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# The Fundamental Theorem of Design Economics

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This Draft:  
August 14, 2002

Our thanks to Masahiko Aoki, Wayne Collier, Sonali Shah, David Sharman, Kevin Sullivan, Jason Woodard, and members of the Negotiations, Organizations and Markets group at Harvard Business School for sharing key insights. We alone are responsible for errors, oversights and faulty reasoning.

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# The Fundamental Theorem of Design Economics

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Every artifact has a design, and thus designs are an important class of information goods. In this paper, we establish the scope of the design valuation methodology based on real options, which we developed in *Design Rules, Volume 1, The Power of Modularity* (MIT Press, 2000). We argue that if an economic process is:

- *ex ante* uncertain;
- *ex post* rankable by outcome;
- *ex post* contingent;
- costly; and
- has non-exclusive outputs;

and if better outcomes have higher financial value (are worth more money), then the value of that process will embed either simple real options (if the process is indivisible) or compound real options (if the process is modular). The real options, in turn, will have a “Q(k)-type structure,” where Q(k) represents the expectation of the maximum of the outcomes of k processes run in parallel. We note that Q(k) is both an order statistic function and a real option function.

All design processes are *ex ante* uncertain; costly; and have non-exclusive outputs. Virtually all designs are *ex post* rankable by outcome within an appropriate functional category. Finally, many designs can be made *ex post* contingent by separating the design process from the production process for the artifact in question. Hence the fundamental theorem applies to a large subset of an important class of information goods.

**Key words:** Technological innovation— real options — information goods — design — modularity — experimentation — modular design evolution .

**JEL Classification:** D83, G31, G34, L22, L23, O31, O34

# 1 The Fundamental Theorem

All artifacts have designs, thus designs are an important class of information goods. Here we establish the scope of the design valuation methodology based on real options, which we developed in *Design Rules, Volume 1, The Power of Modularity* (MIT Press, 2000).

To begin with, we need two definitions:

Define an *economic process* as a method that converts inputs, tangible and intangible, into outputs that have economic value. In other words, the outputs of the process can be sold for money. Define a *value representation* as a mathematical formula that sums up the financial costs and benefits of an economic process, using market prices for all inputs and outputs, and asset prices for all future cash flows.

**Lemma.** If an economic process is indivisible, then  $k$ , the number of processes to run in parallel will be an *ex ante* decision variable.

**Proof.**

An economic process is a method, and thus it can be enacted more than once. How many times to enact the process is an *ex ante* decision variable. But if the process is divisible, then the decision variable is a vector: the decision maker must decide how many times to enact each subprocess.

**Proposition 1.** (The Fundamental Theorem.) If an economic process is:

- indivisible;
- *ex ante* uncertain;
- *ex post* rankable by outcome;
- *ex post* contingent;
- costly; and
- has non-exclusive outputs.

and if better outcomes have higher financial value (are worth more money), then:

a. the value representation of the  $k$  processes will have the form:

$$V(k) = Q(k) - C(k)$$

where  $Q(k)$  is the present value of the expectation of the maximum of the outcomes of the  $k$  parallel processes:

$$Q(k) \equiv V_0[E(X^* | X^* = \max(X_1 \dots X_k))] ;$$

and  $C(k)$  is the cost of the inputs to the  $k$  parallel processes. Note that  $Q(k)$  is both an order statistic expression and a real option expression.<sup>1</sup>

b. *Ex ante* optimization of these processes takes the form of finding and selecting  $k^*$  such that:

$$V(k^*) = \max_k [Q(k) - C(k)] .$$

c. Optimal  $k$  may be greater than one.

**Proof:**

1a. Assume the decision maker runs  $k$  processes. After the fact, the processes will have  $k$  outcomes,  $\langle X_1, \dots, X_k \rangle$ .

By the fact that the outcomes are rankable, there exists a highest  $X$ .

By the fact that the processes are contingent, the outputs which have the highest  $X$  can be used or supplied or applied to the purpose needed.

By the fact that the outputs are non-exclusive, those with the highest  $X$  can be used as many times as they are needed: the outputs of the other processes may be discarded.

The expected outcome obtained from running the  $k$  processes and selecting the best will be:

$$E[X^* | X^* = \max(X_1 \dots X_k)] .$$

By the definition of economic process (see above), this outcome has a set of financial values in the future, that is a set of cash flows. If the cash flows are uncertain, they have expectations, and the expectations are well-defined functions of  $k$ . The present value of a series of expected cash flows in the future is a well-defined asset price. It, too, is a function of  $k$ , and we denote it  $Q(k)$ :

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<sup>1</sup> Cf. Lindgren, 1968; Merton, 1973.

$$Q(k) \equiv V_0\{E[X^* | X^* = \max(X_1 \dots X_k)]\} ;$$

Here the valuation operator,  $V_0$ , denotes both the mapping of outcome  $X^*$  to cash flows, and the conversion of future cash flows into a present value (an asset price).

Finally, by value additivity, we can subtract the present value of costs, denoted  $C(k)$ , and represent the total value as:

$$V(k) = Q(k) - C(k) .$$

**1b.** We have shown that  $V(k)$  is a well-defined function of the integer variable  $k$ . Therefore,

$$\max_k V(k)$$

is well-defined. The highest economic value in the present is obtained by selecting  $k^* = \operatorname{argmax} V(k)$ . (Note that while there is only one  $\max V(k)$ , there may be several  $\operatorname{argmax}$ es of the function. Selecting any  $\operatorname{argmax}$  suffices for optimization.)

**1c.** If an economic process is *ex ante* uncertain, rankable, contingent, non-exclusive and *not costly*, then there is no *ex ante* upper bound to the number of times it should be run. The expectation of the maximum of  $k$  trials of the process is then strictly increasing in  $k$ .

But if the process is costly, then there may be an upper bound to the number of times the process should be run. Indeed, there exist cost structures that can serve to make any number of trials, from 0 to infinity, the optimal number of trials. One such cost structure is:

$$\begin{aligned} C(k) &= k\varepsilon && \text{where } \varepsilon < Q(k^*+1) - Q(k^*) , && 0 \leq k \leq k^* ; \\ C(k) &> kQ(1) , && && k > k^* . \end{aligned}$$

Under this cost structure, the optimal number of trials is  $k^*$ , which can be 0 or any positive integer. The cost structure works because  $Q(k+1) - Q(k)$  is strictly decreasing in  $k$ . Thus a per-trial cost less than  $Q(k^*+1) - Q(k^*)$  justifies investment in trials 1 through  $k^*$ ; while a per-trial cost greater than  $Q(1)$  makes all trials unprofitable. **QED.**

All design processes are *ex ante* uncertain; costly; and have non-exclusive outputs. Virtually all designs are *ex post* rankable by outcome within an appropriate functional category. And many designs can be made *ex post* contingent by separating the design process from the production process for the artifact in question.

However, many large design processes can be split into smaller units of activity, and thus design processes are not in general indivisible.

## 2 The Fundamental Theorem in Modular Systems

*Modularization* is the process of splitting a large design into coherent units (“modules”) that can function together as an integrated whole. By definition, *modules* are indivisible units of design activity within a larger, divisible and hierarchical system.

Design processes focused on individual modules generally conform to the premises of the fundamental theorem, hence their value representations have the simple  $Q(k) - C(k)$  structure described above. However, moving up to the next level of aggregation, *modular systems* offer many complex and interesting ways of combining and recombining modules.

In *Design Rules, Volume 1*, we attempted to capture those opportunities via six modular operators:

- splitting;
- substituting;
- excluding;
- augmenting;
- inverting; and
- porting.

The operators are logical actions that are applicable to individual modules and/or subsets of modules in a system of designs.

We went on to derive a generic value representation for each of the operators. Not surprisingly, one or more  $Q(k)$ -type functions appeared in the value representation of each operator.

The value of a system of modular design processes can thus be represented as a complex aggregation of  $Q(k)$ -type real options. However, the combinatorial properties of the operators quickly outpace the computational capacity of any known information-processing entity. As a

result, even in small modular systems it is impossible to calculate, much less optimize, the value of the system over all possible alternative paths. In effect, sheer complexity mandates both decentralized decision-making and less-than-perfect optimization of an evolving modular system of designs.

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