

# Fiscal Risk and the Portfolio of Government Programs\*

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## Abstract

This paper proposes a new approach to social cost-benefit analysis using a model in which a benevolent government chooses risky projects in the presence of market failures and tax distortions. The government internalizes market failures and therefore perceives project payoffs differently than do individual private actors. This gives it a “social risk management” motive – projects that generate social benefits are attractive, particularly if those benefits are realized in bad economic states. However, because of tax distortions, government financing is costly, creating a “fiscal risk management” motive. Government projects that require large tax-financed outlays are unattractive, particularly if those outlays tend to occur in bad economic times. At the optimum, the government trades off its social and fiscal risk management motives. Frictions in government financing create interdependence between two otherwise unrelated government projects. As in the theory of portfolio choice, the fiscal risk of a project depends on how its fiscal costs covary with the fiscal costs of the government’s overall portfolio of projects. This interdependence means that individual projects should not be evaluated in isolation.

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# 1 Introduction

In modern economies, a significant fraction of economy-wide risk is borne indirectly by taxpayers via the government. Governments have significant liabilities associated with retirement benefits, unemployment insurance, and financial system backstops. These liabilities are large—the amount of credit risk explicitly recognized on the balance sheet of the US federal government now exceeds \$3 trillion, and implicit or off-balance sheet liabilities are even larger. For instance, off-balance sheet guarantees on mortgage-backed securities account for another \$5 trillion. And, during the financial crisis, total off-balance sheet financial system backstops temporarily reached more than \$6 trillion (Geithner, 2014). Moreover, the government’s overall risk is not idiosyncratic but varies systematically with economic conditions. For example, the US debt-to-GDP ratio rose from 38% to 72% between 2007 and 2013 due to falling tax revenue and increasing expenditures on government programs that automatically expand in a recession.

Given the sheer magnitude of these exposures, it is critical to understand how the government should choose, price, and manage the set of risks it bears. The standard approach to government cost-benefit analysis essentially ignores these issues. Common practice in government budget accounting—enshrined in U.S. budgetary law under the Federal Credit Reform Act of 1990 (FCRA)—is to determine whether individual programs in isolation have positive net present values, when their costs are discounted at the government’s borrowing rate, essentially the riskless rate.

This common practice is incorrect for two reasons. First, basic finance theory dictates that the cost of capital for a project should depend on the attributes of the project—the outlays it requires and the payoffs it generates. Second, since taxation is distortionary, it is costly for the government to raise financing through taxes. Large literatures in public economics study each of these factors. There are vast literatures studying the outlays and benefits of large government programs like unemployment insurance and social security. And an equally vast literature studies optimal government financing holding fixed the set of programs the government wishes to undertake.

In this paper, we bridge the gap between these two literatures, studying the ways in which distortionary government financing interacts with the set of projects a benevolent government should choose to undertake. The result is a rich framework for conducting social cost-benefit analysis in a stochastic environment with distortionary taxation that provides novel implications for government risk management and risk pricing.

Our setup differs from the Ricardian framework, where the government is simply a veil for the taxpayer, in two important ways. First, we assume that government taxation is distortionary, so that a dollar of government revenue costs society more than a dollar. One can think of this as a short-hand for a variety of frictions associated with government involvement in the private economy. Second, we assume that government projects can generate social benefits that private actors are not able to generate on their own. We model these social benefits in reduced form, but think of them as arising from the fact that the government often has unique technologies for addressing market failures. For instance, the government may be able to use regulations to correct externalities, mandate contributions to address public goods problems, or mandate purchases to

address market-unravelling. In this setting, we can characterize the social welfare maximizing scale of a government program, taking the payoffs from the project as given. Equivalently, we characterize the optimal pricing (i.e., cost of capital) for the program, taking its scale as given.

In the finance literature, corporate hedging and risk management activities can enhance firm value if it is costly for firms to raise financing from external investors (Froot, Scharfstein, and Stein, 1993). Relatedly, financing frictions have implications for the scale and composition of optimal firm investment (Bolton, Chen, and Wang, 2011; Bolton, Chen, and Wang, 2013; Campello, Graham, and Harvey, 2011; Fazzari, Hubbard, and Petersen, 1988; Kaplan and Zingales, 1997). In our setting, the distortionary costs of taxation play a similar role, making it costly for the government to raise funds from taxpayers through taxation. The distortionary costs of taxation induce the government to behave as though it is more risk-averse than the taxpayers it represents, just like financing frictions can make firms behave as though they are more risk-averse than shareholders.

In our model, the government has two motives for risk management: a “social risk management” motive and a “fiscal risk management motive”. The government’s social risk management motive arises from the fact that the benefits of a government program—the additional output created from mitigating market failures—may vary cyclically. For instance, the financial stability benefits of deposit insurance may largely accrue from preventing large-scale bank runs in recessions, making it a valuable social hedge. Similarly, the welfare benefits from reducing labor market frictions or from fiscal stimulus may largely accrue during recessions.

The government’s fiscal risk management motive arises from the desire to avoid costly tax distortions. By taking on projects that are tax hedges, by reducing government outlays when government deficits would otherwise be high, the government can reduce the average tax distortions. Moreover, since the distortionary costs of government financing depend on the total scale of government outlays, the fiscal risk management motive generates interdependence between the optimal scale of two otherwise unrelated government projects. The intuition for this interdependence corresponds closely to classic precepts from the theory of portfolio choice (Markowitz, 1952; Tobin, 1958; Sharpe, 1964; Linter, 1965). Specifically, the fiscal risk of a program depends on how its fiscal costs covary with the fiscal costs of the total government portfolio. This means that costs of individual programs cannot be evaluated in isolation. And when the government fiscal burden is elevated, the cash-flow profile of a given project in relation to the rest of the government portfolio is critical.

A key issue we highlight is the interaction between the government’s social and fiscal risk management motives. These motives may conflict because programs like deposit insurance that have strong social risk management benefits tend to involve government expenditures and hence higher tax distortions in bad times, creating more fiscal risk. On the other hand, the two risk management motives may reinforce each other if social risk management preserves private output in a downturn—reducing the distortionary impact of a given dollar amount of taxes.

We illustrate the novelty of our framework by considering an example in which the government chooses between a “fiscally safe” ex-ante technology and a “fiscally risky” ex-post technology for correcting a market failure. For concreteness, we consider the context of interventions aimed at promoting financial stability. Regulation of financial intermediaries is an ex-ante technology

for promoting financial stability: it is fiscally safe in the sense that government expenditures on regulation vary little across states of the world. By contrast, capital injections or lender of last resort operations (i.e., “bailouts”) are ex-post technologies. They are fiscally risky because government expenditures associated with them vary significantly across states. In our framework, it is optimal for the government to pursue some regulation and some ex-post bailouts. A key insight is that the relative attractiveness of these two technologies for producing financial stability depends on the total fiscal burden of the government and how it varies across states. When the government’s fiscal burden is low, ex-post bailouts are a relatively attractive way to provide financial stability. However, as the fiscal burden rises, it is optimal to substitute toward the less fiscally risky technology, namely, regulation. This corresponds to the classic portfolio choice intuition that investors who face higher level of “background risk” should choose more conservative financial portfolios (Merton, 1973; Campbell and Viceira, 2003). In addition, we show that when the distortionary costs of taxation rise, the optimal quantity of fiscally risky ex-post bailouts falls. This corresponds to the standard finance intuition that the optimal portfolio allocation to risky assets falls as risk aversion rises.

Our paper sits at the intersection of public finance, corporate finance, and asset pricing. Work in public finance often considers static government interventions in isolation. There is, for instance, one strand of literature studying the market failures that justify the use of unemployment insurance and a separate strand studying the market failures that justify the use of deposit insurance. However, in a non-Ricardian world, these two problems are not separable. Our portfolio approach to government finance tells us that the cost of capital should be higher for a program with larger expenditures in bad fiscal times, when spending on other programs is already high and taxes are therefore elevated.

Work in macroeconomics recognizes that government expenditures may be cyclical and, assuming that tax-smoothing is imperfect, that the tax burden will therefore be cyclical as well. However, this work typically treats the government as an exogenously given collection of programs. By contrast, our approach shows that the cyclicity of government expenditures has important implications for the set of programs that should be undertaken by the government.

The plan for the paper is as follows. In Section 2 we develop the general model and characterize the optimal scale of a single potentially welfare-improving government program in the presence of distortionary taxation. Section 3 explores several special cases of the general model that help clarify the key intuitions developed in Section 2. Section 4 extends the model to consider portfolios of multiple government programs. In particular, we explore the choice between a fiscally safe and a fiscally risky program for addressing a given market failure. Section 5 considers a variety of additional extensions of the basic model, and Section 6 concludes.

## 2 The General Model

### 2.1 Setup

Exogenous private income is  $Y_t$ . By correcting some market failure, a government project or program may generate a social payoff of  $W_t$  in the form of additional private income. For instance, if  $W_t = 0$ , there are no social benefits associated with the project. In contrast, deposit insurance may create broad financial stability benefits, so it would have  $W_t > 0$ . Suppose the government does some quantity  $q$  of this program. We assume this changes private income to  $Y_t + qW_t$  and requires government outlays of  $qX_t$ . We assume that the government can issue default-free bonds on any maturity. Specifically, let  $D_t^{(j)}$  denote the amount of  $j$ -period zero coupon bonds issued at time  $t$  and due at  $t + j$ . Similarly, let  $R_t^{(j)}$  denote the gross  $j$ -period interest rate on these bonds.

The government's flow budget constraint is given by

$$T_t + \sum_{j=1}^{\infty} D_t^{(j)} = \sum_{j=1}^{\infty} (R_{t-j}^{(j)})^j D_{t-j}^{(j)} + G_t + qX_t, \quad (1)$$

where  $T_t$  is taxes,  $G_t$  is exogenous government expenditure, and  $qX_t$  is the endogenous level of expenditure associated with the program under consideration. Equation (1) says that the government's source of funds from taxes and the proceeds from new debt issues must equal its uses of funds for maturing debt and spending.<sup>1</sup>

We assume that taxation is distortionary. Thus, the household budget constraint equating household expenditures with household income is given by

$$C_t + T_t + \sum_{j=1}^{\infty} D_t^{(j)} = Y_t + qW_t - h(T_t) + \sum_{j=1}^{\infty} (R_{t-j}^{(j)})^j D_{t-j}^{(j)}, \quad (2)$$

where  $C_t$  is private consumption. Here  $h(\cdot)$  represents the distortionary costs or excess burden of taxation with  $h'(\cdot) > 0$  and  $h''(\cdot) > 0$ . We will frequently choose  $h(T_t) = (\eta/2)T_t^2$  for simplicity.

Household utility is

$$U = E_0[\sum_{t=0}^{\infty} \beta^t u(C_t)], \quad (3)$$

where  $0 < \beta \leq 1$ ,  $u'(\cdot) > 0$ , and  $u''(\cdot) \leq 0$ . Isolating private consumption and using the government budget constraint to substitute out debt and taxes yields:

$$C_t = Y_t + q(W_t - X_t) - h(T_t) - G_t. \quad (4)$$

The government's problem is to choose  $q$  and the path of taxes (equivalently the path of government debt) to maximize household utility

$$U = E_0[\sum_{t=0}^{\infty} \beta^t u(Y_t + q(W_t - X_t) - h(T_t) - G_t)], \quad (5)$$

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<sup>1</sup>Note that  $W_t$  must be interpreted as the incremental payoff relative to other government projects embedded in  $Y_t$ . For instance, in the case of deposit insurance,  $W_t$  represents incremental social benefits of deposit insurance, taking as given the existence of a lender of last resort.

taking the path of  $\{Y_t, W_t, X_t, G_t\}$  as given. In other words, the government faces a stochastic endowment  $\{Y_t\}$  and production technology  $\{W_t, X_t\}$  as well as a set of stochastic government expenditures  $\{G_t\}$ .

### 2.1.1 Examples

To make the setup concrete, we briefly discuss two examples. Appendix A provides a more explicit formal mapping of these examples into our model setup. The first example comes from financial economics and considers the value of government interventions to prevent economically destabilizing fire sales, using Stein’s (2012) model as a baseline. In that model, runs by short-term creditors can result in fire sales. The real cost of these fire sales is that outside investors must use capital to purchase liquidated assets, rather than investing in productive new projects. In this setting, consider a program where the government steps in and guarantees the short-term debt in order to prevent runs. For simplicity, assume these guarantees ensure there is no run, and further assume the guarantee is unanticipated so that it has no ex-ante effects. In our notational conventions, the government’s outlays  $X$  are the realized fiscal costs of the guarantee, which will be positive if the government has to pay to make the short-term debt whole. The social payoff  $W$  from this program is the gain in net private income created by the fact that outside investors can use their capital for productive new projects.

Our second example considers unemployment insurance, taking the Rothschild-Stiglitz (1976) model as our baseline. In that model, there are two types of agents with different probabilities of becoming unemployed.<sup>2</sup> It is well-known that when agents know their types, pooling competitive equilibria may not exist in this setting. In the separating equilibrium that can survive, agents with a low probability of future unemployment must be less than fully insured to prevent agents with a high probability of future unemployment from mimicking them and purchasing cheap insurance. In this setting, government-run mandatory insurance can improve overall welfare by enforcing pooling. The social payoff  $W$  from mandatory insurance is the welfare gain associated with the move from the separating equilibrium to the pooling outcome. And the net government outlay  $X$  is the difference between government insurance payouts and premia collected.

### 2.1.2 Discussion of Setup

Several features of the model deserve discussion. First, note that since we are working in a representative agent framework, all income here is “social” in the sense that it flows to the representative agent. Social benefits  $W_t$  ultimately flow to the representative agent even though they may not be taken into account when individual agents make decisions. Note that if market failures create scope for the government policies to affect Pareto improvements, a representative agent may fail to exist. In the case of simple incomplete markets problems like pollution externalities, a representative agent may exist, but in other cases where market failures are generated by information problems among heterogeneous agents, it may not exist (Huang and Litzenberger, 1988; Duffie,

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<sup>2</sup>As we discuss further below, there is some tension in our use of a representative agent framework in situations where there are multiple types of agents.

2001). In cases where no formal representative agent exists, our framework can be viewed as a short-hand for a planner maximizing a more complicated social welfare function. Our key simplifying assumption then, which may not always be satisfied, is that aggregate consumption is a sufficient statistic for social welfare. The intuitions that emerge from our model should still apply in more complex settings where such simpler aggregation fails. Programs that generate social benefits when all agents have low consumption will be valuable social hedges. Similarly, programs that increase the government tax burden at such times will be fiscally risky. However, our framework will be unable to capture intuitions where distributional considerations are important. For instance, in the case of deposit insurance, there is no sense in which banks and their shareholders, rather than consumers, are paying insurance premia in our setup.

A second feature of the model setup is that distortions apply to the level of taxes as opposed to the tax rate. Formally, this means that taxes are distortionary despite the fact that they are modeled as lump-sum in our setup. We adopt this specification solely for analytical tractability. This is a simple modeling device that allows to abstract from the types of complex optimal taxation problems studied in the public finance literature (Stokey and Lucas, 1983; Aiyagari, Rao, Sargent, and Seppala, 2002; Golosov, Tsyvinski, and Werning, 2006; Acemoglu, Golosov, and Tsyvinski, 2008, among many others), which are not our focus, and enables us to obtain closed-form solutions. And as we show below, the exact same basic forces obtain in a model in which distortions are a function of tax rates. Furthermore, although we refer to distortions  $h(T)$  as “tax distortions,” they serve as a short-hand for a host of costs that are incurred when taxation is large or when the government faces a significant fiscal burden (e.g., frictions associated with the risk of sovereign default).

A third feature of the set up is that the government makes a one-shot choice about program scale and can credibly commit to this scale indefinitely. Thus, the model is best seen as applying to non-discretionary budgeting where, whether for reasons of efficiency, fairness, or political economy, program scale is stable over time. In practice, we will assume that these decisions are made from the vantage point of the economy’s steady-state. However, in Section 5 we briefly explore the other extreme of a pure discretionary policy where the government is free to adjust the desired scale each period.

Finally, the setup largely abstracts from the notion that the presence of a government project can change private behavior. Formally, by taking  $Y_t$  as fixed, we are assuming that the private sector does not react to the government’s choice of project scale  $q$ . This is potentially important since many critiques of government programs, particularly guarantee programs, argue that they distort private sector behavior. For instance, deposit insurance may create a moral hazard problem, increasing bank risk taking beyond the social optimum. In the context of the model, one could think of these kinds of effects as being captured in the  $W$ s and  $X$ s, but they are not explicitly modeled here.

## 2.2 The Key First Order Condition

We now solve the model, deriving the first order condition for the optimal scale of the project  $q$ . The first order condition for optimal issuance of  $t$ -period bonds at time 0 is

$$0 = u'(C_0) h'(T_0) - E_0[\beta u'(C_t) h'(T_t) R_0^{(t)}] \quad (6)$$

for all  $t \geq 1$ . To see this, fix a time path of government borrowing and taxation. Consider a deviation in which the government issues more debt riskless  $t$ -year debt  $D_0^{(t)}$  and reduces taxes  $T_t$  by the same small amount. This deviation reduces tax distortions by  $h'(T_0)$  at time 0, which raises utility at time 0 by  $u'(C_0) h'(T_0)$  at the margin. Since this deviation raises future taxes by  $(R_0^{(t)})^t$  at time  $t$ , it raises future tax distortions by  $h'(T_t) (R_0^{(t)})^t$  at time  $t$ , which lowers discounted expected utility by  $E_0[\beta u'(C_t) h'(T_t) (R_0^{(t)})^t]$  at the margin. Equation (6) says that, along the optimal path of debt and taxes, such a deviation must have zero effect on discounted expected utility.

Letting

$$M_{0,t} = \frac{\beta^t u'(C_t)}{u'(C_0)} \quad (7)$$

denote the stochastic discount factor for  $t$ -period ahead payoffs at time 0 and recalling that  $R_0^{(t)}$ , the gross yield on  $t$ -period riskless borrowing from time 0 to  $t$ , satisfies  $(R_0^{(t)})^t = 1/E_0[M_{0,t}]$ , we can rewrite the first order condition for borrowing as

$$h'(T_0) = \frac{E_0[M_{0,t} h'(T_t)]}{E_0[M_{0,t}]} \quad (8)$$

for all  $t \geq 1$ .

We next solve for the optimal scale  $q$  of the government project. Note that the effect of changing  $q$  on household consumption at time  $t$  is given by

$$\frac{\partial C_t}{\partial q} = W_t - X_t (1 + h'(T_t)).$$

Increasing project scale  $q$  directly alters the contribution of the government project to consumption by  $(W_t - X_t)$  and increases tax distortions  $h'(T_t) \frac{\partial T_t}{\partial q}$ . The main analytical benefit from specifying tax distortions as a function of the level of taxes is that  $\frac{\partial T_t}{\partial q}$  takes a simple form: it is simply equal to  $X_t$ . Overall, the net impact of increasing the scale of the government project is  $W_t - X_t (1 + h'(T_t))$ .

The optimal amount of government activity,  $q^*$ , satisfies

$$0 = \sum_{t=0}^{\infty} \beta^t E_0[u'(C_t) \frac{\partial C_t}{\partial q}],$$



which we can rewrite as

$$0 = \sum_{t=0}^{\infty} E_0[M_{0,t} (W_t - X_t (1 + h'(T_t)))]. \quad (9)$$

**Proposition 1** *The optimal scale of government intervention  $q^*$  is implicitly defined by*

$$0 = \underbrace{\sum_{t=0}^{\infty} \frac{E_0 [W_t - X_t]}{(R_0^{(t)})^t}}_{\text{Expected cashflow cost/benefit}} + \underbrace{\sum_{t=0}^{\infty} Cov_0 [M_{0,t}, W_t - X_t]}_{\text{Cash flow risk premium}} \quad (10)$$

$$- \underbrace{\sum_{t=0}^{\infty} \frac{E_0 [h'(T_t) X_t]}{(R_0^{(t)})^t}}_{\text{Expected tax cost/benefit}} - \underbrace{\sum_{t=0}^{\infty} Cov_0 [M_{0,t}, h'(T_t) X_t]}_{\text{Tax risk premium}},$$

where we have used the fact that  $E_0[M_{0,t}] = 1/(R_0^{(t)})^t$  where  $R_0^{(t)}$  is gross yield on  $t$ -period riskless bonds from time 0 to  $t$ .

**Proof.** Expand (9) and use the identity  $E_0 [M_{0,t} Z_t] = (R_0^{(t)})^{-t} E_0 [Z_t] + Cov_0 [M_{0,t}, Z_t]$ . ■

We can use (10) to pin down the optimal scale of some government program, taking its (possibly stochastic) fiscal costs  $X_t$  and social benefits  $W_t$  as given. However, as discussed below, we can also use (10) to pin down the optimal “pricing” of a government program, taking a desired program scale as given.<sup>3</sup>

### 2.2.1 Interpretation

First, we note that if  $W_t = 0$  so there is no social payoff from the government project and  $h'(T_t) = 0$  so there are no tax distortions, then expression (10) collapses to the standard asset pricing equation

$$0 = \sum_{t=0}^{\infty} E_0 [M_{0,t} X_t] = \sum_{t=0}^{\infty} (R_0^{(t)})^{-t} E_0 [X_t] + \sum_{t=0}^{\infty} Cov_0 [M_{0,t}, X_t]. \quad (11)$$

Under these assumptions, our model collapses to the frictionless benchmark where the government evaluates projects just as the private sector would. Relative to the frictionless benchmark, our model contemplates the possibility that government projects may deliver social benefits by mitigating market failures ( $W_t > 0$ ), but do so in the presence of distortionary taxation ( $h'(T_t) > 0$ ).

There are four terms in expression (10) for the optimal scale of a government program which we now discuss in turn.

**Expected cashflow cost/benefit** The first term in expression (10) is the expected net cash flow benefit from the government program, discounted at the risk-free rate. This term encapsulates the way the cost of credit and guarantee programs is accounted for in the Federal budget.

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<sup>3</sup>This is similar to asset pricing where the same Euler equation can be used (i) to pin down optimal asset holdings taking returns as given or (ii) to pin down equilibrium returns taking asset supply as given.

Specifically, under the Federal Credit Reform Act (FCRA) of 1990,<sup>4</sup> the expected net present value of government outlays discounted at the risk-free rate for such programs should be zero. In the notation of the model, the expected NPV of  $X_t$  for guarantee programs should be zero. The model shows that the total effect, including the social gain from the program ( $W_t$ ) and the direct cash flows associated with the program ( $X_t$ ), should be taken into account from a cost-benefit perspective. In particular, the effect of the program on the unconditional path of private output could be positive if the program fixes some unconditional market failure ( $E_0 [W_t] > 0$ ).

**Cash flow risk premium** The second term in (10) is the risk premium associated with these cash flows. It is commonly argued (e.g., Lucas, 2012) and the citations within) that the government should charge the same risk premium as the private sector because it is acting on behalf of risk-averse tax payers. Specifically, for bearing the risk of cash outlays  $X_t$ , the government should charge the risk premium  $Cov_0 [M_{0,t}, X_t]$  just as private investors would. Again, our model shows that the risk premium should be associated with the total social gain or loss from the project. Specifically, the term  $Cov_0 [M_{0,t}, W_t]$  captures the government’s “social risk management” motive. In particular, for some government programs like deposit insurance, the social gains from correcting market failures may accrue primarily in bad times. Despite the fact that the government takes cash flow losses from deposit insurance in bad times, the program also generates broad social and economic gains at exactly those times. The first effect makes deposit insurance risky for the government, but the second effect makes it desirable to offer. Essentially, deposit insurance is a desirable social hedge,  $Cov_0 [M_{0,t}, W_t] > 0$ , which reduces the risk premium that the government should charge for it.

One subtle point regarding this term, is that the stochastic discount factor  $M_{0,t}$  and, hence, risk premia depend on the scale of the government program  $q$  and taxes. This can be seen clearly if we substitute (4) into (7) to obtain

$$M_{0,t} = \frac{\beta^t u' (Y_t + q (W_t - X_t) - h (T_t) - G_t)}{u' (Y_0 + q (W_0 - X_0) - h (T_0) - G_0)}.$$

Intuitively, the consumption of the representative household is affected by the existence of government programs and tax distortions. If government policies reduce (increase) the volatility of consumption and, hence, the volatility of marginal utility, risk premia will be lower (higher) than they would in the corresponding *laissez faire* economy where  $q = 0$ .

**Expected tax cost/benefit** The third term in (10) captures the government’s “fiscal risk management” motive. When  $h' (T_t) > 0$ , tax distortions induce the government to act as if it is more risk-averse than the taxpayers it represents. Intuitively, because project outlays  $X_t$  flow through the government budget, they affect taxes and therefore tax distortions. Specifically, since  $E_0 [h' (T_t) X_t] = E_0 [h' (T_t)] E_0 [X_t] + Cov_0 [h' (T_t), X_t]$ , this term makes programs less desirable to the extent they tend to raise taxes on average ( $E_0 [X_t]$  is large) or tend to raise taxes at times

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<sup>4</sup>See <http://www.fas.org/sgp/crs/misc/R42632.pdf>

when taxes would already be elevated ( $Cov_0 [h' (T_t), X_t]$  is large). By contrast, if  $h' (T_t) = 0$ , the model collapses to the Ricardian case in which the government is a veil for taxpayers.

A simple way to see the government’s fiscal risk management motive is to note that the discount factor for the additional private income generated by a government program is  $M_{0,t}^W = M_{0,t}$ , while the discount factor for the government outlays/inlays associated with the program is  $M_{0,t}^X = M_{0,t} (1 + h' (T_t))$ . Since  $M_{0,t} > 0$  and  $h' (T) > 0$  for all  $T > 0$ , it follows that we have  $M_{0,t}^W < M_{0,t}^X$ . This means that, all else equal, the government should apply a lower discount rate to government outlays  $X_t$  than to social benefits  $W_t$ . The fact that  $M_t^W < M_t^X$  reflects the fiscal conservatism that is built into our framework due to the distortionary costs of taxation.

As a second illustration of this motive, consider a traditional fiscal stimulus program. Assume that this program requires government spending of  $X_t > 0$  and generates additional private output of  $W_t = f_t X_t > 0$  where  $f_t$  is the “fiscal multiplier.” In a Ricardian world, the government should undertake the program if  $W_t - X_t > 0$ , which is equivalent to  $f_t > 1$ . However, with distortionary taxation, it should only undertake the program if  $W_t > (1 + h' (T_t)) X_t$ , which corresponds to  $f_t > 1 + h' (T_t) > 1$ . Intuitively, fiscal multipliers must be greater than one in order to compensate for the distortionary costs of taxation.

A critical tension that emerges from our framework is the conflict between the social risk management motive captured by the second term in (10) and the fiscal risk management motive captured by the third term. Programs like deposit insurance and automatic stabilizers that have significant social risk management benefits tend to involve government expenditures and, hence, higher tax distortions in bad times, creating more fiscal risk.

**Tax risk premium** The final term in (10) is the risk premium associated with the cyclical-ity of tax distortions. If a program leads to increased tax distortions in bad economic times, these distortions reduce private consumption precisely when it is most valuable. Thus, the term  $Cov_0 [M_{0,t}, h' (T_t) X_t]$  reflects the interaction between the “social risk management” and “fiscal risk management” motives.

**A Taxonomy of Government Programs** The four forces captured in our key first order condition (10) provide a taxonomy of government programs along two dimensions. The first is whether the associated government outlays carry aggregate risk. In the terminology of our model, programs with systematically risky outlays have  $Cov [M_t, X_t] \neq 0$ . This type of risk is closely related to though not exactly the same as fiscal risk, which is systematic if  $Cov [M_t, h' (T_t) X_t] \neq 0$  in the terminology of the model. The second dimension along which programs differ is whether the social payoffs they generate for society vary systematically. In our terminology, programs with systematic social cash flows have  $Cov [M_t, W_t] \neq 0$ .

At one end of the spectrum are programs that have idiosyncratic government outlays and idiosyncratic social cash flows. Programs in this category include government flood insurance, government terrorism insurance, and government agricultural insurance. The incidences of floods, terrorist attacks, and crop failures are largely uncorrelated with the behavior of the macroeconomy as a whole. It is sometimes argued that government involvement in these areas is necessary because

private insurers become decapitalized after a disaster, making it difficult for consumers and firms to purchase new insurance after the disaster. However, such benefits are not strongly correlated with the state of the economy and are therefore idiosyncratic. Though these programs may provide idiosyncratic social benefits, neither their impact on the government budget nor their social benefits vary systematically with aggregate consumption.

For these types of programs, our model shows that a modified version of the Arrow-Lind (1970) theorem holds. As in Arrow-Lind, the government should use the risk-free rate as its cost of capital. However, in contrast to Arrow-Lind, our model shows that this discount rate should be applied to the social net cash flows generated by the program, including the uninternalized social cash flows and the marginal tax distortions it generates. In the notation of the model, the relevant cash flows are given by  $(W_t - X_t(1 + h'(T_t)))$ .

At the other end of the spectrum are programs with systematic government outlays and systematic social benefits. Programs in this category include financial stabilizers like deposit insurance and lender of last resort activities, as well as automatic stabilizers for the real economy including unemployment insurance. Such programs require government outlays in bad times but may also deliver significant social benefits at those times. For instance, as documented by Calomiris and Gorton (1990) among others, the risk of bank runs is highest during economic downturns. Deposit insurance and the lender of last resort activities prevent the inefficient early liquidation of long-term projects (e.g., Diamond and Dybvig, 1983) at such times, raising overall social output in bad states of the world. Thus, these activities provide strong macroeconomic hedging benefits. However, these social hedging benefits come at a cost. Government expenditures on these activities are strongly countercyclical, which means that the government is incurring direct costs as well as the costs of marginal tax distortions in bad states of the world.

For these types of programs, the Arrow-Lind (1970) theorem does not hold, even in a modified form. The government must charge a risk premium. This risk premium may be either positive or negative. It is negative if the social hedging benefits of the program outweigh the riskiness of the direct costs and tax distortions. Otherwise it is positive. Again, in the notation of the model, the relevant cash flows are  $(W_t - X_t(1 + h'(T_t)))$ .

Finally, there are intermediate programs that require systematically risky government outlays but generate idiosyncratic social cash flows. Take for example the case of student loan guarantees. The standard rationale for these guarantees is that they provide unconditional social benefits; since human capital cannot easily be pledged, the private supply of student loans may be lower than the first best level. The benefits that government guarantees provide in terms of expanding the supply of student loans accrue continually and do not have a strong correlation with the business cycle. Thus, the social benefits of guarantees are idiosyncratic. However, since student loan defaults tend to happen in recessions, the private cash flows from the program are risky.

### 2.3 Optimal Program Scale

In the case where  $h(T) = (\eta/2)T^2$ , we can derive closed-form expressions for taxes, debt, and the optimal scale of government involvement  $q$ . We begin by outlining the standard solution for

optimal taxation (equivalently optimal debt management) in an infinite horizon setting when taxes are distortionary.

Let  $E_t^*[\cdot]$  denotes expectations under the risk-neutral measure. Specifically, for any random variable  $Z$  we define  $E_0^*[Z_t] = E_0[(M_{0,t}/E_0[M_{0,t}])Z_t] = E_0[M_{0,t}Z_t]R_0^{(t)}$ , which implies that  $E_0[M_{0,t}Z_t] = (R_0^{(t)})^{-t}E_0^*[Z_t]$ . Straightforward arguments show that the government's lifetime budget constraint at time  $t$  is

$$\sum_{j=0}^{\infty}(R_t^{(j)})^{-j}[\sum_{i=j+1}^{\infty}(R_{t-i}^{(i)})^i D_{t-i}^{(i)}] = \sum_{j=0}^{\infty}(R_t^{(j)})^{-j}E_t^*[T_{t+j} - (G_{t+j} + qX_{t+j})]. \quad (12)$$

Equation (12) simply says that the market value of outstanding government debt (the left hand side) equals the discounted risk-adjusted present value of future primary surpluses. If  $h(T) = (\eta/2)T^2$ , then (8) implies that

$$T_t = E_t^*[T_{t+j}] \quad (13)$$

for all  $j \geq 1$ . In other words, tax-smoothing implies that tax distortions and, hence, taxes are a random walk under the risk-neutral measure. Combining (12) and (13), we see that the optimal level of taxes is

$$T_t = \frac{\sum_{j=0}^{\infty}(R_t^{(j)})^{-j}[\sum_{i=j+1}^{\infty}(R_{t-i}^{(i)})^i D_{t-i}^{(i)}] + \sum_{j=0}^{\infty}(R_t^{(j)})^{-j}E_t^*[G_{t+j} + qX_{t+j}]}{\sum_{j=0}^{\infty}(R_t^{(j)})^{-j}}. \quad (14)$$

Since the denominator of (14) is the price of a perpetuity that pays one unit each period, equation (14) states that taxes should equal the annuitized value of the accumulated debt burden plus the discounted risk-adjusted present value of future expenditures.

With this solution for optimal taxes in hand, we can express the optimal scale of the government  $q$  in closed form. Specifically, using (13), we can rewrite the first order condition (9) as

$$0 = \sum_{t=0}^{\infty}(R_0^{(t)})^{-t}E_0^*[W_t - X_t] - \eta \sum_{t=0}^{\infty}(R_0^{(t)})^{-t}E_0^*[X_t]T_0 - \eta \sum_{t=0}^{\infty}(R_0^{(t)})^{-t}Cov_0^*[X_t, T_t]. \quad (15)$$

Substituting (14) into (15) and solving, we obtain an expression for the optimal level of  $q$ :

$$q^* = \left\{ \begin{array}{l} \frac{\sum_{t=0}^{\infty}(R_0^{(t)})^{-t}E_0^*[W_t - X_t]}{-\eta \sum_{t=0}^{\infty}(R_0^{(t)})^{-t}E_0^*[X_t] \left( \frac{\sum_{t=0}^{\infty}(R_0^{(t)})^{-t} \left[ \sum_{i=t+1}^{\infty}(R_{0-t}^{(i)})^i D_{0-t}^{(i)} \right] + \sum_{t=0}^{\infty}(R_0^{(t)})^{-t}E_0^*[G_t]}{\sum_{t=0}^{\infty}(R_0^{(t)})^{-t}} \right)} \\ -\eta \sum_{t=0}^{\infty}(R_0^{(t)})^{-t}Cov_0^* \left[ X_t, \frac{\sum_{j=0}^{\infty}(R_t^{(j)})^{-j} \left[ \sum_{i=j+1}^{\infty}(R_{t-i}^{(i)})^i D_{t-i}^{(i)} \right] + \sum_{j=0}^{\infty}(R_t^{(j)})^{-j}E_t^*[G_{t+j}]}{\sum_{j=0}^{\infty}(R_t^{(j)})^{-j}} \right] \end{array} \right\} \quad (16)$$

$$\div \left\{ \eta \sum_{t=0}^{\infty}(R_0^{(t)})^{-t} \left( \frac{\sum_{t=0}^{\infty}(R_0^{(t)})^{-t}E_0^*[X_t]}{\sum_{t=0}^{\infty}(R_0^{(t)})^{-t}} \right)^2 + \eta \sum_{t=0}^{\infty}(R_0^{(t)})^{-t}Cov_0^* \left[ X_t, \frac{\sum_{j=0}^{\infty}(R_t^{(j)})^{-j}E_t^*[X_{t+j}]}{\sum_{j=0}^{\infty}(R_t^{(j)})^{-j}} \right] \right\}.$$

Equation (16) is the ratio of the two terms in curly braces.<sup>5</sup> Although (16) looks complicated

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<sup>5</sup>Since the SDF depends on  $q$ , this is not a closed-form since both the risk-neutral measure as well as riskless interest rates are potentially an implicit function of  $q$ . Furthermore, future realizations of the value of outstanding

at first blush, this formula bears a close resemblance to standard portfolio choice expressions in asset pricing (see e.g., Campbell and Viceira, 2003).

The denominator of (16) depends on the sum of the square of the lifetime average risk-adjusted outlays plus the covariance outlays with future average lifetime outlays, all times the strength of distortions  $\eta$ . The denominator is the analog of the return variance times risk aversion in standard asset pricing models. Here, marginal tax distortions  $\eta$  induce a form of fiscal risk aversion which pins down the optimal scale of government programs. (This is distinct from the household’s risk aversion which is embedded in the risk neutral expectations  $E_0^*[\cdot]$ ) Loosely speaking, since tax distortions are convex and  $E_0^*[X_t^2] = (E_0^*[X_t])^2 + Var_0^*[X_t]$ , all else equal, the government dislikes projects with large average expenditures or with highly volatile expenditures.

The numerator of (16) contains three terms. First,  $\sum_{t=0}^{\infty} (R_0^{(t)})^{-t} E_0^*[W_t - X_t]$  is the expected present value of the net social payoff to the project—the social payoff minus the associated government outlays—which is the analog of the expected excess return in standard portfolio choice settings. The next two terms capture the government’s fiscal risk management motive. The first of these captures the fact that, while the government always dislikes projects with high levels of expected lifetime outlays, this dislike becomes more pronounced when (i) the accumulated debt burden is high or (ii) when exogenously given lifetime government outlays are high. The second of these captures the fact that, all else equal, the government dislikes programs that add to the tax burden in states when the burden would otherwise already be high. These terms are analogous to those that appear in portfolio choice settings in which investors are endowed with “background risk” (e.g., labor income risk).

## 2.4 Distortions as a Function of the Tax Rate

Before studying the above model in greater detail, we first confirm that specifying distortions as a function of the level of taxes as opposed to the tax rate has little effect on our basic conclusions. Obviously, a vast literature in public economics shows that income taxation and other forms of proportional taxation give rise to excess burden because they inevitably distort behavior in inefficient ways. By contrast, lump sum taxation creates no such excess burden in conventional models. Thus, we want to be sure that this modelling convenience made for the sake of analytical tractability has little effect on our overall conclusion.<sup>6</sup> Here we show that a very similar set of trade-offs emerges if we instead model distortions as a function of the tax rate.

The government’s budget constraint is as above in (1). However, we now assume that government levels a proportional tax  $\tau_t$  on total private income which raises tax revenues of

$$T_t = \tau_t (Y_t + qW_t). \tag{17}$$

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debt will also depend implicitly on  $q$ .

<sup>6</sup>Why does the  $h(T) = (\eta/2)T^2$  assumption buy us significant analytical tractability relative to a  $h(\tau) = (\eta/2)\tau^2$  assumption? In short, the  $h(T) = (\eta/2)T^2$  assumption makes it possible to obtain closed-form solutions for the optimal path of taxes, which can then be used to obtain closed-forms for  $q^*$ . By contrast, with the  $h(\tau) = (\eta/2)\tau^2$  assumption it is very difficult to obtain closed-form solutions for tax rates (Bohn, 1990).

Critically, we now assume that the household budget constraint is

$$(Y_t + qW_t)(1 - h(\tau_t)) + \sum_{j=1}^{\infty} (R_{t-j}^{(j)})^j D_{t-j}^{(j)} = C_t + T_t + \sum_{j=1}^{\infty} D_t^{(j)}, \quad (18)$$

where  $h(\cdot)$  represents the distortionary costs or excess burden of taxation with  $h'(\cdot) > 0$  and  $h''(\cdot) > 0$ . This specification for the excess burden of taxation follows Barro (1979) and Bohn (1990). As above, a natural choice here is  $h(\tau) = (\eta/2)\tau^2$ .

In this setting, the first order condition for  $D_0^{(t)}$  implies that optimal taxation satisfies

$$h'(\tau_0) = \frac{E_0[M_{0,t}h'(\tau_t)]}{E_0[M_{0,t}]} \quad (19)$$

for  $t \geq 1$ . As above, tax-smoothing implies that distortions are a random walk under the risk-neutral measure. Similarly, the optimal amount of government activity satisfies

$$0 = E_0[M_{0,t}W_t(1 + h'(\tau_t)\tau_t - h(\tau_t))] - E_0[M_{0,t}X_t(1 + h'(\tau_t))] \quad (20)$$

or

$$\begin{aligned} 0 = & \underbrace{\sum_{t=0}^{\infty} (R_0^{(t)})^{-t} E_0[W_t - X_t]}_{\text{Expected cashflow cost/benefit}} + \underbrace{\sum_{t=0}^{\infty} Cov_0[M_{0,t}, W_t - X_t]}_{\text{Cash flow risk premium}} \\ & + \underbrace{\sum_{t=0}^{\infty} (R_0^{(t)})^{-t} E_0[W_t(h'(\tau_t)\tau_t - h(\tau_t)) - X_t h'(\tau_t)]}_{\text{Expected tax cost/benefit}} \\ & + \underbrace{\sum_{t=0}^{\infty} Cov_0[M_{0,t}, W_t(h'(\tau_t)\tau_t - h(\tau_t)) - X_t h'(\tau_t)]}_{\text{Tax risk premium}}. \end{aligned} \quad (21)$$

Comparing (21) with (10) from above, we see that the same basic set of forces emerges where we assume that distortions are tied to the level of taxes or tax rates.

However, there are some minor differences that are worth noting. Specifically, the discount factor the government applies to the private income generated by a program is  $M_{0,t}^W = M_{0,t}(1 + h'(\tau_t)\tau_t - h(\tau_t))$ , and the discount factor applied to the government inlays/outlays associated with the program is  $M_{0,t}^X = M_{0,t}(1 + h'(\tau_t))$ . Since  $0 < h'(\tau_t)\tau_t - h(\tau_t) < h'(\tau_t)$  for all  $\tau_t \in (0, 1]$  by the convexity of  $h(\cdot)$ , it follows that we have  $M_{0,t} < M_{0,t}^W < M_{0,t}^X$  for all  $\tau_t \in (0, 1]$ . This means that the government should apply a lower discount rate to  $X_t$  than to  $W_t$ . As above, the fact that  $M_{0,t}^W < M_{0,t}^X$  reflects the fiscal conservatism built into our framework due to the distortionary costs of taxation.<sup>7,8</sup> How-

<sup>7</sup>Obviously,  $\partial M_{0,t}/\partial \tau_t > 0$ ; because tax distortions reduce consumption, private agents place greater weight on states with high tax rates. However, we have  $\partial(M_{0,t}^W/M_{0,t})/\partial \tau_t > 0$  and  $\partial(M_{0,t}^X/M_{0,t})/\partial \tau_t > 0$ , so a benevolent government evaluating the impact of a program on private output and government outlays places even more weight on high tax states.

<sup>8</sup>As above, consider a traditional fiscal stimulus program that requires government spending of  $X_t > 0$  and generates private output of  $W_t = f_t X_t > 0$ . In a Ricardian world, the government should undertake the project if  $f_t > 1$ . However, with distortionary taxation, the government should only undertake the project if  $f_t > [1 + h'(\tau_t)]/[1 + h'(\tau_t)\tau_t - h(\tau_t)] > 1$ . The numerator  $(1 + h'(\tau_t))$  reflects that fact that government outlays are costlier than they would be in a Ricardian world since they must be financed with distortionary taxes. The denominator  $(1 + h'(\tau_t)\tau_t - h(\tau_t))$  reflects the fact that, when distortions are linked to the tax rate as opposed to

ever, when tax distortions depend the tax rate, the government should also apply a lower discount rate to the social benefits of the program than private agents would (i.e.,  $M_{0,t} < M_{0,t}^W$ ).

What is the intuition for this result? And how does this compare to the solution in the baseline model? Above we had  $M_{0,t}^W = M_{0,t}$  and  $M_{0,t}^X = M_t(1 + h'(T_t))$ , so that  $M_{0,t}^W < M_{0,t}^X$  for  $T_t > 0$ . Thus, relative to the case where distortions are linked to the level of taxes, when distortions are a function of the tax rate, the government places additional weight on private income generated by government programs,  $W_t$ . Intuitively, there is an additional incentive to manage social risk: this keeps the tax base high, which keeps tax rates low, which reduces tax distortions. In effect, the desire to manage fiscal risk amplifies the desire to manage social risk.<sup>9</sup>

With this single caveat in mind, we conclude that little of substance is changed by our modeling convenience of linking distortions to the level of taxes as opposed to tax rates. And, as we shall see in the next section, the payoff is that we can obtain simple closed-forms that provide deeper intuition into the key trade-offs we are after.

### 3 Special Cases of the General Model

To simplify the analysis and build intuition, we consider two special cases of the general model where the assumption that  $h(T) = (\eta/2)T^2$  in combination with other assumptions enables us to obtain a full closed-form solution. These solutions in turn allow us to easily characterize the key comparative statics of the model. The first special case is a two-period version of the model allowing for general household risk aversion. The second special case is an infinite horizon version of the model, imposing risk neutrality and convenient AR(1) specification for the evolution of all exogenous state variables.

#### 3.1 Two-Period Model with Risk Aversion

##### 3.1.1 Set Up and Solution

The first special case we consider is a simple two-period version of the model. In this case, the government chooses program scale  $q$  as well as the amount of 1-period debt to be issued  $D_0$  at time 0; all debt must be repaid at time 1. For simplicity, we assume  $h(T) = (\eta/2)T^2$ . We also assume, without loss of generality, that the government begins time 0 with no debt.

In the two-period model, the condition for optimal time 0 borrowing is  $T_0 = E_0^*[T_1]$ . Using the accounting definitions of taxes  $T_0 = -D_0 + G_0 + qX_0$  and  $T_1 = RD_0 + G_1 + qX_1$ , this implies

$$D_0 = \frac{G_0 - E_0^*[G_1] + q(X_0 - E_0^*[X_1])}{1 + R}, \quad (22)$$

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level, the government wants to expand the tax base because this allows it to keep the tax rate and thus distortions down. However, the net effect here is always towards fiscal conservatism.

<sup>9</sup>Note, however, that this additional weight on  $W_t$  only applies to government programs that alter *taxable* private income. Examples here might include macro-prudential policies aimed at controlling financial crises or fiscal stimulus programs or automatic stabilizers that have a multiplier effect.



where  $R = (E_0[M_{0,1}])^{-1}$  is the gross interest rate between time 0 and 1. In other words, the government chooses  $D_0$  to smooth risk-adjusted expenditures  $G_t$  and  $X_t$  across periods. The solution for optimal borrowing can be substituted into the accounting definitions of taxes to obtain

$$T_0 = \frac{G_0 + R^{-1}E_0^*[G_1] + q(X_0 + R^{-1}E_0^*[X_1])}{1 + R^{-1}} \quad (23)$$

$$T_1 = \frac{G_0 + R^{-1}G_1 + q(X_0 + R^{-1}X_1) + (G_1 - E_0^*[G_1]) + q(X_1 - E_0^*[X_1])}{1 + R^{-1}}, \quad (24)$$

which clearly satisfy the optimality condition for debt and taxes since  $T_0 = E_0^*[T_1]$ .

In the two-period case, the key Euler equation (9), which determines the optimal scale of government involvement in the project, reduces to

$$0 = [W_0 - X_0(1 + \eta T_0)] + R^{-1}E_0^*[W_1 - X_1(1 + \eta T_1)]. \quad (25)$$

Finally, substituting the above expressions for taxes into the optimality condition for  $q$  and solving implies

$$q^* = \frac{(W_0 - X_0) + R^{-1}E_0^*[W_1 - X_1] - \eta \frac{(G_0 + R^{-1}E_0^*[G_1])(X_0 + R^{-1}E_0^*[X_1])}{1 + R^{-1}} - \eta R^{-1}Cov_0^*[X_1, G_1]}{\eta \left( \frac{(X_0 + R^{-1}E_0^*[X_1])^2}{1 + R^{-1}} + R^{-1}Var_0^*[X_1] \right)}. \quad (26)$$

It is easy to see that equation (26) is the precise 2-period analog of the more complicated solution for the infinite period model given in equation (16).

### 3.1.2 Comparative Statics

We now derive comparative statics in the two-period model. Here we state the comparative statics, taking  $M_{0,1}$  as given so that we do not take into account how an endogenous change in the scale of the government program impacts risk premia via changes in the SDF. This is equivalent to assuming that the project is small relative to private consumption, which is a reasonable assumption for some, but not all, government programs. In particular, if we wanted to capture the idea that the financial stability benefits of deposit insurance lower risk premia in general, or the Pastor and Veronesi (2013) idea that uncertainty about government actions creates its own risk premium, we could not take  $M_1$  as given.

Formally, our comparative statics holding the SDF fixed but allowing net household consumption to change are analogous to substitution effects in a textbook demand context. Conversely, the effects holding net household consumption constant but shifting the SDF are analogous to income effects. Since our setting is analogous to traditional consumption-saving decision, income effects will generally have the opposite sign of substitution effects. As is always the case, without restrictions on preferences, anything goes, and either income or substitution effects can dominate. Of course, we usually think that substitution effects dominate in consumption-saving decisions (e.g., people save more when expected returns rise), and this will be true under various parametric

restrictions. Loosely speaking, this is always true in our model so long as risk aversion isn't too high. This is clear since in the limiting case of risk neutrality, the SDF is independent of  $q$  (it is constant) so there are no income effects. The Internet Appendix provides a more detailed analysis of these issues.

The following proposition states the effects of changing various parameters on the optimal scale of the project  $q^*$ .

**Proposition 2** *In the two-period model, assume that  $h(T) = \frac{1}{2}\eta T^2$  and  $G_0 + R^{-1}E_0^*[G_1] \geq 0$ . Then we have the following comparative statics (holding fixed the SDF):*

- $\partial q^*/\partial W_0 > 0$  and  $\partial q^*/\partial E_0^*[W_1] > 0$ ,
- $\partial q^*/\partial X_0 < 0$  and  $\partial q^*/\partial E_0^*[X_1] < 0$ ,
- $\partial q^*/\partial G_0 \propto -\eta(X_0 + R^{-1}E_0^*[X_1])/(1 + R^{-1})$ ,
- $\partial q^*/\partial \eta \propto -T_0((X_0) + R^{-1}E_0^*[X_1]) - R^{-1}Cov_0[X_1, T_1^*] = -\eta^{-1}((W_0 - X_0) + R^{-1}E_0^*[W_1 - X_1])$
- $\partial q^*/\partial Corr_0^*[X_1, G_1] < 0$ .
- *In general, the sign of  $\partial q^*/\partial Var_0^*[X_1]$  depends on sign( $q^*$ ) and  $Corr_0^*[X_1, G_1]$ . However, in the common case where  $q^* > 0$  and  $Corr_0^*[X_1, G_1] > 0$ , we have  $\partial q^*/\partial Var_0^*[X_1] < 0$ .*

**Proof.** Straightforward differentiation of equation (26). ■

The first two comparative statics are intuitive. Raising the social payoff to the project at time 0,  $W_0$ , or the risk-adjusted expected externality return at time 1,  $E_0^*[W_1]$ , increases the optimal scale of the project. Since  $R^{-1}E_0^*[W_1] = R^{-1}E_0[W_1] + Cov[M_1, W_1]$ , the discounted risk-adjusted expected externality return at time 1 can be raised either by increasing expected social payoff  $E_0[W_1]$  under the physical measure or making the project a better social hedge by increasing  $Cov_0[M_1, W_1]$ . Similarly, raising the required government outlays for the projects at time 0,  $X_0$ , or the risk-adjusted outlays at time 1,  $E_0^*[X_1]$ , lowers the optimal scale of the project. And, of course,  $E_0^*[X_1]$  can be raised either by increasing expected outlays  $E_0[X_1]$  under the physical measure or by having outlays that occur at bad economic times (i.e., raising  $Cov_0[M_1, X_1]$ ).

When  $\eta > 0$ , the effect of initial government spending  $G_0$  depends on whether the program is expected to generate net government outlays or inlays in risk adjusted terms. In particular, if we have  $X_0 + R^{-1}E_0^*[X_1] > 0$ , then the program is expected to entail net outlays for the government in risk-adjusted terms. The optimal scale of positive-outlay programs declines with  $G_0$ . Intuitively, increasing  $G_0$  increases the government's fiscal burden and therefore tax distortions. By decreasing the scale of a positive outlay program, the government can reduce the need for taxation partially offsetting the effect of increasing  $G_0$ . In contrast, if we have  $X_0 + R^{-1}E_0^*[X_1] < 0$  so that programs entail risk-adjusted expected inlays, then the optimal scale of the project rises as  $G_0$  rises. As the government's fiscal burden rises, we want to increase the scale of activities that reduce to that burden.

Similar logic applies to the effect of the severity of marginal tax distortions,  $\eta$ . The comparative static has two terms. The first,  $-T_0 \cdot ((X_0) + R^{-1}E_0^*[X_1])$ , depends on the risk-adjusted expected cash outlays of the program and time 0 taxes. If the program requires positive outlays, then decreasing the scale of the program reduces the tax burden. Reducing the tax burden is particularly valuable when the level of taxes,  $T_0$ , is already high because marginal tax distortions are large at high levels of taxation. The second term in the comparative static,  $-R^{-1}Cov_0^*[X_1, T_1]$ , indicates that the optimal scale of the program also decreases if it tends to typically require outlays at times when the time 1 tax burden will be high. As tax distortions increase, programs that covary with other fiscal risks become less attractive to the government. At the optimum,  $\partial q^*/\partial \eta$  is also proportional to  $-\eta^{-1}((W_0 - X_0) + R^{-1}E_0^*[W_1 - X_1])$ , so an increase in distortions  $\eta$  leads the government to cut back on attractive projects with large discounted, risk-adjusted net benefits.

Finally, the government does less of programs whose  $t = 1$  outlays are (i) highly correlated with  $G_1$  and (ii) more variable (under regularity conditions).

We next consider comparative statics on the price that the government should be willing to pay for different projects. Specifically, we can fix the supply of the project at  $\bar{q}$  and then allow the initial price of the project  $P = X_0$  to adjust so that demand  $q^*$  equals supply  $\bar{q}$ . Specifically, using (25), we have

$$P_0^* = \frac{W_0 + R^{-1}E_0^*[W_1 - X_1(1 + \eta T_1)]}{1 + \eta T_0}.$$

The following proposition characterizes the behavior of government program prices.

**Proposition 3** *In the two-period model, assume that  $W_0 = 0$  and  $h(T) = \frac{1}{2}\eta T^2$ . Then we have the following comparative statics (holding fixed the SDF):*

- $\partial P^*/\partial W_0 > 0$  and  $\partial P^*/\partial (E_0[W_1]) > 0$
- $\frac{\partial P^*}{\partial G_0} \propto -(\eta/(1 + R^{-1}))(P^* + R^{-1}E_0^*[X_1])$ ,
- $\frac{\partial P^*}{\partial \eta} \propto -T_0^*(P^* + R^{-1}E_0^*[X_1]) - R^{-1}Cov_0^*[X_1, T_1^*]$ , and
- $\frac{\partial P^*}{\partial \bar{q}} \propto -\eta((P^* + R^{-1}E_0^*[X_1])^2/(1 + R^{-1}) + R^{-1}Var_0^*[X_1]) < 0$ .
- $\partial P^*/\partial Corr_0^*[X_1, G_1] < 0$ .
- *In general, the sign of  $\partial P^*/\partial Var_0^*[X_1]$  depends on  $Corr_0^*[X_1, G_1]$ . However, in the common case where  $Corr_0^*[X_1, G_1] > 0$ , we have  $\partial P^*/\partial Var_0^*[X_1] < 0$ .*

**Proof.** See Appendix B. ■

The first four comparative statics here are similar to those on quantities. If a change in parameter values implies that the government would like to do more of a project, then if the supply of the project is fixed at  $\bar{q}$ , price must rise for equilibrium to be reached. The comparative static with respect to the quantity supplied  $\bar{q}$  is negative. Regardless of the nature of the program, its price at the margin must fall as the government increases its scale. The reason for this is that any program, whether it generates positive or negative expected risk-adjusted cash flows, increases

the volatility of the government budget. Since there are convex costs of tax distortions, as the size of the program increases, this volatility effect drives up the expected costs of tax distortions. The price of the program must fall to compensate the government for these costs. Finally, the government is willing to pay a lower price for programs whose  $t = 1$  outlays are (i) highly correlated with  $G_1$  and (ii) more variable (under regularity conditions).

### 3.1.3 Sub-cases

Two special cases of the two-period model are worth considering.

**No tax distortions** First, consider the case where there are no tax distortions so that  $\eta = 0$ . In this case, the government can now frictionlessly convert output from  $Y_t$  to  $Y_t + q(W_1 - X_1)$ . Since we assume that there are no diminishing returns to the project, the optimal solution is to always go to a corner by either setting  $q$  to zero or its maximum feasible value. Specifically, the government will set  $q$  to the maximum value if

$$(W_0 - X_0) + R^{-1}E_0^*[W_1 - X_1] > 0$$

and will set  $q = 0$  if

$$(W_0 - X_0) + R^{-1}E_0^*[W_1 - X_1] < 0.$$

We consider the case of diminishing project returns in one of the extensions below.

**No net social benefits** The second special case to consider is one where there are tax distortions but no net social benefits from the project—i.e., where  $W_1 - X_1 = W_0 - X_0 = 0$ . When the project has no net social benefits, the only reason for the government to undertake it is for tax-hedging motives as in Bohn (1990). Specifically, with zero net social benefits, the government's optimal choice is the weighted average of the solutions in two limiting cases

$$q^* = - \left( \frac{\frac{(X_0 + R^{-1}E_0^*[X_1])^2}{1+R^{-1}}}{\frac{(X_0 + R^{-1}E_0^*[X_1])^2}{1+R^{-1}} + R^{-1}Var_0^*[X_1]} \right) \left( \frac{G_0 + R^{-1}E_0^*[G_1]}{X_0 + R^{-1}E_0^*[X_1]} \right) - \left( \frac{R^{-1}Var_0^*[X_1]}{\frac{(X_0 + R^{-1}E_0^*[X_1])^2}{1+R^{-1}} + R^{-1}Var_0^*[X_1]} \right) \left( \frac{Cov_0^*[X_1, G_1]}{Var_0^*[X_1]} \right).$$

First, if the project is riskless (i.e.,  $Var_0^*[X_1] = 0$ ),  $q^* = -(G_0 + R^{-1}E_0^*[G_1]) / (X_0 + R^{-1}E_0^*[X_1])$ , which implies  $0 = q^*(X_0 + R^{-1}E_0^*[X_1]) + (G_0 + R^{-1}E_0^*[G_1])$ . Assuming that  $G_0 + R^{-1}E_0^*[G_1] > 0$ , the government wants to undertake projects that generate discounted risk-adjusted inlays (i.e.,  $X_0 + R^{-1}E_0^*[X_1] < 0$ ) in order to lower households overall tax burden. Second, suppose that  $X_0 + R^{-1}E_0^*[X_1] = 0$ —i.e., the project is zero NPV. This would be the case if the project's cashflows represent an asset that is priced by households. In this case, we have  $q^* = -Cov_0^*[X_1, G_1] / Var_0^*[X_1] = \arg \min_q Var_0^*[T_1] = \arg \min_q Var_0^*[G_1 + qX_1]$ . In other words, the government simply chooses  $q$  to minimize the variability of time 1 taxes.

### 3.1.4 Numerical Example

We next present a simple numerical example to demonstrate that fiscal risk can have a quantitatively meaningful effect on the optimal size of a government project. We work with the two-period model for simplicity. Assume that  $Y_0 = 100$  and  $G_0 = 30$  so that initial government spending is 30% of GDP. At  $t = 1$ , there are two possible states: a high state, which occurs with probability 90%, and a low state, which occurs with probability 10%. In the high state, the stochastic discount factor takes value 0.8, and in the low state it takes value 2.5. These values imply that the risk-free rate is 3%, and the maximum Sharpe ratio in the economy is 0.525. Assume  $h(T) = 0.25\% \cdot T^2$ .

We consider a government program that has the profile of a financial stability program. We assume that the program generates slightly negative social returns at  $t = 0$ ,  $W_0 = -0.5$ , and at  $t = 1$  if the high state is realized,  $(W_1)^H = -0.5$ . There are no government cash flows at  $t = 0$  or if the high state is realized at  $t = 1$ :  $X_0 = 0$  and  $(X_1)^H = 0$ . If the low state is realized, the government incurs significant costs  $(X_1)^L = -5$ , but the program generates substantial social benefits  $(W_1)^L = 10$ . The optimal quantity  $q^*$  of such a program can be interpreted as representing the expansiveness of government protection of the financial system. Specifically, larger values of  $q^*$  can be interpreted as the protection of a larger set of financial intermediaries or a larger set of financial sector liabilities (e.g., deposits, commercial paper, repo, etc).

To emphasize the importance of fiscal risk, we consider two paths of other government expenditure at time 1. In the baseline, government expenditure is acyclical and we have  $G_1^H = G_1^L = 35$ . In this case, the optimal size of the financial stability program is  $q^* = 4.1$ . This implies that in the low state, the government is incurring costs of 20.3, or approximately 20% of GDP. In contrast, suppose government expenditure is cyclical with  $G_1^H = 30$  and  $G_1^L = 40$ . Then the optimal size of the program is  $q^* = 2.6$ , 35% smaller. Thus, the effect of fiscal risk is quite large, even though the realized value of tax distortions is quite small in this example – it averages 2% of time 0 GDP.

## 3.2 Risk-Neutral Infinite Horizon Model with AR(1) Dynamics

### 3.2.1 Set up and Solution

The second special case of the model we consider is one where government spending  $G_t$ , project social benefits  $W_t$ , and outlays for the project  $X_t$  all follow AR (1) processes. In addition, we assume risk neutrality and  $h(T) = (\eta/2)T^2$  for analytical tractability. Relative to the two-period model, this parameterization allows us to study the dynamic implications of the model. Specifically, we assume

$$\begin{aligned} G_t - \bar{G} &= \rho_G (G_{t-1} - \bar{G}) + \varepsilon_{G,t} \\ X_t - \bar{X} &= \rho_X (X_{t-1} - \bar{X}) + \varepsilon_{X,t} \\ W_t - \bar{W} &= \rho_W (W_{t-1} - \bar{W}) + \varepsilon_{W,t}. \end{aligned} \tag{27}$$

Let  $\sigma_G^2 = Var[\varepsilon_{G,t}]$ ,  $\sigma_X^2 = Var[\varepsilon_{X,t}]$ ,  $\sigma_W^2 = Var[\varepsilon_{W,t}]$ , and  $\rho_{XG} = Corr[\varepsilon_{X,t}, \varepsilon_{G,t}]$ .

Since households are risk neutral, the term structure of bonds is trivially  $R_t^{(n)} = 1/\beta = R$  for all

$t$  and  $n$ . Thus, we can, without loss of generality, assume that the government is only borrowing short-term, rolling over 1-period debt from one period to the next. Under these assumptions, optimal taxes are

$$\begin{aligned} T_t &= \frac{RD_{t-1} + \sum_{j=0}^{\infty} R^{-j} E_t [G_{t+j} + qX_{t+j}]}{\sum_{j=0}^{\infty} R^{-j}} \\ &= (\bar{G} + q\bar{X}) + (R-1) \left( D_{t-1} + \frac{G_t - \bar{G}}{R - \rho_G} + q \frac{X_t - \bar{X}}{R - \rho_X} \right). \end{aligned} \quad (28)$$

Naturally, we have  $\frac{\partial T_t}{\partial G_t} = (R-1)/(R-\rho_G) < 1$  and  $\partial T_t/\partial X_t = q(R-1)/(R-\rho_X) \leq q$  and  $\partial^2 T_t/\partial G_t \partial \rho_G > 0$  and  $\partial^2 T_t/\partial X_t \partial \rho_X > 0$ . Current taxes respond less than one-for-one to changes in current government outlays. However, they respond more aggressively when shocks to outlays are more persistent. In the limit where shocks to outlays are a random walk, taxes must respond one-for-one. Debt evolves as

$$\begin{aligned} D_t &= RD_{t-1} + G_t + qX_t - T_t \\ &= D_{t-1} + (G_t - \bar{G}) \left( \frac{1 - \rho_G}{R - \rho_G} \right) + q(X_t - \bar{X}) \left( \frac{1 - \rho_X}{R - \rho_X} \right), \end{aligned} \quad (29)$$

so  $\partial D_t/\partial G_t = (1 - \rho_G)/(R - \rho_G) < 1$  and  $\partial D_t/\partial X_t = q(1 - \rho_X)/(R - \rho_X) < q$  and  $\partial^2 D_t/\partial G_t \partial \rho_G = -(R-1)/(R-\rho_G)^2 < 0$  and  $\partial^2 D_t/\partial X_t \partial \rho_X = -q(R-1)/(R-\rho_X)^2 < 0$ .<sup>10</sup> Debt is used to smooth taxes when there is a shock to government outlays. Current debt responds less than one-for-one to changes in current government outlays. However, debt responds less aggressively when shocks to outlays are more persistent. In summary, permanent shocks to outlays have a large impact on taxes, whereas transitory shocks to outlays are smoothed using borrowing and have little impact on taxes.

We now characterize the optimal level of  $q$ . The optimal level of  $q$  satisfies

$$0 = \sum_{t=0}^{\infty} R^{-t} E_0 [W_t - X_t] - \eta \sum_{t=0}^{\infty} R^{-t} E_t [X_t] T_0 - \eta \sum_{t=0}^{\infty} R^{-t} Cov_0 [X, T_t], \quad (30)$$

which is the analog of (15). Suppose we start in the steady state at  $t = 0$  with debt  $D_{-1}$ . Since we start in the steady state, we have

$$0 = \frac{\bar{W} - \bar{X}}{1 - R^{-1}} - \eta \frac{\bar{X} (\bar{G} + q\bar{X}) + (R-1) \bar{X} D_{-1}}{1 - R^{-1}} - \eta \sum_{t=0}^{\infty} R^{-t} Cov_0 [T_t, X_t].$$

Using these expressions, as shown in the Appendix, we can solve directly for the optimal size of the program  $q^*$  :

$$q^* = \frac{(\bar{W} - \bar{X}) - \eta \bar{X} (\bar{G} + (R-1) D_{-1}) - \eta \frac{1 - \rho_G \rho_X}{R - \rho_X} \frac{R-1}{R - \rho_G} Cov [X_t, G_t]}{\eta (\bar{X})^2 + \eta \frac{1 - \rho_X^2}{R - \rho_X} \frac{R-1}{R - \rho_X} Var [X_t]}, \quad (31)$$

<sup>10</sup>Obviously,  $\partial T_t/\partial G_t + \partial D_t/\partial G_t = 1$  and  $\partial T_t/\partial X_t + \partial D_t/\partial X_t = q$ .

where  $Var [X_t] = \sigma_X^2 / (1 - \rho_X^2)$  is the unconditional variance of  $X$ , and  $Cov [X_t, G_t] = \rho_{XG} \sigma_X \sigma_G / (1 - \rho_X \rho_G)$  is the unconditional covariance of  $X$  and  $G$ .

### 3.2.2 Comparative Statics

**Proposition 4** *In the infinite-horizon model, assume that  $W$ ,  $X$ , and  $G$  follow AR(1) processes, agents are risk-neutral, and  $h(T) = (\eta/2)T^2$ . For simplicity, also assume that  $q^* > 0$ ,  $\bar{X} > 0$ ,  $\bar{G} > 0$ ,  $\rho_X < R^{-1}$ , and  $\rho_G < R^{-1}$ . We then have the following comparative statics results:*

- $\partial q^* / \partial \rho_X < 0$  either holding fixed  $\sigma_X^2$  or  $Var [X_t] = \sigma_X^2 / (1 - \rho_X^2)$ .
- $\partial q^* / \partial Var [X_t] < 0$
- $\partial q^* / \partial \bar{X} < 0$
- $\partial q^* / \partial \rho_G < 0$  either holding fixed  $\sigma_G^2$  or  $Var [G_t] = \sigma_G^2 / (1 - \rho_G^2)$ .
- $\partial q^* / \partial Cov [X_t, G_t] < 0$
- $\partial q^* / \partial \bar{G} < 0$ .
- $\partial q^* / \partial \eta < 0$

**Proof.** Straightforward differentiation of equation (31). ■

The optimal scale of the government program decreases with the mean, variance, and time-series persistence of the outlays it requires. The optimal scale of the project also decreases with the mean and time-series persistence of other government spending  $G_t$ , as well as the covariance between other government spending and program outlays. All of these factors increase the amount and variability of taxes when the government undertakes the project. Since taxes are distortionary, these factors make the project less attractive to the government, all else equal.

## 4 Portfolios of Programs

A key feature of our approach is that it easily extends to the case where we have a portfolio of many government projects. The same basic tradeoffs between social and fiscal risk management that we derived above will apply here, but they now inherit a portfolio management flavor. For simplicity, we study the optimal portfolio of government programs in the context of the two-period model developed above in Section 3.1.

Let projects be indexed by  $j = 1, \dots, J$ . For  $t = 0, 1$ , the government outlays for project  $j$  are  $X_{j,t}$ , social payoffs are  $W_{j,t}$ , and the total quantity of project  $j$  is  $q_j$ . Household consumption is

$$C_t = Y_t + \sum_{j=1}^J q_j (W_{j,t} - X_{j,t}) - (\eta/2) (T_t)^2 - G_t = Y_t + (\mathbf{w}_t - \mathbf{x}_t)' \mathbf{q} - (\eta/2) (T_t)^2 - G_t \quad (32)$$

for  $t = 0, 1$ , where we use the vector notation  $[\mathbf{x}_t]_j = X_{j,t}$ ,  $[\mathbf{w}_t]_j = W_{j,t}$ , and  $[\mathbf{q}]_j = q_j$ . The government chooses  $\mathbf{q}$  and  $D_0$  to maximize  $u(C_0) + \beta E_0 [u(C_1)]$ .

As above, the condition for optimal time 0 borrowing is  $T_0 = E_0^*[T_1]$ . Using the accounting definitions of taxes  $T_0 = -D_0 + G_0 + \mathbf{q}'\mathbf{x}_0$  and  $T_1 = RD_0 + G_1 + \mathbf{q}'\mathbf{x}_1$ , this implies that

$$D_0 = \frac{G_0 - E_0^*[G_1] + \mathbf{q}'(\mathbf{x}_0 - E_0^*[\mathbf{x}_1])}{1 + R}, \quad (33)$$

where  $R = (E_0[M_{0,1}])^{-1}$  is the gross interest rate between time 0 and 1. The solution for optimal borrowing can be substituted into the accounting definitions of taxes to obtain

$$T_0 = \frac{G_0 + R^{-1}E_0^*[G_1] + \mathbf{q}'(\mathbf{x}_0 + R^{-1}E_0^*[\mathbf{x}_1])}{1 + R^{-1}} \quad (34)$$

$$T_1 = \frac{G_0 + R^{-1}G_1 + \mathbf{q}'(\mathbf{x}_0 + R^{-1}\mathbf{x}_1) + (G_1 - E_0^*[G_1]) + \mathbf{q}'(\mathbf{x}_1 - E_0^*[\mathbf{x}_1])}{1 + R^{-1}}, \quad (35)$$

which clearly satisfy the optimality condition for debt and taxes since  $T_0 = E_0^*[T_1]$ .

Thus, the system of first order conditions that define the optimal scale of these projects is given by

$$\mathbf{0} = (\mathbf{w}_0 - \mathbf{x}_0(1 + \eta T_0)) + R^{-1}E_0^*[(\mathbf{w}_1 - \mathbf{x}_1(1 + \eta T_1))], \quad (36)$$

which is simply the vector analog of equation (9) above. Plugging in the solutions for optimal taxes and solving for  $\mathbf{q}$ , we obtain

$$\mathbf{q}^* = \underbrace{\eta^{-1} \left[ \frac{(\mathbf{x}_0 + R^{-1}E_0^*[\mathbf{x}_1])(\mathbf{x}_0 + R^{-1}E_0^*[\mathbf{x}_1])'}{1 + R^{-1}} + R^{-1}Var_0^*[\mathbf{x}_1] \right]^{-1}}_{\text{Portfolio weights absent fiscal hedging motives}} \underbrace{\left[ (\mathbf{w}_0 - \mathbf{x}_0) + R^{-1}E_0^*[\mathbf{w}_1 - \mathbf{x}_1] \right]}_{\text{Fiscal hedging motive}},$$

$$- \underbrace{\left[ \frac{(\mathbf{x}_0 + R^{-1}E_0^*[\mathbf{x}_1])(\mathbf{x}_0 + R^{-1}E_0^*[\mathbf{x}_1])'}{1 + R^{-1}} + R^{-1}Var_0^*[\mathbf{x}_1] \right]^{-1}}_{\text{Fiscal hedging motive}} \left[ \frac{(G_0 + R^{-1}E_0^*[G_1])(\mathbf{x}_0 + R^{-1}E_0^*[\mathbf{x}_1])}{1 + R^{-1}} + R^{-1}Cov_0^*[\mathbf{x}_1, G_1] \right],$$

which is just the natural vector analog of equation (26) above.

## 4.1 Application: Ex-post bailouts versus ex-ante regulation

We can use this setup to think about the desirability of ex-post bailouts versus ex-ante regulations for financial stability, sometimes called the “lean vs. clean” debate.<sup>11</sup> Both bailouts and regulation have large financial stability benefits because they help to mitigate severe financial crises. Regulation can be used to lean against asset bubbles ex-ante, reducing the probability and severity of financial crises. But this can come at the cost of inefficiently reducing economic growth if regulation dampens welfare-improving innovations or unnecessarily constricts credit supply. Ex-post bailouts leave ex-ante growth unfettered but require a larger use of government balance sheet capacity in the event of a financial crisis. A key point that emerges from our framework is that the relative quantity of each intervention will vary with the degree of tax distortions and the existing

<sup>11</sup><http://www.federalreserve.gov/newsevents/speech/stein20131018a.htm>



government debt burden.

To examine this formally in our framework, consider the two-period ( $t = 0, 1$ ) version of the model. Denote ex-ante interventions with  $j = 1$  and ex-post bailouts with  $j = 2$ . Suppose that  $W_{j,0} = 0$  and  $X_{j,0} = 0$  for  $j = 1, 2$  in order to focus on period 1 cash flows.<sup>12</sup> Suppose that ex-ante regulation ( $j = 1$ ) requires the government to incur some small constant costs through its government balance sheet so that we have constant  $X_{1,1} = k > 0$ . These costs can be thought of as the costs of paying regulators and conducting bank examinations.

Given the assumptions, the optimal choices are given by

$$\begin{aligned} \begin{bmatrix} q_1^* \\ q_2^* \end{bmatrix} &= \frac{1 + R}{\eta k} \left( \frac{E_0^* [W_{1,1}]}{k} - 1 \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &+ \frac{1}{\eta k} \frac{E_0^* [X_{2,1}]}{Var_0^* [X_{2,1}]} \left( \frac{E_0^* [W_{2,1}]}{E_0^* [X_{2,1}]} - \frac{E_0^* [W_{1,1}]}{k} \right) \begin{bmatrix} -E_0^* [X_{2,1}] \\ k \end{bmatrix} \\ &- (G_0 + R^{-1} \gamma E_0^* [g_1]) \begin{bmatrix} \frac{R}{k} \\ 0 \end{bmatrix} - \gamma \frac{Cov_0^* [g_1, X_{2,1}]}{Var_0^* [X_{2,1}]} \begin{bmatrix} -\frac{E_0^* [X_{2,1}]}{k} \\ 1 \end{bmatrix}. \end{aligned}$$

This formula illustrates the forces that determine  $q_1^*$  and  $q_2^*$ . First, an increase in the risk-adjusted efficacy of ex-ante regulation,  $E_0^* [W_{1,1}] / k$ , increases  $q_1^*$ . Second, an increase in the differential efficacy of ex-post bailouts versus ex-ante regulation,  $E_0^* [W_{2,1}] / E_0^* [X_{2,1}] - E_0^* [W_{1,1}] / k$ , leads the planner to substitute from ex-ante regulation towards ex-post bailouts. Specifically, the government will only choose  $q_2^* > 0$  if  $E_0^* [W_{2,1}] / E_0^* [X_{2,1}] > E_0^* [W_{1,1}] / k$ —i.e., if the risk-adjusted bang-for-buck is greater for ex-post interventions. Given the desire the smooth taxes, this substitution is greater when  $Var_0^* [X_{2,1}]$  is small. The final two terms capture the way that background fiscal risk impact the choices of  $q_1$  and  $q_2$ .

The following proposition describes the behavior of the optimum. In order to consider the effects of scaling all other government expenditure up or down, we write  $G_1 = \gamma g_1$  and compute comparative statics with respect to  $\gamma$ .

**Proposition 5** *Suppose that ex-ante regulation and ex-post bailouts have the cash flow characteristics assumed above. We have the following comparative statics:*

- $\partial q_1 / \partial G_0 = -R/k < 0$
- $\partial q_2 / \partial G_0 = 0$ ,
- $\partial q_1 / \partial \gamma = -k^{-1} (E_0^* [g_1] - E_0^* [X_{2,1}] \cdot (Cov_0^* [X_{2,1}, G_1] / Var_0^* [X_{2,1}]))$
- $\partial q_2 / \partial \gamma = -Cov_0^* [g_1, X_{2,1}] / Var_0^* [X_{2,1}]$ , and

<sup>12</sup>This is without loss of generality since, as show above in (37), all that matters for  $j = 1, 2$  are the NPV of net benefits minus outlays ( $(\mathbf{w}_0 - \mathbf{x}_0) + R_f^{-1} E_0^* [\mathbf{w}_1 - \mathbf{x}_1]$ ), the NPV of outlays ( $x_0 + R_f^{-1} E_0^* [\mathbf{x}_1]$ ), time 1 variance of outlays ( $Var_0^* [\mathbf{x}_1]$ ), and covariance of outlays with background spending ( $Cov_0^* [\mathbf{x}_1, G_1]$ ).

- $\partial q_1^* / \partial \eta = - \{ (1 + R) / (\eta^2 k) \} \{ E_0^* [W_{1,1}] / k - 1 \}$   
 $+ \{ (E_0^* [X_{2,1}])^2 / (Var_0^* [X_{2,1}] (\eta^2 k)) \} \{ E_0^* [W_{2,1}] / E_0^* [X_{2,1}] - E_0^* [W_{1,1}] / k \}$
- $\partial q_2^* / \partial \eta = - \{ E_0^* [X_{2,1}] / ((\eta k)^2 Var_0^* [X_{2,1}]) \} \{ E_0^* [W_{2,1}] / E_0^* [X_{2,1}] - E_0^* [W_{1,1}] / k \}$

The comparative statics have strong analogies to portfolio choice logic. As the level of initial government expenditures  $G_0$  rises, all adjustment takes place by reducing the amount of ex-ante regulation. Since it always incurs constant cost  $k$ , regulation is essentially a riskless technology, so when riskless government expenditures  $G_0$  go up, all the adjustment comes from reducing regulation. This is analogous to the portfolio choice logic that dictates that with CARA utility, the total dollar amount invested in risky technologies is constant in total wealth. Only the amount invested in riskless technologies varies.

When the amount of government expenditure at time 1,  $G_1 = \gamma g_1$ , is scaled up, the comparative statics depend on  $Cov_0^* [G_1, X_{2,1}]$ , the (risk-adjusted) covariance between other government expenditures and bailout cash flows. It is natural to assume that  $Cov_0^* [G_1, X_{2,1}] > 0$  since the government is likely to be spending a lot on other expenditures like unemployment benefits (high  $G_1$ ) at times when it is making bailout payments (i.e., when  $X_{2,1}$  is high). In this case, we have  $\partial q_2 / \partial \gamma < 0$ . Raising other government expenditures makes ex-post interventions less attractive because they are correlated with other government budget expenditures. This raises the government's total tax burden and thus the total distortions incurred. As in portfolio choice problems, the degree of adjustment in the quantity of bailouts  $q_2$  depends on the (risk-adjusted) beta of government expenditures with respect to bailout payoffs.

When the amount of government expenditure  $G_1 = \gamma g_1$  rises, there are two effects on the quantity of regulation,  $q_1$ . To see the first effect, suppose  $E_0^* [g_1] = 0$ . Then

$$\frac{\partial q_1}{\partial \gamma} = \frac{E_0^* [X_{2,1}] Cov_0^* [X_{2,1}, G_1]}{k Var_0^* [X_{2,1}]} > 0.$$

Thus, at low levels of other government expenditures, increases in  $\gamma$  make bailouts less attractive, so we substitute towards ex-ante regulation. However, there is a second countervailing effect when  $E_0^* [g_1] > 0$ . When the expected tax burden is too high, increases in  $\gamma$  make all government expenditure less attractive. If this effect overwhelms the substitution effect, it may be optimal to cut back on both ex-post bailouts and ex-ante regulation as  $\gamma$  rises.

These results can also be interpreted as motivating "financial repression" at high levels of government debt. Reinhart and Sbrancia (2011) argue that at high levels of government debt, financial regulation is used to force financial intermediaries to hold government debt, providing a captive buyer. Our model makes the point that financial repression may be optimal in high debt situations, not just because the government needs a buyer of debt but because the government cannot afford the costs of the alternative financial stability policy – bailouts.

Similar logic applies to the comparative statics on the severity of tax distortions  $\eta$ . We have

$$\begin{aligned}\frac{\partial q_1^*}{\partial \eta} &= -\frac{1}{\eta^2} \frac{1+R}{k} \left( \frac{E_0^*[W_{1,1}]}{k} - 1 \right) + \frac{1}{\eta^2 k} \frac{(E_0^*[X_{2,1}])^2}{\text{Var}_0^*[X_{2,1}]} \left( \frac{E_0^*[W_{2,1}]}{E_0^*[X_{2,1}]} - \frac{E_0^*[W_{1,1}]}{k} \right) \\ \frac{\partial q_2^*}{\partial \eta} &= -\frac{1}{\eta^2} \frac{E_0^*[X_{2,1}]}{\text{Var}_0^*[X_{2,1}]} \left( \frac{E_0^*[W_{2,1}]}{E_0^*[X_{2,1}]} - \frac{E_0^*[W_{1,1}]}{k} \right).\end{aligned}$$

To the extent that ex-post bailouts are highly attractive relative to ex-ante regulation on a risk-adjusted basis (i.e.,  $E_0^*[W_{2,1}]/E_0^*[X_{2,1}] - E_0^*[W_{1,1}]/k$  is large), an increase in distortions pushes the government away from ex-post bailouts and towards ex-ante regulation (i.e., we have  $\partial q_2^*/\partial \eta < 0$  and  $\partial q_1^*/\partial \eta > 0$ ). However, to the extent that ex-ante regulation is highly attractive (i.e.,  $E_0^*[W_{1,1}]/k - 1$  is large), there is an offsetting effect: in this case regulation expenditures are already quite large, so the increase in distortions leads the government to cut back on regulation as well.

We have discussed comparative statics in terms of quantities here. As pointed out above, if we fix the quantities of regulation and bailouts, comparative statics in terms of their prices will simply have the opposite signs. Thus, the prices charged for bailouts should rise as other government expenditures  $\gamma$  and tax distortions  $\eta$  increase.

## 4.2 Application: Optimal budget cuts

We can use the same basic setup to study how the government should react to fiscal shocks. Consider two government projects. The first, denoted with  $j = 1$ , has expenditures that do not covary with other government programs. Spending by the National Science Foundation might be an example of such a project. As above, we model this project as having constant time 1 expenditures  $X_{1,1} = k$ . The second project, denoted  $j = 2$ , has expenditures that do covary with other government spending. Spending on automatic stabilizers would be an example of such a project. We denote these expenditures  $X_{2,1}$ .

Our model highlights two different types of fiscal shocks, which the government should respond to differently. First, there could be an unexpected shock at time 0, which raises government spending at that time,  $G_0$ , without affecting expectations of the future. As shown by the comparative statics derived in Proposition 5,  $\partial q_1/\partial G_0$  and  $\partial q_2/\partial G_0$ , such shocks should be fully adjusted to by reducing the riskless project  $j = 1$ . The intuition here is that the government should try to offset one-time shocks rather than incur additional tax distortions by adjusting the overall path of expenditures.

In contrast, suppose the fiscal shock increases expectations of future government expenditure. Then the comparative statics  $\partial q_1/\partial \gamma$  and  $\partial q_2/\partial \gamma$  in Proposition 5 show that there are both income and substitution effects. The government should substitute towards projects with lower fiscal risk, i.e., those that covary less with future government expenditure. Income effects push towards shrinking all programs.

## 5 Extensions

### 5.1 Nonlinear Government Technology

In the main model we assume that there are constant returns to scale in the projects the government is considering. Of course, in reality there may be increasing or decreasing returns to scale for different projects undertaken by the government. This idea can be easily incorporated into our basic framework by assuming that the social returns to a project of scale  $q$  are  $f(q)W_t$  rather than  $qW_t$ , where  $f' > 0$  and  $f'' < 0$  in the diminishing returns case and  $f'' > 0$  in the increasing returns case. Similarly, we assume that outlays are  $g(q)X_t$  rather than  $qX_t$  where  $g' > 0$  and  $g'' > 0$  in the diminishing returns case and  $g'' < 0$  in the increasing returns case. Under these assumptions, household consumption is

$$C_t = Y_t + f(q)W_t - g(q)X_t - (\eta/2)(T_t)^2 - G_t$$

and the optimal amount of government activity,  $q^*$ , now satisfies

$$0 = \sum_{t=0}^{\infty} (R_0^{(t)})^{-t} E_0^* [f'(q^*)W_t - g'(q^*)X_t] \quad (38)$$

$$\begin{aligned} & -g'(q^*)\eta \sum_{t=0}^{\infty} (R_0^{(t)})^{-t} E_0^* [X_t] T_0 \\ & -g'(q^*)\eta \sum_{t=0}^{\infty} (R_0^{(t)})^{-t} Cov_0^* [X_t, T_t]. \end{aligned} \quad (39)$$

Obviously, this expression collapses to (15) when  $f(q) = g(q) = q$ .

For instance, one can use this extension to think about the *Powell doctrine* for government financial stability interventions. The Powell doctrine is the idea that it is optimal to use overwhelming force when dealing with financial crises. A powerful show of government intervention early in a crisis can restore confidence and prevent amplification mechanisms like fire sales from taking hold. In our setup, one could think of representing this idea with increasing returns to scale for financial bailouts over some region. Specifically, if  $f'' > 0$  over some region then it could be the case that small interventions are negative NPV, so that evaluating the government's first order condition at 0 we have

$$0 > \sum_{t=0}^{\infty} (R_0^{(t)})^{-t} E_0^* [f'(0)W_t - g'(0)X_t(1 + \eta T_t)].$$

However, since  $f'' > 0$ , it can still be the case that the optimal scale is  $q^* > 0$ .

It may also be interesting to use our setup to think about how the Powell doctrine interacts with uncertainty about the necessary size of the bailout. Increasing returns to scale make the case for a large intervention, but if the size of the financial sector's capital hole is uncertain, there is the risk of ending up like Ireland instead of the US in the recent financial crisis. In the language of the model, one would be trading off the increasing returns to scale to bailouts against increasing expected distortionary costs.

A weaker version of the Powell doctrine that may apply more generally is that government inefficiency may raise the hurdle rate for government interventions.

## 5.2 Discretionary Policies

We now consider the case of a discretionary fiscal policy that can be varied over time. Specifically, we assume that at time  $t$  the government chooses the size of the program that will prevail at time  $t + 1$ . However, the relevant realizations of  $\{W_{t+1}, X_{t+1}, G_{t+1}\}$  are not known for certain at time  $t$  when this policy is chosen. For simplicity, we assume that households are risk-neutral and that  $h(T) = (\eta/2)T^2$ . The government's problem is

$$\max_{\{D_t, q_t\}} \sum_{j=0}^{\infty} R^{-j} E_0 \left[ q_{t-1} (W_t - X_t) - (\eta/2) (G_t + RD_{t-1} + q_{t-1}X_t - D_t)^2 \right], \quad (40)$$

taking the path of  $\{W_t, X_t, G_t\}$  as given.

The first order condition for  $q_t$  is

$$E_t [W_{t+1} - X_{t+1}] = \eta E_t [X_{t+1} T_{t+1}] \quad (41)$$

and the first order condition for  $D_t$  implies that  $T_t = E_t [T_{t+j}]$  for all  $j > 0$ . The lifetime government budget constraint as of time  $t$  is  $RD_{t-1} = \sum_{j=0}^{\infty} R^{-j} E_t [T_{t+j} - G_{t+j} - q_{t+j-1} X_{t+j}]$ . Combining this with the fact that  $T_t = E_t [T_{t+j}]$  implies that optimal taxes at time  $t$  are

$$T_t = (R - 1) D_{t-1} + \frac{R - 1}{R} \left( \sum_{j=0}^{\infty} R^{-j} E_t [G_{t+j} + q_{t+j-1} X_{t+j}] \right). \quad (42)$$

Plugging this solution for  $T_t$  into  $D_t = G_t + q_{t-1} X_t + RD_{t-1} - T_t$  and rearranging, we see that taxes tomorrow equal taxes today plus the innovation to expectations of lifetime spending

$$T_{t+1} = T_t + (E_{t+1} - E_t) \frac{R - 1}{R} \sum_{j=0}^{\infty} R^{-j} (G_{t+1+j} + q_{t+j} X_{t+j+1}), \quad (43)$$

which implies that

$$q_t^* = \frac{1}{\eta} \frac{R}{R - 1} \frac{E_t [W_{t+1} - X_{t+1}]}{\text{Var}_t [X_{t+1}] + R^{-1} (E_t [X_{t+1}])^2} \quad (44)$$

$$\frac{E_t [X_{t+1}] \left( RD_{t-1} + \sum_{j=0}^{\infty} R^{-j} E_t [G_{t+j}] + q_{t-1}^* X_t + R^{-1} \sum_{j=1}^{\infty} R^{-j} E_t [q_{t+j}^* X_{t+j+1}] \right)}{\text{Var}_t [X_{t+1}] + R^{-1} (E_t [X_{t+1}])^2}$$

$$\frac{\sum_{j=0}^{\infty} R^{-j} \text{Cov}_t [X_{t+1}, G_{t+1+j}] + \sum_{j=1}^{\infty} R^{-j} \text{Cov}_t [X_{t+1}, q_{t+j}^* X_{t+j+1}]}{\text{Var}_t [X_{t+1}] + R^{-1} (E_t [X_{t+1}])^2}.$$

Thus, assuming that  $q_t^* > 0$ , the discretionary program is large today when:

- $E_t [W_{t+1} - X_{t+1}]$  is large, so the project is expected to generate large net social benefits next period.
- $\text{Var}_t [X_{t+1}] + R^{-1} (E_t [X_{t+1}])^2$  is small, so the project adds little to either the expected level or volatility of taxes tomorrow.

- The sum of past accumulated debt, expected future non-discretionary spending, past discretionary spending, or expected future discretionary spending are low. Each of these reduces  $E_t [T_{t+1}]$  for a given choice of  $q_t$  which makes discretionary spending more attractive in the presence of distortionary taxes.
- $\sum_{j=0}^{\infty} R^{-j} Cov_t [X_{t+1}, G_{t+1+j}]$  is small so shocks to discretionary spending at  $t + 1$  are not strongly correlated with news about future non-discretionary spending.
- $\sum_{j=1}^{\infty} R^{-j} Cov_t [X_{t+1}, q_{t+j}^* X_{t+j+1}]$  is small so shocks to discretionary spending at  $t + 1$  are not strongly correlated with news about future discretionary spending.

## 6 Conclusion

We present a model in which the distortions associated with taxation make financing costly for the government. We explore the consequences of this assumption for the set of programs that the government chooses to undertake. As in the literature on costly external finance in corporate finance, we show that distortionary taxation impacts the optimal scale and pricing of government programs. In particular, the government has both social and fiscal risk management motives. The social risk management motive arises from the fact that some government programs deliver large benefits when marginal utility is high. The fiscal risk management motive arises from the government's desire to avoid raising distortionary taxes both on average and in particular when marginal utility is high. Fiscal risk gives the government a hedging motive and means that individual programs cannot be evaluated in isolation – the fiscal risk of a particular program depends on the other programs and expenditures the government is undertaking.

We highlight the interaction between the social and fiscal risk management motives. These motives may conflict if programs with strong social risk management benefits involve government expenditures and hence higher tax distortions in bad times, creating more fiscal risk. On the other hand, they may reinforce each other if social risk management preserves private output in a downturn—reducing the distortionary impact of a given dollar amount of taxes.

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# A Mapping Examples into the Model Setup

To make the setup concrete, we present two simple examples.

## A.1 Financial stabilization

The first is an example from finance that considers the value of government interventions to prevent fire sales. We take the Stein (2012) model of fire sales as our starting point. In the model, there are three periods  $t = 0, 1, 2$ . Levered intermediaries hold assets  $I$  and back fraction  $d$  with riskless short-term debt. Bad news at the interim date ( $t = 1$ ) means that there is probability  $(1 - q)$  that the assets are worth zero at  $t = 2$  and probability  $q$  that they are worth  $\lambda I/q$  at  $t = 2$ . Thus, if there is bad news at the interim date, the assets are worth  $\lambda I$  in expectation. To render the short term debt riskless, intermediaries must ensure that the fire sale value of the assets at the interim date can cover repayment of the debt. That is, we must have

$$dI = R_f^{-1}k\lambda I$$

where  $0 < k \leq 1$  is the fire sale discount at the interim date. The discount is pinned down by the capital  $W$  and outside opportunities  $g(\cdot)$  of patient investors, who will purchase assets fire sold by levered intermediaries at  $t = 1$ . Specifically, we have

$$\frac{1}{k} = g'(W - dIR_f).$$

The real cost of the fire sale is that patient investors only invest  $W - dIR_f$  in new outside projects rather than  $W$  if the fire sale takes place.

Now suppose there is bad news at  $t = 1$  so that there is a fire sale in the absence of government intervention. Consider a government program where the government steps in and guarantees the short-term debt. For simplicity, think of the government as assuming all the assets and liabilities of the intermediaries and guaranteeing their short-term debt so that it does not run at the interim date. This assumption ensures that there are no inflows or outflows of cash at  $t = 1$ . Further assume the guarantee is unanticipated so that it has no ex-ante effects. However, this guarantee prevents the fire sale from taking place at  $t = 1$ .

In our notational conventions, the government's outlays at  $t = 2$  are  $X_2 = dI$  if the outcome is bad, and the assets are worth zero (probability  $1 - q$ ) or  $X_2 = dI - \lambda I/q$  if the outcome is good and the assets recover (probability  $q$ ). The externality cash flow from this government program is the gain in net output relative to the counterfactual with no government guarantee

$$W_1 = [g(W) - W] - [g(W - dIR_f) - (W - dIR_f)].$$

## A.2 Automatic stabilizers

Our second example considers unemployment insurance, taking the Rothschild-Stiglitz (1976) model as our baseline.<sup>13</sup> There are two types of individuals, those ( $L$ -types) with low probability of being unemployed  $p_L$  and those ( $H$ -types) with high probability of being unemployed  $p_H$ . Agents know their types, and fraction  $f$  are high types, and  $(1 - f)$  are low types. Both types

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<sup>13</sup>As we discuss further below, there is some tension in our use of a representative agent framework in situations where there are multiple types of agents.

earn income  $E_1$  when they are employed and income  $E_2 < E_1$  when they are unemployed. The insurance contract specifies that the agent pays the insurer  $\alpha_1$  if he is employed and receives  $\alpha_2$  if unemployed. With insurance premia set by risk-neutral intermediaries, insurance is actuarially fair, and we have

$$\alpha_2 = \frac{1-p}{p}\alpha_1$$

where  $p$  is the probability of being unemployed by those agents who purchase insurance. It is well-known that no pooling equilibrium exists in this setting.

The separating equilibrium that can survive is one where high types fully insure so their marginal utility is equated across states

$$u'(E_1 - \alpha_{1,H}) = u'\left(E_2 + \frac{1-p_H}{p_H}\alpha_{1,H}\right),$$

and low types receive so little insurance that the high types do not wish to deviate:

$$(1-p_H)u(E_1 - \alpha_{1,H}) + p_H u\left(E_2 + \frac{1-p_H}{p_H}\alpha_{1,H}\right) \geq (1-p_H)u(E_1 - \alpha_{1,L}) + p_H u\left(E_2 + \frac{1-p_L}{p_L}\alpha_{1,L}\right).$$

Expected welfare in the private market separating equilibrium is

$$W_{priv} = (1-f) \left[ \underbrace{(1-p_L)u(E_1 - \alpha_{1,L}^*) + p_L u\left(E_2 + \frac{1-p_L}{p_L}\alpha_{1,L}^*\right)}_{\text{EU for } L\text{-types}} \right] + f \left[ \underbrace{(1-p_H)u(E_1 - \alpha_{1,H}^*) + p_H u\left(E_2 + \frac{1-p_H}{p_H}\alpha_{1,H}^*\right)}_{\text{EU for } H\text{-types}} \right].$$

Suppose the government mandates insurance where the employed pay  $\alpha_1^{**}$  and the unemployed receive  $\alpha_2^{**}$ . Furthermore, we now introduce aggregate risk by having  $\bar{p}$ , the average probability of being unemployed, vary over time. We assume that  $\alpha_1^{**}$  and  $\alpha_2^{**}$  do not vary with  $\bar{p}$ , so the net government expenditure associated with unemployment insurance will vary as a function of  $\bar{p}$ . Expected welfare is

$$W_{govt} = (1-\bar{p})u(E_1 - \alpha_1^{**}) + \bar{p}u(E_2 + \alpha_2^{**}),$$

so that

$$W_1 = W_{govt} - W_{priv}.$$

And net government outlays are

$$X_1 = \bar{p}\alpha_2^{**} - (1-\bar{p})\alpha_1^{**} = \bar{p}(\alpha_2^{**} - \alpha_1^{**}) - \alpha_1^{**},$$

which rises with  $\bar{p}$  so long as  $\alpha_2^{**} > \alpha_1^{**}$ . In this context, assuming a broad program of unemployment insurance with a given level of benefits  $\alpha_2^*$ , one could use our program to solve for the optimal employment insurance  $\alpha_1^*$ .

## B Omitted Proofs

### B.1 Proof of Proposition 3

The Euler equation defining  $P^*$  is

$$0 = f(P^*, \theta) = W_0 - P^*(1 + \eta T_0^*) + R^{-1}E_0^*[W_1 - X_1(1 + \eta T_1^*)].$$

For any parameter  $\theta$ , comparative statics follow from

$$\begin{aligned} 0 &= f_P(P^*, \theta) \frac{\partial P^*}{\partial \theta} + f_\theta(P^*, \theta) \\ \Rightarrow \frac{\partial P^*}{\partial \theta} &= -\frac{f_\theta(P^*, \theta)}{f_P(P^*, \theta)}. \end{aligned}$$

Since

$$\frac{\partial T_0^*}{\partial P} = \frac{\partial T_1^*}{\partial P} = \frac{\bar{q}}{1 + R^{-1}},$$

we have

$$f_P(P^*, \theta) = -\left(1 + \eta \frac{G_0 + R^{-1}E_0^*[G_1] + 2\bar{q}(P^* + R^{-1}E_0^*[X_1])}{1 + R^{-1}}\right) < 0$$

In a neighborhood of zero-subsidy price  $P^* + R^{-1}E_0^*[X_1X_1] = 0$ , we always have  $f_P(P^*, \theta) < 0$ . Thus, we will assume that  $f_P(P^*, \theta) < 0$  so that we have

$$\frac{\partial P^*}{\partial \theta} = -\frac{f_\theta(P^*, \theta)}{f_P(P^*, \theta)} \propto f_\theta(P^*, \theta).$$

1. **Effect of  $\eta$ :** We have

$$\frac{\partial P^*}{\partial \eta} \propto -T_0^*(P^* + R^{-1}E_0^*[X_1]) - R^{-1}Cov_0^*[X_1, T_1^*].$$

2. **Effect of  $G_0$ :** We have

$$\frac{\partial P^*}{\partial G_0} \propto -\frac{\eta}{1 + R^{-1}}(P^* + R^{-1}E_0^*[X_1]).$$

3. **Effect of  $W_0$  and  $E_0^*[W_1]$ :** We have

$$\frac{\partial P^*}{\partial W_0} > 0 \text{ and } \frac{\partial P^*}{\partial (E_0^*[W_1])} > 0.$$

4. **Effect of  $\bar{q}$ :** We have

$$\frac{\partial P^*}{\partial \bar{q}} \propto f_{\bar{q}}(P^*, \bar{q}) = -\eta \left( \frac{(P^* + R^{-1}E_0^*[X_1])^2}{1 + R^{-1}} + R^{-1}Var_0^*[X_1] \right) < 0.$$

## B.2 Proof of Proposition 4

Recall that

$$T_t = (\bar{G} + q\bar{X}) + (R - 1) \left( D_{t-1} + \frac{G_t - \bar{G}}{R - \rho_G} + q \frac{X_t - \bar{X}}{R - \rho_X} \right)$$

and

$$D_t = D_{t-1} + (G_t - \bar{G}) \left( \frac{1 - \rho_G}{R - \rho_G} \right) + q (X_t - \bar{X}) \left( \frac{1 - \rho_X}{R - \rho_X} \right).$$

Since we start in the steady state at  $t = 0$  with debt  $D_{-1}$ , we have

$$0 = \frac{\bar{W} - \bar{X}}{1 - R^{-1}} - \eta \frac{\bar{X} (\bar{G} + q\bar{X}) + (R - 1) \bar{X} D_{-1}}{1 - R^{-1}} - \eta \sum_{t=0}^{\infty} R^{-t} Cov_0 [T_t, X_t].$$

Thus, to solve for  $q^*$ , we simply need to evaluate  $\sum_{t=0}^{\infty} R^{-t} Cov_0 [T_t, X_t]$ .

One subtle point here is that the optimal level of taxes at time  $t$  depends on the amount of accumulated debt  $D_{t-1}$ . And the amount of accumulated debt is a random variable which depends on the past path of  $X_t$  and  $G_t$ . We need to take this path-dependency into account when computing  $\sum_{t=0}^{\infty} (R_f)^{-t} Cov_0 [T_t, X_t]$ . Fortunately, this is easy given the assumed  $AR(1)$  dynamics. Specifically, accumulated debt at  $t - 1$  is

$$D_{t-1} = D_{-1} + \left( \frac{1 - \rho_G}{R - \rho_G} \right) \sum_{j=1}^{t-1} (G_{t-j} - \bar{G}) + q \left( \frac{1 - \rho_X}{R - \rho_X} \right) \sum_{j=1}^{t-1} (X_{t-j} - \bar{X}),$$

so we have

$$\begin{aligned} T_t &= (\bar{G} + q\bar{X}) + (R - 1) D_{-1} \\ &+ \left( \frac{R - 1}{R - \rho_G} \right) \left( (1 - \rho_G) \sum_{j=1}^{t-1} (G_{t-j} - \bar{G}) + (G_t - \bar{G}) \right) \\ &+ q \left( \frac{R - 1}{R - \rho_X} \right) \left( (1 - \rho_X) \sum_{j=1}^{t-1} (X_{t-j} - \bar{X}) + (X_t - \bar{X}) \right), \end{aligned}$$

which implies

$$\begin{aligned} Cov_0 [T_t, X_t] &= \left( \frac{R - 1}{R - \rho_G} \right) \left( (1 - \rho_G) \sum_{j=1}^{t-1} Cov_0 [G_{t-j}, X_t] + Cov_0 [X_t, G_t] \right) \\ &+ q \left( \frac{R - 1}{R - \rho_X} \right) \left( (1 - \rho_X) \sum_{j=1}^{t-1} Cov_0 [X_{t-j}, X_t] + Var_0 [X_t] \right). \end{aligned}$$

Now since  $X_t = \bar{X} + \sum_{j=0}^{t-1} \rho_X^j \varepsilon_{X,t-j}$  and  $G_t = \bar{G} + \sum_{j=0}^{t-1} \rho_G^j \varepsilon_{G,t-j}$ , we have

$$\begin{aligned} Var_0 [X_t] &= \left(1 - (\rho_X^2)^t\right) \frac{\sigma_X^2}{1 - \rho_X^2} \\ Cov_0 [X_t, G_t] &= \left(1 - (\rho_G \rho_X)^t\right) \frac{\varrho_{XG} \sigma_G \sigma_X}{1 - \rho_G \rho_X} \\ Cov_0 [G_{t-j}, X_t] &= \rho_X^j Cov_0 [G_{t-j}, X_{t-j}] = \rho_X^j \left(1 - (\rho_G \rho_X)^{t-j}\right) \frac{\varrho_{XG} \sigma_G \sigma_X}{1 - \rho_G \rho_X} \\ Cov_0 [X_{t-j}, X_t] &= \rho_X^j Var_0 [X_{t-j}] = \rho_X^j \left(1 - (\rho_X^2)^{t-j}\right) \frac{\sigma_X^2}{1 - \rho_X^2}. \end{aligned}$$

Combining these facts we have

$$\begin{aligned} Cov_0 [T_t, X_t] &= \left(\frac{R-1}{R-\rho_G}\right) \frac{\varrho_{XG} \sigma_G \sigma_X}{1 - \rho_G \rho_X} \left( (1 - \rho_G) \sum_{j=1}^{t-1} \rho_X^j \left(1 - (\rho_G \rho_X)^{t-j}\right) + (1 - (\rho_G \rho_X)^t) \right) \\ &\quad + q \left(\frac{R-1}{R-\rho_X}\right) \frac{\sigma_X^2}{1 - \rho_X^2} \left( (1 - \rho_X) \sum_{j=1}^{t-1} \rho_X^j \left(1 - (\rho_X^2)^{t-j}\right) + (1 - (\rho_X^2)^t) \right) \\ &= (1 - \rho_X^t) \left[ \left(\frac{R-1}{R-\rho_G}\right) \frac{1 - \rho_G \rho_X}{1 - \rho_X} \frac{\varrho_{XG} \sigma_G \sigma_X}{1 - \rho_G \rho_X} + q \left(\frac{R-1}{R-\rho_X}\right) (1 + \rho_X) \frac{\sigma_X^2}{1 - \rho_X^2} \right] \\ &= (1 - \rho_X^t) \left[ \left(\frac{R-1}{R-\rho_G}\right) \frac{1 - \rho_G \rho_X}{1 - \rho_X} Cov [X_t, G_t] + q \left(\frac{R-1}{R-\rho_X}\right) (1 + \rho_X) Var [X_t] \right] \end{aligned}$$

where  $Var [X_t] = \sigma_X^2 / (1 - \rho_X^2)$  is the unconditional variance of  $X$  and  $Cov [X_t, G_t] = \varrho_{XG} \sigma_X \sigma_G / (1 - \rho_X \rho_G)$  is the unconditional covariance of  $X$  and  $G$ . Thus, we have

$$\sum_{t=0}^{\infty} R^{-t} Cov_0 [T_t, X_t] = \frac{R}{R-1} \frac{1 - \rho_X}{R - \rho_X} \left[ \left(\frac{R-1}{R-\rho_G}\right) \frac{1 - \rho_G \rho_X}{1 - \rho_X} Cov [X_t, G_t] + q \left(\frac{R-1}{R-\rho_X}\right) (1 + \rho_X) Var [X_t] \right]$$

which implies that the optimal value of  $q$  satisfies

$$q^* = \frac{(\bar{W} - \bar{X}) - \eta \bar{X} (\bar{G} + (R-1) D_{-1}) - \eta \frac{1 - \rho_G \rho_X}{R - \rho_X} \frac{R-1}{R - \rho_G} Cov [X_t, G_t]}{\eta (\bar{X})^2 + \eta \frac{1 - \rho_X^2}{R - \rho_X} \frac{R-1}{R - \rho_X} Var [X_t]}.$$

### B.3 Proof of Proposition 5

Suppose that  $w_0 = x_0 = 0$ ,  $G_1 = \gamma g_1$ , and  $X_{11} = k$ . Then the general expression for  $\mathbf{q}^*$  in the two-period model reduces to

$$\begin{aligned} \begin{bmatrix} q_1^* \\ q_2^* \end{bmatrix} &= \frac{1+R}{\eta k} \left( \frac{E_0^* [W_{1,1}]}{k} - 1 \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\eta k} \frac{E_0^* [X_{2,1}]}{Var_0^* [X_{2,1}]} \left( \frac{E_0^* [W_{2,1}]}{E_0^* [X_{2,1}]} - \frac{E_0^* [W_{1,1}]}{k} \right) \begin{bmatrix} -E_0^* [X_{2,1}] \\ k \end{bmatrix} \\ &\quad - (G_0 + R^{-1} \gamma E_0^* [g_1]) \begin{bmatrix} \frac{R}{k} \\ 0 \end{bmatrix} - \gamma \frac{Cov_0^* [g_1, X_{2,1}]}{Var_0^* [X_{2,1}]} \begin{bmatrix} -\frac{E_0^* [X_{2,1}]}{k} \\ 1 \end{bmatrix}. \end{aligned}$$

Note that we then have

$$\begin{aligned}
\begin{bmatrix} \partial q_1^* / \partial G_0 \\ \partial q_2^* / \partial G_0 \end{bmatrix} &= - \begin{bmatrix} \frac{R}{k} \\ 0 \end{bmatrix} \\
\begin{bmatrix} \partial q_1^* / \partial \gamma \\ \partial q_2^* / \partial \gamma \end{bmatrix} &= - \begin{bmatrix} E_0^* [g_1] / k - \frac{Cov_0^* [g_1, X_{2,1}]}{Var_0^* [X_{2,1}]} E_0^* [X_{2,1}] / k \\ \frac{Cov_0^* [g_1, X_{2,1}]}{Var_0^* [X_{2,1}]} \end{bmatrix} \\
\begin{bmatrix} \partial q_1^* / \partial \eta \\ \partial q_2^* / \partial \eta \end{bmatrix} &= - \frac{1+R}{\eta^2 k} \left( \frac{E_0^* [W_{1,1}]}{k} - 1 \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \frac{1}{\eta^2 k} \frac{E_0^* [X_{2,1}]}{Var_0^* [X_{2,1}]} \left( \frac{E_0^* [W_{2,1}]}{E_0^* [X_{2,1}]} - \frac{E_0^* [W_{1,1}]}{k} \right) \begin{bmatrix} -E_0^* [X_{2,1}] \\ k \end{bmatrix}
\end{aligned}$$