case being that CEO marginal product scales proportionally with firm size.

We show that if the marginal product of effort is constant across firm size, then incentives are determined solely by the dollar change in CEO wealth per dollar change in firm value. This measure, first estimated by Jensen and Murphy (1990) and labeled “b”,<sup>2</sup> falls very quickly in our model as firm size rises (since the variance of firm value explodes with firm size), leaving CEOs of large firms with trivial incentives relative to small-firm CEOs. Although this model is common in the literature, it is appropriate only for thinking about the class of decisions made by CEOs whose marginal products do not scale with firm size. An example would be the purchase of a corporate jet. A CEO that owns 1 percent of the firm can buy a corporate jet at a 99 percent discount and a CEO that owns 10 percent of the firm can buy the jet at a 90 percent discount. This discount to the CEO is invariant to firm size. Only the percent owned matters.

The opposite polar case is when the marginal product of effort scales proportionately with firm size. Examples of actions that scale with firm size might be a corporate reorganization

---

<sup>2</sup> Throughout this paper, we will loosely refer to “b” as the “percent owned.” This is strictly correct if the CEO owns only stock. If the CEO also owns stock options or has cash pay that is sensitive to firm value, then b will be larger than percent ownership. In this paper, we ignore the sensitivity of annual salary and bonus changes to performance, but we are careful to include the incentive effects of executive stock options. Since, for a given change in stock price, the Black-Scholes value of one dollar’s worth of stock option changes more than the value of one dollar’s worth of stock, b is larger than the CEO’s equity stake in the firm (the value of stock and options) divided by firm value. There is also a question about whether executives value options at their Black-Scholes value; we return to this question below.

or a change in the strategic focus of the firm. In cases such as these, good (or bad) actions by the CEO affect the entire firm. In the words of Rosen (1992), the CEO's actions have a "chain-letter like" effect on the value of the firm. We show that in the polar case of proportional scaling (the elasticity of  $\ln V$  with respect to firm size is one), the strength of incentives can be inferred from the CEO's equity stake in the firm,  $b$  times the value of the firm.<sup>3</sup> The intuition for this is straightforward: if the marginal product of actions scale proportionally with firm size, then CEO actions affect the *percent change*, not the dollar change, in firm value. In this case, the strength of CEO incentives can be measured by the CEO's equity stake in the firm (which will be multiplied by any percentage point change in firm value).

We do not consider one other possible variable that might affect the strength of CEO incentives: the fraction of the CEO's wealth that is tied up in the firm. We ignore this variable for two reasons. First, we have no way to measure, with the data available, the non-firm wealth of the CEOs in our sample. But we consider this lack of data to be only a modest drawback for the following reason. Almost all of the CEOs in our sample have stakes worth millions of dollars. We believe that a very high percentage of our CEO's wealth is inside the firm and furthermore believe that the variation in percentage of wealth inside the firm varies far less with firm size than either their percentage ownership (which declines dramatically with firm size) or their dollar stakes (which increase dramatically with firm size). We argue that any cross-sectional differences in the marginal utility of income between CEOs with different wealth levels will not be significant when compared to the variations in incentives that come from either their percentage ownership or the size of their stakes.<sup>4</sup>

We use our model, a range of assumptions about CEO risk aversion, and two data sets on CEO pay and equity ownership to estimate the relationship between the marginal product of effort ( $\ln V$ ) and firm size. The data strongly rejects either of the two polar cases. Our estimate of the elasticity of CEO marginal product with respect to firm size is approximately

---

<sup>3</sup> Again, if the CEO owns only stock, then  $b$  is the percent owned and the value of the CEO's stake equals  $b$  times firm value. However, since virtually all of our CEOs hold options, the value of their stock and option holdings is less than  $b$  times firm value. Throughout the paper, we refer to  $b$  times firm value as the CEO's stake in the firm.

<sup>4</sup> We do believe that this variation in the fraction of wealth at stake may change a CEO's propensity to take risks. Analysis of this effect is beyond the scope of this paper.

0.4, rejecting an elasticity of zero ( $\epsilon$  is constant across firm size) and one ( $\epsilon$  increases proportionately with firm size). We interpret this as evidence that CEOs do a range of activities, the marginal product of which scales with size in varying degrees.

With regard to total incentives and size, we find that incentives are roughly constant or fall slightly as firms become larger. This is because total incentives—the marginal returns to effort—equals  $b$  times  $e$ . As firms become larger,  $b$  falls strongly and  $e$  rises sharply. The combined effect in most of our estimates is zero or a small decline. Thus incentives may fall as firm size increases, but not nearly as drastically as many models predict. The large dollar equity stakes of large-company CEOs do matter.

Our results have a number of related implications, which we summarize here and detail in Section V.

- We argue that both the empirical and theoretical literatures have mischaracterized the relationship between firm size and managerial marginal products, and as a result have provided misleading estimates of pay sensitivity.
- We explain the existence of expensive corporate staffs in large firms as the response to the high marginal product of CEO effort in these firms.
- The low (but optimal)  $b$ 's in large firms relative to those in smaller firms induce greater agency problems for some activities than others. More specifically, activities with the same dollar impact (positive or negative) on large and small firms will create greater agency problems for large firms, necessitating more monitoring by large firms than small.
- We explain the existence of bureaucratic “red tape” in large firms.

The paper proceeds as follows. In the next section, we lay out our model of managerial production and compensation, including our strategy for estimating the marginal product of CEO effort. In Section III, we describe our data sources and summarize the data. Section IV contains our estimates of the elasticity of CEO marginal products with respect to firm size, and our estimates of incentive differences across firms of different sizes. Section V discusses implications of our simple model, and develops a richer multi-task model that allows a reinterpretation of the results of Section IV. Section VI concludes.

## II. Model

In order to understand variation in pay sensitivities across firms of different sizes, we need a model that can accommodate differences in the managerial production function. We construct a model that does this, allowing the effects of managerial action on value to differ for different size firms.

Consider a firm value production function that maps managerial actions on to firm value. We assume that firm value has components that are affected by managerial effort, and components that are independent of managerial effort.

$$V_{i,t+1} = (V_{it}) a_{it} + V_{it} + (V_{it}),$$

where:

$V_{i,t+1}$  is the value of firm  $i$  at the beginning of period  $t+1$ ,

$V_{it}$  is the value of firm  $i$  at the beginning of period  $t$ ,

$(V_{it})$  is the random component of firm value in period  $t$ .  $(V_{it})$  is a normally distributed random variable with mean 0 and a standard deviation  $(V_{it})$ .  $(V_{it})$  varies with the size of the firm.

$a_{it}$  is the effort of the CEO of firm  $i$  in period  $t$ ,

$(V_{it})$  is the marginal product of managerial effort. It also varies with the size of the firm.

CEOs have disutility for effort. We assume that all CEOs share the same dislike for their effort, or at least that CEOs of larger firms are not systematically different from those of smaller firms with respect to their cost of effort.<sup>5</sup>

$$C(a_{it}) = \frac{a_{it}^2}{2}$$

---

<sup>5</sup> This is a potentially troubling assumption, since more able CEOs, who could be thought of as having lower cost of effort, may tend to be selected into larger firms. We deal with this issue by subsuming cross-sectional differences in effort cost into the difference in the marginal product of effort. Thus we consider CEOs with low effort cost to have high marginal products, and vice versa. This assumption has no effect on our theoretical or empirical analysis. See the discussion at the end of this section.

CEOs are risk averse, with negative exponential utility for wealth, and additively separable cost of effort. This implies that the expected utility for monetary rewards can be captured with the mean and the variance of wealth. CEOs maximize their expected utility at the end of period  $t$ :

$$E[U_i(W_{i,t+1}, a_{it})] = E(W_{i,t+1}) - \frac{1}{2} \rho_i \sigma_{W_{it}}^2 - C(a_{it}),$$

where:

$W_{i,t+1}$  is CEO  $i$ 's wealth at the beginning of period  $t+1$  (the end of period  $t$ ),

$E(W_{i,t+1})$  is the expected value of CEO  $i$ 's wealth at the end of period  $t$ ,

$\rho_i$  is the CEO's measure of absolute risk aversion,

$\sigma_{W_{it}}^2$  is the variance of the CEO's wealth in period  $t$ .

The utility function that we use has no "wealth effects." That is, rich CEOs trade off money and effort in the same way that poor CEOs do (although they may have different risk tolerances). As discussed above, we believe that this simplification does not change the interpretation of our results very much.

CEO rewards in the model come from a fixed salary and the change in wealth resulting from changes in the value of stock and option holdings. Jensen and Murphy (1990) and Hall and Liebman (1998) have shown that variations in the value of stock and options are very large relative to variations in salary and bonus. So we restrict our attention to this component of CEO rewards, and ignore the sensitivity of salary and bonus to changes in firm value.<sup>6</sup> We abstract from the non-linear nature of the value of options, and assume that the relationship between CEO wealth and firm value can be modeled as a simple linear function. Thus, the CEO's wealth at the beginning of period  $t+1$  is:

$$W_{i,t+1} = W_{it} + S_{it} + b_i(V_{i,t+1} - V_{it}),$$

where:

---

<sup>6</sup> We also ignore differences in "sensitivity" that might come from differential likelihood of firing. Jensen and Murphy's (1990) estimates of the incentive effect of such actions, however, suggest that they are not large. We also have no prior on how they vary with firm size. The effects of other governance differences (tighter monitoring by the board, more rigid control systems) are addressed in Section V.

$b_i$  is the CEO's effective ownership percentage,<sup>7</sup>

$S_{it}$  is a fixed component of CEO rewards, independent of firm performance.

Given this compensation scheme, the CEO chooses effort to maximize utility. The first-order condition for this effort choice is:

$$(1) \quad a_{it} = b_i (V_{it})$$

In this model, the CEO's effort decision depends on the strength of the pay-for-performance relation ( $b_i$ ), and on the marginal product of effort,  $\gamma_i$ . (We now drop the explicit functional notation for  $a_{it}$  and  $S_{it}$ , and refer to  $(V_{it})$  as  $v_i$  and  $(V_{it})$  as  $v_i$ ).

The optimal  $b_i$  involves the trade-off standard in the literature: bigger  $b$ 's mean better incentives for effort, but impose more risk on the CEO. This simple model yields the following familiar expression for the optimal slope of the compensation scheme:

$$(2) \quad b_i^* = \frac{\gamma_i^2}{\gamma_i^2 + 2\rho_i\sigma_i^2}$$

Note that, in this expression, the more important CEO actions are relative to the amount of random variation in firm value (that is, the size of  $\gamma_i$  relative to  $\sigma_i$ ), the larger will be the optimal sensitivity of pay-to-performance.

We could use this model to predict how  $b^*$  will change with firm size. To do this, however, would require that we know how  $\gamma_i$ ,  $\rho_i$  and  $\sigma_i$  change with firm size. The latter two parameters are not a problem. If we assume constant relative risk aversion, then absolute risk aversion is inversely proportional to the CEO's wealth. If we can estimate how a CEO's wealth changes with firm size, then we can infer how a CEO's absolute risk aversion changes with firm size. In addition, the relationship between a firm's size and the standard deviation of its returns is well known. However, we do not know how the marginal product of a CEO's effort changes with firm size.

---

<sup>7</sup> We recognize that most of an executive's stock is held voluntarily, in the sense that the firm cannot formally prevent an executive from selling stock. However, formal rules (such as stock ownership guidelines), informal pressures (including concern over market signals) and the implicit threat to renegotiate salary and bonus terms, give boards much control over CEO stock holdings. See Core and Guay (1999) for evidence that boards attempt to bring executives to a target level of equity incentives.

The model yields useful predictions, however, on what determines CEO incentives based on different possible relationships between  $\beta$  and firm size. The elasticity of  $\beta$  with respect to firm size is the critical parameter. If this elasticity is zero, then  $\beta$  is invariant with firm size. Under this assumption (common in the theoretical literature, and the implicit one in Jensen and Murphy (1990) and the many subsequent empirical papers), the dollar change in CEO wealth for each dollar change in firm value—that is,  $b$ —determines the strength of CEO incentives. It does not matter whether the CEO owns 1 percent of a \$100M firm, or 1 percent of a \$10B firm; an action that wastes \$1M in one firm (say, the purchase of a corporate jet) also wastes \$1M in the other, and both cost him \$10,000. Absent differences in the marginal utility of income, he will make the same choice even though the first CEO has \$1M at stake, while the second has \$100M at stake.

Consider now the opposite extreme: that the elasticity of  $\beta$  with respect to firm size is one. Such an assumption is implicit in much of the empirical literature (for instance, Joskow, Rose and Shepard (1993) and Gibbons and Murphy (1990); for a discussion, see Joskow and Rose (1994)). In this case, an action that has a \$5M effect on a \$100M firm will have a \$500M effect on a \$10B firm. An example of this type of action might be the reorganization of the firm or a change in strategic direction. For this type of activity, the CEO's actions affect firm *percentage returns*—the same action increases (or decreases) the value of the firm by 5 percent. Under this assumption, CEO incentives are determined not solely by  $b$ : a CEO with a 1 percent stake in a small firm has less incentive than a CEO with a 1 percent stake in a large firm.

Reference to Equation (1) above shows why. Since effort is determined by  $b_i$ , and  $\beta_i$  increases proportionally with firm size, the strength of incentives is also proportional to  $b_i V_i$ . This result helps explain, and partly justify, the intuition that the dollar value of a CEO's holdings of stock and options matters to his incentives. If the CEO owns only stock, so that  $bV$  is equal to his stake,<sup>8</sup> and  $\beta$  is proportional to  $V$ , then the CEO's stake is a good measure of the strength of his incentives. Table 1 summarizes the differences between a model that assumes that the elasticity of  $\beta$  with respect to firm size is zero and one that assumes that this elasticity is one.

---

<sup>8</sup> As discussed in footnote 4, if the CEO also holds options with positive exercise prices, then the value of his holdings will be less than  $bV$ .

*Estimating the marginal product of CEO effort*

Since we do not know how the marginal product of CEO effort changes with respect to firm size, we cannot predict how the optimal  $b$  changes with firm size. Instead, we take the opposite approach. We assume that the  $b$ 's that we observe in the data are optimal (or at least not differentially biased with respect to firm size; more on this later), and use this assumption to estimate the elasticity of  $\rho_i$  with respect to firm size. Solving Equation (2) for  $\rho_i$  yields:

$$(3) \quad \rho_i = \sqrt{\frac{2 b_i^* \rho_i \sigma_i^2}{1 - b_i^*}}$$

For each firm in the sample, we know  $b^*$  and  $\rho_i$  in a year. We must only get a handle on how  $\rho_i$  (the measure of absolute risk aversion) varies with firm size. There is reason to suspect that it might: CEOs of larger firms are likely to be wealthier (and thus have lower levels of absolute risk aversion) than the CEOs of smaller firms. We make the assumption that all CEOs have the same level of relative risk aversion (although we examine this assumption in Section IV), and then use multiple estimates of how CEOs' wealth changes with firm size.

Absolute risk aversion is equal to:

$$\rho_i = \frac{\alpha}{W_i}$$

where  $\alpha$  is the coefficient of relative risk aversion.

Since we do not know CEO wealth, we make three different assumptions about the relationship between firm size and CEO wealth.

1. CEO wealth is proportional to total annual compensation (salary, bonus and the market value of stock and options, measured by Black and Scholes (1973) and Merton (1973)).
2. CEO wealth is proportional to the CEO's wealth in the firm (the sum of the value of CEO stock and stock option holdings).
3. CEOs of large firms are neither richer nor poorer than CEOs of small firms.

Using data on firm size, the standard deviation of returns and each of these assumptions about the wealth of CEOs, we calculate a value for  $\beta$  for each firm in the sample. Since we are only interested in how  $\beta$  changes with firm size, we will not worry about the absolute size of  $\beta$  (whose units are arbitrary anyhow: dollars of firm value per unit of effort). Rather, we plot how  $\beta$  varies with firm size and use regression analysis to estimate the elasticity of  $\beta$  with respect to firm size.

In understanding our methodology, it is important to recognize that the actual level of relative risk aversion ( $\gamma$ ), as well as the multipliers that we use (for our estimates of the wealth of the CEO) are irrelevant to our estimates of how  $\beta$  varies with firm size. Note that any change in the scale of  $\beta_i$  has no effect on the ratios of  $\beta$ 's, nor does it affect elasticity estimates. Since what we are investigating is how  $\beta$  changes with size, only the relationship between  $\beta$  and size matters.

It is now possible to see why we can subsume cross-sectional differences in the *cost of effort* into our parameter for the *marginal product of effort*. Note that if we had assumed that CEOs' cost of effort varied with firm size, our expression for  $\beta^*$  would change. Everywhere that the current expression has  $\beta_i^2$ , we would need to replace it with  $\beta_i^2/C_i''$ , where  $C_i''$  is the second derivative of the cost of effort function. Such a change would alter none of the analysis—it would only change the parameter that we are estimating. Instead of estimating the marginal product of CEO effort in a firm, we would be estimating the ratio of this marginal product to the square root of effort cost for the CEO in this firm. This is the sense in which a CEO with a low cost of effort can be thought of as working in a firm with a high marginal product of effort.<sup>9</sup>

### III. Data Sources and Description

We use two different sources of compensation data for our empirical estimation. Our first source is the dataset on CEO stock and option holdings used by Hall and Liebman (1998). We use the most recent year (1994) and the earliest year (1980) in their sample. The

---

<sup>9</sup> Taken literally, our model predicts that if a particular CEO moves from a small firm to a large one, his marginal product will take a large jump. While this is probably true to some extent, our estimates of  $\beta$  will tend to overstate this change if  $C''$  falls with firm size. It would be possible to exploit this, and possibly disentangle the effect of a CEO's ability from the effect of a change in his marginal product by analyzing changes in incentive arrangements around CEO transitions.

advantage of this data is that it contains very precise and detailed measures of how the value of a CEO's stock and stock option holdings change with changes in firm value. Hall and Liebman (1998) used the history of stock and stock option grants (and exercises and sales) to determine the details about each CEO's portfolio of stock and stock option holdings. This approach is particularly important for options since the value and sensitivity of an option cannot be determined with the Black-Scholes formula without knowing the characteristics of each option held—remaining maturity, dividend rate, volatility, stock price and exercise price. The details of the methodology used to calculate CEO stock and stock option holdings are contained in the appendix of Hall and Liebman (1998).

While the advantage of the Hall-Liebman dataset is its precision (in measuring  $b$  and CEO wealth in the firm), the disadvantage is that it contains only a sample of the largest publicly traded firms (approximately half of the companies in the *Forbes* annual survey of the largest companies). This includes 320 firms in 1994 and 304 firms in 1980 after (35 and 29, respectively) founders are dropped.<sup>10</sup> Because of this, we also use a second dataset, Standard and Poor's *ExecuComp* for the year 1996. This dataset contains a much larger sample (1,125) of firms,<sup>11</sup> and a wider range of firms in terms of firm size.

The disadvantage of using this larger dataset is the loss in precision in measuring the value and the sensitivity of the stock option packages. *ExecuComp* reports only the number of in-the-money options held by the CEO and we do not have the precise characteristics of each CEO's stock option holdings at any point in time. Nevertheless, the dataset has a reasonably good measure of the total number of shares and (in-the-money) options held by each CEO and we adjust option holdings into share equivalents by multiplying each option

---

<sup>10</sup> We drop founders throughout because we are analyzing how owners (principals) use incentives to motivate top managers (agents) and founder-CEOs are both owners and managers. As a robustness check, we re-ran all of the regressions in the paper including founders (which have much higher  $b$ 's on average since they have large stock holdings). Although the coefficients are typically measured with less precision when founders are included, none of the substantive results of the paper are changed and so we do not report them.

<sup>11</sup> Although *ExecuComp* does not designate founders, we removed companies whose CEOs owned more than 5 percent of the company since it is very atypical for executives to accumulate such large company stock holdings absent being founders. But again, our results are not sensitive to different cutoff points, or to inclusion of all of these firms.

by 0.7, which approximates the median delta in the Hall-Liebman data.<sup>12</sup> For the non-compensation variables—market value and volatility—we use CRSP data. The volatility measures we use (the standard deviation and variance of changes in market value) are annualized and based on monthly data for the previous 36 months, consistent with the standard practice.

Summary statistics for each of the variables used in this paper, and for each year, are shown in Table 2. In addition to the variables already described, these variables include CEO age, CEO tenure, annual firm sales and CEO total compensation (which is defined as salary, bonus and the annual value of stock and options grants). All of these variables are from *ExecuComp* (or from the proxies in the case of the Hall-Liebman data) with the exception of firm sales, which was taken from Compustat.

#### IV. Empirical Results

Our empirical results fall into two main categories: the relationship between the marginal product of effort ( $\gamma$ ) and firm size, and the relationship between incentive strength ( $b^*$ ) and firm size. We estimate each for three measures of  $\gamma$  (based on the three assumptions about the relationship between firm size and CEO wealth noted above) and three time periods: 1980, 1994 (both using Hall-Liebman) and 1996 (using *ExecuComp*).

##### *CEO marginal productivity ( $\gamma$ ) and firm size*

In order to see the basic trends in the data, we begin by plotting the  $\gamma$ 's against firm size as measured by the market value of the company. Since we are interested in estimating the elasticity of  $\gamma$  with respect to firm size, we plot the log of  $\gamma$  against the log of firm size.<sup>13</sup> We do this for each of the three measures of the  $\gamma$ -size relationship described above (now denoted as  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$ , respectively). Figure 1 shows the plots for the year 1996. The

---

<sup>12</sup> The median delta is about 0.6 for new (typically at-the-money) options, but the median option held by CEOs is in-the-money, which leads to a higher delta.

<sup>13</sup> Of course, regressing logs on logs yields an accurate estimate of the elasticity only for small changes in the variables. For large changes (like those in this study), we must convert our logs-on-logs estimate back into an elasticity. If  $\beta$  is the logs-on-logs estimate, then the true elasticity for a given percentage size change  $\Delta s/s$  is equal to  $(1 + \beta \Delta s/s) - 1$ .

plots for 1994 and 1980 (not shown) look quite similar, though with fewer data points. In all cases, there appears to be strong and positive relationship between  $\beta$  and firm size. Note that  $\beta$  rises fastest with size in the (probably unrealistic) case where CEO wealth is assumed to be constant across firm size.

In order to estimate the elasticity of  $\beta$  with respect to firm size, we regress the log of  $\beta$  on the log of size for each of the three years and each of the three measures of  $\beta$ . In order to ensure that our results are not driven by outliers, we report robust regressions<sup>14</sup> as well as OLS estimation. For each regression, we report the coefficient and t-tests against the null hypotheses of the two polar cases we have described: that the elasticities are equal to zero (no relationship between  $\beta$  and size) and one (that there is a proportional relationship between  $\beta$  and size).

The results are reported in Table 3. The point estimates for the elasticity are approximately 0.4, ranging from 0.33 to 0.5 for the 1 and 2 cases. In the 3 cases, the elasticities are, as expected, a bit higher and slightly above 0.6. In all cases, both null hypotheses are strongly rejected at conventional levels of significance. For all years and all specifications, the elasticity estimates are neither zero nor one. In addition, the fit of the model is surprisingly good. The adjusted  $R^2$  in most of the regressions are quite high. Firm size appears to be an important determinant of the marginal product of CEO effort.

We conduct several robustness checks, which are reported in Table 4. In these and all subsequent regressions, we report only robust regressions, but all of the results are substantively similar under OLS specifications. Our first robustness check involves using sales rather than market value as our measure for the size of the firm. Annual sales is arguably a less noisy measure than market value as a proxy for the “scale” of the firm. The results, with the log of sales replacing the log of market value, are shown in the first column of Table 4. The elasticity estimates are, on average, a bit lower than our earlier estimates. But they are still generally in the range of 0.4 and precisely estimated as before, easily rejecting the hypothesis that the elasticity is either zero or one.

---

<sup>14</sup> This is the RREG command in Stata, version 6.0. RREG first performs an initial screening to eliminate gross outliers according to Cook (1977) and then performs Huber iterations followed by bi-weight iterations (Li, 1985).

A second set of robustness checks addresses concerns that our results may be driven by differences between the CEOs themselves, or by differences in the firms other than size. In particular, newly appointed and younger CEOs are known to have lower levels of stock and option ownership than CEOs with longer tenures. Perhaps our assumption that the  $b$ 's are optimal only holds for CEOs with enough tenure that they have had time to accumulate an optimal stock and option portfolio. In addition, differences in incentive patterns across industries may be driving our results. In order to address concerns of this sort, we re-run the regressions only on CEOs with above-median tenures (column 2), and for CEOs with above-median ages (column 3). We include industry dummies in these regressions as well.<sup>15</sup> The coefficient on firm size goes up slightly, but still remains far from the polar cases of zero or one.

The coefficients on the industry dummies are interesting in and of themselves. Are there some industries where the marginal product of effort seems to be systematically and significantly larger than in others? We re-run the regression on the full sample, controlling for age and tenure, and including a full set of industry dummies for each of the 59 two-digit SICs in our sample. The results are shown in the next two columns (specification 4) of the table. First, the results show that the coefficient on firm size is highly significant and around 0.4. Second, only one of the industry dummies, that for regulated utilities (SIC 49), is consistently significant across specifications.<sup>16</sup>

We report the size of the coefficient on the industry dummy for SIC 49 in the fifth and last column of the table. The coefficient is large, negative and (in seven of nine cases) highly significant. The magnitude of the (significant) coefficients range from  $-0.44$  to  $-1.67$ , which implies that the marginal product of CEO effort in this industry is somewhere between 36 percent and 81 percent lower than the average marginal products of CEOs in other industries. Such a finding is consistent with the hypothesis suggested by Joskow, Rose and Shepard (1993) that regulations constrain the discretion of the CEOs in regulated

---

<sup>15</sup> The results are substantively similar with and without industry controls.

<sup>16</sup> Since we want to compare the individual industry coefficients to the coefficient of the median industry, we estimated each equation, found the SIC code with the median coefficient, and dropped the SIC code with the median coefficient. Thus, the size of the coefficients and all tests of significance are relative to the median industry for each of the nine specifications.

industries—which in turn lowers the marginal product of their effort—and with Palia's (2000) finding that regulated firms attract lower quality managers.

We conduct one final robustness check. As noted earlier, we use changes in the market values of stock and options (as valued by Black-Scholes) to measure executive incentives. But the market value of executive portfolios generally represents an overestimate of the value of stock and options from an executive's perspective since the executive—in violation of the assumptions of Black-Scholes—cannot trade the options on market exchanges or otherwise hedge their risk. Thus, risk-averse and undiversified executives rationally value their non-tradable stock and options at less than their market values.<sup>17</sup> As a result, measures of *incentives* based on market values—changes in the value of executive portfolios in response to changes in firm value—also generally represent an overestimate of the incentives facing executives holding non-tradable stock and options.

To the extent that any bias in our measures of  $b$ , and therefore  $\beta$ , is correlated with firm size, our estimates of the elasticity of  $\beta$  with respect to firm size will be similarly biased. In order to investigate this possibility, we recalculated our  $b$ 's by measuring incentives as the change in the certainty equivalent (or *executive value*)<sup>18</sup> of executive portfolios (in response to a \$1,000 change in firm value) rather than the change in the market value of executive portfolios. The measures of  $b^*$  based on executive values lead to new measures of  $\beta$ , as calculated by Equation (3). Because of data limitations, we reestimate the  $\beta$ -size elasticity

---

<sup>17</sup> For evidence and analysis on this point, see Lambert, Larcker and Verrecchia (1991), Hall and Murphy (2000, 2002) and Meulbroek (2001).

<sup>18</sup> This certainty equivalence approach follows Lambert, Larcker and Verrecchia (1991) and Hall and Murphy (2000, 2002). Under this approach, the *executive value* of a portfolio is the dollar amount of cash that would be required to make an executive indifferent between holding the cash and the executive's portfolio of company stock and options. Under the assumptions below, the executive value is virtually always below the market value of an executive's portfolio. This requires a utility function and assumptions regarding a number of parameters. Following Hall and Murphy (2000, 2002) and Hall and Knox (2002), we use measures of volatility, dividend rates and betas based on company-specific, historical data. We assume executives have constant relative risk-aversion of 2.5 and have approximately one-half of their total wealth in the firm. We assume a risk-free rate of return of 6 percent and an equity-premium of 6.5 percent. Following Hall and Knox (2002), we use executive and company-specific data on the details of executive stock and options portfolios (remaining time to maturity, stock prices, exercise prices, number of options, etc.). The model allows executives to optimize the fraction of their non-firm wealth in bonds or equities (i.e., the market portfolio) and allows the executive's fraction of wealth in the firm to vary—within the range of 0.1 to 0.9 while preserving the 0.5 average—according to the executive-specific relationship between firm-specific wealth and compensation. See Hall and Knox (2002) for more details on the assumptions and technique used to measure the executive value of executive portfolios.

for our 1996 sample only.<sup>19</sup> The results are shown in column 5 of Table 4. The estimates are quite similar to those of Table 3, with precisely estimated elasticities in the range of 0.4 to 0.5, and higher when  $\beta_3$  is used. This suggests that any bias resulting from our use of market (rather than executive) values to measure incentives is minor.

There are three final points worth mentioning regarding our estimates of the  $\beta$ -size relationship. First, comparison of the estimated elasticities under  $\beta_3$  with those under  $\beta_1$  and (particularly)  $\beta_2$  provide a nice implicit test of how sensitive our results are to changes in assumptions about risk aversion. The assumption underlying the  $\beta_3$  estimates (essentially that CEOs have constant absolute risk aversion) is surely an overstatement of the risk aversion of large company CEOs. However, the assumption underlying the  $\beta_2$  estimate is that the non-firm wealth of CEOs goes up as quickly with firm size as the value of their stock and option packages. We find this assumption almost equally unlikely. Thus, we are probably underestimating the risk aversion of large firm CEOs with the assumptions underlying the estimates on  $\beta_2$ . Yet, while the estimated elasticities change in the appropriate directions in moving from  $\beta_2$  specifications to  $\beta_3$  specifications, they do not change dramatically and are all many standard deviations from either zero or one. Our basic finding that the elasticity is between zero and one is therefore robust to widely different assumptions about executive risk aversion.

Second, we emphasize that our basic results still hold even if the  $b$ 's are a noisy measure and even a downward (or upward) biased measure of the optimal  $b$ 's. As noted, our basic  $\beta$ -size elasticity estimates are still valid so long as the magnitude of the bias in  $b$  is not correlated with firm size. However, there is reason to believe that this correlation may be present. Jensen and Murphy (1990) have suggested that public and private political forces reduce the pay-to-performance sensitivity of CEOs. To the extent that they are correct, we suspect that these forces would be stronger in larger firms, creating a downward bias in  $b$  that is larger for larger firms. In this case, the true  $\beta$ -size elasticity is larger than the one we have estimated, since optimal  $b$ 's higher than what we observe would imply a higher  $\beta$ .

---

<sup>19</sup> We use the data, and executive value measures of the incentives created from executive portfolios, from Hall and Knox (2002). Their sample spans 1996 to 2000 and is based on *ExecuComp*. The sample size drops from 1125 to 859 executives since multiple years of prior data, not present for all executives, is required to build up executive portfolio holdings with sufficient detail to simulate executive values.

Although we have no way of testing this formally, we are confident that this bias does not change the basic finding in this paper that the  $\beta$ -size elasticity is between zero and one.<sup>20</sup>

Third, it is interesting that our most plausible estimate of the  $\beta$ -size elasticity is about 0.4, and between 0.3 and 0.4 when sales is the proxy for firm size. These estimates are strikingly close to the 0.3 estimates of the elasticity on *the level of CEO pay* with respect to size (Murphy 1985, Rosen 1992). Rosen's explanation for the increase in CEO pay with firm size is that the marginal product of the CEO rises with size. Of course, the marginal product that we are estimating in this paper (the marginal product of CEO effort) is different from one that determines compensation in the market for CEOs (which presumably is determined by the marginal value of CEO ability). Although beyond the scope of this paper, we find this to be an intriguing finding and one worth further study.

#### *Incentive strength ( $\gamma*b$ ) and firm size*

As Rosen (1992) and Holmstrom (1992) have argued, the relationship between firm size and incentives has not been carefully analyzed. One exception is Schaefer (1998), who estimates that pay-to-performance sensitivities (as measured by "b") are inversely proportional to the square root of firm size. The inverse relationship between pay-to-performance and firm size is also strongly present in our data. We regress  $\ln(b)$  on  $\ln(\text{size})$  and estimate an elasticity of  $-0.48$  for the 1996 *ExecuComp* data and  $-0.55$  for the 1994 Hall-Liebman data. Both estimates are highly significant. Our elasticity estimate of  $-0.5$  is equivalent to Schaefer's result.

However, the strongly declining pay-to-performance sensitivity does not imply that "incentive strength" declines in a similar way, although pay-to-performance sensitivities are sometimes loosely interpreted as measures of incentive strength. Since the marginal product of CEO effort ( $\beta$ ) rises with firm size, while  $b$  falls with firm size, the effect of increasing firm size on CEO effort is unclear. Recall from the model (Equation (1)) that managerial effort is determined by  $b*$ . Thus, the relationship between incentive strength and firm size is determined by the comparison of declining  $b$  with the rising  $\beta$ .

---

<sup>20</sup> To illustrate, suppose that the betas for the largest firms were biased downward by a factor of 10. In this case, the estimated elasticity for  $\beta = 1$  (in the 1996 data) would rise from 0.46 to only about 0.6.

In order to examine this issue, we conduct a similar set of tests as above, with  $b^*$  replacing  $b$ . Figure 2 shows plots of  $\ln(b^*)$  versus the log of firm size (analogous to Figure 1) for 1996. Table 5 shows regressions analogous to those in Table 4.

In Figure 2, the plots show no strong relationship between incentive strength and firm size. (As before, the plots for the other years look similar and are not shown.) The estimates in the regressions, which control for CEO tenure, CEO age and SIC code, are consistent with the plots in that they do not reveal a consistent relationship between incentive strength and firm size. While more of the coefficients are negative and significant, three of the nine are statistically insignificant and one is positive and significant.<sup>21</sup>

These findings underscore the importance of the distinction between pay-to-performance and pay-to-effort (incentive strength) in cross-sectional data. In estimating pay-to-performance, researchers have used different specifications, and (partially as a result), have generated very different estimates of pay sensitivities. Underlying these different specifications and different interpretations of incentive strength has been a different set of assumptions about how managerial actions affect firm value.

As an illustration, Figure 3 shows how two different definitions of pay sensitivity, common in the empirical literature, vary with firm size. The first definition is the “dollar-on-dollar” measure of pay sensitivity: how much does CEO wealth change for each dollar change in firm value? Consistent with Schaefer (1998), this measure drops dramatically with firm size.

The second measure of pay sensitivity shows how CEO wealth changes for each 1 percent change in firm value (implicit in regressions with returns on the right hand side). This measure *increases* dramatically with firm size! These two measures of pay sensitivity implicitly embed the two polar assumptions discussed above about how CEO marginal products vary with firm size.<sup>22</sup> Note the different implications of these two implicit

---

<sup>21</sup> The results for 1996 are quite similar when  $b$ 's and  $b^*$ 's are based on executive rather than market values of executive portfolios and are therefore not reported.

<sup>22</sup> They also correspond to Holmstrom's (1992) two models of pay sensitivity. Holmstrom (1992) claims that what he calls the “geometric form” (in which the elasticity of  $b$  with respect to firm size is one, as contrasted with the “arithmetic form” in which the elasticity is zero) is better, based in part on its fit with the data.

assumptions. If pay-to-performance measures are interpreted as measures of incentive strength, then specifications with changes in dollar value on the right hand side lead to the conclusion that incentives are much weaker in large firms, while specifications with percent changes on the right hand side lead to the opposite conclusion.

Our analysis suggests that neither is correct. Both measures in Figure 3 reflect extreme assumptions about the relationship between CEO productivity and firm size. When the appropriate intermediate relationship between CEO productivity and firm size is taken into account, the relationship between incentive strength and firm size is shown to be between the two extremes shown in the graph: in the aggregate, incentives are roughly constant, or fall somewhat, as firm size increases.

## V. Implications and Extensions

We turn now to examine the implications of these results for firm structure, and the design of control systems. To fully understand the results, we need to develop the theoretical model more fully, using a multi-task framework.

### *Firm size and structure*

If the marginal product of CEO effort for a large firm is many times that for a CEO of a small firm, what implications does this have for organization structure? Since neither person has more than 24 hours available in the day, the value to firms of CEOs “leveraging” their time is greater for large firms than for small ones since the CEO’s marginal productivity is much higher. Large staffs and the hiring of high-priced consultants by large-company CEOs are a likely consequence.

If it is true, as our elasticity estimate of about 0.4 suggests, that the marginal product of CEO effort is 16 times higher for the CEO of IBM (with a market value of \$150B) than for a small company with a market value of \$150M, then it is no surprise that the CEO of IBM has a large staff, while the CEO’s of smaller companies typically have much smaller staffs.

Of course, it is important to recognize that the estimates of marginal product that we provide in Section IV are really *averages* across a CEO’s many activities. In fact, CEOs take many actions, some of which affect firm value in dollars (like the purchase of a jet), and some of which affect the value of the firm in percentage terms (like conceiving a new

corporate strategy). In order to explore this insight more fully, we develop a multi-tasking model of CEO effort and reinterpret our empirical results in light of this model.

*A multi-task model of CEO value creation*

We enrich the model developed in Section II by introducing a large number of tasks in which a CEO engages. For each task, the marginal product may differ, and the relationship between the marginal product and firm size may differ. Thus, in this model, CEOs can engage in both “jet-like” and “strategy-like” activities.

Consider the following modification to the model presented above.

$$V_{i,t+1} = \sum_{j=1}^n \gamma_j (V_{it}) a_{itj} + V_{it} + (V_{it}),$$

where:

$V_{i,t+1}$  is the value of firm  $i$  at the beginning of period  $t+1$ ,

$V_{it}$  is the value of firm  $i$  at the beginning of period  $t$ ,

$(V_{it})$  is the random component of firm value in period  $t$ .  $(V_{it})$  is a normally distributed random variable with mean 0 and a standard deviation  $(V_{it})$ .  $(V_{it})$  varies with the size of the firm.

$a_{itj}$  is the effort of the CEO of firm  $i$  in period  $t$  on task  $j$ . There are  $n$  tasks that the CEO engages in.

$\gamma_j(V_{it})$  is the marginal product of managerial effort on task  $j$ . There are  $n$  marginal products to go along with the  $n$  tasks. Some of these marginal products could be zero. Each marginal product also varies with the size of the firm.

CEOs have disutility for effort on each task. Again, we assume that all CEOs share the same dislike for their effort on each task, but we could include a task- and firm-specific cost-of-effort term, that would affect the optimal effort choice. Such a firm- and task-specific cost term can again be subsumed into the marginal product of effort. The total cost of effort is:

$$C(a_{it1}, a_{it2}, \dots, a_{itn}) = \sum_{j=1}^n \frac{a_{itj}^2}{2}$$

CEOs' risk aversion and utility for wealth are identical to the model above:

$$U_i(W_{it}, a_{it}) = E(W_{it}) - \rho_i \sigma_{W_{it}}^2 - C(a_{it1}, a_{it2}, \dots, a_{itn}),$$

where:

$W_{it}$  is CEO  $i$ 's wealth in period  $t$ ,

$E(W_{it})$  is the expected value of CEO  $i$ 's wealth in period  $t$ ,

$\rho_i$  is the CEO's measure of absolute risk aversion,

$\sigma_{W_{it}}^2$  is the variance of the CEO's wealth in period  $t$ .

In this model, the CEO chooses effort levels on each task to maximize utility. There are  $n$  first order conditions:

$$(4) \quad a_{itj} = b_i \gamma_j(V_{it}) \quad j = \{1, 2, \dots, n\}$$

In this model, the CEO's effort decision on each task depends on the strength of the pay-for-performance relation ( $b_i$ ), and on the marginal product of his effort on this task,  $\gamma_j$ .

As in Section II, the optimal  $b_i$  involves the trading off incentives and risk. This model yields the following expression for the optimal slope for the compensation scheme:

$$(5) \quad b_i^* = \frac{\sum_{j=1}^n \gamma_{ij}^2}{\sum_{j=1}^n \gamma_{ij}^2 + 2\rho_i \sigma_i^2}.$$

In the multi-tasking model, instead of  $b^*$  being a function of *the* marginal product of effort, it is a function of the marginal products of effort on each task. The more tasks that a CEO performs, and the more that each task affects firm value, the higher will be the optimal  $b$ . The "importance" of CEO actions relative to the randomness in firm value is now measured by the sum of squared marginal products.

With this model in mind, we can now reinterpret the results in Section IV. Nothing in the estimation procedure nor the reporting of the results need change: only the interpretation of the marginal product. Now, instead of estimating the relationship between size and  $\gamma$ , we are estimating the relationship between size and the term that replaced  $\gamma$  in the numerator

and denominator of Equation (5):  $\sqrt{\sum_{j=1}^n \gamma_{ij}^2}$ .

This multi-task model yields an interesting new set of predictions about how CEO effort will vary across different types of tasks in firms of different sizes. Note that, since the firm can choose only one  $b^*$ , the CEO's effort is allocated across tasks according to the marginal product of effort on that task (see Equation (4)). This implies that CEOs of large firms, with optimally small  $b$ 's, will tend to devote little attention to tasks whose marginal products do not scale with firm size compared to CEOs of small firms. Symmetrically, for those tasks whose marginal product grows proportionally with firm size, CEO effort in large firms will be much greater than that for CEOs of small firms. An example will help to illustrate.

Consider a simplification of the above model with two only two tasks—one that does not scale with firm size, and one that scales proportionally. There are two firms, one with market value of \$1B, the other with market value of \$10B. According to our estimates in Section IV,  $\sqrt{\gamma_{ij}^2}$  should be about 2.6 times larger in the large firm, while  $b^*$  should be about 3.16 times smaller. Using these two estimates as parameters, we calculate the marginal products of effort on the two tasks in the two firms, and calculate the CEO's effort choices in each firm. Our assumptions and simulation results are shown in Table 6.

Note that, in the small firm, relative CEO effort is higher on the tasks that do not scale, while relative CEO effort in the large firm is greater for the tasks that do. This suggests that CEOs of large firms (relative to CEOs of small firms) will spend more of their time worrying about activities that have system-wide effects, and will spend less time worrying about those that do not. Similarly, CEOs of small firms will “sweat the details” more than the CEOs of large firms, and will expend relatively less effort on tasks that have more wide-ranging effects.

#### *Firm size, control and bureaucracy*

This multi-task model also makes predictions about how large and small firms will differ in their design of management control systems, and about the optimality of bureaucratic “red tape” in larger firms.

Consider the relative magnitude of the agency problem for different tasks faced by large firms. For tasks whose marginal products scale with firm size, the low  $b^*$  in the large firm will not be too great a problem: the high margin on the CEO's effort for these tasks will keep her attentive to this sort of activity. The CEO understands that if the development of a

new corporate strategy raises the value of the firm by a small percentage, this represents a large amount of value creation, and therefore, a large wealth increase to the CEO (even with a small  $b^*$ ). However, for tasks whose marginal products do not scale (such as perquisite consumption) with firm size, the very low  $b^*$  in the large firms will induce significant agency costs. The cost of a corporate jet is not very great to a CEO who owns 0.2 percent of the firm.

In large firms, with low (but optimal)  $b$ 's, boards will find it worthwhile to design control systems that monitor certain types of activities by CEOs (i.e., perquisite purchases) very carefully. Such systems are likely to filter down throughout the organization, appearing in the form of spending sign-offs, capital and operating budgeting systems and other forms of red tape. Such systems will thus tend to be much more prevalent in large firms than in small ones. In this sense, bureaucracy increases optimally with firm size and is a necessary cost of being large. Monetary incentives designed to tie an executive's wealth to firm value simply will not solve the problem of inefficient perquisite consumption in large firms.

## **VI. Conclusion**

Both the theory and practice of executive pay are hampered by disagreement and confusion over the appropriate measure of CEO incentives. We show that this confusion, shared by academics and practitioners alike, derives from hidden assumptions about the nature of CEO actions and their effects on the value of their firms. These hidden assumptions become confounding when one attempts to measure "incentive strength" in firms of dramatically different sizes.

This paper attempts to clarify and answer questions about incentive strength in firms of very different sizes. Specifically, we ask: who has stronger incentives, the CEO of a large firm with a tiny fractional ownership but an equity stake worth tens of millions of dollars, or the CEO of a small firm who owns a significant share of company stock, but whose stake is worth much less?

Our theoretical results show that the answer to this question depends on how CEO actions affect firm value. If CEO actions have constant dollar effects across firms of different sizes, then a CEO's "percent owned" is the appropriate measure of incentives. If, on the other hand, CEO actions have a chain-letter-like impact on companies and affect percentage returns on firm value, then the value of the CEO's "equity stake" is the correct measure of

CEO incentive strength. We show that the critical parameter in establishing the relevance of different measures of incentives is the elasticity of the marginal product of CEO effort with respect to firm size. One of two polar case assumptions about the magnitude of this elasticity (zero—the marginal product is invariant with firm size; and one—the marginal product of effort scales proportionally with firm size) is implicit in most analyses of CEO pay and incentives.

Our empirical results indicate that neither polar case assumption is correct. We estimate that the elasticity of CEO marginal product with respect to firm size is roughly 0.4. Thus CEO marginal products rise strongly, but not proportionally, with firm size. One implication of this finding is that CEO incentives do not fall dramatically with firm size. Indeed, we estimate that CEO incentives are roughly constant or decline slightly as firm size increases.

Several conclusions emerge from these findings. First, it is useful to distinguish between pay-to-performance and what might be called “pay-to-effort” measures of incentive strength. Doing so highlights the importance of cross-sectional variation in CEO marginal products, and avoids confusion about the interpretation of pay sensitivity estimates. Second, the appropriate specification for estimating both pay sensitivity and incentive strength in cross-sectional data must account for these differences in CEO marginal products across the sample. While we have estimated these differences for firms of different sizes, and have begun to do so for firms in different industries, one could imagine extending this methodology to investigate differences in managerial marginal products for firms with different capital intensities, with different levels of diversification<sup>23</sup> or with different organizational structures.

We also show that estimating the overall level of pay sensitivity for a CEO obscures an important point about CEO incentives for different types of activities. Since large firms (optimally) have much smaller  $b$ 's, the incentives provided to executives for activities with relatively small dollar effects (such as perquisite consumption) will be weak. CEOs of large firms will thus be inclined to overspend on these types of activities, relative to CEOs of smaller firms. This does not imply, however, that the CEOs of larger firms face incentives that are weaker across the board. Indeed, since the dollar effects of their “system-wide”

---

<sup>23</sup>Rose and Shephard (1997) find evidence that CEOs of more highly diversified firms have higher ability.

activities can be so enormous, they will tend to exert more effort on these types of activities than CEOs of smaller firms.

Understanding top management incentives, and making sense of the welter of empirical results that exist in the literature, requires both a more flexible model of CEO production and a less rigid view of the determinants of CEO pay. The models and results in this paper are, we hope, a step in this direction.

## References

- Aggarwal, Rajesh and Andrew A. Samwick, "The Other Side of the Tradeoff: The Impact of Risk on Executive Compensation," *Journal of Political Economy* 107, pp. 65-105, 1999.
- Black, F. and M. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, pp. 637-659, 1973.
- Core, John E. and Wayne R. Guay, "The Use of Equity Grants to Manage Optimal Equity Incentive Levels," *Journal of Accounting and Economics*, Vol. 28, 1999.
- Cook, R.D., "Detection of Influential Observations, High Leverage Points, and Outliers in Linear Regression," *Technometrics*, 1977.
- Garen, John E., "Executive Compensation and Principal-Agent Theory," *Journal of Political Economy* 102, No. 6, 1994.
- Gibbons, Robert and Kevin J. Murphy, "Relative Performance Evaluation for Chief Executive Officers," *Industrial and Labor Relations Review* 43, No. 3, February 1990.
- Hall, Brian J. and Thomas A. Knox, "Managing Option Fragility," Working Paper, Harvard Business School, May 2002.
- Hall, Brian J. and Jeffrey B. Liebman, "Are CEOs Really Paid Like Bureaucrats?" *Quarterly Journal of Economics* 112, No. 3, pp. 653-691, August 1998.
- Hall, Brian J. and Kevin J. Murphy, "Optimal Exercise Prices for Executive Stock Options," *American Economic Review* 90, pp. 209-214, May 2000
- Hall, Brian J. and Kevin J. Murphy, "Stock Options for Undiversified Executives," *Journal of Accounting and Economics* 33, No. 2, pp. 3-42, 2002.
- Haubrich, Joseph G., "Risk Aversion, Performance Pay and the Principal-Agent Problem," *Journal of Political Economy* 102, No. 2, 1994.
- Holmstrom, Bengt, "Comments," in *Contract Economics*, edited by Lars Werin and Hans Wijkander, Blackwell, pp. 211-14, 1992.
- Jensen, Michael C. and Kevin J. Murphy, "Performance Pay and Top-Management Incentives," *Journal of Political Economy* 98, No. 2, 1990.
- Joskow, Paul, Nancy L. Rose and Andrea Shepard, "Regulatory Constraints on CEO Compensation," *Brookings Papers on Economic Activity - Microeconomics* 1, 1993.
- Joskow, Paul L. and Nancy L. Rose, "CEO Pay and Firm Performance: Dynamics, Asymmetries, and Alternative Performance Measures," NBER working paper 4976, December 1994.

- Lambert, R.A., D.F. Larcker, and R.E. Verrecchia, "Portfolio Considerations in Valuing Executive Compensation," *Journal of Accounting Research* 29, pp. 129-149, 1991.
- Li, G., "Robust Regression," in *Exploring Data Tables, Trends and Shapes*, edited by D.C. Hoaglin, F. Mosteller and J. W. Tukey, John Wiley and Sons, 1985.
- Merton, R.C., "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science* 4, pp. 141-183, 1973.
- Meulbroek, Lisa K., "The Efficiency of Equity-Linked Compensation: Understanding the Full Cost of Awarding Executive Stock Options," *Financial Management* 30, No. 2, pp. 5-30, 2001.
- Murphy, Kevin J., "Corporate Performance and Managerial Remuneration: An Empirical Analysis," *Journal of Accounting and Economics* 7, No. 1, April 1985.
- Murphy, Kevin J., "Executive Compensation," in *Handbook of Labor Economics* 3, edited by O. Ashenfelter and D. Card, North Holland, 1999.
- Palia, Darius, "The Impact of Regulation on CEO Labor Markets," *Rand Journal of Economics* 31, No. 1, Spring 2000.
- Rosen, Sherwin, "Contracts and the Market for Executives," in *Contract Economics*, edited by Lars Werin and Hans Wijkander, Blackwell, pp. 181-211, 1992.
- Rose, Nancy and Andrea Shepard, "Firm Diversification and CEO Compensation: Managerial Ability and Executive Entrenchment," *Rand Journal of Economics* 8, No. 3, pp. 489-514, Autumn 1997.
- Schaefer, Scott, "The Dependence of Pay-Performance Sensitivity on the Size of the Firm," *Review of Economics and Statistics* 80, No. 3, 1998.
- White, H. A., "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," *Econometrica* 48, 1980.

**Table 1**

Comparison of Different Assumptions About the Elasticity of  $\eta$  (the marginal product of CEO effort) with Respect to Firm Size.

	Two Polar Cases	
	0	1
Elasticity of $\eta$ with respect to firm size:		
The marginal product of CEO actions on firm value:	Is invariant to firm size	Scales proportionally with firm size
CEO actions affect:	Shareholder wealth in dollars	Shareholder returns
Examples of type of managerial action:	<ul style="list-style-type: none"> <li>• Buying a corporate jet</li> <li>• Investing in a pet project</li> <li>• Selling an underutilized asset</li> </ul>	<ul style="list-style-type: none"> <li>• Reorganizing the firm</li> <li>• Developing a new corporate strategy</li> <li>• Designing a new accounting or compensation system</li> </ul>
Appropriate measure of incentives:	b: dollars of CEO wealth per dollar of firm value created	bV: CEO “stake in the firm”

**Table 2**  
Summary Statistics

	<i>Mean</i>	1 <sup>st</sup> Percentile Cutoff	10 <sup>th</sup> Percentile Cutoff	<i>Median</i>	90 <sup>th</sup> Percentile Cutoff	99 <sup>th</sup> Percentile Cutoff
<b>1996 (ExecuComp)</b>						
n = 1,125						
# Founders dropped=158						
Market Value	\$5.1B	\$54M	\$224M	\$1.3B	\$10.9B	\$62.1B
Variance of Mkt Value	\$7.5e+18	\$5.6e+14	\$5.8e+15	\$1.4e+17	\$6.2e+18	\$1.5e+20
Firm Sales	\$4.4B	\$21M	\$203M	\$1.2B	\$10.1B	\$54.6B
Total Compensation	\$2.76M	\$233K	\$530K	\$1.44M	\$5.11M	\$19.4M
CEO Wealth in the Firm	\$28.3M	\$38.5K	\$863K	\$7.7M	\$51M	\$410M
b	.0145	.0002	.0013	.0086	.0356	.0784
CEO Age	55.4	38	47	55	63	71
CEO Tenure	7.65	2	2	6	15	28
<b>1994 (Hall-Liebman)</b>						
n = 320						
# Founders dropped= 35						
Market Value	\$5.0B	\$94M	\$696M	\$2.4B	\$11.0B	\$39.0B
Variance of Mkt Value	\$4.1e+18	\$1.7e+15	\$2.5e+16	\$3.2e+17	\$6.5e+18	\$1.5e+20
Firm Sales	\$6.4B	\$252M	\$868M	\$3.2B	\$14.0B	\$53.8B
Total Compensation	\$2.53M	\$375K	\$595K	\$1.49M	\$4.72M	\$18M
CEO Wealth in the Firm	\$13.8M	\$149K	\$787K	\$4.88M	\$30M	\$326M
b	.0131	.00005	.0005	.0044	.0181	.2855
CEO Age	57.6	41	50	58	64	70
CEO Tenure	7.64	1	2	6	16	26
<b>1980 (Hall-Liebman)</b>						
n = 304						
# Founders dropped=29						
Market Value	\$1.0B	\$24M	\$62M	\$413M	\$1.8B	\$9.3B
Variance of Mkt Value	\$2.2e+17	\$6.5e+13	\$3.1e+14	\$1.2e+16	\$2.8e+17	\$6.0e+18
Firm Sales	\$3.0B	\$102M	\$266M	\$1.2B	\$6.3B	\$26.6B
Total Compensation	\$801K	\$194K	\$273K	\$596K	\$1.43M	\$3.28M
CEO Wealth in the Firm	\$4.86M	\$69.6K	\$303K	\$1.59M	\$11.2M	\$46.6M
b	.0161	0	.000151	.00188	.0360	.240
CEO Age	56.8	40	48	58	64	68
CEO Tenure	7.18	1	2	6	15	31

Notes: b is the dollar change in CEO wealth for each dollar change in firm value.  
Total compensation is defined as salary and bonus, plus the Black-Scholes value of stock options.  
The table shows the mean and distribution—median and various percentile cutoffs—of each variable separately.

**Table 3**

The Elasticity of CEO Marginal Product ( ) with Respect to Firm Size

The dependent variable is  $\ln(\ )$  and the independent variable is  $\ln(\text{firm size})$ .

	OLS			Robust Regressions			
	Elasticity	t test: elasticity =0	t test: elasticity =1	Adj. R <sup>2</sup>	Elasticity	t test: elasticity =0	t test: elasticity =1
<b>1996</b>							
n = 1,125							
ln 1	.46	33.45	39.82	.51	.44	31.87	40.16
ln 2	.33	32.54	66.77	.54	.34	36.20	70.65
ln 3	.63	35.31	20.51	.53	.64	40.22	22.43
<b>1994</b>							
n = 320							
ln 1	.44	11.88	15.00	.32	.44	13.42	17.14
ln 2	.42	13.22	18.64	.49	.46	18.92	22.69
ln 3	.62	15.28	9.56	.41	.60	16.77	11.03
<b>1980</b>							
n = 304							
ln 1	.50	11.19	11.27	.29	.50	11.79	11.87
ln 2	.48	22.11	23.65	.71	.48	20.38	22.05
ln 3	.62	13.50	8.22	.36	.63	13.24	7.75

Notes: 1 assumes CEO wealth is proportional to CEO annual total compensation (salary, bonus and stock options valued at Black-Scholes).

2 assumes CEO wealth is proportional to CEO wealth in the firm (the value of stock and stock options).

3 assumes CEO wealth is constant across firm size.

Firm size is measured as the market value of the firm.

The t tests in the OLS regressions are based on heteroskedasticity-consistent standard errors (White, 1980).

Robust regressions are from the RREG command in STATA.

**Table 4**

The Elasticity of CEO Marginal Product ( ) with Respect to Size

The dependent variable is  $\ln(\ )$ .

	(1)	(2)	(3)	(4)	(5)	
	Coeff. on $\ln(\text{sales})$	Coeff. on $\ln(\text{mktval})$ (tenure > median*)	Coeff. on $\ln(\text{mktval})$ (age > median**)	Coeff. on $\ln(\text{mktval})$	Coeff. on Regulated Utilities***	Coeff. On $\ln(\text{mktval})$ ****
<b>1996</b>						
n = 1,125						
$\ln$ 1	0.31 (0.02)	0.48 (0.02)	0.51 (0.02)	0.47 (0.01)	-0.91 (0.23)	.50 (0.01)
$\ln$ 2	0.28 (0.01)	0.37 (0.01)	0.37 (0.01)	0.37 (0.01)	-0.44 (0.18)	.39 (0.01)
$\ln$ 3	0.48 (0.02)	0.68 (0.02)	0.72 (0.02)	0.69 (0.01)	-1.33 (0.13)	.71 (0.01)
<b>1994</b>						
n = 320						
$\ln$ 1	0.43 (0.04)	0.39 (0.05)	0.35 (0.05)	0.41 (0.03)	-1.10 (0.29)	
$\ln$ 2	0.41 (0.03)	0.45 (0.05)	0.40 (0.04)	0.44 (0.03)	-0.32 (0.19)	
$\ln$ 3	0.63 (0.05)	0.56 (0.05)	0.52 (0.05)	0.58 (0.03)	-1.67 (0.12)	
<b>1980</b>						
n = 304						
$\ln$ 1	0.45 (0.05)	0.52 (0.06)	0.37 (0.08)	0.46 (0.04)	-1.18 (0.56)	
$\ln$ 2	0.42 (0.04)	0.45 (0.05)	0.43 (0.06)	0.45 (0.03)	-0.12 (0.43)	
$\ln$ 3	0.59 (0.06)	0.66 (0.06)	0.52 (0.09)	0.60 (0.05)	-1.61 (0.35)	
SIC Dummies	No	Yes	Yes	Yes		Yes
Age and Tenure Controls	No	No	No	Yes		Yes

Notes: 1 assumes wealth is proportional to total compensation.

2 assumes wealth is proportional to wealth in the firm (the value of stock and stock options).

3 assumes wealth is constant across firm size.

All regressions are robust regressions (RREG in STATA).

Standard errors are in parentheses.

\*Includes only observations where CEO tenure is greater than the median CEO tenure in that year.

\*\* Includes only observations where the CEO age is greater than the median CEO age in that year.

\*\*\* Defined as SIC code 49.

\*\*\*\*The dependent variable is  $\ln(\ )$  where  $\ )$  is based on  $b$ 's that are defined as changes in *executive values* (rather than market values) of executive stock and option portfolios in response to \$1,000 changes in firm value. The dependent variable is described in greater detail in the text and in Hall and Knox (2002).

**Table 5****The Relationship Between Incentive Strength (b or Effort) and Firm Size**

The dependent variable is  $\ln(b^*)$ .

	Coefficient on $\ln(\text{mktval})$
<b>1996</b>	
n = 1,125	
$\ln(b^* 1)$	0.044 (0.031)
$\ln(b^* 2)$	-0.064 (0.024)
$\ln(b^* 3)$	0.265 (0.033)
<b>1994</b>	
n = 320	
$\ln(b^* 1)$	-0.188 (0.072)
$\ln(b^* 2)$	-0.164 (0.057)
$\ln(b^* 3)$	-0.029 (0.077)
<b>1980</b>	
n = 304	
$\ln(b^* 1)$	-0.239 (0.118)
$\ln(b^* 2)$	-0.210 (0.085)
$\ln(b^* 3)$	-0.103 (0.121)

Notes: 1 assumes CEO wealth is proportional to CEO annual, total compensation (salary, bonus and plus stock options valued at Black-Scholes).

2 assumes CEO wealth is proportional to CEO wealth in the firm (the value of stock and stock options).

3 assumes CEO wealth is constant across firm size.

Firm size is equal to the market value of the firm.

Standard errors are in parentheses.

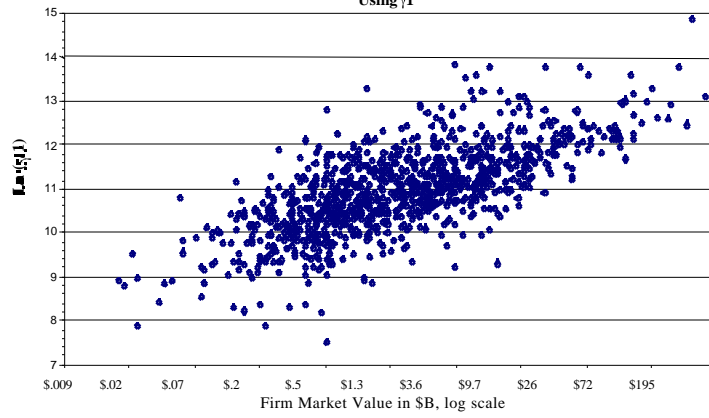
All regressions include controls for tenure, age and industry (not shown). Regressions are robust regressions (RREG in STATA).

**Table 6**

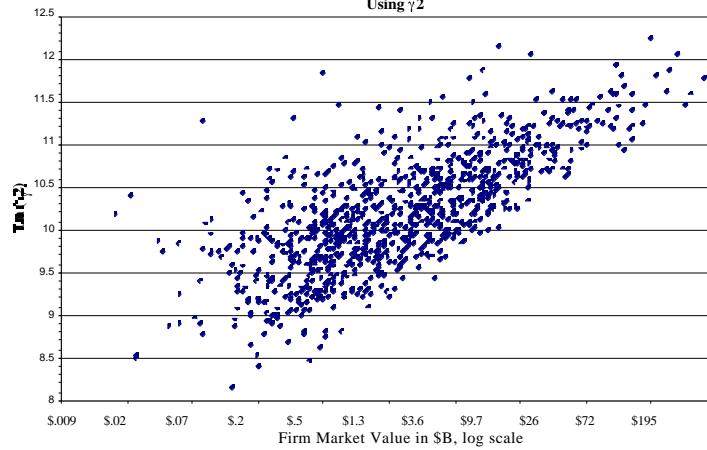
## Simulation of Effort Choices Across Two Tasks in Firms of Different Sizes

	Firm 1	Firm 2
Firm Size	\$1B	\$10B
Marginal Product on task 1 (does not scale with firm size)	100	100
Marginal Product on task 2 (scales with firm size)	24	240
$\sqrt{\gamma_j^2}$	103	259 (=103*10 <sup>4</sup> )
b*	.01	.0032 (=1*10 <sup>-5</sup> )
Effort on task 1 (non-scaling task)	1	.32
Effort of task 2 (scaling task)	.24	.77
b * $\sqrt{\gamma_j^2}$ (effort intensity)	1.03	.83

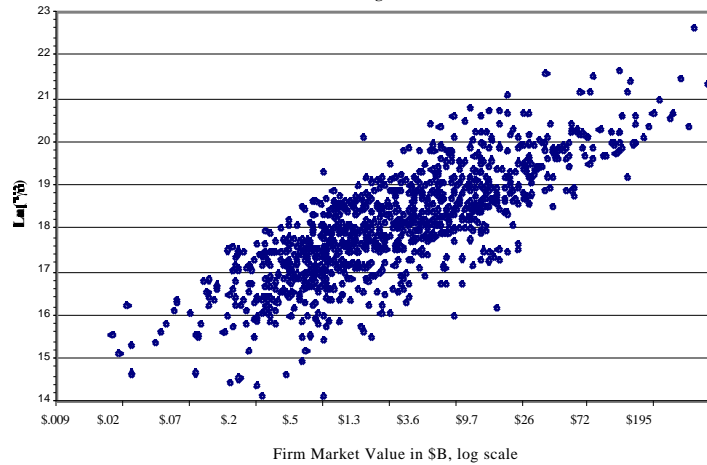
**Figure 1**  
**Relationship Between Marginal Product ( $\gamma$ ) and Firm Size**  
 Using  $\gamma_1$



Using  $\gamma_2$

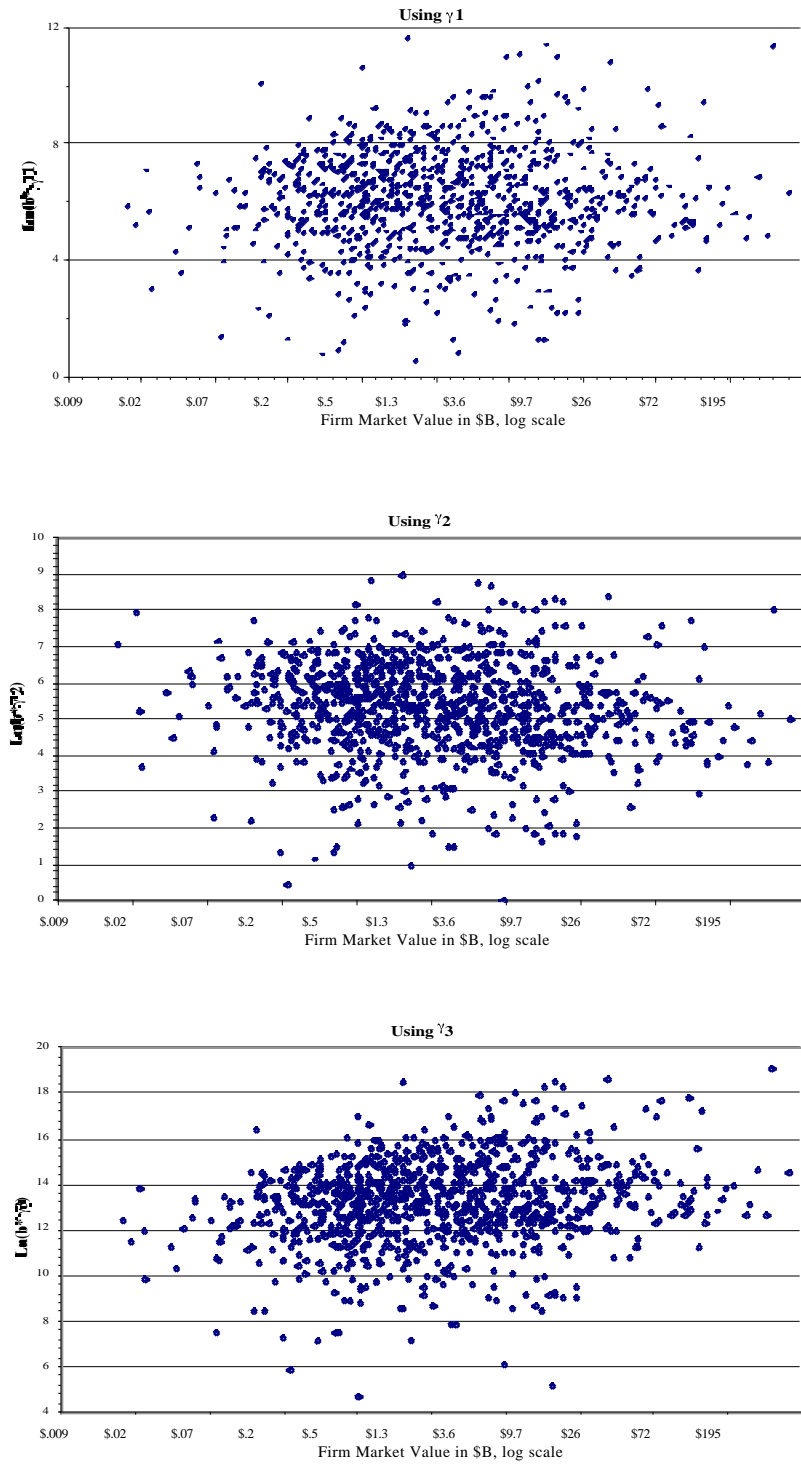


Using  $\gamma_3$



Note: Data are from *ExecuComp* in 1996. 1 assumes that CEO wealth is proportional to compensation; 2 assumes that CEO wealth is proportional to the CEO's stake in the firm; 3 assumes that CEO wealth is constant.

**Figure 2**  
**Relationship Between CEO Incentives ( $b^*\gamma$ ) and Firm Size**



Note: Data are from *ExecuComp* in 1996. 1 assumes that CEO wealth is proportional to compensation; 2 assumes that CEO wealth is proportional to the CEO's stake in the firm; 3 assumes that CEO wealth is constant.