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## Abstract

The last twenty years have witnessed the rise of disaggregated "clusters" or "networks" of firms. In these clusters the activities of R&D, product design, production, logistics and selling may be split up among hundreds or even thousands of firms. Different firms will design and produce the different modules of a complex artifact (like the processor, peripherals, and software of a computer system), and different firms will specialize in different stages of a complex production process. This paper considers the pricing behavior and profitabililty of such clusters. In particular, we investigate a possibility hinted at in prior work: that pressures to raise prices across complementary-goods markets can offset pressures to reduce prices within oligopolistic differentiated-goods markets. In this paper, we isolate the offsetting price effects and show how they might operate in large as well as small clusters. We argue that it is theoretically possible for a "modular cluster" of firms to mimic the pricing behavior and profitability of "one big firm."

**Key words:** oligopolistic pricing; vertical integration; modularity; cluster

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# 1 Introduction

The last twenty years have witnessed the rise of disaggregated "clusters" or "networks" of firms in a number of industries including computers, telecommunications and pharmaceuticals. In these clusters the activities of R&D, product design, production, logistics and selling may be split up among hundreds or even thousands of firms. Different firms will design and produce the different modules of a complex artifact (like the processor, peripherals, and software of a computer system), and different firms will specialize in different stages of a complex production process.[1]

A number of researchers have argued that a disaggregated supply chain or a cluster of firms, in which complementary activities are carried out by different enterprises, can be a more efficient and / or more innovative than a single integrated firm.[2] However, in such work, the effect of the cluster form of industrial organization on the prices of end products, hence the profitability of the cluster, is usually not addressed. In this note, we will address the pricing issue directly and ask: how large are the profits (or "quasi-rents") that a cluster can capture?[3] In particular, is it possible for a cluster of firms to generate enough quasi-rents to fuel sustained investments in innovation or fixed capital over time?

The question of cluster rents is important for the economics of innovation because expenditures on innovative products and processes are usually *sunk costs* by the time the products are offered for sale. Thus if a group of product markets cannot deliver a stream of quasi-rents to the firms competing in those

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[1] For evidence on the formation of clusters and networks and their economic importance, see among others, Porter 1990), Langlois and Robertson (1992), Saxenian (1994), Gomes-Casseres (1996), Baldwin and Clark (1997, 2000), Baker, Gibbons and Murphy (2002b), Sturgeon (2002), Fallick, Fleischman and Rebitzer (2003), and Bresnahan and Gambardella (2003).

[2] On the benefits of clusters and / or vertical disintegration, see for example, Stigler (1951), Langlois and Robertson (1992), Garud and Kumaraswamy (1995), Sanchez and Mahoney (1996), Baldwin and Clark (1997), Fine (1998), Schilling (2000), Aoki (2001), Aoki and Takizawa (2002), Baker, Gibbons and Murphy (2002a), Langlois (2002), Sturgeon (2002), Fallick, Fleischman and Rebitzer (2003). Many of these works discuss the benefits of "modularity" and modular forms of organization. However, we must emphasize that the organizational economics question, "Is a cluster of firms likely to be more efficient or innovative than one big firm?" is distinct from the engineering design question, "Is a modular architecture likely to be more efficient or flexible than an interdependent architecture for a particular design or production process?" One big firm can use a modular product and process architecture, and many do. Conversely, a cluster of firms is not precluded from using an interdependent product and process architecture, although in practice such architectures tend to have high "mundane" transaction costs (Baldwin and Clark, 2002a).

[3] Quasi-rent is income from an asset or enterprise that serves as the return to sunk-cost capital (Klein, Crawford and Alchian, 1978). In financial theory and practice, future quasi-rents are equivalent to future "free cash flows." In a competitive capital market, a sunk-cost investment is worth undertaking if and only if the present value of expected future free cash flows (quasi-rents) discounted by an appropriate risk-adjusted cost of capital is greater than or equal to the cost of the investment.

markets, the economic incentives to invest in new product development and process improvements in that arena will be greatly diminished.

A fundamental tenet of economic theory is that more competition, that is an increase in the number of sellers of a good, reduces equilibrium prices and increases the quantities sold relative to a monopoly. Another less-well-known result, first derived by Cournot (1838), is that *splitting the supply of complementary goods across multiple monopolies increases prices and decreases the quantities sold* relative to the corresponding vertically integrated monopoly. In the case of multiple monopolies, if the complementary goods constitute the successive stages of a supply chain, the latter effect is known as "double marginalization" or the "chain of monopolies" effect (Spengler, 1950; Tirole, 1988, pp. 174–175). But, as Nalebuff (2000), Ruffin (2001) and Baumol (2002) have observed, the effect applies not only to the stages of a production process, but to *any set* of complementary goods and services that get combined into a composite good or a complex system. Thus the goods subject to Cournot's "chain of monopolies" effect can be the successive stages of a production process; a set of components that get assembled into a finished product; or a set of products that end-users purchase and combine into a system.

In all these cases, the complementary goods are *modules*, that is, they are separate units that function together within a larger, more complex whole, called the *system*. Thus, Cournot's result applied to modular systems suggests that distributing the decision rights and ownership of modules among several suppliers ("module monopolies") should cause prices to go up and quantities to go down relative to the prices and quantities chosen by a single, integrated firm.

In a nutshell, therefore, the "horizontal" splitting of a market among several firms reduces prices, while the "vertical" splitting of a market into several distinct *module monopolies* increases prices. In both instances, the sum of the profits of all firms is less than the profit of a single monopoly that spans the whole market. However, the opposite direction of these two effects suggests that in some cases they might offset one another. If so, a horizontally and vertically disaggregated *modular cluster* of firms might mimic the prices of "one big firm" that spanned the whole system. Assuming that its costs were the same, the cluster would be as profitable as the corresponding "one big firm" and would have a commensurate quasi-rent stream. In this paper, we will show when and how that can happen.

The rest of this paper is organized as follows. Section 2 locates this paper in the literatures of industrial and organizational economics and points to related works. Section 3 explains the thought experiment we plan to conduct. Section 4 lays out the assumptions of our formal model. Section 5 derives the main results, and Section 6 presents an illustrative example. Section 7 relates our detailed results to those found in prior work. Section 8 concludes.

# 2 Related Literature

There are vast literatures on oligopoly pricing, vertical integration, and contracting between vertically related firms. In general, however, the literature on oligopoly pricing has focused on single-stage production processes; the recent literature on vertical integration has focused primarily on the design of optimal bilateral contracts or on issues of vertical control and foreclosure; and the contracting literature has focused on designing incentives to improve the allocation of effort or to elicit the disclosure of private information.[4] For the most part, however, the theoretical papers in these literatures do not consider the possibilities of simultaneous vertical and horizontal disaggregation nor do they look at the pricing behavior of large groups of firms.

An important exception to this rule is a seminal paper by Economides and Salop (1992). Their analysis suggested that a 2x2 cluster (two firms in each of two component markets) might be able to mimic the pricing behavior, and hence the profitability, of one big firm. But the precise circumstances that would give rise to that outcome were difficult to discern from their model. We will discuss their modeling approach in relation to ours in Section 4 below.

Except for Economides and Salop, we do not know of any paper that directly addresses the questions we are asking. Notwithstanding that fact, our results are foreshadowed by results found in several other models. For example, Rey and Stiglitz (1995) explored the impact of marketing agreements that effectively set up two layers of oligopolistic competition linked by vertical contracts. They showed that industry profits increase when horizontal and vertical price effects offset one another. More recently Nalebuff (2000) and Ruffin (2001) constructed models of horizontally and vertically disaggregated industries and investigated prices, strategic behavior, and profits within such industries. We will relate

[4] On oligopolistic pricing, see, Tirole (1988) and Vives (1999). On bilateral contracting and vertical foreclosure, see Rey and Tirole (2003). On vertical contracting, see the foundational work of Klein, Crawford and Alchian (1978); Williamson (1985); Grossman and Hart (1986); as well as the recent synthesis by Baker, Gibbons and Murphy (2002a).

our model and detailed results to these other models in Section 7 below.

Our goal in this paper is to draw attention to results that are implicit in these prior works but sometimes overlooked. We will also show how the findings can be generalized to apply to large aggregations—or clusters—of firms.

# 3 Our Thought Experiment

In this section, we offer two examples of the problem we wish to address and explain the thought experiment we plan to conduct. Our purpose is to give readers an intuitive understanding of the research question before laying out a formal model. One example involves a beach resort, which is a somewhat prosaic, “low-tech” setting. The other, shorter example involves the greater computer industry, including the makers of all the components, hardware, software, and services that go into a complete computer system. We have chosen these examples, first, because many readers will have had direct experience (as consumers) with beach resorts and computers, and, second, because modular clusters appear to be viable forms of industrial organization in both these settings.

## 3.1 The Beach Resort

Consider a beach resort that has many hotel rooms, many restaurants serving many meals, many bars, swimming pools, beach umbrellas, taxis, sports activities, etc. A complete vacation includes (1) transportation to and from the resort; (2) transportation within the resort; (3) lodging (e.g., a hotel room for 6 nights); (4) food; (5) drinks; and (6) various types of recreation; and (7) leisure in pleasant surroundings. For purposes of the thought experiment, we will consider the capital stock of the resort to be fixed so that the number of vacationers can vary between zero and a very large number (e.g., 40,000 per week or 2 million per year). However, there are variable costs associated with supplying all the goods and services that go into a single vacation (a person-week at the resort).

The prices set at the resort will depend on the *configuration* of the resort as an industry. Configuration in this context refers to the allocation of profit incentives and decision rights over prices. One classic configuration is for *one big firm* to own all the facilities and set all the prices. That firm, by definition, is a monopolist with respect to the resort, and its pricing decisions can serve as a benchmark in measuring the prices and profits of other configurations. Another classic configuration is to “slice” the beach into some number of *full-span oligopolies*, each of which performs all the functions of one big firm, but serves only a fraction of the market. Still another classic configuration is to “partition” the goods and services provided at the resort into a set of complementary *module monopolies*. Finally, we could both “slice” and “partition” the goods and services—the result would be what we are calling a *modular cluster* of firms.

## 3.2 Beach Resort Technology and External Demand

Let us now look at the technology of the beach resort more closely. Of necessity, a vacation involves many subsidiary components. Some of these components appear to the vacationers as goods and services consciously selected and purchased—for example, a hotel room, a meal, a boat ride. Other components are intermediate goods and services, like maid service at the hotel, or the food bought by the restaurant. Still other, smaller components are groups of resources, tasks and decisions within a subsidiary unit. For example, the contents of the maids’ trolleys or the restaurant’s refrigerator are resources; room cleaning and cooking are tasks; and hiring and scheduling are decisions made in the housekeeping department or at the restaurant. The point is that, whether the vacationer is aware of them or not, *all* of these technological and operational components—the resources, tasks and decisions that go into each and every good and service the resort provides—are inputs to the vacation.

In general, the technological and operational components involved in a complex system of production are nested. Thus if one “opens up” one component, say, a restaurant, one will find in it another set of components in the form of resources, tasks and decisions. In principle (and in practice) the process of opening up components can continue until one arrives at a set of primitive resources, tasks, and decisions.[5]

The specific arrangement of nested technological and operational components in a complex system is called the *architecture* of that system. The details of this structure are of interest mainly to the designers and the operating managers of the various parts. Quite often, therefore, no comprehensive map

[5] The practice of “opening up,” analyzing, and rearranging primitive technological components is the essence of the activity known as “process engineering” (or “re-engineering”). In a low-tech setting, such as a beach resort, it is a responsibility of managers. In other settings, it is the task of engineers.

of the whole architecture exists. But the fact that no map exists does not mean that the structure itself is non-existent, arbitrary, or unimportant. It is this structure—a specific arrangement of technological and operational components—that makes the resort a coherent system that can produce enjoyable, whole vacations from which no necessary part is missing.

The building blocks of a complex system are called *modules*. Loosely speaking, a module is a group of components—resources, tasks and decisions—that "belong together" because they depend on one another more than they depend on other groups of components.[6] Intuitively, the airline flight to the resort is a module; a taxi-plus-driver is a module; each restaurant and each shop is a module; the swimming pool operation is a module; the bike rental establishment is a module; and the beach itself may be divided into modules (each with umbrellas, chairs, towels, a snack bar, etc.). Moreover, we have already said that these "large modules" can be subdivided into smaller modules and then even smaller components. For example, each night (or each hour) in a hotel room is a component of the hotel stay; maid service and laundry service are components of hotel service; landscaping is a component; the towel, the chaise longue and pool maintenance are all components, and so on. On this view, the total number of "primitive production components," denoted $\Xi$, in the technology of supplying a beach resort vacation is very large.

To a first approximation, we can think of each vacationer as consuming a bundle consisting of *one of each primitive component* in the resort's production system. This does not mean, by the way, that the vacationers must all do exactly the same thing or have the same experiences: below, we shall see that there may be many *variants* within each component category. However, the resort basically supplies a composite good — the week-long vacation experience. That experience, in turn, is created by combining a large set of primitive production components in proportions that are essentially fixed (at least in the short run). In this sense, the vacation is made up of $\Xi$ (the very large number of) primitive components, and every vacationer purchases and consumes a composite good with the same number of primitive components.

Even if every vacationer consumes one of each of the primitive components, their vacations do not have to be identical. Within each component category, the actual goods may be differentiated: for

[6] On the definition of modularity and the delineation of modules within a complex system, see Baldwin and Clark (2000), Chapter 4.

example, each hotel has a unique location, décor and staff: some vacationers may prefer one hotel and some another. Even within a hotel, the east-facing rooms differ from the west-facing ones. Some vacationers love Italian food, while others go for steak. To capture these variegated aspects of the beach resort's system of production, we assume that within each of the primitive component categories there is a very large range of *variants*. Thus, in the category of laundry service (for sheets, towels, etc.), Hotel A might have a laundry in the basement. There might also be a laundry down the street that specializes in bleaching, and another one two blocks away that will replace torn linens at no cost. These three laundries, which by our earlier assumption all have the same cost, offer *variants* in the category of laundry service.

Let us now turn to the issues of demand, pricing and cost. We assume that vacationers choose this resort based on their assessment of the price they (will) pay for the whole vacation. This total price amounts to the sum of the prices of all the things each vacationer gets charged for during his or her trip. We will call this the "system price" and denote it P. We assume that the demand for vacations anywhere at this resort (the number of vacationers coming each week) is a function, Q(P), that depends on the system price only. (Implicitly, this means that firms at the resort do not have the ability to price-discriminate amongst vacationers based on their willingness-to-pay.) In addition, we have already said that all vacationers consume *one of each of the primitive components*. Hence their vacations do not vary in terms of cost.

### 3.3 Modular Partitions and Alternative Groupings of Components

Our basic thought experiment involves carving up the components of the beach resort system of production and assigning ownership, pricing decision rights and profits in different ways. In describing the production system of the resort, we have gone to some lengths to make sure that the architecture is finely divisible, both horizontally (as variants) and vertically (as modules comprised of primitive components). In reality, the industrial organization of a real resort (or any complex industrial system) must take account of the *natural modular structure* of that system—the natural groupings of resources, tasks and decisions that make sense for the specific technologies involved. For example, it is natural to separate transport services from hotel services, and there is a natural breakpoint between taxi service and hotel service at the curbside of the hotel. Often, economic transactions are located at these breakpoints—they are the so-called "thin crossing points" of the natural modular structure. One pays the taxi driver, and checks into the hotel. But a natural breakpoint in the modular structure does not have to be utilized for transactions: many hotels provide transportation to guests as part of their service.[7]

Within a complex system of production, a module is both a thing and a group of things. On the one hand, a module is a building block of the complex system, and in that sense it is a single thing. On the other hand, a module is also a grouping of components—it is a *set* of resources, tasks and decisions that "belong together." A large module, in turn, can be made up of smaller modules. But as one opens up the successive layers of a modular structure, one eventually comes to modules that cannot be subdivided—their primitive components are simply too interdependent. For example, taxi service requires a car and a driver: without both, it is not taxi service, though it may be something else (a Zip car, a chauffeur). In addition, real modular structures are always asymmetric, involving big modules and small ones, and different degrees of hierarchy and nesting of some modules within others.[8]

Nevertheless, for the purpose of analyzing prices and profits under different industry configurations, it is useful to suppress the indivisibilities and asymmetries inherent in real modular structures and to look at a range of *symmetric modular partitions* of the basic components of the system of production. From a theoretical standpoint, we can do this by making a convenient assumption about the architecture of the system: that every primitive component is itself a module. Under this assumption, no primitive component "needs" to be grouped with any other. As a result, any subdivision or symmetric (re)grouping of primitive components will be technologically admissible. And if (as we have assumed) there is a very large number, $\Xi$, of primitive components / modules, then a correspondingly large number of symmetric modularizations are possible. (For example, if $\Xi = K!$, then one can create symmetric partitions made up of 2 modules, 3 modules, … J modules, and K modules.)

## 3.4 Configurations of the Resort

Under this simplifying architectural assumption, the canonical configurations we shall examine are: (1) One Big Firm; (2) a set of N "full-span" oligopolies; (3) a chain of J complementary module

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[7] On the cost-effective locations for transactions in a large system of production, see Baldwin and Clark (2002a).

[8] On the mapping of artificial systems including modular and semi-modular structures, see, for example, Simon (1962), Steward (1982), Eppinger (1991), Ulrich (1995), Baldwin and Clark (2000), Sullivan *et. al.* (2002), and Sharman *et. al.* (2002).

monopolies; and, finally (4) a JxN modular cluster, in which N firms compete within each of J module markets. In subsections below, we shall explain what each of these canonical configurations means in the context of the beach resort.

### *One Big Firm*

It is easy to imagine all the facilities at the resort being owned and managed by One Big Firm. Because One Big Firm internalizes all subsidiary pricing decisions, if we hold technology and costs fixed, we know that its profits will be the highest that *any* configuration of firms can possibly attain. Given our assumption that each and every primitive component is essential to the vacation, in order to set prices rationally, One Big Firm has only to calculate its profits in terms of "system price" and "system cost": that is, it must price a whole vacation relative to the cost of supplying a whole vacation.[9] We will derive One Big Firm's optimal price and profit formally in the next section of the paper. Its price and profit level in turn will serve as a benchmark by which to gauge the prices and profitability of the other configurations.

### *Full-Span Oligopolies*

The next canonical configuration is for two or more firms to own different "slices" of the beach. For example, one firm might own all the facilities—hotels, restaurants, taxis, bicycles, umbrellas, etc.—on the north end of the beach while another owned everything on the south end. Being in competition with one another, we would expect these duopolists to charge lower prices than the aforementioned One Big Firm. And since they would not be pricing "optimally" relative to external demand, the sum of their profits will also be lower than One Big Firm's profit. Carrying on in this vein, we can imagine "slicing" the beach into ever finer resort-firms, each of which is a small, but complete resort. Under symmetry, if there were N such firms, then *each one would own, set prices for, and garner profits from* a fraction $\alpha_N = 1 / N$ of the full set of primitive production components.

If each full-span resort-firm's products were perfect substitutes for all the others', then the price

[9] This result was proved by Economides and Salop (1992). In reality, for purposes of managing its complex production system, controlling theft, deterring over-consumption, rationing scarce facilities, and fine-tuning the resort's design, One Big Firm may want to charge vacationers for some of the individual items they consume. From our perspective, however, how it chooses to itemize the bill is a second-order issue. The particular way in which the bill is itemized does not enter into a vacationer's decision as to whether to come the resort, nor into One Big Firm's decision as to how much to charge per person-week.

competition amongst them would be fierce. Theory then says that, absent collusion, system prices would fall to marginal cost. But, for our theoretical agenda, we would like to model a gentler form of price competition. Therefore we assume (Hotelling style) that the resort-firms' products are imperfect substitutes. The rationale for this assumption lies in existence of *variants within each component category*. Because primitive components can vary, *modules* made up of primitive components can vary, hence *products* can and do vary within their categories.

We assume that price competition among variants within categories works in the following way. Suppose, as stipulated, there are two firms, North and South. Each owns and sets prices for 1/2 of all the variants in each module category. Each competes *by module* with the other for *market share*, knowing that its market share is somewhat, but not entirely responsive to price. For example, in the module category "Restaurants," North believes that if it lowers the prices at all its restaurants, a certain fraction of vacationers will change their dinner plans, and it will gain restaurant-market-share relative to South. But North does not believe that changing its restaurant prices will cause *all* the vacationers to flock to its restaurants: some will still prefer the food or ambience on the south side of the beach. South has symmetric beliefs with respect to North.

We will derive North's and South's equilibrium prices and profits in terms of a formal model and generalize the results to an N-firm oligopoly in the next section. In the context of the model, we will show that, not surprisingly, the sum of the oligopolists' profits is always lower than the profit of One Big Firm and declines as the number of oligopolists goes up.[10]

### *Module Monopolies*

The third canonical configuration is to partition the productive activities between two or more monopolies that supply different, but complementary goods and services. For example, one firm might provide all the "customer-facing" goods and services: it would own and staff all the hotels, bars, restaurants, taxis, airlines, and recreational facilities. Another firm might own and staff all the

[10] Interestingly, this result does not depend on the *technological integration* of modules nor on any restrictions of consumer choice. North and South do not have to sell full-week vacation "bundles" nor restrict the vacationers' experiences. The vacationers can stay in North's hotels and eat in South's restaurants and vice versa. The decline in prices and profits depends solely on the fact that North and South control and garner profits from *a fractional share of a full set of modules*. For this reason, we have labeled configurations in which each firm controls a fractional share of all modules "full-span oligopolies" rather than "fully integrated oligopolies."

infrastructure of the beach resort: the laundries, the food warehouses, the trucks, the airport, the phone, water and electrical systems. In supply chain terminology, the customer-facing firm would be "Downstream," and the infrastructure firm would be "Upstream." Upstream and Downstream would not compete with one another as North and South did above. Instead Upstream and Downstream would each own, set prices for, and garner profits from complementary parts of the resort's overall system of production.

Other partitions of the production system are also possible: for example, one firm might take care of everything having to do with Food; another might manage Shelter; a third Transportation; and a fourth Recreation. Each of these four firms might have a whole supply chain that was fully independent of the other three. There would then be no transactions between them (as there are transactions between an Upstream supplier and a Downstream customer). But Food, Shelter, Transportation and Recreation would still be supplying complementary goods within a larger system of production. Indeed these two hypothetical partitions are different ways of organizing the ownership of *modules* (or, more precisely, groups of modules) within the architecture of the resort's overall production system.

If one firm supplies and sets prices for all variants of a module, then it is a monopoly with respect to that module. This explains our name for this configuration: module monopolies. Module monopolies subsume both successive monopolies in a vertical chain of production, and complementary goods monopolies wherein consumers select different components and combine them to make a composite good, e.g., a vacation.

What kinds of prices will module monopolies charge? As first shown by Cournot, and subsequently generalized by Economides and Salop (1993), Nalebuff (2000), Ruffin (2001) and Baumol (2003), module monopolies will generally charge higher prices than One Big Firm. To see this, imagine that there are several module monopolies, each supplying a different, but essential, piece of a larger, composite system. The key intuition is that if the price of the system declines through the price cut of *one* module monopoly, the demand for *all* modules will go up because each is an essential part of the system. The result is an imbalance of costs and benefits: the individual module monopoly will bear the full brunt of a prospective price cut, but will garner only a fraction of the revenue gained from it. Most of the incremental revenue and profit will go to the other module monopolies. However, in the absence of contracts, a single module monopolist who thinks of cutting its price cannot demand compensation from the other module makers whose sales and profits will go up as a result of its price reduction. Nor can one module monopolist assume that the others will cut their prices *pari passus*.[11]

This imbalance of cost and benefit causes the equilibrium system price set by module monopolies (the sum of the module prices) to be higher than the system price set by One Big Firm and to increase as the number of separate module monopolies goes up. But the higher system price implicitly charged by the module monopolies is inferior in terms of system profit maximization. Hence the total profit garnered under this configuration is less than the profit of One Big Firm and declines with the number of module monopolies. (Below we shall derive these results in the context of a formal model.)

### *A Modular Cluster*

The fourth and last canonical configuration is the one we are calling a *modular cluster*. In this case ownership and pricing are split both within and across modules. Thus, in a 2x2 modular cluster at the beach resort, there would be a North Upstream, North Downstream, a South Upstream and South Downstream, for a total of four firms in all. North Upstream would compete in its module market with South Upstream; similarly North Downstream would compete with South Downstream. Assuming that any symmetric partition of modules and variants is feasible, we can generalize this idea to allow for J module markets and N firms per module market—a JxN modular cluster.

The question that interests us is the following. Holding technology and external demand constant, how does modular cluster pricing compare with pricing under the other canonical configurations? In the next section, we will present a formal model of equilibrium prices under each canonical configuration. Assuming differentiated goods competition and no price collusion, we will show that a modular cluster comprising JxN firms has an equilibrium system price (the sum of the module prices) that is (1) higher than the system price of N full-span oligopolies, but (2) lower than the system price of J module monopolies.

Figure 1 shows this result graphically for J=2 and N=2. It depicts the *profit functions* perceived by a representative firm in equilibrium under each of the four canonical configurations. In each case, consistent with the assumptions of a Nash equilibrium, the representative firm takes the other firm's

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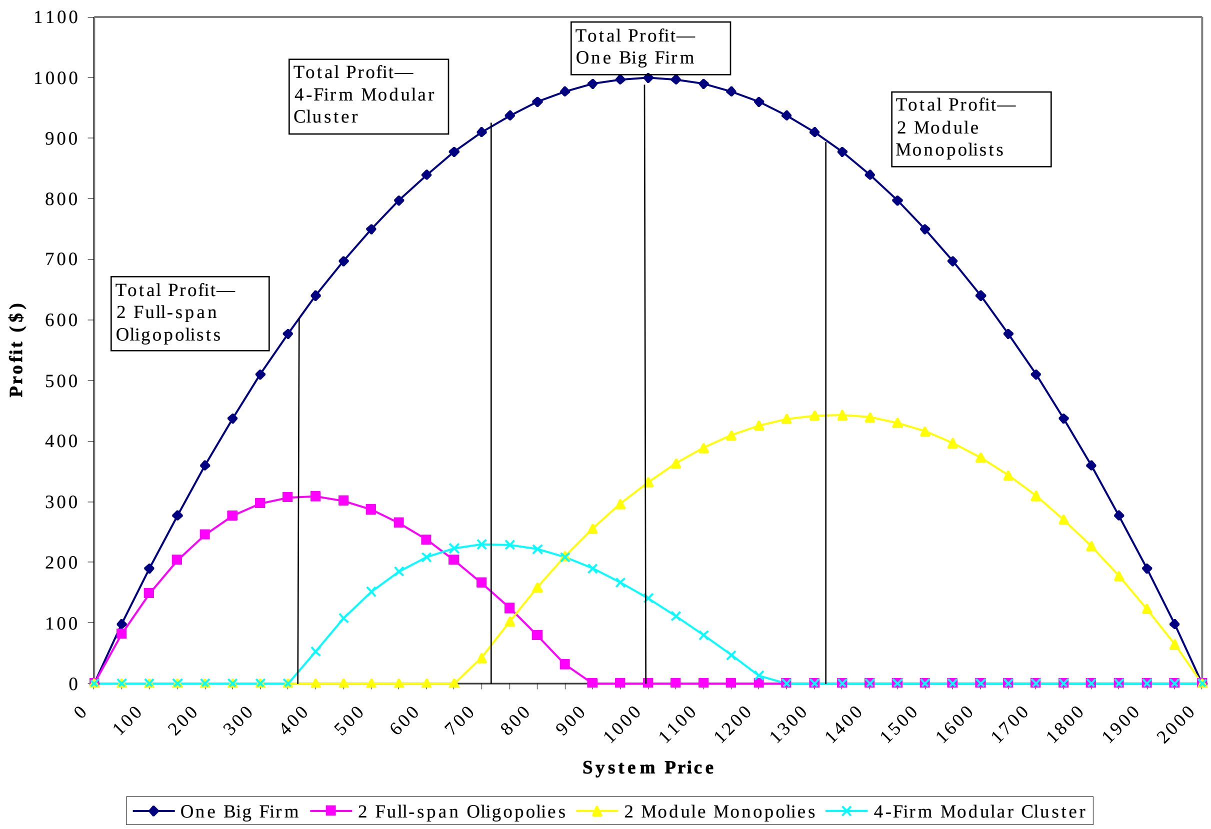
[11] This problem can be addressed via optimal two-part contracts, the focus of much recent investigation. However, optimal contracts are difficult to arrange in a large cluster with many modules and firms.

prices as fixed, and chooses a price that maximizes its own profit. In the figure, we have aligned the peak of each profit function with the *system price* implied by profit maximization. One can thus see that (1) the two full-span oligopolies set a system price that is lower than One Big Firm's system price; (2) the two module monopolies set a system price (the sum of their individual prices) that is higher than One Big Firm's price; and (3) the four modular cluster firms set a system price (the sum of two module prices) that is between the system prices of the full-span oligopolies and the module monopolies.

**Figure 1**
**Profit Maximization and System Prices under Four Canonical Configurations**




As it happens, in this example, the system price of the modular cluster is closer to the profit-maximizing system price than either of the system prices under the other two configurations. This means that, as long as a JxN modular cluster suffers no cost disadvantage relative to the other configurations, *it will be more profitable than N full-span oligopolies or J module monopolies*. Indeed, we shall see below that, in theory at least, some modular-cluster configurations may arrive at the same system price, hence be as profitable, as One Big Firm.

## 3.5 The Greater Computer Industry

A computer is a composite good that is made up of many subsidiary components. This has been true since the very first working computers were constructed in the mid-1940s. However, early in the artifact's history, the components of a computer were technologically very interdependent. Computer architectures were not modular, and as a result, the design and production of a working computer system had to be tightly controlled and technologically integrated. Computers were then perforce supplied by vertically integrated firms, whose activities spanned all of the components of computer design and production.

The architecture of computers changed in the decade between 1965 and 1975. The two key events marking the transition were (1) the introduction and extraordinary commercial success of IBM's System / 360, the "first modular computer design"; and (2) the publication of Gordon Bell and Allen Newell's book *Computer Structures*, which explained how to create a modular computer system. Thereafter, virtually all important computer designs were modular. As a result, subsets of components (the resources, tasks and decisions) of computer design and production became "loosely coupled" modules within the architecture of larger computer systems. Such modules could be designed and produced by firms that were not vertically integrated.

Indeed, between 1970 and 2000, several thousand firms entered the computer industry. Most of these entrants did not make whole computer systems, but specialized in supplying modules. In some cases, the modules they focused on were specific pieces of hardware or software—disk drives, microprocessors, memory chips, operating systems, application programs. In other cases, the modules were specific stages of the design or production process—chip fabrication, contract manufacturing, system integration and testing, tech support, system maintenance and servicing. But the history of the industry (at least through the year 2000) is one of ever-finer modular partitioning of the components that go into designing, making and selling computers. Thus, today in 2003, across the greater computer industry, there are tens thousands of *modules* that serve as the basis for products and transactions throughout the industry.

There are also many variants within each module-product category. For example, a quick search of the website of an Internet PC retailer revealed 80 different models of external hard drive, supplied by twelve manufacturers in the price range $140 to $250.[12] The drives differed in terms of features, appearance, terms of delivery, and the brand reputation of the manufacturer. Thus, in PC external hard drives, as in most product categories in the computer industry, there is scope for variety and competition between differentiated goods.

Like a vacation, then, a computer system is a composite good. Although consumers have some choice as to what to include in their systems, they must acquire a basic set of essential hardware and software components in order to have a functioning whole computer system. In addition, the demand for computers has historically been affected by the overall system price. Moreover, since the 1970s, computers have had modular architectures. Therefore, like the beach resort, the computer industry can in theory support any of the four canonical configurations of industrial organization described above.

That said, the greater computer industry is more complex than a beach resort. The greatest difference lies in the computer industry's higher rates of innovation and the resulting patterns of investment. Beach resorts do change and improve over time—there were few televisions and air-conditioners and no Internet connections at beach resorts fifty years ago. But most of the resorts that existed fifty years ago still exist. Indeed, some hotels, restaurants and other businesses at those resorts have been in continuous existence during that time period. In contrast, computers have changed almost beyond recognition over the same five decades.

To pursue the analogy, if the computer industry were a beach resort, some geological process would be raising up new beach frontage all the time. As the new beach emerged, new structures would be built on it. Older structures would rapidly lose their usefulness and be abandoned. At the same time, the "length" of the virtual beach would keep expanding, as the range and variety of product offerings increased.

In this paper, we will not attempt to address formally the dynamics of investment or innovation.

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[12] The search was conducted at: http://www.macconnection.com/scripts/searchresults.asp?SR=1&ER=40&TR=132&MarketID=92&ST=AS&plattype=P&Features=51646&sortval=Price on July 28, 2003.

However, as we noted in the introduction, rational investments, including investments in new products and processes, must be based on a realistic expectation that quasi-rents will be captured in the future. The existence and magnitude of quasi-rents, in turn, depends on industrial organization—specifically, the configuration of markets for both intermediate and final goods, and the nature of competition in those markets. If, as appears to be the case, the computer industry has a modular cluster configuration,[13] then the industry's aggregate quasi-rents, hence its prospective returns on investment, will be influenced by cluster-type pricing. A deeper understanding of pricing behavior in modular clusters, which is the purpose of this paper, may thus shed light on the future profitability of this industry.

# 4 A Formal Model of Pricing under Different Configurations

As indicated, the purpose of this paper is to try to better understand how equilibrium prices and aggregate profits in a modular cluster compare to prices and profits under the other three canonical configurations. In this section, we will construct a formal model that addresses this question. The model does not purport to capture the complexity of real pricing moves in a technologically dynamic environment. Its virtue lies in the fact that it draws attention to two basic forces at work in modular clusters and shows how these forces interact. The two forces highlighted by the model are (1) price competition amongst differentiated goods; and (2) pricing externalities amongst complementary goods.

The next three sections lay out the detailed assumptions of the model.

## 4.1 Technology

End users purchase a composite good, which, by definition, contains many subsidiary components. These components are technologically and operationally related via a architecture that specifies both the design of the composite good and its system of production. To facilitate the analysis of a range of symmetric configurations, we assume that the architecture contains a large number, $\Xi$, of

[13] Since the mid-1980s more than 50% of the market value of the industry has been accounted for by firms making modules, not whole computer systems. (Baldwin and Clark, 2000,) Most of these firms face differentiated product competition within their module product markets. Thus the greater computer industry appears to have the form of a modular cluster.

primitive components. The primitive components can be grouped into larger sets, called modules, but from a technological perpsective, no primitive component "needs" to be grouped with any other.

A *symmetric modular partition* is a set of symmetric modules that contains all the primitive components that make up the composite good. There are symmetric partitions made up of <1, 2, 3,… J, K, …> modules.

Every primitive component, hence every module, is essential to the composite good. None can be omitted. For convenience and simplicity, we assume that the variable cost of each primitive component is the same. Thus let $c_{\Xi}$ denote the cost per primitive component; we assume that: $c_{\Xi} = C/\Xi$. This implies that in a configuration with J modules, the cost per module is C/J, and the cost of the composite good is always C.

Primitive components, hence modules which are made up of primitive components, can vary within their categories. All variants cost the same amount. However, some consumers may have idiosyncratic preferences for one variant over another. The variants of the primitive components within a particular module can also be split up into different symmetric sets. These sets will correspond to the *product lines* of firms that compete in a given module market. We assume that the set of all variants of all primitive components, thus the union of all product lines of all firms is the same in all circumstances.

A *configuration* of the industry is a one-to-one mapping of module-level product lines to firms. Configurations are indexed by two numbers, JxN, where J is the number of modules in the configuration and N is the number of firms competing within each module market. For simplicity, we will consider symmetric modular partitions and symmetric product lines only. Given such symmetry, the canonical configurations, already discussed, are:

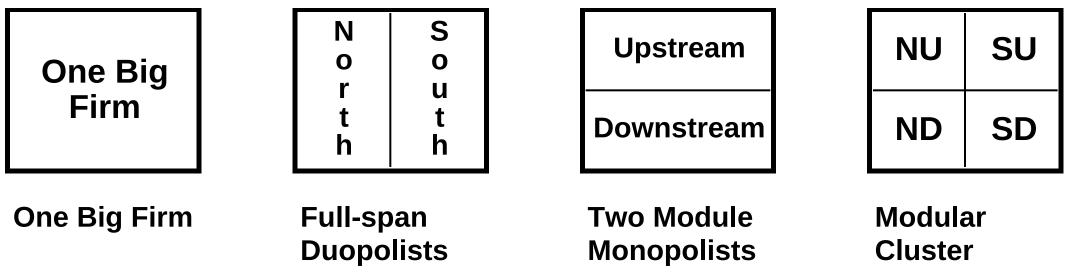
1x1 One Big Firm;
Jx1 J Module Monopolies;
1xN N Full-Span Oligopolies; and
JxN N Firms Competing in each of J Module Markets.

In a given configuration, each firm sets prices for its own module-product line, that is, for the variants of its module that it produces. We assume that firms cannot price discriminate, and all variants cost the same amount, thus each firm will set the same price for all of its variants.

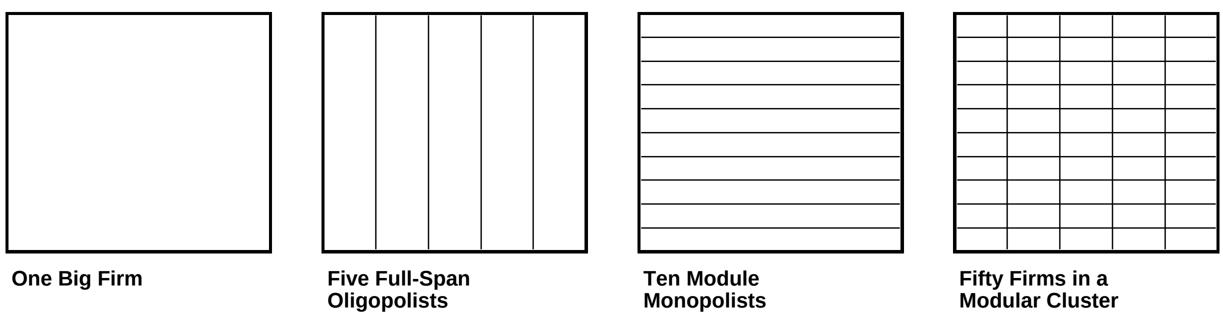
Different industry configurations can be visualized in terms of a checkerboard layout with varying numbers of rows, columns and squares. For example, Figure 2 shows the four canonical configurations for J= 2, N= 2 and J=10, N=5 respectively. Note that in the former case, there are four firms in the modular cluster; in the latter case there are fifty.

**Figure 2**
**The Four Canonical Configurations**

**J=2, N=2**




**J= 10, N= 5**




## 4.2 Demand and Pricing

The next set of assumptions, involving demand and prices, needs some motivation. The key input to a pricing model is a specification of the demand function: how will the quantities sold respond to the pricing decisions of a particular firm or group of firms? In this paper, we want to model demand in a way that captures the price interdependencies of a modular cluster, yet at the same time is understandable and tractable.

One way to characterize demand in a modular cluster is to specify all the "systems" that can be constructed by combining different modules. (For example, think of all the different experiences that can be constructed in one week at the beach resort or all the different personal computer systems one might buy.) Each of those systems is a unique product with a "system price" equal to the sum of the prices charged for each module. In principle, each of these unique products has a corresponding a demand function: the quantities purchased of unique system "A" will vary with its own price and the prices of the other unique systems, B, C, D, etc. And from the demand for each unique system, one can, in theory, obtain the *derived demand* for each variant of each module that appears in that unique system.

This was the modeling approach taken by Economides and Salop in their seminal paper in 1992. The problem is that as the number of modules, J, and variants, N, grows, the number of unique systems grows as $N^J$, in other words, it explodes. And to get the demand for the products of a single firm, one has to sum demands over the $(N-1)^J$ systems in which those products appear! Thus when the number of firms in a modular cluster is large, modeling the demand for unique systems and the derived demand for each firm's modules is analytically and computationally intractable.

After examining the unique system approach in detail, we elected to take a different route. Accordingly, we assume that a representative firm (the $i^{th}$) in a modular cluster perceives itself to have a share, $\alpha_i$, of its own module market. It also perceives a demand function for whole systems, denoted Q(P), which depends on the system price only. Both market share and system price are affected by the $i^{th}$ firm's pricing of its own product line. Specifically, the $i^{th}$ firm's own demand, $q_i$, has the following functional form:

$$q_i \; = \; \alpha_i(p_i\,;\, \ldots)\; Q[P(p_i\,;\, \ldots)] \quad . \tag{1}$$

In other words, demand for the $i^{th}$ firm's product equals (1) its share of its own module market; times (2) system demand. Both market share and system demand in turn are affected by the $i^{th}$ firm's prices.

However, market share and system demand are also affected by things the $i^{th}$ firm does not control: that is the significance of the "…" after the semicolon in each term. In particular, we will assume that the $i^{th}$ firm's market share is affected *by the pricing decisions of other firms in the same module market*, while the system price is affected *by the pricing decisions of firms making complementary modules*.

Consistent with classic Nash equilibrium assumptions, we assume that in setting its own prices, the $i^{th}$ firm takes all the other firms' prices as fixed. Assuming that all firms act in this way, we can then then solve for the equilibrium prices and profits using standard oligopolistic pricing methods. The virtue of this approach is that a representative firm's profit maximization problem has a general form under all configurations. Thus we can solve the profit maximization problem once for all configurations; obtain a general functional form; and using that functional form, then go on to compare equilibrium system prices and aggregate profits across different configurations.

Before proceeding, however, we need to characterize the functions α(.) and Q(.). In general, in any configuration, a representative firm will interact with firms making variants of its own module and firms making complements, that is, other modules. For a given firm, denoted by the subscript i, let firms making variants of its products be denoted by the subscript v, and let firms making complements of its products be denoted by the subscript j. In principle, α(.) and Q(.) may be functions of all prices charged by all firms:

$$\alpha_{i,JxN}(.) \quad = \quad \alpha_{i,JxN}(p_i \,;\, \mathbf{p}_v \,,\, \mathbf{p}_j) \quad ; \qquad (2a)$$

$$Q(.) \quad = \quad Q(p_i \,;\, \mathbf{p}_v \,,\, \mathbf{p}_j) \quad ; \qquad (2b)$$

where $p_i$ is the $i^{th}$ firm's own price and $\mathbf{p}_v$ and $\mathbf{p}_j$ are vectors denoting respectively the prices of all variants of its products and the prices of all complements of its products.

We assume that the functions α(.) and Q(.) as well as their first and second derivatives are continuous in all their arguments. This will allow us to apply standard tools of calculus to the $i^{th}$ firm's pricing problem.

The subscript, <i, JxN> attached to the function α(.) in equation (2a) indicates that the *market share function* of a representative firm depends on the configuration in which it is lodged. For example, if N=1, the representative firm will be a monopoly in its module market. Its market share will then equal one regardless of the price it chooses. Thus we are positing the existence of a family of market share functions, one for each configuration. Below, we will impose additional regularity assumptions on the members of this family.

### *The Market Share Family of Functions,* $\alpha_{i,JxN}(.)$

The function $\alpha_{i,JxN}(.)$ denotes the $i^{th}$ firm's share of its own module market under the configuration JxN. We assume that under any configuration:

$$\partial \alpha_i(p_i ; \ldots) / \partial p_i < 0 \; ; \tag{3a}$$

$$\partial \alpha_i(p_i ; \ldots, p_v, \ldots) / \partial p_v > 0 \; , \quad v \; ; \tag{3b}$$

$$\partial \alpha_i(p_i ; \ldots, p_v, \ldots) / \partial p_i \partial p_v \leq 0 \; , \quad v \; ; \tag{3c}$$

$$\partial \alpha_i(p_i ; \ldots, p_j, \ldots) / \partial p_j = 0 \; , \quad j \; . \tag{3d}$$

The first three expressions reflect the standard assumptions of oligopolistic price competition. Expression (3a) indicates that, *ceteris paribus*, Firm i's market share will decline if it raises its price; (3b) indicates that Firm i's market share will increase if any variant firm raises its price; and (3c) indicates that if both Firm i and a variant both raise their prices, then Firm i's market share will either stay the same or decline.

The fourth expression, (3d), reflects our simplified approach to cluster pricing. Here we assume that Firm i's *share* in its own module market is not affected by any change in the price of any complementary product. This crucial, simplifying assumption rules out "special complementarities" between variants in different modules. Special complementarities exist when Firm i's share of its own module market depends on the price charged by a *specific* complementor. For example, in the context of a beach resort, suppose a restaurant was located on the pier from which a particular boat ride departed. If the boat ride firm raised the price of its rides, then (we predict) fewer people would take them. Fewer people would then find themselves hanging around the restaurant on the pier, and the restaurant's sales and market share might suffer as a result. This is an example of a "special complementarity." Restaurants and boat rides are generic complements in the production system of the beach resort, but a particular restaurant and and a particular boat ride might be "special complements." Assumption (3d) rules out such special complementarities, not because they do not exist in reality, but because we think it makes sense to treat them as second-order phenomena in the context of this model.

Under any configuration, the market shares of all variants in a module market must sum to one:

$$\alpha_i + \Sigma \alpha_v = 1 \; . \tag{4}$$

Note that under symmetry in equilibrium the market shares of all firms in the same module market will be the same and equal to 1 / N (a configuration parameter):

$$\alpha_{i,JxN}* = \alpha_{v,JxN}* = 1/N \quad .$$

Here, the stars "*" denote values achieved in equilibrium.

### *The Total Demand Function, Q[P(.)]*

The function Q[P(.)] denotes the total quantity of all systems demanded as a function of the "system price," P. The system price, in turn, equals the sum of the prices of all the modules that make up the system.[14] Thus for the representative Firm i, we define the system price to be the sum of its own price and the average prices of its complements:

$$P(p_i) = p_i + \sum_{m-1}^{J-1} \left\{ \left[ \sum_{j=1}^{N} p_{j,m} \right] / N \right\} \quad .$$

Recall that the subscript j indicates firms making complementary modules. Let the complementary module markets be indexed by "m". In a JxN modular cluster, there are (J–1) complementary markets with N firms in each one. Hence this condition says that, for purposes of optimization, the representative firm calculates the system price as its own price plus the average price charged in each of its complementary module markets. It follows from this definition that:

$$\partial P / \partial p_i = 1 \quad . \qquad (5)$$

A dollar's increase in the representative firm's module price equates to a dollar's increase in the system price.

[14] It can be claimed that in reality there is no single system price: there are only derived prices for unique systems. Nevertheless the concept of a system price and of total system demand are useful abstractions that allow us to cut through the intractable problem of estimating a demand function for each unique system and the derived demand for each module.

We assume that an increase in the system price causes the demand for systems to decline. From this and equation (5):

$$\partial Q(P)/\partial P = \partial Q(P)/\partial p_i < 0 .$$

An increase in the price of the $i^{th}$ firm's module causes the demand for systems to decline commensurately.

### *Technical Assumptions*

Finally, we need to make three technical assumptions. First, we assume that, in all configurations, Firm i's revenue function has a single peak with respect to its own price:

$$\partial^2[p_i\,\alpha_i(p_i\,; \ldots)\;Q[P(p_i\,; \ldots)]/\partial p_i^{\,2} < 0 \text{ , in every configuration.}$$

This requirement is satisfied automatically if the functions $\alpha_i()$ and Q() are linear or concave in $p_i$. Many but not all convex functions and combinations of concave and convex functions satisfy this requirement as well. As is well known, if the revenue function has more than one peak, then the pricing strategies of individual firms become extremely complicated (they must choose between two or more local maxima). We will not be able to derive equilibrium prices for a cluster under those circumstances.

For purposes of the thought experiment, we also need to place restrictions on the family of market share functions. In the first place, suppose we fix the number of modules, J, and vary the number of firms in each module market, N. As we vary N, we do not want the market share functions of adjacent configurations to cross one another. If the functions do cross, that leads to an implicit contradiction: in the neighborhood of the crossing point, introducing a competitor (increasing N by one) can make the market share of the representative firm, hence all firms go up. This is mathematically possible, but economically improbable.

There are many mathematical ways to prevent the market share functions from crossing. We will make a simple assumption that is stronger than necessary. Specifically, we assume that the representative firm's market share function depends only on the average price of its variants, and that, holding the average price constant, the market share functions are parallel. More precisely, let $\alpha_{JxN}(.)$ and $\alpha_{JxM}(.)$ be the market share functions associated with the JxN and JxM configurations, respectively, and let $2 \leq M < N$. Also, let $p_v^{avg}$ denote the average price of variants in a given configuration. Then:

$$\alpha_{i,\,JxM}(p_i;\, p_v^{avg}) \;=\; \alpha_{i,\,JxN}(p_i;\, p_v^{avg}) \;+\; \Delta_{M,N} \quad . \qquad (6)$$

The difference, $\Delta_{M,N}$ depends on M and N, but not on prices.[15]

Next suppose we fix N, the number of firms in each module market, and vary the number of modules. As we move from one symmetric configuration to another (e.g., from JxN to KxN), we will in effect be repartitioning the module markets and reallocating the Ξ primitive components. In the real world these repartitionings would be constrained by the technological interdependencies and transactions costs: some configurations would be feasible and others would not. But for purposes of our thought experiment, we want to make all symmetric partitions both feasible and equivalent.

Equivalent in this context means that we do not want the market share functions to change simply because the modules are bigger or smaller. For example, consider a transition from a two-module to a three-module configuration (holding N fixed). If each module maker in the 3xN configuration charges 2/3 of the price charged by each module maker in the 2xN configuration (adding up to the same system price), we would like the market shares of the 3xN firms to be the same as the market shares of the 2xN firms in their respective module markets. In other words, the repartioning of modules and markets requires a reallocation of the system price (to three modules instead of two), but we want market shares functions to be conserved under this reallocation. Specifically, let JxN and KxN be two different configurations, and let $<p_i';\, p_v^{avg}>$ be a combination of own and other prices in the JxN configuration. We assume that:

$$\alpha_{i,\,JxN}(p_i';\, p_v^{avg}) \;=\; \alpha_{i,\,KxN}[(J/K)p_i';\, (J/K)\, p_v^{avg}] \quad . \qquad (7)$$

In effect, just as assumption (3-d) ruled out special complementarities across firms, assumption (7) rules out special complementarities within firms. Examples of special complementarities within firms

[15] Under this formulation it is possible for the market share functions to be negative for some combinations of prices. A more precise statement of the assumption is therefore:

$$\alpha_{i,\,JxM}(.) \;=\; \max[0,\, \alpha_{i,\,JxN}(.) + \Delta_{M,N}] \quad .$$

Also, note that under symmetry, the difference, $\Delta_{M,N}$, must equal $[1/M - 1/N]$. However, these technical details do not affect any of the proofs in our argument.

would be (1) umbrella brands, wherein some components derive market-share benefits from being in the same firm as other components; and (2) natural bundles, wherein consumers prefer to purchase a group of components in one transaction rather than separately. Umbrella brands and natural bundles do exist, but ruling them out greatly simplifies our analysis. (We will discuss the effects of relaxing this assumption and the previous one after we derive our main results.)

### 4.3 Profit Maximization, System Price and Aggregate Profit

Our goal is to compare equilibrium outcomes across different configurations. In so doing, we need to keep track of four items:

- the price charged by a representative firm;
- the system price;
- the profit obtained by a representative firm; and
- the aggregate profit obtained by all firms in the configuration.

In general we will use stars “ * ” to denote prices and profits that are obtained in equilibrium, and employ the subscripts, e.g., “JxN”, to associate equlbrium prices and profits with different configurations. We will use lower-case p and $\pi$ to refer to the prices and profits of representative firms, and upper-case P and $\Pi$ to refer to system prices and aggregate profits. Finally, if we need to index the individual firms in a specific configuration we will use a subscript such as <i, JxN> to mean the “a representative firm in a configuration involving J module markets with N firms in each one”.

Under the assumptions detailed above, the profit function of the $i^{th}$ firm can be written as:

$$\pi_i = p_i\, q_i - c_i\, q_i \qquad (8)$$
$$= p_i\, \alpha_i\, Q - c_i\, \alpha_i\, Q\ .$$

(To simplify notation, we have suppressed the arguments of $\alpha_i$ and Q.)

If, as we have assumed, the Firm i’s revenue function has a single peak in its own price, and its total costs are linear in the quantity demanded, then its profit function has a single global optimum. Furthermore, its optimal price can be obtained by taking the partial derivative of $\pi_i$ with respect to its own price, setting this marginal profit, $\partial\pi_i / \partial p_i$, equal to zero, and solving for the implied $p_i$*. Finally, if there are N firms competing in each of J module markets, the Nash equilibrium for this *symmetric modular cluster* can be obtained by solving the resulting system of NJ equations for the NJ unknown prices of all the firms in the cluster.

Proceeding along these lines, Firm i's marginal profit and the resulting first order condition is:

$$\frac{\partial \pi_i}{\partial p_i} = \alpha_i Q + (p_i^* - c_i)\left\{ Q \frac{\partial \alpha_i}{\partial p_i} + \alpha_i \frac{\partial Q}{\partial p_i} \right\} = 0 \qquad (9)$$

Solving for the optimal $p_i^*$, we obtain:

$$p_i^* = \frac{-\alpha_i Q}{\left\{ Q \frac{\partial \alpha_i}{\partial p_i} + \alpha_i \frac{\partial Q}{\partial p_i} \right\}} + c_i \quad ; \qquad (10)$$

Also, under symmetry, the following identities define "system price" and "aggregate profit" for a generic JxN configuration:

**System Price:** $$P_{JxN} = \sum_{i=1}^{J} p_{i,\,JxN} = J\, p_{JxN} \quad ; \qquad (11)$$

**Aggregate Profit:** $$\Pi_{JxN}^* = \sum_{i=1}^{JN} \pi_i(p_{JxN}^*) = J\, N\, \pi_{JxN}^* \quad . \qquad (12)$$

Note that the system price is a sum over modules while aggregate profit is a sum over firms.

Equations (8)–(12) will be our basic building blocks as we analyze and compare configurations.

# 5 Equilibrium Prices and Profits under Different Canonical Configurations

## 5.1 One Big Firm—1x1

We begin our analysis by looking at the optimal pricing behavior and profit of One Big Firm—the 1x1 canonical configuration. If one firm supplies and sets prices for all variants and modules in the market, then, we have said, its profit maximization problem can be framed in terms of the system price alone. This, by the way, does not mean that One Big Firm will set only a system price. For managerial purposes, it may also want to set prices for particular components or groups of components. It is free to do so as long as (1) all variants of in a given module category have the same price; and (2) the sum of component prices adds up to the profit-maximizing system price. (If every component is essential to the system, then each consumer will purchase one of each. Hence every consumer will end up paying the system price for his or her total package or bundle.)

The fact that there is only *One* Big Firm implies that $\alpha_i = 1$ and $\partial\alpha_i / \partial p_i = 0$ in this configuration. Substituting these values into Equation (9), and noting that $p_i = P$ and $c_i = C$, obtains the first order condition characterizing the 1x1 equilibrium:

$$\frac{\partial \pi_{1x1}}{\partial p_i} = Q(P_{1x1}^*) + [P_{1x1}^* - C] \frac{\partial Q(P_{1x1}^*)}{\partial P} = 0 \quad . \tag{13}$$

Intuitively, a monopolist sets a price that equates the marginal revenue from a price increase with the foregone profit on lost sales. Rearranging terms in equation (13) obtains an expression for the the equilibrium price:

$$P_{1x1}^* = \frac{-\ Q(P^*)}{\partial Q(P^*) / \partial P} + C \quad ; \tag{14}$$

In this expression, both the demand function, Q(.), and its partial derivative, $\partial Q(.) / \partial P$, are understood to be evaluated at the equilibrium price, $P_{1x1}^*$, which, in this configuration, is both a firm-level price and a system price. Because $\partial Q / \partial P < 0$ (the demand curve is downward sloping), the first term in this expression is positive. The monopoly maximizes profit by charging a price equal to its per-unit cost plus a markup. These are familiar results from elementary microeconomics.

The firm-level profit is also the aggregate profit of the 1x1 configuration. Of necessity, holding costs fixed, this is the highest aggregate profit that any configuration can obtain. As such, it provides a benchmark against which we can measure the profit performance of other configurations. Proposition 1 formalizes the idea of a benchmark and will be useful later on.

**Proposition 1.** Let A' and A" be two configurations with associated equilibrium system prices P' and P" and aggregate profits Π' and Π" respectively. If P" < P' < $P_{1x1}$* or P" > P' > $P_{1x1}$*, then Π" < Π' < $Π_{1x1}$*.

**Proof.** By assumption, aggregate profit is a continuous, single-peaked function of the system price. $P_{1x1}$* is the system price that maximizes aggregate profit. Thus, as the system price P approaches $P_{1x1}$* from above or below, aggregate profit increases and approaches the maximum, $Π_{1x1}$*. The proposition follows immediately. QED.

## 5.2 Module Monopolies—Jx1

Next we shall look at the equilibrium pricing and aggregate profit of a set of J "module monopolies." Recall that this configuration consists of J firms, each of which makes an essential component of the system, and each of which is a monopoly in it own module market. Module monopolies may supply components that users assemble into a system, for example, the meals and lodging on a vacation or the hardware and software on a computer. Alternatively, module monopolies may be linked as vertical stages of a production system (the so-called "chain of monopolies" analyzed by Spengler, 1950). Indeed our first task in this section is to demonstrate that, in our model as in others, these two seemingly different forms are equivalent:

**Lemma.** Under the assumptions of our model, in a configuration with at least two modules (J≥2), equilibrium module prices, hence the system price and aggregate profit, are the same whether the modular elements constitute successive stages of a production process or are separate goods purchased and assembled by end-users.

**Proof.** See the Appendix. The strategy of proof is to impose a change of variables on the profit functions of firms in the successive-stages regime, and show that under this change, the profit functions of the two regimes are identical. The proof makes use of assumption (3d), that, in each module market, market share functions are not affected by the prices of complementary modules.

Thus, as we indicated earlier in the paper, configurations with two or more modules may embody to two, quite different, technological regimes: (1) a disaggregated supply chain or (2) a set of complementary module suppliers. Indeed, a given configuration may contain both supply-chain elements and complementary-goods elements. From the standpoint of price setting, the two technologies are the equivalent.

We now turn to the question of how prices will be set in the Jx1 canonical configuration. For a representative module monopoly, as for One Big Firm, $\alpha_i = 1$ and $\partial\alpha_i / \partial p_i = 0$. Standard methods obtain the first order condition characterizing the equilibrium:

$$\frac{\partial\pi_{Jx1}}{\partial p_i} = Q(P_{Jx1}^*) + \left( p_{Jx1}^* - \frac{C}{J} \right) \frac{\partial Q(P_{Jx1}^*)}{\partial P} = 0 \quad . \tag{15}$$

This expression makes use of the fact that $\partial P / \partial p_i = 1$.

For each firm in the set of module monopolies, equation (15) defines the peak of that firm's profit function, holding the prices of all other modules fixed. Equilibrium is obtained at the unique point where the equation holds for all firms in the set. However, we are interested in how the individual firm's chosen prices and the resulting system price change as the number of firms changes.

**Proposition 2.** Consider two module monopoly configurations, Jx1 and Kx1, where $1 \le J < K$. Under the assumptions of this model, $P_{Jx1}^* < P_{Kx1}^*$. In other words, the equilibrium system price is strictly *increasing* in the number of module monopolies.

**Proof.** The details of the proof are in the Appendix. The strategy and intuition of the proof are as follows.[16] We evaluate the Jx1 marginal profit function at the point $p_i = (K/J)p_{Kx1}^*$. The significance of this point is that if all firms in the Jx1 configuration were to charge this price, the Jx1 system price would then equal the Kx1 *equilibrium* system price. We show that, in the Jx1 configuration, the representative firm's marginal profit, $\partial\pi_{Jx1} / \partial p_i$, is *negative* at this point. Thus individual firm prices consistent with the Kx1 equilibrium price *lie on the downward slope* of the Jx1 representative firm's profit function. Since the profit function is (by assumption) single-peaked, the Kx1 equilibrium system price must be higher than the Jx1 equilbrium system price.

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[16] This strategy is an adaptation of methods used in Baumol (2002), Chapter 7.

**Proposition 3.** Consider two module monopoly configurations, Jx1 and Kx1, where $1 \leq J < K$. Under the assumptions of this model, $\Pi_{Jx1}^* > \Pi_{Kx1}^*$. In other words, aggregate profit is strictly decreasing in the number of module monopolies.

**Proof.** By Proposition 2, $P_{Kx1}^* > P_{Jx1}^* > P_{1x1}^*$. Proposition 1 then implies that $\Pi_{Kx1}^* < \Pi_{Jx1}^* < \Pi_{1x1}^*$. QED.

### 5.3 Differentiated, Full-span Oligopolies—1xN

We now turn to the third canonical configuration: differentiated, "full-span" oligopolies. In this case, we assume, there are N firms, each of which spans the full set of components, but makes only a fraction, $\alpha_{1xN}$, of the variants of each one. Each firm views its own market share as somewhat sensitive to its own pricing decisions, but takes the other firms' prices as fixed. Under these assumptions, the first-order condition and optimal price for the representative firm are given by equations (9) and (10) respectively. However, because each firm is a "full-span" oligopoly, the prices set by the individual firms are system prices (not module prices).

**Proposition 4.** Consider two differentiated, full-span oligopoly configurations, 1xN and 1xM, where $1 \leq M < N$. Under the assumptions of this model, $P_{1xM}^* > P_{1xN}^*$. In other words, the system price is strictly *decreasing* in the number of full-span oligopolies.

**Proof.** See the Appendix. The strategy of proof is the same as in Proposition 2. The proof makes use of the technical assumption that the market share functions are parallel.

**Proposition 5.** Consider two full-span oligopoly configurations, 1xM and 1xN, where $1 \leq M < N$. Under the assumptions of this model, $\Pi_{1xM}^* > \Pi_{1xN}^*$. In other words, aggregate profit is strictly decreasing in the number of full-span oligopolies.

**Proof.** By Proposition 4, $P_{1xN}^* > P_{1xM}^* > P_{1x1}^*$. Proposition 1 then implies that $\Pi_{1xN}^* < \Pi_{1xM}^* < \Pi_{1x1}^*$. QED.

Before proceeding to look at equilbrium prices and aggregate profit in a modular cluster, let us pause to take stock of these results. We can think of individual configurations being mapped into the cells of a spreadsheet, as shown in Figure 3. As we move out along any row in this array, the number of firms in each module market increases. Symmetrically, as we move down any column, the number of modules and module markets increases.

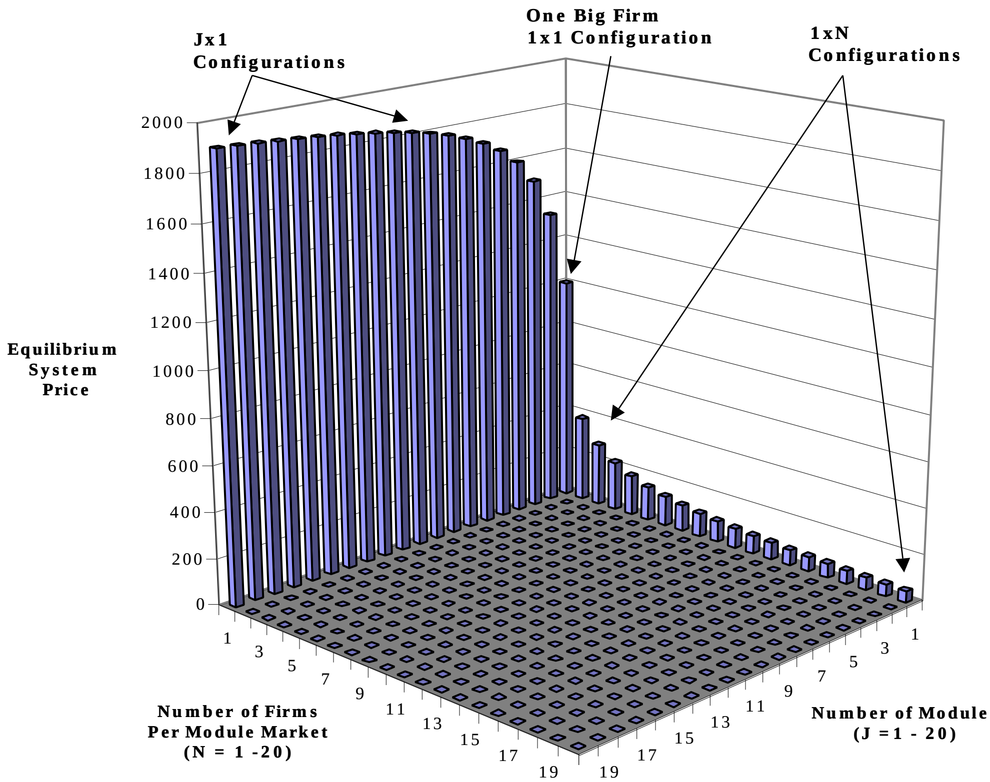
**Figure 3**
**The Array of Configurations**

**More Firms in Each Market**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1x1 | 1x2 | 1x3 | 1x4 | … | 1xM | 1xN | … |
| More | 2x1 | 2x2 | 2x3 | 2x4 | … | 2xM | 2xN | |
| Module | 3x1 | 3x2 | 3x3 | 3x4 | … | 3xM | 3xN | |
| Markets | 4x1 | 4x2 | 4x3 | 4x4 | … | 4xM | 4xN | |
| | … | | | | | | | |
| | Jx1 | Jx2 | Jx3 | Jx4 | … | JxM | JxN | |
| | Kx1 | Kx2 | Kx3 | Kx4 | … | KxM | KxN | |
| | … | | | | | | | … |

Although this array may appear superficially like the "checkerboards" in Figure 2, it represents something quite different. Each cell in Figure 3 corresponds to *a whole board* in Figure 2. Thus as we move from cell to cell in Figure 3, we are moving from one "checkerboard" pattern to another. Each configuration, therefore each cell in Figure 3, has an associated equilibrium system price, and aggregate profit. Our objective is to map out the full array. Propositions 1 through 5 have given us the results we seek for the first row and the first column. From these propositions, we know that the lowest system prices are associated with the *top right corner* of the array, e.g., the 1xN cell. System prices increase as we move leftward along the row to the 1x1 cell, and they continue to increase as we turn the corner and move down the column. The highest system prices are associated with the lower left corner, e.g., the Kx1 cell. Figure 4 illustrates this pattern.

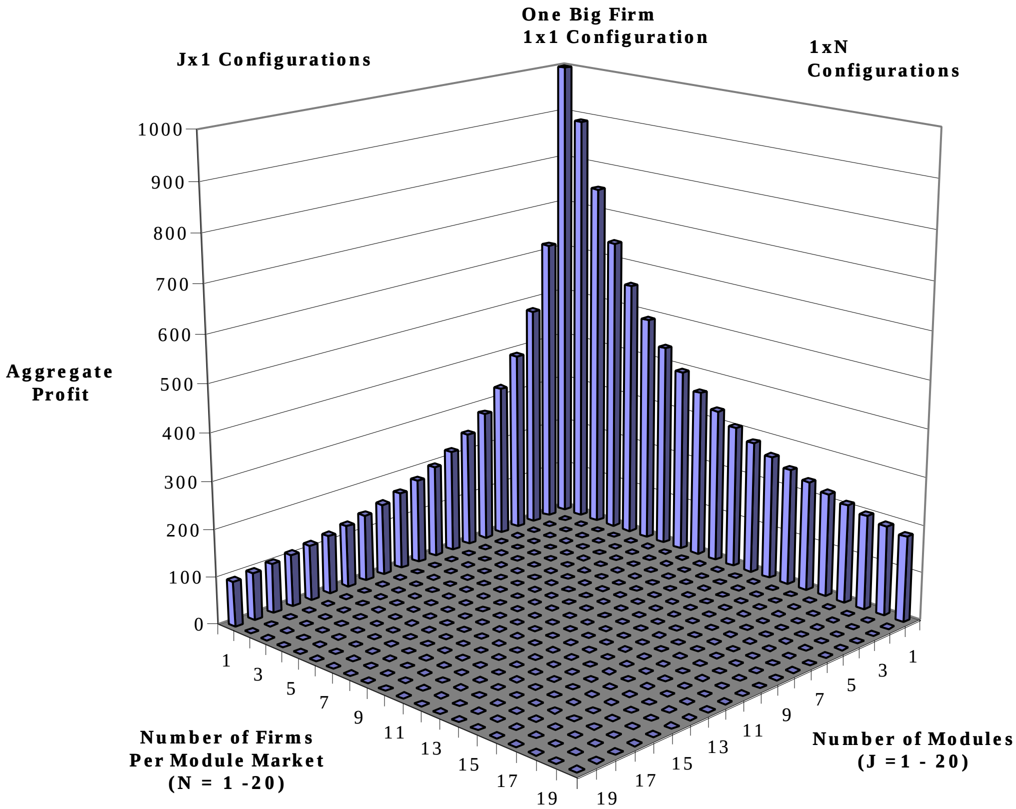
**Figure 4**
**Equilibrium System Prices For Various Configurations**




The highest aggregate profit is associated with *top left corner*: the 1x1 configuration consisting of One Big Firm. Then, as shown in Figure 5, aggregate profits go down, monotonically, as we move rightward across the top row or down the first column. (Figures 4 and 5 were generated using the results of a specific numerical example, which we will describe later in the paper.)

In order to fill in the rest of the array, we must consider pricing within and across modular clusters.

**Figure 5**
**Aggregate Profit for Various Configurations**




## 5.4 Modular Clusters—JxN

In a JxN modular cluster, there are J complementary modules and N firms competing in each module market. Each firm has a fractional market share, $\alpha_{i,\,JxN}$, of its own module market. It views its market share as somewhat, but not totally sensitive to its own price, and takes the prices set by other firms in its own module market as given. Conversely, each firm supplies only a part of a whole system. Each perceives that the demand for whole systems is sensitive to the system price, which is the sum of module prices. But, as we discussed above, there is a pricing externality: a price drop by one module maker increases the demand, hence profit of the other module makers. For this reason, module makers do not have the same incentives to decrease prices that full-span oligopolies do.

Again, the first-order condition and optimal price for the representative firm are given by equations (9) and (10). The main results of our analysis are in the following proposition.

**Proposition 6. (a)** Let the number of firms in each module market be N. Consider two modular cluster configurations, JxN and KxN, where 1≤ J < K. Under the assumptions of this model, $P_{JxN}* < P_{KxN}*$. In other words, for any N, prices are strictly increasing in the number of module makers.

**(b)** Let the number of module makers be J. Consider two modular cluster configurations, JxN and JxM, where 1 ≤ M ≤ N. Under the assumptions of this model, $P_{JxM}* > P_{JxN}*$. In other words, for any J, prices are strictly decreasing in the number of firms competing in each module market.

**Proof.** See the Appendix. The proofs of parts (a) and (b) correspond exactly to the proofs of Propositions 2 and 4, although the algebra is slightly more complex.

In terms of the array of configurations in Figure 3, Proposition 6 says that system prices will increase down every column and decrease across every row. What then happens to aggregate profit? Let us think about starting in the first column, in the Jx1 cell. By Proposition 2, the equilibrium system price in this configuration is higher than the system price charged by One Big Firm. But as we move across the row to configurations Jx2, Jx3, Jx4, etc., according to Proposition 6(a), the equilibrium system price will fall. Within the row, *aggregate profit will be highest in the configuration whose equilibrium system price turns out to be closest to the system price charged by One Big Firm*.[17] It is likely that the highest profit configuration will not be the Jx1 configuration, but will be one of the cluster configurations, Jx2, Jx3 … .

The same reasoning applies if we start in the first row, in the 1xN cell, and move down a column. By Proposition 4, the equilibrium system price in the 1xN configuration is lower than the system price charged by One Big Firm. But as we move down the column, by Proposition 6(b), the equilibrium system price will rise. The highest aggregate profit accrues to the configuration which most closely approximates the pricing behavior of One Big Firm. This in turn is likely to be one of the cluster configurations, 2xN, 3xN, … .

Note that in any row or column, we do not know which configuration will be most profitable.

[17] This statement is not precisely true because the One Big Firm profit function may not be symmetric to the left and the right of its highest point. More precisely stated, aggregate profit will be highest in one of the two configurations whose system prices bracket the equilibrium system price of One Big Firm.

Indeed, we cannot *absolutely* guarantee that the Jx1 or the 1xN configuration will not be the most profitable for some choice of demand and market share parameters. However, as J increases or N increases, Propositions 2 and 4 state that the prices set by module monopolies or full-span oligopolies will tend to move farther away (in opposite directions) from One Big Firm's optimal price.

In that context, bringing competition to the module monopolies tends to move those system prices down toward One Big Firm's system price. Conversely, modularizing the full-span oligopolies tends to move their system prices up toward One Big Firm's system price. In this fashion, modular clusters can arrive at system prices closer to One Big Firm's system price than the corresponding module monopolies or the full-span oligopolies. As a result, clusters can be more profitable in aggregate than either module monopolies or full-span oligopolies. This is especially true if J is large or N is large, for then the module monopolies and the full-span oligopolies will arrive at a system prices that are far away from One Big Firm's optimal system price. Moreover, as we will show in the next section, it is possible for some modular clusters to get very close to the system price hence the profitability of One Big Firm.

In any case, the fact that system prices decrease across each row, and increase down each column implies that the profit "landscape" of the array has a single profit peak in each row or column. If the technical assumptions governing the α(.) functions do not hold, then system prices may not fall strictly down each row, nor rise strictly up each column. In such cases, the profit landscape will be "rugged," in the sense of having many local peaks.

## 5.5 Module Pricing and Product Bundles

In our analysis thus far, we have assumed that each firm in a modular cluster will set the price of "its module," as denoted by the symbol $p_{i,JxN}$. It is easy to leap from that assumption to the view that firms must sell "their" modules as indivisible bundles. However, from both a theoretical and a practical perspective, that is not the case. Practically speaking, as we said in the introduction, a large module that comprises all the activities of a single firm can be "opened up," and many technologically separable, smaller modules may be found within the larger one. Thus a single beach resort firm can own a hotel, a restaurants and a taxi service; a single computer company can make make printers, laptop computers, and application software. These technologically separable activities can and usually do carry separate prices, and consumers can and usually do mix and match product offerings from different vendors. Even in a supply chain setting, a manufacturer can control some stages of the overall production process and outsource others.

For analytic purposes, until our main results were in hand, it was easier to treat firm-level modules as bundles that carried a single "module price" $p_{i,JxN}$. Each representative firm in a JxN modular cluster could then be said to set a single price, subject to the prices established by rivals and complementors. In this way, we were able to avoid the significant complexity of modeling multi-product firms. But we introduced a presumption that a firm would have to bundles of *all* its primitive components into a single product. This bundling assumption, although analytically convenient, is not only counterfactual, but is overly restrictive and unnecessary in the context of our model.

To see this, first recall that each firm in a JxN modular cluster spans (or contains) $\Xi / J$ primitive components. Hereafter, we shall call the number of primitive components within a firm, that firm's "modular span." Note that modular span is the inverse of the number of modules in a given configuration.

The module price selected by a firm in the context of our model can be viewed as a *price per primitive component,* $p_{\Xi}$, times the firm's modular span:

$$p_{i,JxN} \equiv (\Xi / J)\, p_{\Xi,\, JxN}$$

On this view, when a firm optimizes its "own price," or the "price of its module," it is at the same time optimizing its price per primitive component, $p_{\Xi,\, JxN}$ in the context of the cluster. Having done that, the firm can assemble products by aggregating primitive components and can set product prices by adding up primitive-component prices. With this mechanism in place, the firms' product-bundles do not have to match their modular spans. Within a modular cluster, there can be multi-product firms, and purchasers can then mix and match different firms' *product offerings* as they wish.

# 6 An Illustrative Example

We can gain additional insight by specializing the functional forms of both the system demand function and the market share functions. Given specific functional forms, we can obtain numerical solutions for the system price and aggregate profit corresponding to each cell in the array of configurations. Those explicit solutions will allow us to "view" the profit landscape. Then, by varying parameters of the functional forms, we can see how the landscape changes.

## 6.1 Assumptions

For simplicity, we will assume that the system demand and market share functions are linear. Also, without loss of generality, we will set the cost of each module to zero.[18] Specifically, we assume:

$$Q(P) = A - bP$$

This implies that:

$$\partial Q / \partial P = -b \quad .$$

We also assume:

$$\alpha_{i,\,J\times N}(p_i;\, p_v^{avg}) = 1 \qquad \text{if } N = 1 \quad ;$$

$$= \frac{1}{N} + \frac{s \sum p_v}{N-1} - s\, p_i \qquad \text{if } N > 1 \quad .$$

Recall that there are N–1 variants of the representative firm's product line in its own module market. Thus equation (19) implies that the market share of the representative firm is a linear function of its own price and the average prices of all its variants. This family of market share functions always sum to one when summed over N; they are parallel with respect to changes in N; and they conserve market share across module reallocations. In addition:

$$\partial \alpha_{i,\,J\times N}(p_i;\, p_v^{avg}) / \partial p_i = 0 \qquad \text{if } N = 1 \quad ;$$

$$= -s \qquad \text{if } N > 1 \quad .$$

In equilibrium, under symmetry, each firm charges the same price. Thus in equilibrium, the market shares of all firms will be the same and equal to 1 / N; and the system price will equal J times the individual module prices:

[18] Under our simple cost assumptions, optimal prices equal cost plus a markup. Thus if costs are greater than zero, equilibrium prices will be simply the prices we compute plus the cost, and aggregate profits will be the profits we compute.

$\alpha_{JxN}* = 1 / N$ ;

$P_{JxN}* = Jp_{JxN}*$ .

Substituting these assumptions into equation (9), the first order condition of the representative firm in a JxN configuration is:

$$\frac{\partial \pi_{JxN}*}{\partial p_i} = A - bJp_{Jx1}* + p_{Jx1}*(-b) = 0 \qquad \text{if } N = 1 \;;$$

$$= (1/N)(A - bJp_{JxN}*) + p_{JxN}*[(A - bJp_{JxN}*)(-s) + (1/N)(-b)] = 0 \qquad \text{if } N > 1 \;.$$

As always, stars " * " denote values at the corresponding equilibrium. We can solve these expressions to obtain equilibrium module prices for N=1 and N>1, respectively. The module prices imply the following equilibrium system prices:

$$P_{JxN}* = \frac{JA}{(J+1)b} \qquad \text{if } N = 1 \;; \qquad (17a)$$

$$= \frac{[(1/N)(J+1)b + sA] - \{[(1/N)(J+1)b + sA]^2 - 4(sbJ)(1/N)A\}^{1/2}}{2(sb)} \qquad \text{if } N > 1 \;. \qquad (17b)$$

## 6.2 Results

From equations (17a) and (17b), system demand and aggregate profit can be computed for any configuration. And by varying the parameters, A, b and s, we can vary the system demand function and the price sensitivity of customers within the module markets. To draw a graph, however, we need to make specific assumptions about A, b and s. To that end, we assume that:

A=2,000,000; b=1,000; and s=.001 .

Under these numerical assumptions, total demand can range from 0 to 2,000,000 units (per year); system prices may range from \$0 to \$2,000; and for every \$1 increase in the price of a module, the module maker's market share declines by .1%. (**Note:** These were the assumptions used to generate Figures 1, 4, and 5.)

Table 1 shows the results for J = 1, 2, … 20 and N = 1, 2, … 20.

**Table 1**
**System Price and Aggregate Profit for Various Configurations**
**J = 1, 2, … 20, N = 1, 2, …, 20**

**Linear Demand, Zero Marginal Cost**

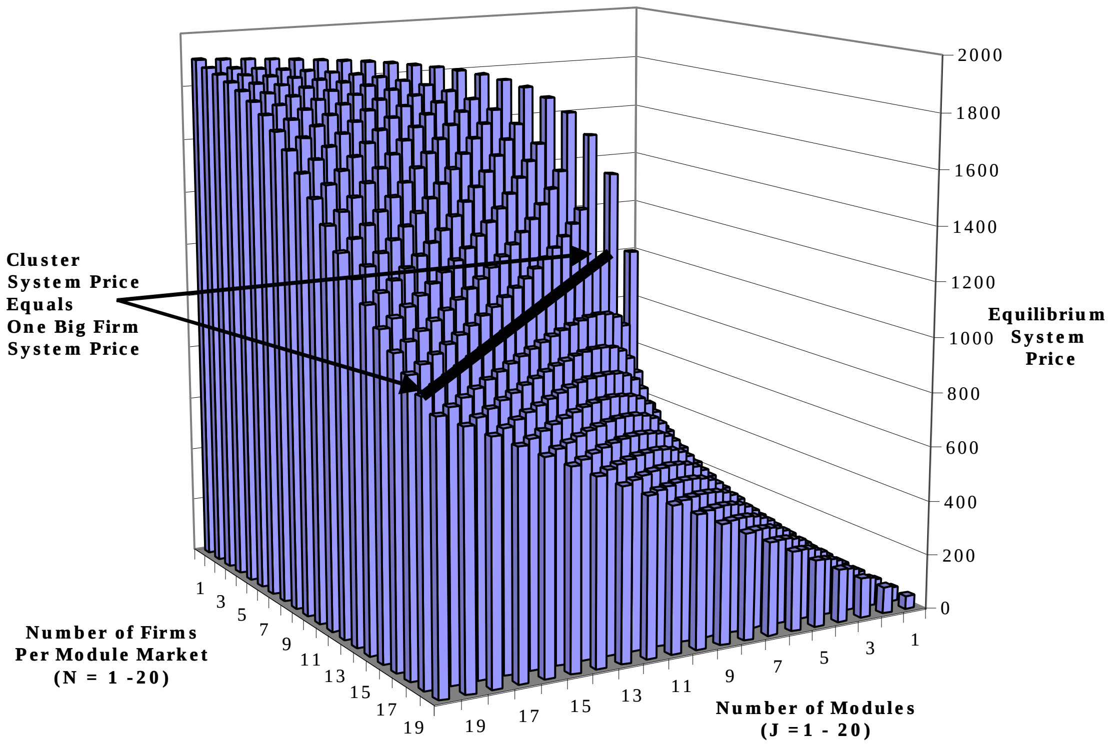
| **Assumptions** | | |
|---|---|---|
| Demand Intercept | A | 2,000,000 |
| Demand Slope | b | 1000 |
| Module Market Sensitivity | s | 0.001 |

| System Price | | Firms in Each Module Market (N) | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| | **1** | **1000** | 382 | 279 | 219 | 180 | 153 | 133 | 117 | 105 | 95 | 87 | 80 | 74 | 69 | 64 | 61 | 57 | 54 | 51 | 49 |
| | **2** | 1333 | 719 | 543 | 431 | 357 | 304 | 264 | 233 | 209 | 190 | 173 | 159 | 148 | 138 | 129 | 121 | 114 | 108 | 102 | 97 |
| | **3** | 1500 | **1000** | 785 | 634 | 528 | 451 | 394 | 349 | 313 | 283 | 259 | 239 | 221 | 206 | 193 | 181 | 171 | 162 | 154 | 146 |
| | **4** | 1600 | 1219 | **1000** | 825 | 694 | 596 | 521 | 462 | 415 | 377 | 345 | 318 | 294 | 274 | 257 | 241 | 228 | 216 | 205 | 195 |
| | **5** | 1667 | 1382 | 1184 | **1000** | 852 | 736 | 646 | 575 | 517 | 469 | 430 | 396 | 367 | 342 | 321 | 301 | 284 | 269 | 255 | 243 |
| Modules (J) | **6** | 1714 | 1500 | 1333 | 1157 | **1000** | 871 | 768 | 685 | 617 | 561 | 514 | 474 | 440 | 410 | 384 | 361 | 341 | 323 | 306 | 291 |
| | **7** | 1750 | 1586 | 1451 | 1293 | 1137 | **1000** | 886 | 793 | 716 | 652 | 598 | 552 | 512 | 478 | 447 | 421 | 397 | 376 | 357 | 340 |
| | **8** | 1778 | 1649 | 1543 | 1407 | 1260 | 1121 | **1000** | 898 | 813 | 741 | 680 | 628 | 584 | 545 | 510 | 480 | 453 | 429 | 408 | 388 |
| | **9** | 1800 | 1697 | 1613 | 1500 | 1368 | 1232 | 1108 | **1000** | 908 | 829 | 762 | 705 | 655 | 611 | 573 | 539 | 509 | 482 | 458 | 436 |
| | **10** | 1818 | 1734 | 1667 | 1575 | 1460 | 1333 | 1210 | 1098 | **1000** | 916 | 843 | 780 | 725 | 678 | 636 | 598 | 565 | 535 | 508 | 484 |
| | **11** | 1833 | 1764 | 1709 | 1634 | 1537 | 1423 | 1304 | 1191 | 1089 | **1000** | 922 | 855 | 795 | 743 | 698 | 657 | 621 | 588 | 559 | 532 |
| | **12** | 1846 | 1788 | 1743 | 1681 | 1600 | 1500 | 1389 | 1278 | 1175 | 1082 | **1000** | 928 | 865 | 809 | 759 | 715 | 676 | 640 | 609 | 580 |
| | **13** | 1857 | 1807 | 1770 | 1719 | 1652 | 1566 | 1465 | 1360 | 1257 | 1161 | 1076 | **1000** | 933 | 873 | 820 | 773 | 731 | 693 | 658 | 627 |
| | **14** | 1867 | 1824 | 1792 | 1750 | 1694 | 1621 | 1532 | 1434 | 1333 | 1238 | 1150 | 1071 | **1000** | 937 | 881 | 831 | 785 | 745 | 708 | 675 |
| | **15** | 1875 | 1838 | 1811 | 1775 | 1728 | 1667 | 1590 | 1500 | 1405 | 1310 | 1221 | 1140 | 1066 | **1000** | 941 | 888 | 840 | 797 | 757 | 722 |
| | **16** | 1882 | 1850 | 1826 | 1796 | 1757 | 1705 | 1638 | 1559 | 1470 | 1378 | 1290 | 1207 | 1131 | 1062 | **1000** | 944 | 894 | 848 | 807 | 769 |
| | **17** | 1889 | 1860 | 1840 | 1814 | 1780 | 1736 | 1680 | 1610 | 1529 | 1442 | 1355 | 1271 | 1194 | 1123 | 1058 | **1000** | 947 | 899 | 855 | 816 |
| | **18** | 1895 | 1869 | 1851 | 1829 | 1800 | 1763 | 1714 | 1653 | 1581 | 1500 | 1416 | 1333 | 1255 | 1182 | 1116 | 1055 | **1000** | 950 | 904 | 862 |
| | **19** | 1900 | 1877 | 1861 | 1842 | 1817 | 1785 | 1743 | 1691 | 1627 | 1553 | 1473 | 1392 | 1314 | 1240 | 1172 | 1110 | 1052 | **1000** | 952 | 908 |
| | **20** | 1905 | 1884 | 1870 | 1853 | 1831 | 1804 | 1768 | 1723 | 1667 | 1600 | 1526 | 1448 | 1371 | 1297 | 1227 | 1163 | 1104 | 1050 | **1000** | 954 |

| Aggregate Profit $l | | Firms in Each Module Market (N) | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| | **1** | **1000** | 618 | 481 | 390 | 328 | 282 | 248 | 221 | 199 | 181 | 166 | 153 | 142 | 133 | 125 | 117 | 111 | 105 | 100 | 95 |
| | **2** | 889 | 921 | 791 | 677 | 586 | 515 | 458 | 412 | 375 | 343 | 316 | 293 | 274 | 256 | 241 | 227 | 215 | 204 | 194 | 185 |
| | **3** | 750 | **1000** | 954 | 866 | 777 | 699 | 632 | 576 | 528 | 487 | 451 | 420 | 393 | 370 | 349 | 330 | 313 | 297 | 283 | 271 |
| | **4** | 640 | 952 | **1000** | 969 | 906 | 837 | 771 | 711 | 658 | 612 | 571 | 534 | 502 | 473 | 448 | 425 | 404 | 385 | 367 | 351 |
| | **5** | 556 | 854 | 966 | **1000** | 978 | 930 | 875 | 819 | 767 | 718 | 675 | 635 | 600 | 568 | 538 | 512 | 488 | 466 | 446 | 427 |
| Modules (J) | **6** | 490 | 750 | 889 | 975 | **1000** | 983 | 946 | 901 | 853 | 807 | 764 | 723 | 686 | 652 | 621 | 592 | 566 | 541 | 519 | 498 |
| | **7** | 438 | 657 | 796 | 914 | 981 | **1000** | 987 | 957 | 919 | 879 | 838 | 799 | 762 | 727 | 695 | 665 | 637 | 611 | 587 | 564 |
| | **8** | 395 | 579 | 706 | 834 | 933 | 985 | **1000** | 990 | 965 | 933 | 898 | 862 | 827 | 793 | 760 | 730 | 701 | 674 | 649 | 625 |
| | **9** | 360 | 514 | 625 | 750 | 865 | 946 | 988 | **1000** | 991 | 971 | 943 | 913 | 881 | 849 | 818 | 788 | 759 | 732 | 706 | 682 |
| | **10** | 331 | 461 | 556 | 670 | 789 | 889 | 956 | 990 | **1000** | 993 | 975 | 952 | 925 | 896 | 867 | 839 | 811 | 784 | 758 | 734 |
| | **11** | 306 | 416 | 497 | 598 | 712 | 821 | 908 | 964 | 992 | **1000** | 994 | 979 | 958 | 934 | 909 | 882 | 856 | 830 | 805 | 781 |
| | **12** | 284 | 379 | 448 | 536 | 640 | 750 | 848 | 922 | 969 | 993 | **1000** | 995 | 982 | 963 | 942 | 919 | 895 | 871 | 847 | 823 |
| | **13** | 265 | 348 | 407 | 483 | 575 | 680 | 783 | 871 | 934 | 974 | 994 | **1000** | 995 | 984 | 968 | 949 | 928 | 906 | 883 | 861 |
| | **14** | 249 | 321 | 372 | 438 | 519 | 615 | 717 | 812 | 889 | 944 | 978 | 995 | **1000** | 996 | 986 | 971 | 954 | 935 | 915 | 894 |
| | **15** | 234 | 298 | 343 | 399 | 470 | 556 | 652 | 750 | 836 | 904 | 951 | 981 | 996 | **1000** | 996 | 987 | 974 | 959 | 941 | 923 |
| | **16** | 221 | 278 | 317 | 366 | 428 | 503 | 592 | 688 | 779 | 857 | 916 | 957 | 983 | 996 | **1000** | 997 | 989 | 977 | 963 | 947 |
| | **17** | 210 | 260 | 295 | 338 | 391 | 458 | 538 | 628 | 721 | 805 | 874 | 926 | 962 | 985 | 997 | **1000** | 997 | 990 | 979 | 966 |
| | **18** | 199 | 245 | 275 | 313 | 360 | 418 | 490 | 573 | 663 | 750 | 827 | 889 | 935 | 967 | 987 | 997 | **1000** | 997 | 991 | 981 |
| | **19** | 190 | 231 | 258 | 292 | 333 | 384 | 447 | 523 | 607 | 694 | 776 | 846 | 901 | 942 | 970 | 988 | 997 | **1000** | 998 | 992 |
| | **20** | 181 | 219 | 243 | 273 | 309 | 354 | 410 | 477 | 556 | 640 | 724 | 799 | 862 | 912 | 948 | 973 | 989 | 998 | **1000** | 998 |

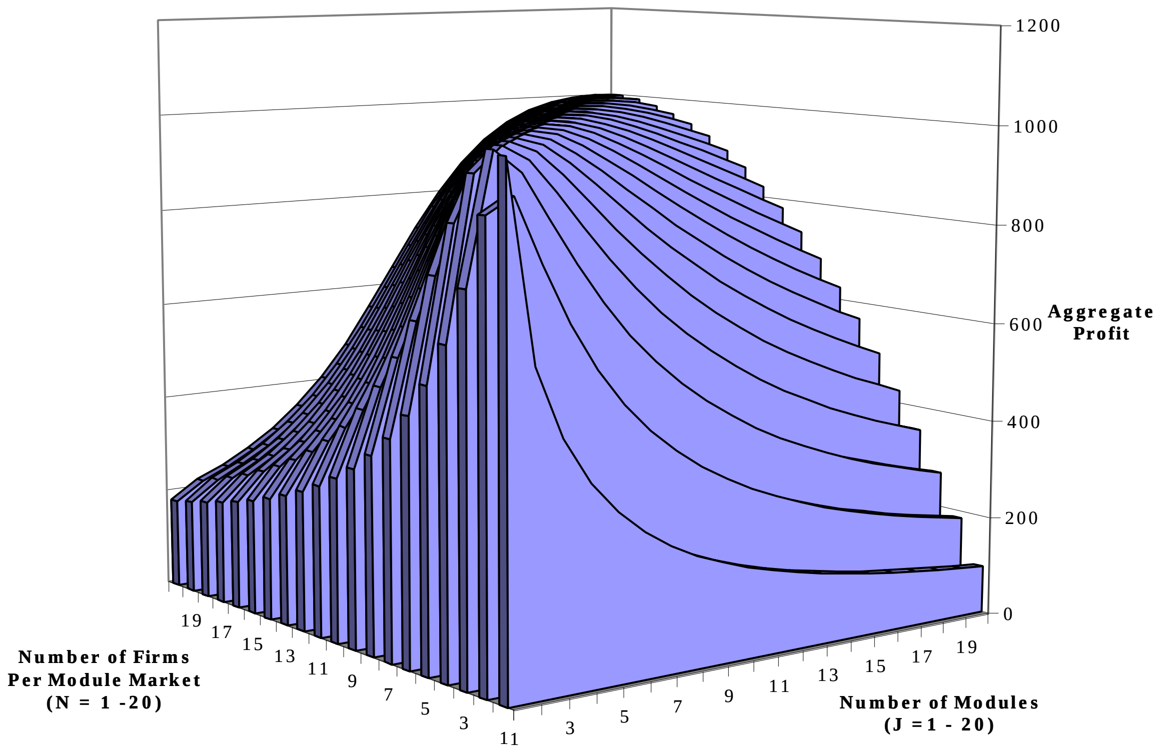
The first panel of Table 1 shows prices consistent with the predictions of Propositions 2, 4 and 6. System prices decline across every row and increase down every column. The prices range from a low of $49 per unit to a high of $1,905 per unit. (Remember that these prices are in fact markups over the system cost, thus no unit is being sold at a loss.) Figure 6 graphs the system prices corresponding to each configuration. Essentially, it fills in the empty cells in the array shown in Figure 4.

**Figure 6**
**Equilibrium System Prices For Various Configurations**




Of particular interest in Table 1 and Figure 6 are a set of configurations wherein the system price arrived at by the firms in a modular cluster equals the system price ($1,000) that is set by One Big Firm. These cells have bold entries in the table and are marked by a heavy black diagonal line in the figure. As can be seen in the second panel of Table 1 and as depicted in Figure 7, this subset of cluster configurations will be as profitable as One Big Firm. (Recall that no configuration has a cost advantage over any other.)

**Figure 7**
**Aggregate Profit for Various Configurations**




The second panel of Table 1 and Figure 7 both show the aggregate profit of different cluster configurations. (For ease of viewing, Figure 7 is shown from a different perpective than Figures 4–6. The 1x1 cell is closest to the viewer in Figure 7, whereas it is farthest from the viewer in the other graphs.) Consistent with Propositions 3 and 5, if we start at the 1x1 cell and move across the first row or down the first column of the array, aggregate profit goes down. However, for N and J greater than 2, this is no longer the case. Along the rows and the columns of the array, profits increase to a peak and then decline. The peaks correspond to those cells wherein the cluster mimics the pricing behavior of One Big Firm.

If one introduces the possibility of moving between configurations by merging or splitting firms, Table 1 and Figure 7 reveal some novel possibilities and some potential traps. First, suppose a vertically integrated industry came to came to have many full-span oligopolists, hence was not very profitable. (Think of the U.S. auto industry in the 1970s and 1980s after the advent of Japanese and European competitors.) To improve the profitability of the industry, the full-span oligopolies could merge with one another. *But there is another possibility:* they might modularize parts of their production systems and then divest the modules, thereby becoming less vertically integrated. In theory, with enough separate module markets, the original firms and their descendants could jointly (but without collusion!) approach the profitability of One Big Firm, even if none of them merges or exits from the industry.

Second, suppose an industry was vertically disintegrated, suffered from the "chain of monopolies" problem, hence was not very profitable. The module monopolies, too, could improve their aggregate profits by merging to form One Big Firm. *But, again, there is another possibility:* each module monopoly might split its variants among several firms that competed with one another in their own (differentiated) module markets. Counterintuitively, price competition in the module markets, as long as it is not too fierce, can benefit the whole industry by forcing system prices down, thereby bringing them closer to the aggregate-profit-maximizing system price of One Big Firm.

Finally, consider an industry that is already in a cluster configuration. Suppose further that in the array of clusters, this particular configuration lay below and to the left of the aggregate-profit-maximizing configuration. In this case, horizontal mergers within module markets would reduce the aggregate profit of the cluster: such mergers would exacerbate the vertical pricing externality and thus drive system prices up. Counter-intuitively, the moves that would enhance the *cluster's* aggregate profit are (1) *vertical* mergers and (2) *horizontal* divestitures. Such moves in theory could drive the cluster as a whole toward a nearby "sweet spot" in the set of configurations. We discuss "sweet spots" in the next section.

## 6.3 "Sweet Spots" among Modular Cluster Configurations

We define the "sweet spots" in a set of configurations of a modular cluster as those combinations of N and J such that the cluster system price equals the system price set by One Big Firm. Under the assumptions of this section—linear demand and zero marginal cost—we can solve for sweet spots analytically. They arise whenever:

$$P_{JxN}^* = \frac{A}{2b} \tag{18}$$

Substituting for $P_{JxN}$* from equation (17b) and solving for J in terms of N, we get:

$$J_{SS} = \frac{sA\ N_{SS}}{2b} + 1 \quad .$$

For the assumptions of our example (A=2,000,000; b=1,000; s=.001), sweet spots arise whenever J = N+1.

However, if we change the demand parameters then the sweet spots will shift. For example, if b=500 and A and s are unchanged, then the sweet spots arise when J = 2N + 1. These results are displayed in Table 2.

**Table 2**
**Moving the Sweet Spots by Changing the Demand Parameters**

**Linear Demand, Zero Marginal Cost**

**Assumptions**

| | | |
|---|---|---|
| Demand Intercept | A | 2,000,000 |
| Demand Slope | b | 500 |
| Module Market Sensitivity | s | 0.001 |

System Price — Firms in Each Module Market (N)

| Modules (J) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2000** | 438 | 306 | 234 | 190 | 160 | 138 | 121 | 108 | 98 | 89 | 82 | 75 | 70 | 66 | 62 | 58 | 55 | 52 | 49 |
| **2** | 2667 | 863 | 607 | 467 | 379 | 319 | 275 | 242 | 216 | 195 | 178 | 163 | 151 | 140 | 131 | 123 | 116 | 110 | 104 | 99 |
| **3** | 3000 | 1268 | 903 | 697 | 567 | 477 | 412 | 363 | 324 | 292 | 266 | 245 | 226 | 210 | 197 | 184 | 174 | 164 | 156 | 148 |
| **4** | 3200 | 1649 | 1192 | 925 | 754 | 635 | 549 | 483 | 431 | 389 | 355 | 326 | 301 | 280 | 262 | 246 | 232 | 219 | 208 | 197 |
| **5** | 3333 | **2000** | 1472 | 1149 | 939 | 792 | 685 | 603 | 538 | 486 | 443 | 407 | 377 | 350 | 327 | 307 | 290 | 274 | 260 | 247 |
| **6** | 3429 | 2314 | 1743 | 1370 | 1122 | 948 | 820 | 722 | 645 | 583 | 532 | 488 | 452 | 420 | 393 | 369 | 347 | 328 | 311 | 296 |
| **7** | 3500 | 2586 | **2000** | 1586 | 1303 | 1103 | 955 | 842 | 752 | 680 | 620 | 569 | 527 | 490 | 458 | 430 | 405 | 383 | 363 | 345 |
| **8** | 3556 | 2814 | 2242 | 1796 | 1482 | 1257 | 1089 | 960 | 859 | 776 | 708 | 650 | 602 | 560 | 523 | 491 | 463 | 438 | 415 | 395 |
| **9** | 3600 | 3000 | 2465 | **2000** | 1658 | 1409 | 1223 | 1079 | 965 | 872 | 796 | 731 | 677 | 630 | 588 | 552 | 521 | 492 | 467 | 444 |
| **10** | 3636 | 3149 | 2667 | 2196 | 1831 | 1560 | 1355 | 1197 | 1071 | 968 | 883 | 812 | 751 | 699 | 654 | 614 | 578 | 547 | 518 | 493 |
| **11** | 3667 | 3268 | 2845 | 2382 | **2000** | 1709 | 1487 | 1314 | 1176 | 1064 | 971 | 893 | 826 | 769 | 719 | 675 | 636 | 601 | 570 | 542 |
| **12** | 3692 | 3363 | 3000 | 2557 | 2164 | 1856 | 1617 | 1430 | 1281 | 1159 | 1058 | 973 | 901 | 838 | 784 | 736 | 694 | 656 | 622 | 591 |
| **13** | 3714 | 3438 | 3131 | 2719 | 2323 | **2000** | 1746 | 1546 | 1386 | 1254 | 1145 | 1054 | 975 | 908 | 849 | 797 | 751 | 710 | 674 | 640 |
| **14** | 3733 | 3500 | 3242 | 2867 | 2475 | 2141 | 1874 | 1661 | 1490 | 1349 | 1232 | 1134 | 1050 | 977 | 914 | 858 | 809 | 765 | 725 | 690 |
| **15** | 3750 | 3551 | 3333 | 3000 | 2620 | 2279 | **2000** | 1775 | 1593 | 1444 | 1319 | 1214 | 1124 | 1046 | 978 | 919 | 866 | 819 | 777 | 739 |
| **16** | 3765 | 3592 | 3409 | 3117 | 2757 | 2413 | 2124 | 1888 | 1696 | 1538 | 1405 | 1294 | 1198 | 1115 | 1043 | 980 | 924 | 873 | 828 | 788 |
| **17** | 3778 | 3628 | 3472 | 3219 | 2883 | 2543 | 2246 | **2000** | 1798 | 1631 | 1491 | 1373 | 1272 | 1184 | 1108 | 1041 | 981 | 928 | 880 | 837 |
| **18** | 3789 | 3658 | 3525 | 3307 | 3000 | 2667 | 2365 | 2110 | 1900 | 1724 | 1577 | 1452 | 1346 | 1253 | 1172 | 1101 | 1038 | 982 | 931 | 886 |
| **19** | 3800 | 3683 | 3569 | 3382 | 3106 | 2785 | 2481 | 2219 | **2000** | 1817 | 1663 | 1532 | 1419 | 1322 | 1237 | 1162 | 1095 | 1036 | 983 | 935 |
| **20** | 3810 | 3706 | 3607 | 3446 | 3200 | 2896 | 2594 | 2326 | 2099 | 1909 | 1748 | 1611 | 1493 | 1391 | 1301 | 1222 | 1153 | 1090 | 1034 | 984 |

Cluster Profit $MM — Firms in Each Module Market (N)

| Modules (J) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2000** | 781 | 565 | 441 | 362 | 307 | 266 | 235 | 210 | 190 | 174 | 160 | 148 | 138 | 129 | 121 | 114 | 108 | 103 | 98 |
| **2** | 1778 | 1353 | 1030 | 825 | 686 | 587 | 512 | 455 | 408 | 371 | 339 | 313 | 290 | 271 | 254 | 238 | 225 | 213 | 202 | 193 |
| **3** | 1500 | 1732 | 1398 | 1151 | 973 | 841 | 739 | 659 | 595 | 542 | 497 | 459 | 427 | 399 | 374 | 352 | 332 | 315 | 299 | 285 |
| **4** | 1280 | 1938 | 1673 | 1422 | 1223 | 1069 | 947 | 849 | 769 | 703 | 647 | 599 | 557 | 521 | 490 | 462 | 437 | 414 | 394 | 375 |
| **5** | 1111 | **2000** | 1861 | 1638 | 1437 | 1271 | 1135 | 1024 | 932 | 854 | 788 | 732 | 682 | 639 | 601 | 567 | 537 | 510 | 485 | 463 |
| **6** | 980 | 1951 | 1967 | 1801 | 1615 | 1447 | 1304 | 1184 | 1082 | 996 | 922 | 858 | 801 | 752 | 708 | 669 | 634 | 603 | 574 | 548 |
| **7** | 875 | 1828 | **2000** | 1914 | 1757 | 1598 | 1454 | 1329 | 1221 | 1128 | 1047 | 977 | 915 | 860 | 811 | 768 | 728 | 693 | 660 | 631 |
| **8** | 790 | 1669 | 1971 | 1979 | 1866 | 1724 | 1585 | 1460 | 1349 | 1251 | 1165 | 1089 | 1022 | 963 | 910 | 862 | 819 | 779 | 744 | 711 |
| **9** | 720 | 1500 | 1892 | **2000** | 1942 | 1826 | 1698 | 1576 | 1464 | 1364 | 1275 | 1195 | 1124 | 1061 | 1004 | 952 | 906 | 863 | 825 | 789 |
| **10** | 661 | 1340 | 1778 | 1981 | 1986 | 1903 | 1792 | 1677 | 1568 | 1468 | 1377 | 1294 | 1221 | 1154 | 1094 | 1039 | 989 | 944 | 903 | 864 |
| **11** | 611 | 1196 | 1643 | 1927 | **2000** | 1958 | 1868 | 1765 | 1660 | 1562 | 1470 | 1387 | 1311 | 1242 | 1179 | 1122 | 1070 | 1022 | 978 | 937 |
| **12** | 568 | 1072 | 1500 | 1845 | 1987 | 1990 | 1927 | 1838 | 1742 | 1647 | 1557 | 1473 | 1396 | 1325 | 1260 | 1201 | 1147 | 1097 | 1050 | 1008 |
| **13** | 531 | 965 | 1360 | 1741 | 1948 | **2000** | 1968 | 1897 | 1811 | 1722 | 1635 | 1552 | 1475 | 1403 | 1337 | 1276 | 1220 | 1168 | 1120 | 1076 |
| **14** | 498 | 875 | 1229 | 1624 | 1887 | 1990 | 1992 | 1943 | 1870 | 1788 | 1705 | 1625 | 1548 | 1477 | 1410 | 1348 | 1290 | 1237 | 1187 | 1141 |
| **15** | 469 | 798 | 1111 | 1500 | 1808 | 1961 | **2000** | 1975 | 1917 | 1845 | 1768 | 1691 | 1616 | 1545 | 1478 | 1416 | 1357 | 1303 | 1252 | 1205 |
| **16** | 443 | 732 | 1007 | 1376 | 1714 | 1915 | 1992 | 1994 | 1954 | 1893 | 1823 | 1750 | 1678 | 1609 | 1542 | 1480 | 1421 | 1365 | 1314 | 1265 |
| **17** | 420 | 675 | 916 | 1257 | 1610 | 1853 | 1970 | **2000** | 1980 | 1932 | 1871 | 1804 | 1735 | 1667 | 1602 | 1540 | 1481 | 1425 | 1373 | 1323 |
| **18** | 399 | 626 | 837 | 1146 | 1500 | 1778 | 1933 | 1994 | 1995 | 1962 | 1911 | 1850 | 1786 | 1721 | 1658 | 1596 | 1537 | 1482 | 1429 | 1379 |
| **19** | 380 | 583 | 768 | 1045 | 1389 | 1692 | 1884 | 1976 | **2000** | 1983 | 1943 | 1890 | 1831 | 1770 | 1709 | 1649 | 1591 | 1535 | 1483 | 1433 |
| **20** | 363 | 545 | 709 | 955 | 1280 | 1599 | 1824 | 1947 | 1995 | 1996 | 1968 | 1924 | 1871 | 1814 | 1756 | 1698 | 1641 | 1586 | 1534 | 1484 |

# 7 Relationship to Other Models

We are not aware of any paper that looks specifically at the pricing behavior and profitability of a large group of firms making both substitutes and complements—the industry structure we are calling a modular cluster. However, there are several models that have obtained related results. We have already described Economides and Salop's (1992) seminal paper, which is a direct predecessor of this work. In this section, we will discuss models crafted by Rey and Stiglitz (1995), Nalebuff (2000); and Ruffin (2001) in relation to this paper. Although these other models were developed to address different questions than the ones we address, one can see corresponding results in these other settings.

Rey and Stiglitz (1995) are interested in the effects of marketing practices on the pricing behavior and profitability of oligopolistic manufacturers. Their base case is model of duopolists selling through a set of perfectly competitive retailers. They show that giving the retailers an intrabrand monopoly (in the form of exclusive territories) "leads to higher retail prices and lower sales. The producers may … [benefit directly] … if it increases the prices they receive" (p. 435), or they may recapture some of the retail profits by charging franchise fees.

The Rey-Stiglitz model is related to ours in the following way. First, note that any JxN modular cluster can be augmented by adding another modular layer, taking J to J+1. *If the new layer is perfectly competitive, the pricing equilibrium will not change.* Thus the Rey and Stiglitz base case with perfectly competitive retailers corresponds to a "one-layer duopoly" or, in our notation, a 1x2 configuration.

When the retailers are endowed with exclusive territories, they no longer face "intrabrand" competition, but only "interbrand" competition. Thus the effect of exclusive territories is to convert each retailer into *a duopolist selling a (retail service) module of a composite good against the corresponding module of the other brand* (see equation 4 in their paper). Thus in terms of our notation, Rey and Stiglitz compare the 1x2 configuration (their base case) to the 2x2 configuration (exclusive territories). Our model and theirs both predict that system (ie., retail) prices and aggregate profits will be higher in the 2x2 case than in the 1x2 case. As we have shown, in terms of pricing and industry profits, a "two-layer duopoly" usually gets closer to the One Big Firm benchmark than a "one-layer" duopoly. In their words: "exclusive territories alter the perceived demand curve, making each producer believe that he faces a less elastic demand curve, thereby inducing an increase of the equilibrium price" ( p. 432). In our words: "partitioning a full-span oligopoly into two or more module markets creates vertical pricing externalities that tend to offset the lower prices caused by oligopolistic competition." Rey and Stiglitz then look to see whether *the producers* (ie. the upstream firms) can be made better off by this change in industry configuration. In their formal setup (which is somewhat different from ours), they show that the producers generally benefit if they can charge franchise fees and sometimes benefit even without them.

Rey and Stiglitz consider symmetric configurations, as we do. In contrast, Nalebuff (2000) and Ruffin (2001) look at asymmetric configurations wherein a single vertically integrated firm competes against a group of component makers. Both of these models, however, start with Cournot's vertical pricing externality. Quoting Nalebuff: "in the case of complementors, when one firm lowers its price, the other firm's sales increase, an externality not taken into account with uncoordinated pricing" (Nalebuff, p. 2). Ruffin calls this a "free rider problem" (Ruffin, p. 5).

Nalebuff and Ruffin model demand in somewhat different ways. Nalebuff constructs a generalized Hotelling product-location model[19] in which every consumer buys a system. Thus his is a pure market-share game in which system demand is fixed. (Our model would ressemble Nalebuff's if we were to set system demand, Q(P), equal to a constant.) Ruffin, in contrast, assumes that system demand is downward sloping in the system price while competition between firms in the same "cohort" (which we call "module market") is Cournot competition in quantities. Thus individual assemblers and components makers set their prices to maximize profits, knowing the prices of their complements and the quantities selected by their rivals. (Our model would ressemble Ruffin's if we set $\alpha_i = 1$, and subtracted the quantities produced by other firms in the same module market from the demand of the representative firm.[20])

Although their models of demand are different, Nalebuff's and Ruffin's thought experiments are quite similar: each asks how an integrated firm fares in competition with a set of uncoordinated firms. They show, not surprisingly, that a single coordinated firm does well in a pricing game against a set of uncoordinated rivals. Interesting for us, however, is the fact vertical integration by one firm causes prices

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[19] Consumers are uniformly located on a unit hypercube, and two variants of each component occupy the positions 0 and 1 respectively. Different systems then correspond to the vertices of the hypercube. Consumers buy the cheapest product subject to a transportation cost.

[20] Formally, we assume that $\pi_i = (p_i - c_i)\,\alpha_i(p_i; p_v^{avg})\,Q(P)$, while Ruffin assumes that $\pi_i = (p_i - c_i)[Q(P) - y]$, where y denotes the quantities produced by the other firms in the same cohort.

to fall. In terms of our array of configurations, one can think of starting with a JxN configuration and proceeding to a 1xN configuration via a series of vertical mergers. In different formal settings, Nalebuff and Ruffin both show that when a vertical merger takes place, system prices decline through the pricing actions of the integrated firm. Thus, although they look at small, asymmetric configurations, their results are consistent with ours: in the presence of oligopolistic competition, more vertical integration reduces system prices (and aggregate profits) below the benchmark standard of One Big Firm. (Nalebuff also shows that a 2x2 configuration is more profitable in aggregate that a 1x2 configuration, consistent with both Rey and Stiglitz and our results.)

# 8 Conclusion

In summary, Economides and Salop's, Rey and Stiglitz's, Nalebuff's and Ruffin's prior models all hint at the fact that a disaggregated cluster of firms might be able get closer to the prices and profitability of One Big Firm than a chain of monopolists or a set of vertically integrated oligopolists. In this paper, we have sought first to clarify and then to generalize the results suggested by these other models. We have done this by constructing a model that is at its core simpler than any of these others, but at the same time is "scalable" in the sense that it can be extended to symmetric clusters of arbitrary size (JxN).

In this last section of the paper, we would like to raise three issues that are more qualitative in nature. First, what are the welfare implications of a modular cluster that mimics the system prices and aggregate profitability of One Big Firm? Second, and relatedly, what are the implications of the cluster form for innovation and industry dynamics? And third, what other types of pricing behavior are possible in a cluster?

## 8.1 Welfare, Investment and Innovation

Turning first to the question of welfare, if a modular cluster can achieve the same prices and profits as One Big Firm, what does that mean for consumers? Should vertical mergers be encouraged in order to "push" the configuration in the direction of lower system prices and aggregate profitability? Should vertical divestitures be forbidden?

Assessing welfare implications requires that cluster pricing be nested in a larger, multi-stage model of investment. As we indicated in the introduction to this paper, rational investments in sunk-cost capital (including R&D) require an expectation of future returns in the form of "free cash flow" or "quasi-rents." Our analysis shows that in some cases the expectation of a substantial stream of quasi-rents can hold for a large and highly disaggregated cluster of firms within an architecture of imperfect substitutes and complements. What then matters for welfare are the conditions of entry into the capital market that allocates ownership of the individual module-product lines. If the capital market is competitive, then every module-product line with a positive net present value will be the target of investment. Firms will be created until the set of positive NPV opportunities is exhausted. Within the cluster, goods will be priced at *long-run* marginal cost, and overall welfare will be high. Conversely, if access to the capital market is restricted, some positive NPV opportunities will go begging, and welfare will be commensurately lower. All of this is to say, that, in the absence of information about conditions of access in the capital market, the impact of a cluster's pricing and profits on welfare cannot be determined.

That said, it is possible that a modular cluster with a high stream of quasi-rents may be very conducive to innovation. The reason is that the targets of substitution in a cluster are relatively small, and "disruptions," that is, costs imposed on complements or substitutes by a particular innovation, are externalities. In effect, one can think of each "square" in a cluster checkerboard as being subject to a winner-take-all design competition for that piece of the overall system of production. In the context of the beach resort, a restaurant may switch from French to Southwestern cuisine if Southwestern cuisine becomes popular; in the computer industry, a company making external drives may introduce a 30 GB drive and simultaneously remove a 10 GB drive from its product line. In a cluster, other restaurants or other drive makers may be forced to react. The result is change and innovation within the cluster.

Indeed, given the decentralized nature of cluster control and decision-making, in a free capital market, one would be hard pressed to stop such changes. In contrast, One Big Firm (1x1) can internalize the disruptive costs of innovation: it might see that an innovation in one part of the system would entail upgrading another part, and then another, until the whole system of production had changed. Seeing this chain of cause and effect triggered by innovation, One Big Firm might not be willing to take the first step.

On the basis of this reasoning, we think it likely that a modular cluster lodged in a free capital market might be more innovative than One Big Firm with the same system prices and aggregate product-market profitability.[21] But we also note that blindness to the externalities of innovation is a two-edged sword: if the social and economic costs of "disruption" are high, then a cluster might overinvest in innovation and reduce welfare in the process.

## 8.2 "Federated" vs. "Portal" Pricing

As indicated, we think the contribution of this paper has been to clarify a result that was implicit in prior work, and to extend the scope of analysis from small (2x2) clusters to large (JxN) ones. However, there is yet another, quite different, way to look at cluster pricing. For ease of reference, we will call the alternate view the "portal" model of industry pricing and ours the "federated" model. In the *federated* model, each firm sets its own price taking account of the average prices of its variants and complements. The system price is then the sum of module prices; and aggregate profit is the sum of individual firm profits. By contrast, in the *portal* model of pricing, system prices are determined by the firms *in one particular module market*. These firms, which are usually taken to be "downstream," are assumed to control their suppliers' access to the end user, hence they are "portals" through which revenue enters the industry. Each portal firm then contracts or bargains with its upstream suppliers, redistributing revenue across the industry. In this setting, the market structure of the "portal layer" of firms determines profits for the whole industry.

For analytic purposes, the portal model of pricing nicely separates the management of end-product demand from the management of contracts and supply. Portal firms can optimize the demand for their products based on competition from other portal firms, and then "turn about" and optimize contracts with their (usually captive) suppliers.

Baker, Gibbons and Murphy's (2002) model of relational contracts within and between firms is an example of a portal-type model: in their setup, a downstream party buys one unit of input from the upstream party, and resells the unit (for different prices conditional on its quality). The downstream party's task is to design an effective contract with the upstream party. In this sense, the downstream party is a portal: Revenue comes into the industry via this channel, and then gets allocated by contract between the upstream and downstream participants. In another recent model of labor market mobility

[21] This argument is similar some of the arguments advanced by Baumol (2003), see especially, Chapter 10.

and innovation in industrial clusters, Fallick, Fleischmann and Rebitzer (2003) also assume portal-type pricing. They posit an industry made up of downstream "manufacturers" and upstream "suppliers". The manufacturers are oligopolistic competitors, whose aggregate profit goes down as their number increases (see Equation 3 and surrounding text). Each manufacturer then splits its own profit with an upstream supplier (the winner of a design contest). Thus the downstream firms determine their profits in competition with one another, and then serve as a portals for transmitting and splitting that profit with upstream suppliers.

In contrast, in our model, as well as those discussed in the previous section, no privileged layer of portal firms controls access to end users. Instead system prices and aggregate profits are determined by the interaction of oligopolistic price competition (within module markets) and vertical pricing externalities (across module markets). Firms make independent decisions, but take account of the actions of other firms in the cluster, hence we have labeled this model "federated" pricing.

The portal and federated models are both plausible models of pricing behavior for firms within a cluster. Both modes of behavior seem to arise in practice, but they lead to very different predictions with respect to system prices and aggregate profitability. Can we say anything about when we are likely to see one or the other?

Portal pricing, we note, presumes a high degree of asset specificity and bilateral immobility between the portal firm and its complementary suppliers (and distributors). Implicitly, in these models, the vertical relationship is the main focus of the analysis, the main problem to be solved. Thus portal pricing seems most likely to occur when modules are technologically incompatible across different systems. In that case, there can be no mixing and matching of modules across systems, hence no competition within module markets. And from the end users' perspective, the main choice is between systems.

In this context, as Nalebuff and Ruffin both show, a vertical set of suppliers that can set a coordinated system price will greatly outperform rivals who cannot set a coordinated system price. Portal pricing behavior is an effective way to set a coordinated system price even if (for whatever reasons) the modules are supplied by different firms. In such cases, for pricing purposes, the configuration is effectively 1xN, where N is the number of systems. And as the number of incompatible systems (each with its own dedicated set of complementary suppliers) becomes large, system prices will fall, and aggregate profits will decline apace. In our model, this follows from Proposition 4, and can be seen by tracking across the first row in each panel of Tables 1 or 2.[22]

In contrast, we have posited an architecture of production that has no "special complementarities" across modules at any level of disaggregation. This assumption in turn implies that each module, indeed each primitive component within each module, is compatible with every other. Mixing and matching of different modules and components is possible essentially without restriction: the only technological constraint we have imposed is that each complete system must have one of each primitive component. In this setting, competition between modules is natural.

Thus when the architecture of a large system of production makes the mixing and matching of compatible modules feasible and pervasive, we would expect "federated" pricing behavior to be the norm. Interestingly, under federated pricing, it is at least theoretically possible for a large modular cluster to mimic the system prices and aggregate profitability of One Big Firm. In contrast, in the same-sized cluster under portal pricing, our model predicts that system prices and aggregate profit would be much lower.

In conclusion, in this paper we have investigated a possibility hinted at in prior work: that pressures to reduce prices within oligopolistic markets can be offset by pressures to increase prices in complementary goods markets. The recognition of this possibility is not new: in fact, it underlies the models and results of Economides and Salop (1992), Rey and Stiglitz (1995), Nalebuff (2000), and Ruffin (2001) among others. Our contribution has been to isolate the offsetting price effects, and show how they might operate in large as well as small clusters of firms.

[22] Nalebuff calls this "bundle against bundle competition," and characterizes it as "ferocious" (p. 7).

# Appendix—Proofs of the Lemma and Propositions 2, 4 and 6

## Lemma

Under the assumptions of our model, in a configuration with at least two modules (J≥2), equilibrium module prices, hence the system price and aggregate profit, are the same whether the modular elements constitute successive stages of a production process or are separate goods purchased and assembled by end-users.

**Proof.** In a configuration with at least two modules (J≥2), consider two regimes. In the first, each firm sells its modules as products directly to the end-users, who then assemble their systems. In the second regime, the modules correspond to successive stages or tasks of a production process: firms in Stage 1 complete a set of tasks and sell their intermediate products to Stage 2 firms; Stage 2 firms complete another set of tasks, and then sell their products to Stage 3 firms, and so on. At the end of the line, the Stage J firms sell complete systems to end-users.

In the first regime, the representative firm's profit function is as shown in equation (8) in the text:

$$\pi_i = p_i\, q_i - c_i\, q_i \qquad (8)$$
$$= (p_i - c_i)\, \alpha_i(p_i; p_v^{avg})\, Q[P(p_i + \Sigma\, p_j^{avg})] \quad .$$

In the second regime, firms at different stages have different profit functions . Consider, for example, firms in the last (the J$^{th}$) stage. The price they charge will be the system price, and the unit cost they incur will be the cost of their own stage of production *plus the average price of goods purchased from firms in the J–1 stage*:

$$\pi_{i,J} = (P_{i,J} - c_i - P_{i,\,J-1}^{avg})\, \alpha_{i,J}(P_{i,J}\,; P_{v,J}^{avg})\, Q[P_{i,J}]$$

Without loss of generality, we define:

$$P_{i,J} \equiv \Delta P_{i,J} + P_{i,\,J-1}^{avg} \quad ;$$

ie, the price set by the J$^{th}$-stage firm can be viewed as the price charged by its predecessor plus an increment. Also by recursive substitution, we have:

$$P_{i,J} \equiv \Delta P_{i,J} + \Delta P_{i,\,J-1}^{avg} + \ldots + \Delta P_{i,\,1}^{avg}$$

Notice that, by the assumptions of a Nash equilibrium, the $J^{th}$-stage firm controls its price increment, but takes the average prices charged by its predecessor firms as fixed.

Substituting these new expressions for $P_{i,J}$, we obtain:

$$\pi_{i,J} = (\Delta P_{i,J} - c_i)\, \alpha_{i,J}(\Delta P_{i,J} + P_{i,J-1}^{avg};\, \Delta P_{v,J} + P_{v,J-1}^{avg})\, Q[\Delta P_{i,J} + \Delta P_{i,J-1}^{avg} + \ldots + \Delta P_{i,1}^{avg}] \quad .$$

By assumption (3d), the market share function of the $J^{th}$-stage firm is unaffected by the prices of complements, hence:

$$\pi_{i,J} = (\Delta P_{i,J} - c_i)\, \alpha_{i,J}(\Delta P_{i,J} ;\, \Delta P_{v,J}^{avg})\, Q[\Delta P_{i,J} + \Delta P_{i,J-1}^{avg} + \ldots + \Delta P_{i,1}^{avg}] \quad .$$

But this profit function is identical in all respects to the profit function of a representative firm in complementary-goods regime. Moreover, a similar change of variables can be imposed on firms at each of the J successive stages. The result will be a set of symmetric profit functions, and the resulting system of equations that determines equilibrium prices will be identical to that obtained in the complementary-goods regime. QED.

## Proposition 2

Consider two module monopoly configurations, Jx1 and Kx1, where $1 \leq J < K$. Under the assumptions of this model, $P_{Jx1} < P_{Kx1}$. In other words, the system price is strictly *increasing* in the number of module monopolies.

**Proof.** From equation (14), substituting K for J, noting that $\partial P / \partial p_i = 1$, and rearranging terms, we obtain the optimal price charged by the representative firm in a Kx1 configuration:

$$p_{Kx1}^* = \frac{-\,Q(P^*)}{\partial Q(P^*) / \partial P} + \frac{C}{K} \quad ; \qquad \text{(A-1)}$$

As always, the demand function and the partial derivative are understood to be evaluated at $P_{Kx1}^*$, the Kx1 equilbrium system price. Also from equation (10) in the text, we have:

$$P_{Kx1}^* = K\, p_{Kx1}^* \qquad \text{(A-2)}$$

The system price implied by (A-1) is simply K times the individual module prices.

Define:

$$p' = (K/J)\, p_{Kx1}^* \quad . \tag{A-3}$$

Applying equation (10) again, we have by substitution:

$$P_{Jx1}[p'] = J\,p' = J\,(K/J)\, p_{Kx1}^* = K\, p_{Kx1}^* = P_{Kx1}^* \quad . \tag{A-4}$$

We can now evaluate the Jx1 representative firm's marginal profit function at the point p'. Intuitively, at this point, the J individual module prices will add up to the Kx1 configuration's equilibrium system price. Thus Q(.) and $\partial Q/\partial P$, which are functions of p through P(.), will be evaluated at $P(.) = P_{Kx1}^*$. Thus:

$$\frac{\partial \pi_{Jx1}}{\partial p} = Q[P_{Kx1}^*] + \left( p' - \frac{C}{J} \right) \frac{\partial Q[P_{Kx1}^*]}{\partial P} = \; ? \quad . \tag{A-5}$$

Here, we are making use of the fact that $\partial P/\partial p = 1$.

Substituting from equation (A-1) to (A-3) and (A-3) to (A-5) and cancelling terms as appropriate, we obtain:

$$(1 - K/J)\, Q(P_{Kx1}^*) < 0$$

K is greater than J, by assumption, hence this expression is *negative*. This implies that the price $p'=(K/J)p_{Kx1}^*$ lies on the downward slope of the Jx1 representative firm's profit function. That profit function, in turn, has only one peak, and thus p' is greater than the Jx1 representative firm's optimal price. Using equation (A-4), it follows immediately that:

$$P_{Jx1}^* = Jp_{Jx1}^* < Jp' = P_{Kx1}^* \quad ; \text{or}$$

$$P_{Jx1}^* < P_{Kx1}^* \quad \text{for all J, K such that } 1 \le J < K \quad .$$

QED.

## Proposition 4

Consider two differentiated, full-span oligopoly configurations, 1xN and 1xM, where $1 \leq M < N$. Under the assumptions of this model, $P_{1xM}^* > P_{1xN}^*$. In other words, the system price is strictly *decreasing* in the number of full-span oligopolies.

**Proof.** The proof involves a certain amount of brute force algebra, but the strategy is relatively simple. As in the proof of Proposition 2 above, we shall "test" the equilibrium price of one configuration (1xN) against the equilibrium first-order condition of the other. We shall find that the 1xM marginal profit function evaluated at the 1xN equilibrium price is positive, hence the 1xN equilibrium price is on the rising slope of the 1xM profit function. From this and the fact that the profit function is single-peaked, it follows that the 1xM equilibrium price is higher than the 1xN equilibrium price.

For full-span oligopolies, the firm-level price equals the system price: p = P. We can simplify notation by making this substitution everywhere. From equation (9) in the text, the optimal price charged by firms in the 1xN configuration is then:

$$P' = P_{1xN}^* = \frac{-\alpha_{1xN}^* Q_{1xN}^*}{\left\{ Q_{1xN}^* \dfrac{\partial \alpha_{1xN}^*}{\partial P} + \alpha_{1xN}^* \dfrac{\partial Q_{1xN}^*}{\partial P} \right\}} + C = \frac{-\alpha_{1xN}^* Q_{1xN}^*}{D} + C \, ; \qquad \text{(A-7)}$$

Here, the stars, " * ", indicate that each function is being evaluated at the 1xN equilibrium price, $P_{1xN}^*$. Thus each term in this expression is a unique number, and $P_{1xN}^*$ is unique number. The symbol P' is an economical way of referring to this number. "D" in the last equation is a convenient way of referring to the denominator of the first term. Note that because both partial derivatives in the denominator are negative, $D < 0$, hence $P_{1xN}^*$ is positive and greater than C.

Using equation (9) in the text, we can evaluate the marginal profit of the representive firm in the 1xM configuration at the point P':

$$\frac{\partial \pi_{1xM}(P')}{\partial P} = \alpha_{1xM}(P')\, Q(P') + (P' - C)\left\{ Q(P') \frac{\partial \alpha_{1xM}(P')}{\partial P} + \alpha_{1xM}(P') \frac{\partial Q(P')}{\partial P} \right\} = \ ? \qquad \text{(A-8)}$$

We now substitute (A-7) into (A-8). After some algebra, we have:

$$\frac{\partial \pi_{1xM}(P')}{\partial P} = \left( \alpha_{1xM}(P') \frac{\partial \alpha_{1xN}^*}{\partial P} - \alpha_{1xN}(P') \frac{\partial \alpha_{1xM}(P')}{\partial P} \right) \frac{[Q(P')]^2}{D} = \ ? \ . \qquad \text{(A-9)}$$

By assumption, the market share functions are parallel:

$$\alpha_{1xM}(.) = \alpha_{1xN}(.) + \Delta_{M,N} ,$$

and $P' = P_{Nx1}^*$, thus:

$$\frac{\partial \alpha_{1xN}^*}{\partial P} = \frac{\partial \alpha_{1xM}(P')}{\partial P} \quad ; \text{and} \quad \alpha_{1xM}(P') - \alpha_{1xN}(P') = \Delta_{M,N} > 0 .$$

Making these substitutions in (A-9), and rearranging terms, we have:

$$\frac{\partial \pi_{1xM}(P')}{\partial P} = \Delta_{M,N} \frac{\partial \alpha_{1xN}^*}{\partial P} \frac{[Q(P')]^2}{D} > 0 . \quad \text{(A-10)}$$

The inequality holds because $\Delta_{M,N} > 0$; $\partial\alpha / \partial P < 0$; $[Q(P')]^2 > 0$; and $D < 0$.

Thus the 1xM marginal profit function evaluated at $P' = P_{1xN}^*$ is positive, indicating that the 1xN equilibrium price, $P_{1xN}^*$, is on the rising slope of the 1xM profit function. The profit function is single-peaked, therefore the 1xM equilibrium price must be higher than the 1xN equilibrium price:

$$P_{1xM}^* > P_{1xN}^* \quad \text{for all M, N such that } 1 \le M < N.$$

QED.

## Proposition 6

**(a)** Let the number of firms in each module market be N. Consider two modular cluster configurations, JxN and KxN, where $1 \le J < K$. Under the assumptions of this model, $P_{JxN}^* < P_{KxN}^*$. In other words, for any N, system prices are strictly increasing in the number of module makers.

**Proof.** From equation (9) in the text, the optimal price charged by firms in the KxN configuration is:

$$p_{KxN}^* = \frac{-\alpha_{KxN}^* Q_{KxN}^*}{\left\{ Q_{KxN}^* \frac{\partial \alpha_{KxN}^*}{\partial p} + \alpha_{KxN}^* \frac{\partial Q_{KxN}^*}{\partial P} \right\}} + \frac{C}{K} = \frac{-\alpha_{KxN}^* Q_{KxN}^*}{D} + \frac{C}{K} ; \quad \text{(A-11)}$$

Here, as before, the stars, “ * ”, indicate that each function is being evaluated at the KxN equilibrium, and “D” in the last equation is simply a convenient way of referring to the denominator of the first term. D<0, hence $p_{KxN}$* is positive and greater than C / K.

From equation (10) in the text, the system price implied by (A-11) is simply K times the module price:

$$P_{KxN}^* = K p_{KxN}^* \qquad \text{(A-12)}$$

As in the proof of Proposition 2, define:

$$p' = (K/J)\, p_{KxN}^* \quad . \qquad \text{(A-13)}$$

Again, we have by substitution:

$$P_{JxN}[p'] = J\,p' = J\,(K/J)\, p_{KxN}^* = K\, p_{KxN}^* = P_{KxN}^* \quad . \qquad \text{(A-14)}$$

We also have from the assumption about the conservation of market share across module reallocations, equation (7) in the text:

$$\alpha_{JxN}(p') = \alpha_{JxN}[(K/J)\, p_{KxN}^*] = \alpha_{KxN}(p_{KxN}^*) = \alpha_{KxN}^* \quad . \qquad \text{(A-15)}$$

We can now evaluate the JxN representative firm's marginal profit function at the point p'. As before, Q(.) and $\partial Q / \partial P$, which are functions of p through P(.), will be evaluated at P(.) = $P_{Kx1}$*. The JxN representative firm's marginal profit function evaluated at this point is:

$$\frac{\partial \pi_{JxN}(p')}{\partial p} = \alpha_{KxN}^*\, Q_{KxN}^* + (p' - C/J)\, \{Q_{KxN}^* \frac{\partial \alpha_{KxN}^*}{\partial p} + \alpha_{KxN}^* \frac{\partial Q_{KxN}^*}{\partial P}\} = \; ? \qquad \text{(A-16)}$$

Again, we are making use of the fact that $\partial P / \partial p = 1$. Note that the term in curly brackets equals D in equation (A-11).

We can now substitute expression (A-11) into equation (A-13) and the result into (A-16). After cancellations, we have:

$$\frac{\partial \pi_{JxN}(p')}{\partial p} = (1 - K/J)\, \alpha_{KxN}^*\, Q_{KxN}^* < 0 \quad . \qquad \text{(A-17)}$$

K is greater than J by assumption, hence marginal profit at this point is negative. Because the profit function is single-peaked, $p_{JxN}^* < p'$, in other words, the optimal module price in the JxN configuration is less than p’. Using equation (A-14), it follows immediately that:

$$P_{JxN}^* \;=\; Jp_{JxN}^* \;<\; Jp' \;=\; P_{Kx1}^* \quad ; \text{or}$$

$$P_{JxN}^* \;<\; P_{KxN}^* \quad \text{for all J, K such that } 1 \leq J < K \;.$$

QED.

**(b)** Let the number of module makers be J. Consider two modular cluster configurations, JxN and JxM, where $1 \leq M \leq N$. Under the assumptions of this model, $P_{JxM}^* > P_{JxN}^*$. In other words, for any J, system prices are strictly decreasing in the number of firms competing in each module market.

**Proof.** The proof runs parallel to the proof of Proposition 4. From equation (9) in the text, the optimal price charged by firms in the JxN configuration is:

$$p' \;=\; p_{JxN}^* \;=\; \frac{-\alpha_{JxN}^* \, Q_{JxN}^*}{\left\{ Q_{JxN}^* \dfrac{\partial \alpha_{JxN}^*}{\partial p} \;+\; \alpha_{JxN}^* \dfrac{\partial Q_{JxN}^*}{\partial P} \right\}} + \frac{C}{J} \;=\; \frac{-\alpha_{JxN}^* \, Q_{JxN}^*}{D} + \frac{C}{J} \;; \qquad \text{(A-18)}$$

Once again, the stars, “ * ”, indicate that each function is being evaluated at the JxN equilibrium, and “D” refers to the denominator of the first term. Recall that $D < 0$, thus $p_{JxN}^* > C / J$.

The system price corresponding to p’ equals the equilibrium system price in the JxN configuration:

$$P(p') \;=\; Jp' \;=\; Jp_{JxN}^* \;=\; P_{JxN}^* \quad .$$

We can evaluate the marginal profit of the representive firm in the JxM configuration at the point p’.

$$\frac{\partial \pi_{JxM}(p')}{\partial p} \;=\; \alpha_{JxM}(p')\, Q_{JxN}^* \;+ (p' - C/J)\left\{ Q_{JxN}^* \frac{\partial \alpha_{JxM}(p')}{\partial p} \;+\; \alpha_{JxM}(p') \frac{\partial Q_{JxN}^*}{\partial P} \right\} = \;? \qquad \text{(A-19)}$$

We now substitute (A-17) into (A-18). After some algebra, we have:

$$\frac{\partial \pi_{JxM}(p')}{\partial p} \;=\; \left( \alpha_{JxM}(p') \frac{\partial \alpha_{JxN}^*}{\partial p} \;-\; \alpha_{JxN}(p') \frac{\partial \alpha_{JxM}(p')}{\partial p} \right) \frac{[Q_{JxN}^*]^2}{D} \;=\; ? \quad . \qquad \text{(A-20)}$$

By assumption, the market share functions are parallel:

$$\alpha_{JxM}(.) = \alpha_{JxN}(.) + \Delta_{M,N} ,$$

and p' = $p_{Nx1}$*, thus:

$$\frac{\partial \alpha_{JxN}^*}{\partial p} = \frac{\partial \alpha_{JxM}(p')}{\partial p} \quad ; \text{and} \quad \alpha_{JxM}(p') - \alpha_{JxN}(p') = \Delta_{M,N} > 0 .$$

Making these substitutions in (A-9), and rearranging terms, we have:

$$\frac{\partial \pi_{JxM}(P')}{\partial p} = \Delta_{M,N} \frac{\partial \alpha_{JxN}^*}{\partial p} \frac{[Q_{JxN}^*]^2}{D} > 0 . \qquad \text{(A-21)}$$

The inequality holds because $\Delta_{M,N} > 0$; $\partial\alpha / \partial p < 0$; $[Q_{JxN}^*]^2 > 0$; and $D < 0$.

Thus the JxM marginal profit function evaluated at p' = $p_{JxN}$* is positive, indicating that the JxN equilibrium price, $p_{JxN}$*, is on the rising slope of the JxM profit function. The profit function is single-peaked, therefore the JxM equilibrium price must be higher than the JxN equilibrium price:

$$P_{JxM}^* > P_{JxN}^* \quad \text{for all M, N such that } 1 \le M < N.$$

QED.

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