The Winner's Curse: Theory and Experiments*

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The Winner's Curse: Experimental Evidence

Abstract

We conduct experiments on common value auctions with rationing. In each auction, the good is randomly allocated to each of the k highest bidders, at the $(k+1)^{st}$ highest price. As the degree of rationing increases, the theoretical winner's curse decreases, and the equilibrium bid function increases. Experimentally, we find that bidders suffer from the winner's curse, and lose money on average. However, the bids in the experiments do adjust in the appropriate direction as the degree of rationing changes, providing support for the comparative statics implications of the theory. We demonstrate the connection between the experimental results and a theoretical measure of the winner's curse. Our results are consistent with subjects having an intuitive understanding of the winner's curse, but being unable to compute the equilibrium bid levels.

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1 Introduction

The winner's curse (that is, the failure of winners in a common value auction to condition on the fact that winning is informative) is well documented with both field and experimental data. Theory predicts that rational agents in standard auctions should understand that winning implies that they have the highest signal, revise their bids accordingly, and thus never fall prey to the winner's curse.[1] In practice, it appears that agents do not compensate for this effect, and lose money in common value auctions. This has been shown with field data by Capen, Clapp and Campbell (1971) and Hendricks and Porter (1988).[2] Kagel and Levin ([2002], Chapter 1) provide an extensive survey of the experimental findings. The winner's curse is pervasive across different auction formats and different bidder populations.

If bidders in common value auctions do fall prey to the winner's curse, it is important to understand why, both for the robustness of the theory and for policy reasons. Many explanations are possible, and these are not mutually exclusive. An extreme explanation is that bidders simply do not understand the information content inherent in winning a common value object. Alternatively, it is possible that they do understand this, but cannot determine the appropriate equilibrium bid. The equilibrium bid is computationally complex: it typically involves a conditional expectation, using the distribution of an order statistic.

In this paper, we describe a series of auction experiments in which we vary the degree of the winner's curse, and hence the equilibrium bid functions. In keeping with much of the existing literature, we find that agents do not compensate enough for the existence of the winner's curse, and pay too much for the object. However, in contrast to the existing literature, we find that, in several cases, they do adjust their bids in the right direction as the size of the winner's curse changes. Thus, if the winner's curse is reduced, they do bid more aggressively but not sufficiently so.

The framework that we use is a common value auction with rationing. Specifically, we solicit sealed bids from a fixed number (n) of potential buyers, and then allocate the good to each of the k highest bidders, with equal probability $\frac{1}{k}$, at the $(k+1)^{st}$ highest price. When $k = 1$, this reduces to the second-price auction as the highest bidder wins the good with probability 1. This case provides us with a benchmark

[1] McAfee and McMillan (1987) and Wilson (1990) survey the theoretical literature.

[2] In a notable exception, Nyborg, Rydqvist and Sundaresan (2002) find evidence in Swedish Treasury bond auctions that bidders behave as if they correctly compensate for the winner's curse.

against which to compare our results to the existing literature. As k varies, the event 'win' becomes less informative. So, in the first price auction, in a symmetric equilibrium with increasing bid functions, the winner knows that his signal is the highest. When k is two, a winner only knows that his signal was one of the two highest, and so forth.

We define two theoretical notions of a winner's curse. The first determines the expected loss of a winner if all bidders naïvely bid the expected value conditional only on their own signals, and excluding the information contained in winning the auction. This conditional winner's curse decreases with k, and increases with n. We then determine an unconditional winner's curse: This is the ex ante expectation of the amount of money an agent will lose, after taking into account the probability of winning the auction (which is essentially $\frac{1}{n}$ with symmetric signals). This second measure also decreases with k, but the effects of n are somewhat ambiguous.

In the experiments, we find that agents do suffer from the winner's curse, and, on average, bids are too high. Having rejected the equilibrium predictions of the theory, we then examine the qualitative implications. Our model allows us to test the comparative statics effects of changes in k and n. We find that subjects do, in fact, compensate in the right direction as k changes. That is, when k increases for a fixed n, bidders bid more aggressively, in accordance with the theoretical predictions. Our results on the comparative statics as n changes are somewhat mixed. For the second-price auction (that is, when $k = 1$), our findings are consistent with those of Kagel and Levin (1986), who find that increasing the number of bidders exacerbates the winner's losses in the first-price auction, and Kagel, Levin and Harstad (1995), who demonstrate the same effects in the second-price auction. Interestingly, we find that when $k = 2$, bidders again adjust their bids in the appropriate direction when n changes.

We demonstrate a relationship between changes in one of our theoretical measures of the winner's curse (the unconditional one), and changes in the experimental bid functions. Given this, our results suggest that agents understand some of the conditioning events of the winner's curse, but cannot compute the equilibrium bid level correctly.

Our theoretical model further predicts that revenue for the seller is increasing in the degree of rationing. However, in the experiments, we find that revenue decreases with rationing. This is in keeping with the notion that our bidders do not sufficiently adjust their bids across different auctions.

We provide some methodological innovations in analyzing our data. First, we test to see whether agents are playing best responses. We do this by calculating the payoffs they would have obtained if, instead of their actual bids, they had made the symmetric equilibrium bid. Since about a third of our subjects are not playing best responses, this helps us rule out asymmetric equilibria. Second, in addition to estimating the parameters of the theoretical bid functions, we test a strong implication of the theory: Observed deviations from equilibrium should be uncorrelated with any exogenous variables. We find that exogenous variables do explain these deviations.

Our experimental design follows the lead of Kagel and Levin (1986). We consider a common value object whose value is drawn from a uniform distribution. Bidders receive private signals that are also drawn from a uniform distribution centered around the true value of the object. Bidders are asked to submit sealed bids via computer terminals. After all bids have been received, the object is allocated with equal probability to one of the k highest bidders, at the $(k+1)^{st}$ highest price.

We test the behavior of our subjects against the Bayesian Nash equilibrium of the one-shot auction with rationing, the symmetric equilibria of which have been characterized by Parlour and Rajan (2002) and Harstad and Bordley (1996). Theoretically, the equilibrium in the mechanism we use is equivalent to one in which the seller divides the object evenly into $\frac{1}{k}$ pieces, and allocates one piece to each of the highest k bidders. Thus, our model draws upon previous work in auction theory, notably Milgrom (1981) and Pesendorfer and Swinkels (1997). Milgrom considers an auction for a common-value object, where the seller has k units for sale and each buyer demands one unit. The k highest bidders are each given one unit, at the $(k+1)^{st}$ highest bid. Pesendorfer and Swinkels (1997) show that the k-unit auction has a unique symmetric equilibrium (albeit under conditions that are stronger than those in our paper) and examine properties of the convergence of the price to the true value of the object.[3]

In Section 2 we describe the model, while the experimental design that we used is detailed in Section 3. Summary results are presented in Section 4, and detailed tests of the theoretical model are presented in Section 5. An interpretation of our results and the winner's curse is presented in Section 6. All proofs appear in Section 7.1. Section 7.2 contains a reproduction of the instructions given to the subjects.

[3]Jackson and Kremer (2001) compare the efficiency and revenue of different auction formats in this setting as n becomes large.

2 Theoretical Framework

Consider the following mechanism: A seller sells an indivisible object to n risk neutral bidders. The value of the object to each agent is denoted by v. We assume that v is uniform on $[v_\ell, v_h]$. Each agent receives a signal s that is drawn from a uniform distribution on $[v-\epsilon, v+\epsilon]$. This model, therefore, exhibits common values (v is the same for all agents) and affiliated signals. Conditional on his signal, each agent submits a sealed bid, which we denote $b(s,k)$.

We define an auction with degree of rationing k, where $1 \leq k \leq n-1$, as one in which the good is allocated with probability $\frac{1}{k}$ to each of the k highest bidders. The winner then pays the $(k+1)^{st}$ highest bid. Thus, for $k=1$ this is equivalent to a 2nd price auction as the highest bidder wins the good and pays the second highest price.

This class of auctions has been analyzed by Harstad and Bordley (1996) and Parlour and Rajan (2002). We use results in these papers to establish properties of the bid functions in symmetric equilibrium. First, let $Y_{j,n-1}$ be the Y_jth order statistic out of $n-1$ draws, where $Y_j > Y_{j+1}$.

Lemma 1 *In the symmetric equilibrium of an auction with degree of rationing k, each bidder bids the expected value of the asset conditional on his signal being equal to the k^{th} highest of the remaining $(n-1)$ players' signals: $b(s;k) = E(v \mid s = Y_{k,n-1})$.*

The proof of Lemma 1 proceeds by showing that the equilibrium in an auction with degree of rationing k is the same as that in the k-unit auction, for which the equilibrium was exhibited by Milgrom (1981).

Given our distributional assumptions, we can solve for the symmetric equilibrium bid functions in closed form. Notice, however, that the functional form of the bid depends on the signal realization. This is because participants in the auction know that the asset value can never be less than v_ℓ or more than v_h. Thus, for bidders receiving low ($s < v_\ell + \epsilon$) or high ($s > v_h - \epsilon$) signals, their posterior beliefs over the underlying value of v are truncated at the asset's upper and lower bounds respectively. Thus, the posterior beliefs over the asset value are no longer uniform.

The signals for which a bidder's posterior beliefs are never truncated we term interior signals. Thus, interior signals are those in the range $[v_\ell + \epsilon, v_h - \epsilon]$. Notice, that if the true value of the asset lies in the range $[v_\ell + 2\epsilon, v_h - 2\epsilon]$, then all the signals that could possibly be generated are interior signals. We refer to such realizations of the common value of the object as interior values.

The bid functions are obtained by integrating over the relevant order statistic distribution. If a signal s is not close to the end points (that is, if s is interior so that $s \in [v_\ell + \epsilon, v_h - \epsilon]$), then the bid function is linear is signal. When $s < v_\ell + \epsilon$ or $s > v_h - \epsilon$, the bids are non linear. In particular,

Proposition 1 *(i) For interior signals, that is,* $s \in [v_\ell + \epsilon, v_h - \epsilon]$*,*

$$b(s;k) = s + \epsilon \left(\frac{2k}{n} - 1 \right). \tag{1}$$

(ii) For signals in the lower corner, $s < v_\ell + \epsilon$*,*

$$b(s;k) \quad = \quad s + \epsilon \left(1 - 2 \sum_{i=0}^{k-1} \frac{x(s)^{n-i}(1-x(s))^i (n-k)!}{i!(n-i)!} \right) \tag{2}$$

where $x(s) = \frac{s - v_\ell + \epsilon}{2\epsilon}$.
(iii) For signals in the upper corner, $s > v_h - \epsilon$

$$b(s;k) \quad = \quad s + \epsilon \left(1 - 2 \frac{(n-k)\left(\frac{1}{n!} - \sum_{i=0}^{k-1} \frac{x(s)^{n-i}(1-x(s))^i}{i!(n-i)!} \right)}{\left(\frac{1}{(n-1)!} - \sum_{i=0}^{k-1} \frac{x(s)^{n-i-1}(1-x(s))^i}{i!(n-i-1)!} \right)} \right) \tag{3}$$

where $x(s) = \frac{s - v_h + \epsilon}{2\epsilon}$.

Intuitively, the bid function is strictly increasing in the degree of rationing (or k), for all signals except the most extreme.[4] Increasing k changes the information content of the event "win." Compare, for example, the case of $k = 1$ (which corresponds to the second price auction) and the case of $k = 2$. If a bidder wins when $k = 1$, he knows that he has received the highest signal and thus takes this into account when he bids. This, of course, is the equilibrium adjustment for the winner's curse. If he wins when $k = 2$, then he knows that he got one of the two highest signals. Thus, with some probability he knows that he got the second highest signal. Hence, the amount by which he shades his bid is less. That is, rationing mitigates the winner's curse, and causes bidders to bid more aggressively.

[4]That is, for all signals except $v_\ell - \epsilon$ and $v_h + \epsilon$, at which points the true value of the asset is known with certainty.

To illustrate the effects of changing k for a fixed n, we plot the bid functions for the case of $n = 8$, and $k = 1, 2, 4$. The parameter values are those that we use in the experiments. In particular, $v \in [100, 300]$, and $\epsilon = 30$. As seen from Figure 1, bids are increasing in k for a fixed signal. Further, the bid function is linear for interior signals (in the range $[100 + \epsilon, 300 - \epsilon]$). Using $k = 1$ as a benchmark, for interior signals, bids are higher by \$7.50 when $k = 2$, and by \$22.50 when $k = 4$.

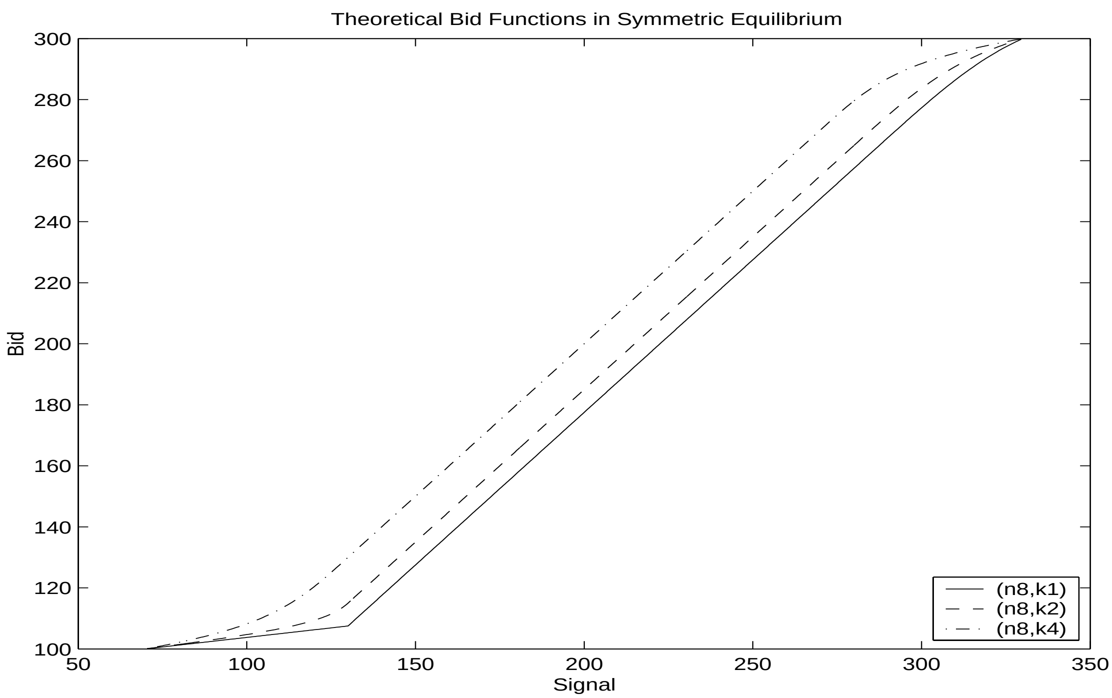


Figure 1: Bid function for n8, k=1,2,4

As a basic test to check that our subjects understood the common value structure of the model, we also consider the equilibrium bids that would obtain were agents to believe that their signals reflected a private consumption value.[5] In this case, it is a weakly dominant strategy for an agent to bid his signal.

Proposition 2 *Suppose agent i believes that his consumption value is s_i. Then, bidding $b_i(s, k) = s_i$ is a weakly dominant strategy for agent i.*

It follows immediately that, if all agents believe they are in a private values environment, then the symmetric equilibrium is for each agent to bid her signal.

[5]In Section 4, we show that the private values model is rejected.

In the experiments, we vary k, n, and ϵ. Each of these changes the size of the winner's curse, but they work in somewhat different ways. Changing k changes the information content of the event "win." In symmetric equilibrium, a higher k means that an agent could win without having the highest signal.[6] Changing n or ϵ changes the expected value of the object, conditional on having one of the k highest signals. Determining the precise expected value of the object under these conditions has a significant computational dimension.

Observe that, if $\frac{k}{n} = \frac{1}{2}$, then, for interior signals, the symmetric equilibrium has agents bidding their signals. Intuitively, if $\frac{k}{n} = \frac{1}{2}$, a bidder in the winning set knows that at worst, half the bidders in the auction had a lower signal and half had a higher signal — thus he is the mean or median signal and, as his posterior over the value of the asset is uniform, the expected value of the asset is just his signal. This logic applies only to interior signals as the posteriors generated by other signals are no longer uniform. Graphically, the highest curve in Figure 1 is the one for which there is no winner's curse, while the vertical difference between the bidding functions represents the adjustment made in each auction.

While we have characterized the symmetric equilibrium, there are also asymmetric equilibria. In particular, there exist a continuum of asymmetric equilibria. For example, if $k = 1$, then if there is an aggressive bidder in the auction he could bid anything larger than v_h and win the auction for sure while the most that a 'passive' bidder would bid is $E[v \mid s = Y_{1,n-1}]$. As we have no ex ante way of sorting agents as aggressive or passive, we test for the symmetric equilibrium.

2.1 Seller's Revenue

Would a seller ever choose to institute rationing? While bidders bid more aggressively for a higher degree of rationing (for the same signal, the bid in an auction with degree of rationing k is lower than the bid in an auction with degree of rationing $k + 1$) the seller awards the good at the bid of a lower signal (at the $k + 2$nd bid instead of the $k+1$st bid). Thus, it is not immediately obvious that a seller would prefer to institute rationing in an auction as he trades off the more aggressive bidding strategy against a lower valuation type.

The revenue to a seller, running a kth order rationing mechanism, for a fixed

[6] The comparative static effects of changing k hold even out of equilibrium, as long as some agents are playing strategies that have bids monotonically increasing in signal.

v, is the expected value of the bid of the $k+1$st order statistic. Thus, $R(v;k) = E_{Y_{k+1,n}}\left(b(s;k) \mid v\right)$.

Given the bid functions, for interior values of v, that is $v \in [v_\ell + 2\epsilon, v - 2\epsilon]$, Harstad and Bordley (1996) show that revenue is

$$R(v;k) = v - 2\epsilon \left\{ \frac{1 - \frac{k}{n}}{n+1} \right\}. \tag{4}$$

Observe that revenue is increasing in k for all n. For corner values of v, we do not have a closed form solution (because of the non-linear bid function when signals are in the corners). However, we can compute the expected revenue.

To illustrate the change in revenue as k changes, we consider $n = 8$, $\epsilon = 30$, and plot the differences between the revenue function for $k = 2$ and $k = 1$, and $k = 4$ and $k = 1$. Figure 2 displays this plot. For interior values of v, going from $k = 1$ to $k = 2$ raises expected revenue by \$0.83, and going from $k = 1$ to $k = 4$ raises expected revenue by \$2.50. The difference in revenue as k changes is, therefore, an order of magnitude smaller than the difference in bids.

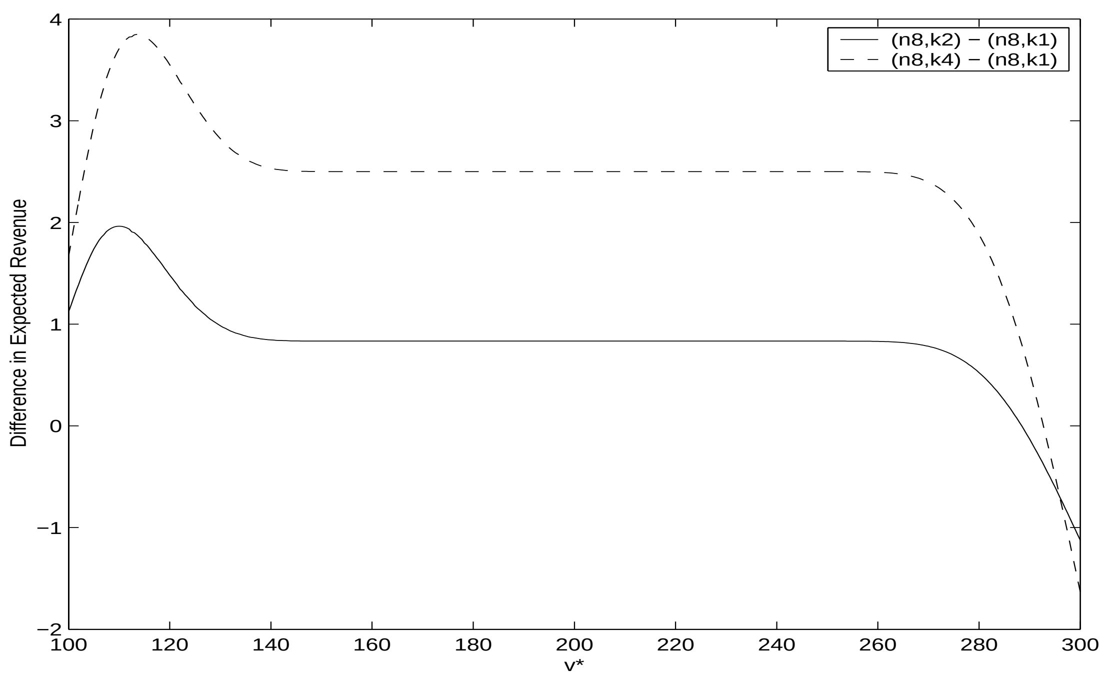


Figure 2: Difference in Revenue Functions for n8, k=1,2,4

In the experimental context, the revenue function above provides a further test of the outcomes of the experiment. That is, it is possible for individual bidders to be incorrect in some way, and yet for the market outcome to accord with theoretical predictions.

2.2 The Winner's Curse

In equilibrium, there is no winner's curse in these (or any other) auctions (that is, the winner expects to earn money, not lose it). Hence, any discussion of the winner's curse is necessarily relative to out-of-equilibrium bidding behavior. Further, any such discussion must be predicated on an assumed bid function. This is because there are two elements to the winner's curse: the expected consumption value of the good to the winner, and the amount he pays for it. For example, a winner who (out of equilibrium) obtained the proceeds of a Treasury bond issue for \$1 would not feel cursed.

In our auction with rationing, agents do not pay their bids, and thus one can separate out a price effect and a consumption value effect. Both of these, of course, depend on beliefs over other bidders' actions. We therefore define a theoretical measure of the winner's curse as follows. Fix an auction format, and a signal s received by some bidder i. Since we consider symmetric equilibria in this paper, postulate a symmetric bid function, $b(\cdot)$ for all agents.[7] Then, the winner's curse faced by a bidder with signal s is independent of the identity of the bidder.

There are two notions of a theoretical winner's curse we consider. First, we define the amount of money a bidder loses conditional on winning the auction. Suppose bidder i receives signal s, and each player uses the bidding function $b(\cdot)$. Then, conditional on winning, bidder i expects to lose

$$W^C(b,s) = E(\text{Price} \mid s, \text{bidder } i \text{ wins}, b) - E(v \mid s, \text{bidder } i \text{ wins}, b) \qquad (5)$$

This is a "curse," in that, if the price is greater than the value of the object, the winner loses money, and $W^C(b,s)$, the winner's curse, is positive. This definition, therefore, is consistent with the discussion of the winner's curse in Kagel and Levin (2001, page 2): "...you win, you lose money, and you curse." The symmetric equilibrium bid demonstrated in Proposition 1 above, b^*, minimizes this quantity.

[7]To extend the definition below to asymmetric bid functions, one can consider a vector $b(\cdot)$, where $b_i(\cdot)$ is the bid function of the i^{th} player. Indeed, the empirical winner's curse is obtained in this manner by examining actual, rather than theoretical, bids.

Second, we consider the amount a bidder expects to lose after allowing for the probability of the event "win," given the bidding functions $b(\cdot)$. This (unconditional) winner's curse is defined as

$$W^U(b,s) \;\; = \;\; \text{Prob(bidder } i \text{ wins) } W^C(b,s). \tag{6}$$

Notice that we incorporate the probability of winning into this definition of the unconditional winner's curse. Intuitively, the more bidders there are, the less likely an agent is to win, and therefore, the less likely she is to suffer from the conditional winner's curse. The paradox, of course, is that the conditional winner's curse increases with the number of agents.[8]

Most intuitive explanations of the winner's curse consider the case of agents bidding naïvely. That is, suppose agents bid just the expected value conditional on their signal alone, without also conditioning on winning the auction. Then, they expect to lose money in auctions in which the good is awarded to the highest bidder. Consider the naïve bid function, $\hat{b}(s) = E(v \mid s)$. Suppose all bidders use this bid function. Then, in the first-price auction, if bidder i wins, she has the highest signal, and our definition of the winner's curse reduces to

$$W^C(\hat{b},s) \;\; = \;\; E(v \mid s) - E(v \mid s, \text{bidder } i \text{ has the highest signal}).$$

This definition corresponds to a common usage of the term "winner's curse" in the literature (see, for example, Nyborg, Rydqvist, and Sundaresan, 2002, and the references therein). In (5) above, we extend this definition to the case of auctions in which the winner does not pay her own bid.

We first determine the extent of the winner's curse, $W^C(\hat{b},s)$, for a given signal and the naïve bidding function. Given that the distributions of both v and s are uniform, the winner's curse is constant for all s in this range. Recall, that in our set-up, for interior signal values, $s \in [v_\ell + \epsilon, v_h - \epsilon]$, the naïve bid function reduces to $\hat{b}(s) = s$.

Proposition 3 *Suppose* $s \in [v_\ell + \epsilon, v_h - \epsilon]$, *and* $\hat{b}(s) = E(v \mid s) = s$. *Then,* $W^C(\hat{b},s) = \frac{\epsilon}{n+1}\,(n - 2k + 1)$, *and* $W^U(\hat{b},s) = \frac{\epsilon}{n(n+1)}\,(n - 2k - 1)$.

[8] Thaler, 1994, page 51, provides a discussion on this.

Observe that both W^C and W^U are decreasing in k, the degree of rationing. However, W^C is always increasing in n, the number of bidders. The unconditional winner's curse, W^U can increase or decrease as n increases.

3 Experimental Design

The experiments were run over a total of seven sessions. Each experimental session consisted of a sequence of auctions. At each round, a single common value item was auctioned simultaneously in at least 2 parallel markets. Table 1 lists the sessions that were run.

n	k	Date	No. Parallel Markets
4	1	Aug. 31, '01	3
4	2	Aug. 31, '01	3
6	1	Sep. 14, '01	2
6	3	Sep. 7, '01	2
8	1	Aug. 31, '01	2
8	2	Sep. 7, '01	2
8	4	Sep. 14, '01	2

Round	Epsilon
1–13	20
14–26	30
27–28	1
29–30	0

Table 1: Description of sessions

As noted from Table 1, in the seven experimental sessions, we had $n = 4, 6$ or 8, and $k = 1, 2, 3$, or 4. In three of the cases ($n = 4, 6, 8$), k was set to 1. Recall that $k = 1$ corresponds to the 2^{nd} price auction; this value was chosen to provide both a benchmark and a comparison to the prior literature. Further, the values of k and n allow comparison for bidding functions across different numbers of bidders. For example, for $(n, k) = (4, 1)$ and $(8, 2)$, the value of $\frac{k}{n}$ is the same (0.25). This therefore, provides a comparison between a second price auction and an auction with degree of rationing 2. From Proposition 1 (i), the interior bid function is the same in these two cases. Finally, for three cases $((n, k) = (4, 2), (6, 3), (8, 4))$, we had $\frac{k}{n} = 0.5$. The interior bid function in this cases reduces simply to $b(s) = s$; that is, the equilibrium bid in the interior is just the signal.

In each session, we had either two or three parallel markets. The number of bidders per market, and the total number of markets, was held constant during each session. In each auction, all bidders in the room were randomly divided into different markets. Thus, bidders did not know ex ante who they were bidding against. This random assignment prevented the auction from mimicking a repeated game when the number of bidders was small.

In each auction, the true value of the object, which we denote $v^\star$ from here on, was drawn from a uniform distribution on $[100, 300]$. Each bidder received a private signal, s, drawn from a uniform distribution on $[v^* - \epsilon, v^* + \epsilon]$. For each auction we varied ϵ, so that $\epsilon \in \{0, 1, 20, 30\}$. The value of ϵ was announced to bidders before each round. Table 1 lists our sessions, and the ϵ that was used on each round.

We used the rounds with $\epsilon = 0, 1$ only as an informal test of whether the subjects understood the relationship between $v^\star$ and signals; in particular, that the precision of their signals varied with ϵ. Most of our analysis focuses on the cases $\epsilon = 20, 30$. We deliberately chose to have the rounds with $\epsilon = 0, 1$ at the end of the experiment. If the early rounds had featured $\epsilon = 0$ or 1, this could potentially have conditioned agents into bidding their signal, since the symmetric equilibrium bid for these values is either exactly the signal (when $\epsilon = 0$), or very close to it (when $\epsilon = 1$).

The number of bidders (n), the number of parallel sessions, the fact that bidders were randomly assigned to markets, the number of potential winners in each market (k), the value of ϵ, and the distributions underlying both $v^\star$ and s were public information.

The chosen parameters for n, k, ϵ provide differences across auctions in both the amount by which bidders adjust their bids (from their signals) in equilibrium, and in the off-equilibrium winner's curse amounts. Recall that the equilibrium adjustment to an interior signal in the bid function is $(\frac{2k}{n} - 1)\epsilon$. Table 2 indicates the differences across auctions. In Section 6, we use this table to interpret our results.

In the instructions to the bidders (see Section 7.2), the language we used was non-technical. The intent here was to facilitate understanding among the subjects, and minimize the possibility that their behavior could be affected by language. For example, we referred to the object to be auctioned as an "envelope," and to $v^\star$ as the "amount of money in the envelope." The signal was referred to as a bidder's "private estimate."

In each session, after the instructions were read aloud, subjects played two practice rounds, to verify that everyone understood how to submit a bid, and how to interpret the information on the screen. The experiment was run using the ComLabGames

			Equilibrium bid	Winner's Curse	
n	k	ϵ	Adjustment	Conditional: W^C	Unconditional: W^U
4	1	20	-10.00	4.00	1.00
		30	-15.00	6.00	1.50
4	2	20	0	-4.00	-1.00
		30	0	-6.00	-1.50
6	1	20	-13.33	8.57	1.43
		30	-20.00	12.86	2.14
6	3	20	0	-2.86	-0.48
		30	0	-4.29	-0.71
8	1	20	-15.00	11.11	1.39
		30	-22.50	16.67	2.08
8	2	20	-10.00	6.67	0.83
		30	-15.00	10.00	1.25
8	4	20	0	-2.22	-0.28
		30	0	-3.33	-0.42

Table 2: Equilibrium bid adjustment and winner's curse for interior signals

software.

On each round, before bidding, each bidder saw her own signal. Bidders were then asked to (i) write down on a piece of paper the amount of money they thought they would make if they won the auction,[9] and (ii) submit a sealed bid, on the computer.

Bids were entered on a computer, and subjects were allowed to change their bid before the auction was closed. Subjects only saw their own bid until the auction was closed. Once the auction was closed, bids could not be changed. All bids had to be rounded to the nearest cent, and had to be greater or equal to zero.

No other instructions or restrictions on a valid bid were given to subjects. This differs from, e.g, the experiments conducted by Kagel and Levin (1986), who required subjects to bid no more than the signal plus epsilon, and also displayed for subjects the

[9]This information was collected because we hoped to tease out strategic effects in bidding, as opposed to mistaken beliefs about the value of the asset. However, it was uncorrelated with bids: Indeed, it was so noisy that we excluded it from our analyses. The point of having them write it on paper was to reinforce the idea that only their bid (entered on the computer) affected their payoff.

smallest and largest possible values of $v^\star$, given their own signal. The first requirement might lead to a downward bias in bids.[10] The second may induce subjects to bid within the displayed range.

After the auction was closed, the item was randomly awarded to one of the k highest bidders in each parallel market. Thus, in each of the parallel markets, each of the k highest bidders had an equal probability, $\frac{1}{k}$, of winning the object. The winner's profit was the value of the item ($v^\star$) less the price, that is, the $(k+1)^{st}$ highest bid. Hence, a bidder's expected payoff in each auction period was:

$$\begin{cases} \frac{1}{k}\left(v^* - b_{k+1}\right) & \text{if among the } k \text{ highest bidders} \\ 0 & \text{otherwise.} \end{cases}$$

The information available to bidders at the end of each auction round included detailed information for the particular market they were in. For this market, they were shown all bids (sorted from highest to lowest), the corresponding bidder's identification number and private estimate (s), the true value $v^\star$ (which was, of course, common for all bidders on that round) and the profit or loss of each bidder on that round. This profit or loss also revealed the identification number of the bidder who was awarded the object (all other bidders had a profit of zero).

In addition, bidders were presented with summary information on all markets that were open in the previous round. This included the price in the auction, and the identification number and profit (or loss) of the winning bidder. The value $v^\star$ was the same in each market in any given round. This information was also recorded on a board in the room, both to provide a short (about 30 seconds or so) break between rounds, and to ensure that subjects knew this information was being provided to them.

None of our subjects had previously participated in auction experiments of any kind. Our subjects were first- and second-year MBA students at the Graduate School of Industrial Administration, Carnegie Mellon University. No special skill or experience was required for participation. Subjects were allowed to participate in only one of the seven sessions.

[10]Kagel and Levin find that, in the first-price auction, subjects overbid, so imposing an upper limit makes their result even more surprising.

3.1 Replicating a one-shot game

We wished to ensure that the sequence of auctions approximated the one-shot model. Therefore, renumeration was not based on cumulative totals, but rather assigned randomly. Each session lasted thirty rounds. At the conclusion of the session, ten rounds were chosen at random, and subjects were paid \$30 plus their net earnings for the selected rounds in cash. Losses from the selected rounds were subtracted from the starting capital, and profits added to it.

The starting balance of \$30 was determined by the prevailing wage rate for MBA students at Carnegie Mellon (\$15 per hour), and the time taken by a session (approximately 2 hours). This amount, then, represents the opportunity cost of time for the subjects.

Subjects were told that (i) if their balance was zero or less they would be paid \$0 for participating, and (ii) the payments would be capped at \$100.[11] The students, therefore, had limited liability. In Subsection 4.1, we discuss this in greater detail.

We randomly selected ten of the thirty rounds to determine payment, rather than accumulate the payments from round to round. This was done for two reasons. First, the theoretical model is a one-period game, and accumulating earnings would introduce an additional dynamic element. Also, empirical evidence suggests that the payoff from previous rounds has an effect on current bidding strategy (see Ham, Kagel, and Lehrer [2000]).

Second, if cumulative earnings are used, there is the possibility of bankruptcy before the experiment is over. In this case, one has to either have fewer bidders in the auction for the remainder of the session (for example, as in Kagel and Levin [1986]), or introduce new bidders. The latter, in turn, can be done by either having extra bidders in each experimental session (as in Levin, Kagel, and Richard [1996]), or replacing missing bidders with computers (as in Goeree and Offerman [2000]). Since our experiments were explicitly about how bidders react as k and n are changed in the auction, none of these designs were appropriate.

[11] This cap was almost, but never, hit: the maximum earned in any session was \$99.63. The next highest payment was \$63.41.

4 Summary of Bidding Behavior and Equilibrium Outcomes

We first present summary statistics of the bidding in each auction. We report results for large ($\epsilon = 20, 30$) and small epsilon ($\epsilon = 0, 1$) separately. There are theoretical and empirical differences between these two cases. In particular, when ϵ is small, the comparative static effects on the bid functions of changing n or k are either non-existent (when $\epsilon = 0$) or very small (when $\epsilon = 1$).

We use the data from rounds 27–30 ($\epsilon = 0, 1$) in two ways: (i) to determine whether agents bid symmetrically, and (ii) to infer whether agents understood that the precision of their signals varies with ϵ.

Table 3 reports summary statistics from rounds 27–30 ($\epsilon = 0, 1$) on the various auctions, and Table 4 reports the corresponding statistics for rounds 1–26 ($\epsilon = 20, 30$). In both tables, the number of bid observations corresponds to the total number of signals across all bidders in that auction. The number of profit observations corresponds to the total number of rounds in that auction times the number of markets open on each round.[12]

When $\epsilon = 0$, signals are fully informative (that is, $v = s$). At $\epsilon = 1$, v must lie within 1 of s. In a symmetric equilibrium, the bids are very close together (they are all equal for $\epsilon = 0$, and within 2ϵ, or 2, when $\epsilon = 1$). Table 3 indicates that the actual bids have a much larger variance, with a minimum bid of 0 across the seven sessions, and a maximum bid of 10,000. The variation in bids suggests that agents are not bidding symmetrically. In the regression tests of the theory in Section 5, we therefore examine error components models, that allow for individual-specific behavior. We note that, for $\epsilon = 0, 1$, a large proportion of bids (86.46%) are within 5% of the theoretical bid.[13]

The variation in bids can be explained by the low costs of deviating. The theoretical expected profits in equilibrium are small when $\epsilon = 1$, and 0 when $\epsilon = 0$. Hence, deviating to a low bid (even a bid of zero) does not reduce a bidder's expected payoff by very much. Conversely, if there are low bids observed, a bidder stands a chance of making money with a high bid. Even if the dispersion of bids is high, the

[12]Due to a computer error, one round of data was lost in the $(n4, k2)$ auction for $\epsilon = 30$, leading to a lower number of rounds and bid observations, as compared to the other auctions.

[13]In absolute dollars, 5% of the theoretical bid lies between 5 (if $b_t = 100$) and 15 (if $b_t = 300$).

Auction		Bids				Profits			
n	k	No.	% in range	Min.	Max.	No.	Min.	Max.	Mean
4	1	48	91.67	22.22	223.24	12	-1.24	-0.01	-0.37
									(0.12)
4	2	48	81.25	90	500	12	-1.76	2.24	-0.06
									(0.35)
6	1	48	93.75	50	223	8	-1.74	0.10	-0.49
									(0.21)
6	3	48	87.50	0	300	8	-0.74	0.59	-0.05
									(0.16)
8	1	64	81.25	0	1,000	8	-1.39	0.26	-0.26
									(0.19)
8	2	64	93.75	100	212	8	-2.90	0.11	-0.52
									(0.36)
8	4	64	78.13	1	10,000	8	-0.74	0.34	-0.09
									(0.14)
TOTAL		384	86.46	0	10,000	64	-2.90	0.58	-0.25
									(0.09)

Notes:
(i) "% in range" = Proportion of actual bids that are within 5% of the theoretical bid.
(ii) Standard errors in parenthesis.

Table 3: Summary statistics for $\epsilon = 0, 1$

economically relevant question then is: how close is the price that sets the bid to the symmetric equilibrium price, or, alternatively, how large are the profits or losses?

The empirical variation in profits is relatively small for $\epsilon = 0, 1$. The average profit per round was $\$-0.25$ (i.e., a loss of 25 cents). Although this is significantly different from zero, as a proportion of v^*, the mean profit per round was just -0.145%.[14] The range of profits was -\$2.90 to \$2.24, so the bid that set the price was within \$3 of the theoretical price in every instance.

We conclude from this that, although some bids do not correspond to the symmetric equilibrium, on the whole it remains a relevant benchmark for analysis. Further,

[14] By contrast, for the $\epsilon = 20, 30$ cases, the mean profit per round was -3.038% of v^*.

Auction		Bids				Winner's Profits			
n	k	No.	% Cons.	Min.	Max.	No.	Min.	Max.	Mean
4	1	312	89.42	0.01	310	78	-38.80	22.94	-4.36
									(1.33)
4	2	300	63.33	30	10,000	75	-60.10	62.70	5.09
									(2.47)
6	1	312	85.89	75	500	52	-41.66	9.00	-11.34
									(1.55)
6	3	312	84.93	95.46	1,555.45	52	-39.25	34.47	-1.32
									(1.72)
8	1	416	83.41	0	1,162.43	52	-37.52	13.64	-11.00
									(1.49)
8	2	416	79.32	1	500	52	-33.17	16.95	-10.02
									(1.58)
8	4	416	85.57	73.03	1,001	52	-32.78	20.75	0.62
									(1.34)
TOTAL		2,484	81.92	0	10,000	413	-60.10	62.70	- 4.06
									(0.73)

Notes:
(i) "% Cons." = Proportion of bids in the range $\max(100, s-\epsilon)$ to $\min(300, s+\epsilon)$.
(ii) A computer-related error led to one Round (round 18) in the $(n4, k2)$ session being dropped.
(iii) Standard errors in parenthesis.

Table 4: Summary statistics for $\epsilon = 20, 30$

agents understand that the precision of their signals varies with ϵ.[15] Finally, we will interpret our results in light of the empirical variation in bids in the $\epsilon = 0, 1$ case.

For the rest of the paper, we restrict our analysis to the case of $\epsilon = 20, 30$. When $\epsilon = 0, 1$, the bids under private or common values are much closer together. Thus, it is difficult to test whether agents behave as if they were in a private value environment, instead of the actual situation they faced. Further, for $\epsilon = 0, 1$, there is no significant computational issue in determining the expected value of the asset (conversely, the cost of an error is minimal). For $\epsilon = 20, 30$, the cost of an error in computing the

[15]In particular, a comparison of profits with $\epsilon = 20, 30$, reported in Table 4 suggests a much larger variation in profits in the latter cases.

expected value is much larger.

How close are actual bids to the theoretical ones, when $\epsilon = 20, 30$? Given a signal s, the true value of the asset, v^* must lie between $\max\{100, s-\epsilon\}$, and $\min\{300, s+\epsilon\}$ (that is, given s, this is the support for the posterior belief over v^*). Agents playing a symmetric equilibrium, but unable to compute the conditional expectation given their posterior over $v^\star$ may bid anywhere in this range. Recall that, in our experiments, we did not either directly inform the agents about the possible lower and upper bounds for asset value, or restrict them to bidding in that interval. Yet, as Table 4 shows, we find that, for $\epsilon = 20, 30$, 81.92% of total bids lay in this range. We describe such bids as *consistent* bids; that is, these bids are consistent with bidding in a symmetric equilibrium.

As we expect from the case of $\epsilon = 0, 1$, there is a wide range of bids (from 0 to 10,000) for $\epsilon = 20, 30$. The bidder who bid 1,555.45 in the $(n6, k3)$ auction immediately called us over and pointed out that that bid was a typo.[16] The volatility in the $(n4, k2)$ auction is exemplified by the high minimum and maximum profits (which, in this case, were earned by the same bidder on two successive rounds).

The range of profits when $\epsilon = 20, 30$ is much larger than when $\epsilon = 0, 1$. The minimum profit was -\$60.10, and the maximum \$62.70 (these were earned on successive rounds by the same bidder in the $(n4, k2)$ auction). The average profit is -\$4.06, again significantly different from zero. The wide range of profits is consistent with agents having trouble in computing the expected value of the asset, given s.

Intuitively, for larger ϵ, uncertainty about the underlying value of the asset is higher. Thus, if agents have trouble computing conditional expected values, or forming beliefs over other players' bids, the difficulty should be more marked for larger ϵ.

4.1 Checking for Limited Liability Effects

As in many previous auction experiments (e.g., Kagel and Levin [1986], Harstad, Kagel, and Levin [1995]), our subjects had limited liability.[17] However, there are

[16] As indicated in Section 3, bidders were given a chance to change their bids before the market closed; this particular bidder informed us about the bid after the market closed.

[17] Hansen and Lott (1991) and Kagel and Levin (1991) discuss the implication of limited liability for bidding strategies in first price common value auctions. Lind and Plott (1991) find the existence of a winner's curse even when controlling for limited liability effects.

several factors that mitigate the effects of limited liability on our results; we argue in this section that these effects were very small in our experiment.

First, limited liability was a feature of all our sessions. Hence, unless it causes differential overbidding in different sessions, our comparative statics results on comparing bid functions as k and n change should remain valid.

Second, we paid no participation fee. Out of 96 subjects, 8 failed to receive a payment. These subjects spent two hours at the experiment, incurring an opportunity cost of about \$30 overall.

Finally, randomization over rounds for potential payoffs implies that the expected loss in any given round was only a third of the actual loss sustained by any agent on that round. For completeness, we computed the expected payoff of agents after each round as if we had accumulated payoffs through the experiment, rather than drawing ten rounds at random at the end. This is what agents could have inferred about their cumulative expected balance after each round.

To compute this cumulative expected balance, we start with the initial balance (\$30), and add a third of an agent's actual gains or losses on a round to her cumulative balance before that round. The 1/3 factor reflects the probability that a particular round would be drawn for payment, at the end of the experiment (since 10 of the 30 rounds were used for payment purposes). It is important to note that this balance was not computed or reported to agents during the experiment itself.

In computing this balance, we restrict attention to rounds 1–26. The last four rounds (with $\epsilon = 0, 1$) had no appreciable effect on payoffs (as shown in Table 3 in Section 4). As mentioned before, we use these primarily as test rounds to make sure our subjects understood the experiment.

The mean cumulative payoff, i.e., including the starting \$30 (across all players in all sessions) was \$24.18, with a standard deviation of 16.36. The min and max were -\$58.60 and \$69.52, respectively. There was, therefore, a wide variety in the outcomes obtained by agents on this criterion, suggesting that agents were behaving as if they had a sufficient economic incentive to earn money in the auction.

At the end of 26 rounds, 90 of the 96 subjects had positive cumulative payoffs, and 6 had a negative one. Table 5 below shows some details about these bidders.

Of the 6 players who faced the possibility of bankruptcy, 5 bid more conservatively after hitting a negative balance (again, we emphasize that the cumulative balance was not computed during the experiment, or reported to bidders). For these 5 subjects,

n	k	Player ID	CB ($)	R_1 (round no.)	EC_b ($)	EC_a ($)
4	1	1482	-1.87	26	-1.27	0.00
4	1	1925	-8.44	20	-2.09	0.16
6	1	1358	- 58.60	10	-4.09	-2.44
8	1	1734	- 23.42	19	-1.72	-2.09
8	2	2569	-4.15	19	-1.68	-0.49
8	2	1573	-17.12	23	-1.59	-0.71

Notes:
CB = Expected Cumulative Balance after round 26
R_1 = Round after which cumulative balance first became negative
EC_b = Expected Payoff per round on rounds 1 through R_1
EC_a = Expected Payoff per round on rounds R_1 through 26.

Table 5: Bidders with negative cumulative payoff after round 26

the expected cumulative balances are higher in the rounds after the first one in which they sink to a negative expected balance (that is, $EC_b < EC_a$). The concern with limited liability is that it will induce risk-taking behavior. Clearly, from Table 5 this effect does not appear to be present in our data. We are not arguing that, if we had paid subjects on the basis of cumulative expected payoff, they would have bid exactly as they did in our experiment. However, we do claim that there is no reason to suspect that, given our experimental design, subjects behaved any more aggressively after they had reached a negative cumulative expected balance.

5 Tests of the Theory

In this section, we provide formal tests of the theory. First, we test the null hypothesis that players behave as if they are in a common value environment, against the alternative that they treat their signals as private values. We reject the private values model in favor of the common values one, suggesting that agents do behave as if they are aware that other bidders possess information relevant to the valuation of the good.

We then test to see if agents are playing strategically: That is, are they playing a best response? This is a weak test, in that we consider whether deviating to the symmetric equilibrium bid, holding fixed the bids of other agents on that round, would have increased a subject's payoff. We find that, for over a third of the agents, we can reject the hypothesis that they are playing a best response to the actual bids of other agents.

Next, we consider a strong implication of the common values model: if agents behave exactly as predicted by the symmetric equilibrium of our model, then deviations from the theoretical predictions should be unexplained by any of the exogenous variables. Importantly, in carrying out this test, we are able to look at all signal values, including those in the corners, since we have a closed form expression for the symmetric equilibrium bid for any signal value. Hence, this deviation can be computed even when the bid function is non-linear in a given signal range.

We find that these deviations are explained by signal and ϵ. A similar test of the revenue function shows that deviations in revenue are, in turn, explained by v^* and ϵ. Together, these reject the strong form of the theory. This confirms the basic analysis of the previous section, which suggests that bidders are bidding too much on average (and hence departing from the theoretical prediction).

We then turn to the comparative statics implications of the theory. Even if agents are bidding in a manner not consistent with the theory, it is interesting to check if they react in the predicted directions as n and k change. We find that they do respond correctly to changes in k. However, the results on n are ambiguous, consistent with prior literature on common value auctions.

Finally, we examine the revenue function for interior values. We find that the coefficients on ϵ and the variable that includes $\frac{k}{n}$ have opposite signs to those predicted. This, too, is consistent with bidders bidding too much on average.

5.1 Methodology

In the analysis of agents' bids, we eliminate some of the observations from our sample. In particular, we ignore all bids greater than or equal to \$1,000. Table 6 list the bids that were eliminated.

Of the 2,484 observations on bids, only 6 were excluded. There were several reasons for this exclusion:

(i) Bids in the experiment were bounded below by zero. Effectively, therefore, no

n	k	Player ID	Signal	Epsilon	Bid
4	2	1573	312.08	30	10,000
6	3	1573	153.56	20	1,555.45
8	1	1482	160.10	30	1,162.43
8	4	1486	297.32	30	1,000
8	4	1486	313.41	30	1,000
8	4	1925	221.68	30	1,001

Table 6: Outlier bids excluded from analysis

bid could have been more than \$100 less than the minimum value of the object. By retaining bids in the \$400-500 range, we are allowing high bids that were up to \$200 more than the maximum value of the object.

(ii) In the experiment, no bids were received strictly between \$500 and \$1,000. An alternative, therefore, was to replace bids of \$1,000 or greater with \$500. Since the number of observations affected is small relative to the sample size, we decided to eliminate the observations altogether.

(iii) As noted from Table 6, 5 of the 6 excluded bids are in auctions in which, for a fixed value of n, k is at its highest in the sample. Excluding these bids, therefore, is conservative in that it biases our results against the theoretical predictions.

(iv) Finally, in one case (the bid of \$1555.45 in the $(n6, k3)$ auction), the bidder concerned reported (unsolicited) immediately after the round that he had made a typographical error.

For all of our regressions on bid functions, we use a two error components model that captures individual bidder-specific effects. That is, the error terms in all reported regressions on bid are of the form

$$\eta_{it} = u_i + \psi_{it}, \tag{7}$$

where i indexes a particular subject, and t a particular round in an auction.

5.2 Common versus private values

We first test to see if the players behave as if the item has a private or common value. This will determine if our model is the appropriate benchmark. When the bid function is linear, the private values case is subsumed in the common values case, from Proposition 2. Thus, we conducted this test on interior signals (where the theoretical bid function is linear); that is, for signals in the range $[100+\epsilon, 300-\epsilon]$. Recall that these are signals for which the posterior over v^* always lies between 100 and 300.

We ran a regression of the form

$$b_{it} = \beta_0 + \beta_1 s_{it} + \beta_2 \epsilon_t + \beta_3 \left(\frac{k_i}{n_i}\right) \epsilon_t + \eta_{it}, \tag{8}$$

where i indexes the subject, and t number of the round in which the bid was made.[18] Since each subject in the sample participated in only one session, and n and k were fixed in each session, we index both of these with i for convenience.

The error terms were modelled as in equation (7). That is, we assumed the error had two terms, an individual specific component u_i and a pure noise component ψ_{it}. This allows for differences in behavior across individual bidders.

In the private values case, the coefficient on ϵ and $\epsilon\frac{k}{n}$ are both 0, while for the common value case, the coefficients are -1 and 2 respectively. The results of the regression are reported in Table 7 below.

From Table 7, the coefficients on all the explanatory variables other than the constant are significant. Since bids are explained by variables other than the signal (in particular, by both ϵ and $\epsilon\left(\frac{k}{n}\right)$, terms that should affect bids, according to the theory), the private values model is rejected. Further, the coefficient on $\epsilon(\frac{k}{n})$ is not significantly different from its predicted value of 2. We thus conclude that the agents do view the auction as a common value one. On the other hand, the coefficients on signal and ϵ in Table 7 above are significantly different from their predicted values. Hence, we expect the common values model to not fit the data exactly.

[18] Epsilon in our sample takes on the values either 20 or 30. We could equivalently use a dummy variable for ϵ. This would amount to a linear transformation on ϵ, that would change the values of the constant and the coefficient on ϵ, without affecting any other coefficients, or the significance level of any of the coefficients other than the constant.

Variable	Estimate	Predicted Value
Intercept	2.258 (5.780)	0
Signal	0.959* (0.018)	1
ϵ	-0.419* (0.212)	-1 (CV), 0 (PV)
$\epsilon\left(\frac{k}{n}\right)$	1.996* (0.450)	2 (CV), 0 (PV)
# Obs R^2 $p-$value	1695 0.631 0.242	

Notes:
(i) * Significantly different from zero at 5% level.
(ii) CV = Common Value, PV = Private Value
(iii) The $p-$value is for the Hausman statistic.

Table 7: Two error components regression of interior bid function for $\epsilon = 20, 30$.

5.3 Are subjects playing best responses?

Given the mixed support for the theory, can we infer from this that agents are not playing best responses? That is, could they change their bidding strategies to increase their payoffs?

To test this, in each round, in each auction, we computed the expected profit to each bidder from his actual bid, if he was among the k^{th} highest. The expected profit on a round is defined as $\frac{1}{k}(v^* - p)$ (where p is the price paid for the object) if the subject is amongst the k highest bidders, and zero otherwise. We compute the expected profit because, in our design, only one bidder was chosen as the winner, and made a profit or loss on that round. However, bidders are modelled as maximizing expected profit.

We then computed, for each bidder, what his expected profit would be were he to deviate to the symmetric equilibrium bid, given his signal. This was done individually for each bidder. That is, for each bidder i, we substituted his theoretical bid (given his signal) for his actual bid, fixed the actual bids of all other agents on that round, and recomputed the k highest bids, and hence the expected profit to bidder i.[19]

This constitutes a weak test of best responses: if a bidder could have made more money by playing her symmetric equilibrium bid, then she could not be playing a best response. Conversely, however, even if playing the symmetric equilibrium would have entailed a lower payoff, there could be other strategies that were superior to her actual one.

Let Δ be the difference between these two payoffs. That is, $\Delta =$ actual expected profit $-$ expected profit from making the symmetric equilibrium bid at each round. If Δ is negative for a subject, that player is not using a best response strategy, given the bids of other players. That is, there exists another strategy which would have allowed him to earn more money. On the other hand, if Δ is positive, we cannot refute the hypothesis that the subject was playing a best response, since we only consider a specific deviation from the actual strategy.

We performed a one-tailed test on Δ by player. The results of this test are reported in Table 8.

Eight of the subjects in the sample earned zero profit on all rounds, and would also have earned zero if they had made their symmetric equilibrium bids instead. For 33 subjects (i.e., 37.5% of the remaining 88 subjects), we can reject the hypothesis that they are playing a best response, at the 5% level of significance. Conversely, there is no subject in the sample for whom we can reject the hypothesis that they are not playing a best response.

This finding suggests that it is unlikely that any asymmetric equilibrium can be supported by the data. In any equilibrium, asymmetric or symmetric, agents must play best responses. However, for over a third of our subjects, we found one strategy (the symmetric equilibrium bid) that would have yielded a higher payoff than the strategy they actually used.

[19]For computational reasons, we did not randomize over the other subjects the bidder could have been matched with on that round.

Auction (n,k)	$H_0: \quad \Delta \geq 0$ $H_a: \quad \Delta < 0$ Rejections at 5% level	$H_0: \quad \Delta \leq 0$ $H_a: \quad \Delta > 0$ Rejections at 5% level
(4,1)	4	0
(4,2)	7	0
(6,1)	5	0
(6,3)	3	0
(8,1)	3	0
(8,2)	8	0
(8,4)	2	0
Total	33	0

Notes:
(i) The table reports the number of players.
(ii) Total sample size=96 players.

Table 8: One tailed t-test of significance of difference between actual expected profit and expected profit to playing symmetric equilibrium bid

5.4 Departures from Equilibrium Bids and Prices

In this subsection, we test the following strong implication of the theory. We have a closed form for the bid function over the entire signal range (as exhibited in Proposition 1). For each bid observation, we computed the theoretical bid, b^*_{it}, in symmetric equilibrium, given the signal, and n, k. Define

$$\nu_{it} \quad = \quad b_{it} - b^*_{it},$$

as the difference between the actual bid by an agent, b_{it}, and the theoretical bid in symmetric equilibrium, b^*_{it}. Then, one prediction of our model is that ν_{it} has mean zero, and is unexplained by any of the exogenous variables in the model.

This method allows us to include data for corner signal values ($s < v_\ell + \epsilon$ and $s > v_h - \epsilon$) in the test. The bid function is non-linear for these signals. Essentially, we handle this non-linearity by allowing the dependent variable (i.e., the left-hand side) to also account for the effect of the theoretical bid function.

Similarly, let p_j be the j^{th} observation on price in the sample; that is, it is the actual price paid in some market for some (n, k) and some round t. Let p_j^* be the price predicted in symmetric equilibrium: This is found by ordering the actual signals, and determining the symmetric equilibrium bid of the agent with the $(k+1)^{th}$ highest signal.[20] The difference between these two is

$$\rho_j = p_j - p_j^*.$$

Then, another implication of the model is that ρ_j should have a mean of zero, and be unexplained by any of the exogenous variables in the model.

First, observe that, on the basic means test, the model is rejected. From Table 9, the mean of ν is positive (so that agents bid too much), and the mean of ρ is also positive (so that agents pay too much for the good). Both these findings are consistent with the previous literature on common value auctions.

Variable	# Observations	Mean
$\nu_{it} = b_{it} - b_{it}^*$	2,478	7.696
		(0.68)
$\rho_j = p_j - p_j^*$	413	6.977
		(0.645)

Table 9: Mean deviations from symmetric equilibrium

Now, consider ν_i. Since the purpose of these regressions was to check if *any* exogenous variables help to explain deviations from symmetric equilibrium, we ran a few different models, with different regressors on the RHS. These were all qualitatively similar. In Table 10 below, we report on one such model.

The form of this regression we exhibit is

$$\nu_{it} = \beta_1 s_{it} + \beta_2 d_{it} + \beta_3 \epsilon_{it} + \beta_4 \frac{k_i}{n_i} + \beta_5 \frac{1}{t} + \eta_{it}. \tag{9}$$

Here, d_{it} is a dummy variable, that is 1 if the agent's signal lies in the interior (i.e., $s_{it} \in [100 + \epsilon, 300 - \epsilon]$) and zero otherwise. The intercept was omitted from this regression, because of collinearity between the constant vector, ϵ, and $\frac{k}{n}$. Since

[20] Since we know all signal realizations, this provides a better prediction on price in any given market and round than considering the expected price over all possible signals.

the theory specifies that coefficients of any right-hand side variables should be zero, the choice of these variables is, to some extent, arbitrary. We also ran a variant of equation (9), with dummy variables for $\epsilon = 20$ ($d_{\epsilon=20}$) and $\epsilon = 30$ ($d_{\epsilon=30}$).

A similar regression was run with ν_{it}^2, the square of the deviation, as the dependent variable. The attempt here was to explain changes in the variance of the deviations in bids.

Dependent Variable	$\nu_{it} = b_{it} - b_{it}^*$		ν_i^2	
s	-0.024*	-0.021*	-0.030	1.748
	(0.009)	(0.010)	(1.8790)	(2.212)
d_{it}	3.757*	3.858*	628.877*	687.424*
	(1.271)	(1.277)	(282.2)	(285.7)
ϵ	0.532*		32.696	
	(0.101)		(20.411)	
$d_{\epsilon=20}\,\epsilon$		0.421		-29.446
		(0.264)		
$d_{\epsilon=30}\,\epsilon$		0.475*		
		(0.168)		(42.971)
$\frac{k}{n}$	-11.24	-7.901	277.335	3.415
	(9.127)	(12.930)	(1712.0)	(26.190)
$\frac{1}{t}$	1.866	2.655	-376.057	199.080
	(3.125)	(3.472)	(679.9)	(778.1)
# Observations	2478	2478	2478	2478
R^2	0.0228	0.0211	0.0096	0.0117
$p-$value	0.941	0.990	0.635	0.9377

Notes:
(i) *denotes significantly different from zero at 5% level (2–tailed test).
(ii) The $p-$value is for the Hausman statistic.

* denotes significantly different from zero at the 5% significance level.

Table 10: Two error components regression of deviations from symmetric equilibrium bids for $\epsilon = 20, 30$

If all subjects bid according to the symmetric equilibrium, all coefficients in Table 10 should be statistically insignificant from zero. However, the coefficients on the intercept, the signal variable, the interior signal dummy and ϵ are statistically significant. Bidders do not adjust their bids enough for changes in signal (so s_i has a negative coefficient). Further, bidders are more likely to overbid when their signals are in the interior. This variable was included in the regression because the inference problem at the corners is more complicated. Contrary to theoretical predictions, bids increase as ϵ increases. Finally, time does not explain deviations from the theoretical bid. Thus, learning does not appear to be present in our data.

Next, consider the regressions of ν_i^2 on the same set of explanatory variables. This yields information about the second moment of ν_i. Only the coefficients on the interior signal dummy is significant, and it is positive. This suggests that, for interior signals, there is greater divergence in bidding across agents.

We also regressed ρ_j (that is, the deviation from equilibrium revenue) and ρ_j^2 on a set of exogenous variables, to identify predictable patterns in the first and second moments of ρ_j. The independent variables in these regressions were v^*, d_v, a dummy for interior values of v^*, ϵ, $\frac{k}{n}$, and $\frac{1}{t}$. Here, $d_v = 1$ if $v^\star \in [100 + 2\epsilon, 300 - 2\epsilon]$, and 0 otherwise. The constant was again omitted, due to multicollinearity.

In the revenue regressions, we could not use an individual-specific error component structure, since we only have one price observation per auction (i.e., per group of n individuals). To allow for bidder heterogeneity, we instead used a session-specific error component in the error term. These regressions also used the two error components model. The results are displayed in Table 11.

Consider first the regressions of ρ_j. The coefficients on v^*, d_v, and ϵ are significantly different from zero (recall that the predicted values are zero). Since the mean value of ρ is positive, the negative coefficient on $\frac{k}{n}$ suggests that subjects bid closer to the theoretical bids as $\frac{k}{n}$ increases. However, the standard error in this case is large. Looking at the regressions of ρ_j^2, the variance of ρ_j increases for interior values of v^*.

5.5 Comparative Statics of Bidding Behaviour

The presence of systematic departures from theoretical predictions constitutes a rejection of the common value symmetric equilibrium. However, we are also interested in the comparative statics and intuitions of the model. In particular, when we vary the degree of the winner's curse by varying k, do agents adjust their bids as predicted by the common value model? We find that they do.

These regressions were run for only interior signal values, that is, signals in the

Dependent Variable	$\rho_j = p_j - p^*$		ρ_j^2	
v^*	-0.028⋆	-0.035⋆	-0.439	-0.391
	(0.009)	(0.010)	(0.305)	(0.321)
d_v	2.378⋆	2.198⋆	89.661⋆	90.918⋆
	(1.160)	(1.158)	(37.841)	(37.954)
ϵ	0.488⋆	18.874⋆		
	(0.116)		(3.919)	
$d_{\epsilon=20}\,\epsilon$		1.165		12.760
		(0.751)		(17.502)
$d_{\epsilon=30}\,\epsilon$		0.893		15.121
		(0.498)		(11.550)
$\frac{k}{n}$	-8.784	-36.57	-608.865	-345.679
	(12.432)	(40.935)	(445.6)	(941.6)
$\frac{1}{t}$	1.310	-0.957	56.967	72.726
	(3.138)	(3.326)	(103.7)	(109.0)
# Observations	413	413	413	413
R^2	0.079	0 071	0.081	0.078
$p-$value	0.419	1.000	0.995	1.000

Notes:
(i) ⋆denotes significantly different from zero at 5% level (2–tailed test).
(ii) The $p-$value is for the Hausman statistic.

⋆ denotes significantly different from zero at the 5% significance level.

Table 11: Two error components regression of deviations from equilibrium revenue for $\epsilon = 20, 30$

range $[100 + \epsilon, 300 - \epsilon]$. For signals in this range, the symmetric equilibrium bid function is linear in signal. As before, we consider only $\epsilon = 20, 30$.

Comparative Statics on k:

Recall that the model predicts that higher k leads to higher bids. To test this, we ran regressions of the form

$$b_{it} = \beta_0 + \beta_1 s_{it} + \beta_2 \epsilon_t + \sum_{n,k} \beta_{n,k}\, d_{n,k}\, \epsilon_t + \eta_{it}, \qquad (10)$$

where $d_{n,k}$ is a dummy variable that is 1 if the observation occurred in the $(n,k)^{th}$ auction and zero otherwise, and η_{it} had the two error components structure of equation (7). The number of such dummies varied, depending on the number of auction types in the regression.

Three such regressions were run, with n fixed in each regression. The following cases were examined:

1. n fixed at 4, $k = 1, 2$.

2. n fixed at 6, $k = 1, 3$.

3. n fixed at 8, $k = 1, 2, 4$.

The first two, therefore, had one dummy variable each, and the third regression had two (for $(n8, k2)$ and $(n8, k4)$).

In discussing the results of these regressions, we compare the actual values of the coefficients to their predicted values. We describe the determination of the predicted values in Case 1 above in detail. The predicted values for the other cases were obtained in similar fashion.

The equilibrium bid function is

$$b_{it}^* \;=\; s_{it} - \epsilon_t + \left(\frac{2k_i}{n_i}\right) \epsilon_t.$$

The two sessions considered in Case 1 were $(n4, k1)$ and $(n4, k2)$. Let $\frac{k}{n} = \frac{1}{4}$ represent the base case, and $d_{(4,2)}$ be a dummy variable that is 1 if an observation is in the $(n4, k2)$ auction, and 0 otherwise. Then, this equilibrium bid function is written as

$$\begin{aligned} b_{it}^* &= s_{it} - \epsilon_t + \left(\frac{2(1)}{4}\right) \epsilon_t + d_{4,2} \left(\frac{2(2-1)}{4}\right) \epsilon_t \\ b_{it}^* &= s_{it} - \frac{1}{2}\,\epsilon_t + \frac{1}{2}\,d_{4,2}\,\epsilon_t. \end{aligned} \tag{11}$$

The regression run was of the form:

$$b_{it} = \beta_0 + \beta_1\, s_{it} + \beta_2\, \epsilon_t + \beta_3\, d_{(4,2)}\, \epsilon_t. \tag{12}$$

Comparing equations (11) and (12), the constant term, β_0, is throughout predicted to be zero. The predicted value of β_1, the coefficient on signal, is 1. For β_2, the coefficient on ϵ, the predicted value is $-\frac{1}{2}$ (in general, this is $(-1+\frac{2}{n})$ in the three regressions). Note that this coefficient captures a combination of the effect of ϵ and of the base values of n and k on bidding behavior.

The coefficient β_3 captures the difference in bid functions between the $(n4, k1)$ and $(n4, k2)$ experiments. Hence, its predicted value is $\frac{2(k2-k1)}{n4}$, or $\frac{1}{2}$ for Case 1. The qualitative prediction of the theory is that this coefficient should be positive, since higher values of k imply a lower winner's curse.

The results of this regression are reported under Cases 1–3 of Table 12. As we are examining the qualitative predictions of the theory, we consider whether the coefficients on the dummy variables have the right sign, and are significantly different from zero.

First, observe that, in these three cases, none of the constants is significantly different from zero. The coefficients on signal, while all in the range 0.958–0.962, are not significantly different from 1. For ϵ, none of the three coefficients are significantly different from zero, and two have a positive sign. All three are higher than their predicted values, indicating that bidders, on average, were bidding too much.

The comparative statics on k are consistent with the qualitative predictions of the theory. Increasing k reduces the extent of the winner's curse. Thus, we expect agents to bid more aggressively as k increases. All the four coefficients are positive, so that bidders do bid higher when k increases. Two of these coefficients are significantly different from zero. For the case of $n = 8$, neither coefficient is significantly different from its predicted value.

Our previous results suggest that winners are unable to quantify the effects of the winner's curse (in particular, they bid too high, and do not compensate for changes in ϵ). However, since they do respond as predicted to changes in k, they seem to have an intuitive understanding of the winner's curse, even though they are unable to quantify it.

Comparative Statics on n:

We next tested for how bids varied with n, keeping k fixed at some level. These regressions were of the form

$$b_{it} = \beta_0 + \beta_1 s_{it} + \beta_2 \epsilon_t + \sum_{n,k} \beta_{n,k}\, d_{n,k}\, \epsilon_t + \eta_{it}, \qquad (13)$$

Base (n,k)	Case 1 (4,1)	Case 2 (6,1)	Case 3 (8,1)	Case 4 (4,1)	Case 5 (4,2)	Case 6 (4,1)	Case 7 (4,2)
Compared to (n,k)	(4,2)	(6,3)	(8,2) (8,4)	(6,1) (8,1)	(8,2)	(8,2)	(6,3) (8,4)
Constant	- 13.622	3.219	8.329	8.591	-1.682	13.804	-12.920
	(15.397)	(8.230)	(7.895)	(7.568)	(14.783)	(9.923)	(10.265)
Pred. Val	0	0	0	0	0	0	0
s	0.958⋆	0.962⋆	0.962⋆	0.941⋆	0.958⋆	0.983⋆	0.972⋆
	(0.045)	(0.026)	(0.025)	(0.024)	(0.042)	(0.032)	(0.031)
Pred. Val	1	1	1	1	1	1	1
ϵ	0.162	0.210	-0.407	-0.278	1.360⋆	-0.741⋆	1.517⋆
	(0.446)	(0.243)	(0.247)	(0.240)	(0.461)	(0.290)	(0.354)
Pred. Val	-0.5	-0.667	-0.75	-0.5	0	-0.5	0
$d_{4,2}$	1.492⋆						
	(0.465)						
Pred. Val	0.5						
$d_{6,1}$				0.429⋆			
				(0.206)			
Pred. Val				-0.167			
$d_{6,3}$		0.125					-0.813⋆
		(0.196)					(0.385)
Pred. Val		0.667					0
$d_{8,1}$				-0.008			
				(0.194)			
Pred. Val.				-0.25			
$d_{8,2}$			0.185		-1.461⋆	0.212	
			(0.214)		(0.462)	(0.179)	
Pred. Val			0.25		-0.5	0	
$d_{8,4}$			0.711⋆				-0.607
			(0.214)				(0.361)
Pred. Val			0.75				0
# Obs.	410	420	865	699	509	521	686
R^2	0.537	0.775	0.639	0.692	0.515	0.662	0.600
$p-$value	0.199	0.314	0.430	0.948	0.060	0	0.155

Notes:
(i) ⋆ denotes significantly different from zero at 5% level (2–tailed test).
(ii) Pred. Val = Predicted Value in Symmetric Equilibrium.
(iii) The $p-$value is for the Hausman statistic.

Table 12: Comparative statics regressions of interior bid functions for $\epsilon = 20, 30$

where $d_{n,k}$ is a dummy variable that is 1 if the observation occurred in the $(n,k)^{th}$ auction and zero otherwise. The number of such dummies varied, depending on the number of auction types in the regression. The predicted coefficient on the dummy term is $2(\frac{k}{n_2} - \frac{k}{n_1})$, where n_1 is the base level of n in the regression, and n_2 the value of n at which the dummy is 1. The predicted value for the coefficient on ϵ is $\left(-1 + \frac{2k}{n}\right)$.

Two such regressions were run:

1. k fixed at 1, $n = 4, 8$.

2. k fixed at 2, $n = 4, 6, 8$.

Cases 4 and 5 of Table 12 report the results of these regressions. The coefficients on signal are in the same range as before (0.941 and 0.958), although in Case 4 it is significantly below 1. The coefficient on ϵ in case 5 is 1.36 (as opposed to a predicted value of -1), and is significantly different from zero.

The results on n are mixed. Case 4 corresponds to the second price auction for $n = 4, 6, 8$. The results here are consistent with those found earlier by Kagel and Levin (1986) and Lind and Plott (1986). That is, agents do not adjust bids downward to account for increases in the size of the winner's curse as n increases. As n increases from 4 to 6, they instead increase their bids.

In Case 5, however, we find that bidders do reduce their bids as n increases from 4 to 8. In particular, they overcompensate: the coefficient of -0.366 is significantly below the predicted value of -0.125. One possible explanation for this is that strategic effects were particularly important in the $(n4, k2)$ auction (the base case in Case 5). This is also reflected in the high value of $(n4, k2)$ dummy term in Case 1 above.

The winners curse in our experiments depends on k, n, and ϵ. Our results on n and ϵ are mostly consistent with previous literature: bidders do not compensate (or compensate in the wrong direction) for changes in the winner's curse. In the light of this, the results on k are striking. In Section 6 below, we interpret our results in terms of the winner's curse.

Comparison of cases with constant $\frac{k}{n}$

Finally, we consider a comparison across auction in which $\frac{k}{n}$ had the same value. The theory predicts that subjects should bid in the same way in these auctions.

We consider two such cases:

(i) $\frac{k}{n} = 0.25$; the auctions included here were $(n4, k1)$ and $(n8, k2)$.

(ii) $\frac{k}{n} = 0.5$. The auctions included here were $(n4, k2), (n6, k3), (n8, k4)$. In this case, the interior bid function reduces to the signal; that is, the equilibrium bid even in the common values case is to bid the signal. Intuitively, this is because exactly half the bidders will be in the set of potential winners.

We tested to see whether the bid functions are the same in all these cases, with a regression of the form

$$b_{it} = \beta_0 + \beta_1 s_{it} + \beta_2 \epsilon_t + \sum_{n,k} \beta_{n,k} d_{n,k} \epsilon_t + \eta_{it}. \tag{14}$$

Cases 6–7 of Table 12 exhibit the results. Again, the signal coefficients are below, but not significantly different from 1. In Case 6, the coefficient on ϵ is significantly below zero, and within one standard deviation of its predicted value. The coefficient on the $(n8, k2)$ dummy is not significantly different from zero. The overall result for this case is, therefore, consistent with the theory, which predicts that behavior in the $(n4, k1)$ and $(n8, k2)$ auctions will be the same for interior signal values.

The overbidding in the $(n4, k2)$ auction is characterized by the high coefficient on ϵ in Case 7. The dummies on the $(n6, k3)$ and $(n8, k4)$ auctions are significantly different from zero here, and negative. Bidders behaved differently in these auctions, therefore, as compared to the $(n4, k2)$ auction. On an F-test of the restriction that $d_{6,3} = d_{8,4}$, we fail to reject the null hypothesis (the p-value is 0.563). This confirms the earlier results of the bidding in the $(n4, k2)$ auction being somewhat anomalous.

Case 6 is the one instance in which the Hausman statistic has a value significantly different from zero. The $p-$value is 0.0001, indicating that the error terms are correlated with the regressors. We therefore ran each of the 7 cases with both (i) no intercept, and (ii) a three error components model, of the form

$$\eta_{it} = u_i + \psi_{it} + \phi_t.$$

The results are qualitatively similar to those displayed in Table 12. For brevity, we do not report these regressions for all cases. However, Table 13 below does show the results for Case 6. The second column replicates Case 6 from Table 12, and columns 3 and 4 indicate the coefficients from alternative specifications of the model. Further, given the $p-$values on the Hausman statistic, we cannot reject the hypothesis that the error terms are uncorrelated with the regressors.

Variable	Intercept Two-component Model	No intercept Two-component Model	Intercept Three-component Model
Constant	13.804		13.779
	(9.923)		(9.925)
Pred. Val	0		0
s	0.983⋆	1.011⋆	0.983⋆
	(0.032)	(0.023)	(0.032)
Pred. Val	1	1	1
ϵ	-0.741⋆	-0.428	-0.735⋆
	(0.290)	(0.232)	(0.291)
Pred. Val	-0.5	-0.5	-0.5
$d_{8,2}$	0.212	0.164	0.207
	(0.179)	(0.249)	(0.188)
Pred. Val	0	0	0
# Obs.	521	521	521
R^2	0.662	0.887	0.662
$p-$value	0.00	0.479	0.580

Notes:
(i) ⋆ denotes significantly different from zero at 5% level (2–tailed test).
(ii) Pred. Val = Predicted Value in Symmetric Equilibrium.
(iii) The $p-$value is for the Hausman statistic.

Table 13: Alternative specifications for Case 6 of Table 12

5.6 Interior Revenue Function

Finally, we test for the linearity of the revenue function for interior values of v, as given by equation (4). The regression run here was of the form

$$p_j = \beta_1 v_j^* + \beta_2 \left(\frac{\epsilon_j}{n_j+1}\right) + \beta_3 \left(\frac{\epsilon_j\, k_j}{n_j(n_j+1)}\right) + \zeta_j,$$

where p_j is the j^{th} observation on price, and $v_j^*, \epsilon_j, n_j, k_j$ the corresponding values of v^*, ϵ, n, k respectively. ζ_j is an error term that is again assumed to have the two error components structure. Here, we allow for a session effect in the regression, across the seven different sessions.

The results of this regression are displayed in Table 14. The significants are all significantly different from zero, and that on v^* is insignificantly different from 1. However, the coefficients β_2 and β_3 have opposite signs to those predicted by the theory.

These results are, therefore, consistent with the notion that bidders are not bidding at the level predicted by symmetric equilibrium.

	$\epsilon = 20, 30$
$v_j^\star$	$1.012^\star$
	(0.0213)
Pred. Val.	1
$\frac{\epsilon}{n+1}$	$4.952^\star$
	(2.152)
Pred. Val.	-2
$\frac{\epsilon(k/n)}{n+1}$	$-11.056^\star$
	(5.375)
Pred. Val.	2
# Observations	179
R^2	0.958
$p-$value	0.720

Notes:
(i) * denotes significantly different from zero at 5% level (2–tailed test).
(ii) Pred. Val = Predicted Value in Symmetric Equilibrium.
(iii) The $p-$value is for the Hausman statistic.

Table 14: Regressions on linearity of interior revenue function

6 Discussion

We summarize our overall results in this section. First, in subsection 5.2, we reject the private values model in favor of the common values one. That is, our bidders behave as if they do understand that other bidders' signals are informative about the value of the object. Next, we check to see if bidders play best responses to the actual bids of other agents in the same auction. We find that, for about $\frac{2}{3}$ of the bidders, we cannot reject the hypothesis that they are playing a best response. However, the results on revenue clearly indicate that bidders bid too high on average.

Further, the strong test of the theory (Subsection 5.4) rejects the quantitative predictions of the model. Thus, while bidders behave as if they understand common value nature of the auction, they depart from equilibrium predictions in the levels of their bids. There are different explanations consistent with such behavior. For example, our assumption about risk-neutrality may be incorrect: Perhaps bidders are risk-loving, leading them to bid too high (relative to risk-neutral levels). Since this is unappealing, we consider an alternative explanation: Bidders are unable to determine the "right" bid function.

There are two aspects to determining best responses: beliefs about other players' strategies, and an ability to compute conditional expectations. As we observe only bid functions (which are jointly determined by these two factors), these are confounded in the data. We find (in Subsection 5.5) that the qualitative predictions of the theory hold. That is, bidders adjust their bids in the right direction, in response to changes in k. They do not always adjust with the right magnitudes: This is consistent with an inability to compute levels. We further find that the outcomes as n changes do not always accord to the theory.

We now consider these results in terms of the winner's curse, defined earlier in Subsection 2.2. Using the information in Table 2, we calculate the changes in the winner's curse across different auction formats below, in Table 15.

		$\epsilon = 20$		$\epsilon = 30$	
Auction	Difference From	ΔW^C	ΔW^U	ΔW^C	ΔW^U
(4,2)	(4,1)	−8.00	−2.00	−12.00	−3.00
(6,3)	(6,1)	−11.43	−1.91	−17.15	−2.85
(8,2)	(8,1)	−4.44	0.56	−6.67	−0.83
(8,4)		−13.33	−1.67	−20.00	−2.50
(6,1)	(4,1)	4.57	0.43	6.86	0.64
(8,1)		7.11	0.39	10.67	0.58
(8,2)	(4,2)	10.67	1.83	16.00	2.75
(8,1)	(4,1)	2.67	−0.17	4.00	−0.25
(6,3)	(4,2)	1.14	0.52	1.71	0.79
(8,4)		1.78	0.72	2.67	1.08

Table 15: Changes in winner's curse across different auctions

As seen from the table, the conditional and unconditional winner's curses respond in different ways across different auctions. In particular, comparing the $(n4, k1)$, $(n6, k1)$ and $(n8, k1)$, there are very small differences in the magnitude of the unconditional winner's curse across these auctions, with larger differences in the magnitude of the conditional winner's curse.

In four cases, we have a relatively large change in the unconditional winner's curse; that is, a change of 2.50 or greater, in absolute value, at an ϵ of 30. Note that this is interpreted as a change, in expectation, in the amount of money lost on a single round as a result of naïve bidding. These cases are:

1. $(n4, k2)$ compared to $(n4, k1)$: W^U falls.

2. $(n6, k3)$ compared to $(n6, k1)$: W^U falls.

3. $(n8, k4)$ compared to $(n8, k1)$: W^U falls.

4. $(n8, k2)$ compared to $(n4, k2)$: W^U rises.

From Table 12, in Cases 1, 3, and 4 above, the experimental results indicate a significant change in the bidding function, in the direction predicted by the theory. That is, when the winner's curse decreases, bidders bid more aggressively, and vice versa.

Our results are, therefore, consistent with the following notion: Bidders understand and respond to the unconditional winner's curse. However, they are unable to determine the appropriate equilibrium levels. When W^U changes by a substantial amount, their behavior changes accordingly. Small changes in W^U typically do not lead to significant changes in behavior, with one exception being $(n6, k1)$ compared $(n4, k1)$. We estimate a positive and significant coefficient on the difference in the bid functions between these two, though the changes in the winner's curse are positive, and relatively small.

Nyborg, Rydqvist and Sundaresan (2002) provide empirical support for the notion that bidders compensate correctly for the winner's curse in first price auctions, using data from Swedish Treasury auctions. Their environment is richer, in that bidders submit prices and quantities. However, they estimate a single bid function similar to ours, in that it contains an (in their case, downward) adjustment to signal, that depends on the precision of the signal. They find that the single bid model, based on the winner's curse, explains bid shading in their data.

Eyster and Rabin (2000) suggest that experimental outcomes in common value auctions can be explained by subjects playing a "cursed equilibrium." In such an equilibrium, players bid a convex combination of the Bayesian Nash equilibrium bid and the naïve bid (i.e., the expected value of the good conditional on their signal alone). Our results are consistent with this.

Overall, we do find support in our experiments for the hypothesis that subjects have an intuitive understanding of the winner's curse. Their ability to react to k is consistent with a theory that they ascribe optimizing behavior to others, and have an understanding of the effects on valuation of being, say, the third highest, instead of the highest, but are unable to compute order statistics.

Although we do not test this experimentally, Parlour and Rajan (2002) demonstrate that, in a $(k+1)^{th}$ price auction, where the good is awarded to the highest bidder at the $(k+1)^{th}$ price, the bid function that obtains with our setup is exactly the same as exhibited in Proposition 1. This is potentially a promising avenue for future research, since it would help draw a parallel to the corresponding literature on private value auctions (see Kagel and Levin [1993]).

7 Appendix

7.1 Proofs

Proof of Lemma 1

This was proved by Parlour and Rajan (2002), Proposition 1. For completeness, we replicate the proof here.

Claim 1. The set of equilibria in the k order rationing mechanism is equivalent to the set of equilibria in an auction in which the seller sells k goods and each of the k^{th} highest bidders wins one unit and pays the $(k+1)^{st}$ highest price.

Consider the decision faced by agent 1 in the k^{th}-order rationing mechanism. Let $b_2, \ldots, b_n$ denote the bids of the other $(n-1)$ agents, in decreasing order. Then, agent 1 chooses a bid b_1 that maximizes

$$\frac{1}{k}\, E(V - b_{k+1} \mid S_1 = s,\ b_1 \geq b_{k+1})\ \mathrm{Prob}(b_1 \geq b_{k+1}).$$

Next, consider a corresponding auction in which k items are sold, and the k highest bidders win and pay the $(k+1)^{st}$ highest bid. In this auction, bidder 1 chooses a bid

$\tilde{b}_1$ to maximize

$$E(V - b_{k+1} \mid S_1 = s,\ b_1 \geq b_{k+1})\ \text{Prob}(b_1 \geq b_{k+1}).$$

That is, the payoffs differ only by a multiplicative constant, $\frac{1}{k}$. Clearly, the set of maximizers is the same in either case. Hence, the best response correspondences of the two mechanisms are identical, as are their equilibria.

Claim 2 The symmetric equilibrium in the k unit auction is $b(s) = E[v \mid s = Y_{k,n-1}]$.

This follows directly from Milgrom (1981).

From Claim 1, it follows that this is also an equilibrium of the k^{th}-order rationing mechanism. ■

Proof of Proposition 1

Harstad and Bordley (1996) showed the bid function for interior signals, and Parlour and Rajan (2002) provide the bid function over the entire signal range. Again, for completeness, we sketch the proof here.

Let $\underline{v}(s)$ and $\bar{v}(s)$ be the possible bounds of v given a signal s. For $s \in (v_\ell + \epsilon, v_h - \epsilon)$, $\underline{v}(s) = s - \epsilon$, and $\bar{v} = s + \epsilon$. Whereas, for $s < v_\ell + \epsilon$, $\underline{v}(s) = v_\ell$, while $\bar{v} = s + \epsilon$ and for $s > v_h - \epsilon$, $\underline{v}(s) = s - \epsilon$, while $\bar{v}(s) = v_h$. Following Milgrom (1981), we have
(i) For $s \in (v_\ell - \epsilon, v_h + \epsilon)$,

$$\begin{aligned} b(s;k) &= \frac{\int_{\underline{v}(s)}^{\bar{v}(s)} vG(s \mid v)^{n-k-1}(1 - G(s \mid v))^{k-1} g(s \mid v)^2 \frac{1}{v_h - v_\ell} dv}{\int_{s-\epsilon}^{s+\epsilon} G(s \mid v)^{n-k-1}(1 - G(s \mid v))^{k-1} g(s \mid v)^2 \frac{1}{v_h - v_\ell} dv} \\ &= \frac{\int_{\underline{v}(s)}^{\bar{v}(s)} vG(s \mid v)^{n-k-1}(1 - G(s \mid v))^{k-1} dv}{\int_{s-\epsilon}^{s+\epsilon} G(s \mid v)^{n-k-1}(1 - G(s \mid v))^{k-1} dv}. \end{aligned}$$

For ease of integration, we change variables. Let $x = \frac{s-(v-\epsilon)}{2\epsilon}$. Then, $v = s + \epsilon - 2\epsilon x$, and $dv = -2\epsilon dx$. Finally, let $\bar{x}(s) = \frac{s-(\underline{v}(s)-\epsilon)}{2\epsilon}$, and $\underline{x}(s) = \frac{s-(\bar{v}(s)-\epsilon)}{2\epsilon}$. Observe that (i) $\underline{x}(s) = 0$ for $s \leq v_h + \epsilon$, $\bar{x}(s) = 1$ for $s \geq v_\ell - \epsilon$.

Then, when $\underline{x}(s) < \bar{x}(s)$,

$$b(s;k) = \frac{-\int_{\bar{x}(s)}^{\underline{x}(s)} (s + \epsilon - 2\epsilon x) x^{n-k-1}(1-x)^{k-1} dx}{-\int_{\bar{x}(s)}^{\underline{x}(s)} x^{n-k-1}(1-x)^{k-1} dx}$$

$$= \quad (s+\epsilon)\frac{-\int_{\bar{x}(s)}^{\underline{x}(s)} x^{n-k-1}(1-x)^{k-1}dx}{-\int_{\bar{x}(s)}^{\underline{x}(s)} x^{n-k-1}(1-x)^{k-1}dx} + 2\epsilon\frac{\int_{\bar{x}(s)}^{\underline{x}(s)} x^{n-k}(1-x)^{k-1}dx}{-\int_{\bar{x}(s)}^{\underline{x}(s)} x^{n-k-1}(1-x)^{k-1}dx}$$

$$= \quad (s+\epsilon) - 2\epsilon\frac{\int_{\underline{x}(s)}^{\bar{x}(s)} x^{n-k}(1-x)^{k-1}dx}{\int_{\underline{x}(s)}^{\bar{x}(s)} x^{n-k-1}(1-x)^{k-1}dx}$$

We use the following two integration formulae which are obtained by repeated integration by parts when γ is a positive integer.

$$\int x^{\alpha}(1-x)^{\gamma}dx \quad = \quad \sum_{i=0}^{\gamma}\frac{\gamma!\,\alpha!\,x^{\alpha+1+i}(1-x)^{\gamma-i}}{(\gamma-i)!\,(\alpha+1+i)!} + A \tag{15}$$

$$\int_0^1 x^{\alpha}(1-x)^{\gamma}dx \quad = \quad \frac{\gamma!\,\alpha!}{(\alpha+1+\gamma)!}. \tag{16}$$

where A in equation (15) is a constant of integration.

Using the integration formulae yields the results.

Proof of Proposition 2

Suppose the consumption value of some player i is s_i. Consider bidding $b = s_i$ versus bidding $\hat{b} > b$. Denote the bids of the other $(n-1)$ agents by $b_{(1)}, \ldots, b_{(n-1)}$, arranged in decreasing order. If $b_{(k)} > \hat{b}$, then the bids b and $\hat{b}$ have the same payoff (zero). Similarly, if $b_{(k)} \leq b$, then b and $\hat{b}$ have the same payoff, $\frac{1}{k}(s_i - b_{(k)})$. If $b_{(k)} \in (b, \hat{b})$, then b earns a payoff of zero, and $\hat{b}$ earns $\frac{1}{k}(s_i - b_{(k)}) < 0$. Finally, if $b_{(k)} = b$, then $b, \hat{b}$ earn the same payoff; if $b_{(k)} = \hat{b}$, b earns zero, and $\hat{b}$ earns $\frac{1}{\hat{k}}(s_i - b_{(k)}) < 0$, where $\hat{k}$ is the number of agents who bid $b_{(k)}$ or higher. Hence, b weakly dominates $\hat{b}$.

A similar argument shows that $b = s_i$ weakly dominates any bid $\hat{b} < s_i$. ■

Proof of Proposition 3

Suppose bidder i receives a signal $s \in [v_\ell + \epsilon, v_h - \epsilon]$, and $s = Y_{j,n}$. She is in the winning set if $j \geq k$. If she wins, she pays $b(Y_{k+1,n})$. The expected price to her, therefore, is $E(\hat{b}(Y_{k+1,n}) \mid Y_{j,n} = s) = E(Y_{k+1,n} \mid Y_{j,n} = s)$, since $\hat{b}(s) = s$. The signal distribution, given v, is uniform, with a range 2ϵ for interior signals. It follows immediately from the order statistics of uniform distributions that

$$E(Y_{k+1}, n \mid Y_{j,n} = s) \quad = \quad s - \frac{2\epsilon\,(k+1-j)}{n+1}.$$

Next, consider $E(v \mid Y_{j,n} = s)$. We have $E(Y_{j,n} \mid v) = v + \epsilon - \frac{2\epsilon\, j}{n+1}$. The posterior distribution over v is also uniform, for $s \in [v_\ell + \epsilon, v_h - \epsilon]$. Again, from the order statistics of a uniform distribution, it follows that $E(v \mid Y_{j,n} = s) = s - \epsilon + \frac{2\epsilon\, j}{n+1}$.

Now, conditional on winning in an auction with a fixed n, k, a bidder knows that $s \geq Y_{k,n}$.[21] Since all distributions are uniform, she is equally likely to be any of the order statistics $1, 2, \ldots, k$. Hence,

$$\begin{aligned}
E(\text{Price} \mid s, \text{bidder } i \text{ wins}, b) &= \frac{1}{k}\sum_{j=1}^{k}\left(s - \frac{2\epsilon\,(k+1-j)}{n+1}\right) = s - \frac{2\epsilon\, k}{k(n+1)} + \frac{2\epsilon}{k(n+1)}\sum_{j=1}^{k} j \\
&= s - \frac{2\epsilon\, k}{k(n+1)} + \frac{2\epsilon}{k(n+1)}\,\frac{k(k+1)}{2} = s - \frac{(k+1)}{n+1}\,\epsilon \\
E(v \mid s, \text{bidder } i \text{ wins}, b) &= \frac{1}{k}\sum_{j=1}^{k}\left(s - \epsilon + \frac{2\epsilon\, j}{n+1}\right) = s - \epsilon + \frac{2\epsilon}{k(n+1)}\sum_{j=1}^{k} j \\
&= s - \epsilon + \frac{2\epsilon}{k(n+1)}\,\frac{k(k+1)}{2} = s - \frac{n-k}{n+1}\,\epsilon.
\end{aligned}$$

$W^C(\hat{b}, s)$ is (by definition) the difference of these two quantities, and hence $W^C(\hat{b}, s) = \frac{\epsilon}{n+1}\,(n - 2k - 1)$.

Now, the distributions of v^* and s given v^* are both uniform, and, for the definition of the winner's curse, we consider all agents bidding naïvely. Hence, for an agent with an interior signal, the probability of being in the set of potential winners is equal to the probability of having one of the k highest signals. That is, it is just $\frac{k}{n}$. Further, the probability of actually winning the object, conditional on being in this set, is $\frac{1}{k}$. Hence, the unconditional probability of winning is the product of these two; that is, $\frac{1}{n}$. Hence, the unconditional winner's curse is $W^U = \frac{\epsilon}{n(n+1)}\,(n - 2k - 1)$. ■

[21] We ignore the possibility of two or more bidders being tied (which would allow $s = Y_{k+1,n}$ conditional on winning) since ties have zero probability.

7.2 Instructions for the (n8, k4) auction

INSTRUCTIONS

This is an experiment in the economics of market decision-making.

In this experiment, 30 different envelopes containing a sum of money will be auctioned in a sequence of trading periods. A SINGLE envelope will be auctioned off in each trading period. There will be thirty trading periods. ***Eight*** players will ***always*** participate in the market for all thirty trading periods. The players are randomly selected and different in every round. If there are more than eight players in the room, the players in the auction you are in will be randomly selected and different in every round. If you win the envelope, your profit is: the amount of money in the envelope minus the price of the envelope (the price of the envelope is the FIFTH highest bid). This can be a gain or a loss. This gain or loss will affect how much money you make. Specifically, at the end of the experiment, ***ten out of thirty*** rounds will be randomly selected and the outcomes of those rounds will determine how much money you will make. Specifically, you receive $30 plus (minus) any profit (losses) you earn from the ten selected rounds. If your net balance is negative, you will receive $0 for participating in the experiment. If you did not have losses or earnings during the experiment, you will receive $30 for participating. Because you do not know which ten rounds will be chosen for payment, you should treat each round as equally important in making your decisions.

What is in the envelope?

There is an amount of money between $100 and $300 in each envelope. Operationally, for each round a computer will randomly generate a new number between $100 and $300 inclusively, with each number in this range equally likely. So, there is never less than $100 in an envelope, and never more than $300. Because the amount in each envelope is independently generated, a high (or low) amount of money in the envelope in one period tells you nothing about the likely value in the next period. The same amount might even appear later.

What do you know about the amount in the envelope?

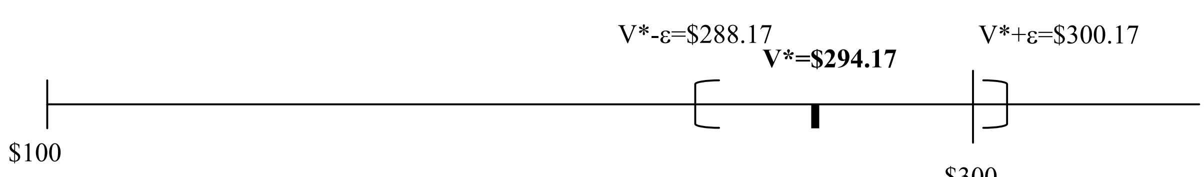
During each round you will not know the amount in the envelope. Instead, you will be given a private estimate of the money in the envelope. This will narrow down the range of possible values for determining your bid. ***Each of you gets a different and independent private estimate.*** We generate the private estimate from the amount in the envelope (V*), and a random variable epsilon (ε). The estimate that you get will be somewhere between (V* - ε) and (V* + ε). Any number in this interval has an equal chance of being drawn. The value of ε changes over different rounds. We will announce it before the beginning of the next round. Depending on the round, epsilon ε will be $0, $1, $20 or $30.

For example, the following eight private estimates were generated in a trial. The estimates are presented in the ascending order:

288.29
291.11
291.82
292.58
295.22
295.83
296.89
300.04

Each player will receive one estimate per round, and has to make a decision how much to bid knowing only his own estimate. The lowest private estimate among the eight players in this example is 288.29, and the highest estimate is 300.04.

In this example, the amount of money in the envelope, V*, is 294.17, and ε is $6 (Note that ε = $6 was chosen to explain the example and the value ε = $6 is not used in our experiment). So, everyone gets a private estimate between $288.17 (V* - ε= $294.17 - $6) and $300.17 (V* + ε= $294.17+6). All the estimates in the example above fall within the lower bound $288.17 (V* - ε= $294.17 - $6), and the upper bound $300.17 (V* + ε= $294.17+6). Note that some private estimates in the example are above the amount of money of the auctioned envelope, and some are below this amount. You may receive a private estimate below $100 or above $300. There is nothing strange about this, it just indicates V* close to $300 (or $100). The diagram below illustrates this.



What is your task?

Your task in each round is twofold:

(1) To write down on the form provided what you think the profit will be if you win the auction.
(2) Decide an amount you are willing to bid for the envelope, and submit a ***bid***.

Note: your profit and your payment depend only on the bids of all eight players, and not on the assessment of the profit

Who wins the envelope?

In each round, EIGHT buyers always participate in the market. We take the FOUR ***highest*** bidders, (or 50% of the market), and randomly assign the envelope to one of them. So if you are one of the highest four bidders, you win the envelope with probability ¼. If you are not among the four highest bidders, you cannot win the envelope.

What do you earn if you are the winner of the envelope?

If you are the winner of the envelope, you make a profit equal to the difference between the amount of money in the envelope and the price of the envelope. The price will be the ***FIFTH highest bid.*** Therefore,

PROFIT = (Amount of Money in the Envelope) – (FIFTH HIGHEST BID)

If the difference is negative, it represents a loss.

If four or more bidders bid the same amount, so that more than four people have bids above the fifth highest bid, all bidders with such bids are potential winners.

What do you earn if you are not the winner of the envelope?

If you don't win the envelope, you get $0 for that round.

How to Bid in the Auction using the Auction program?

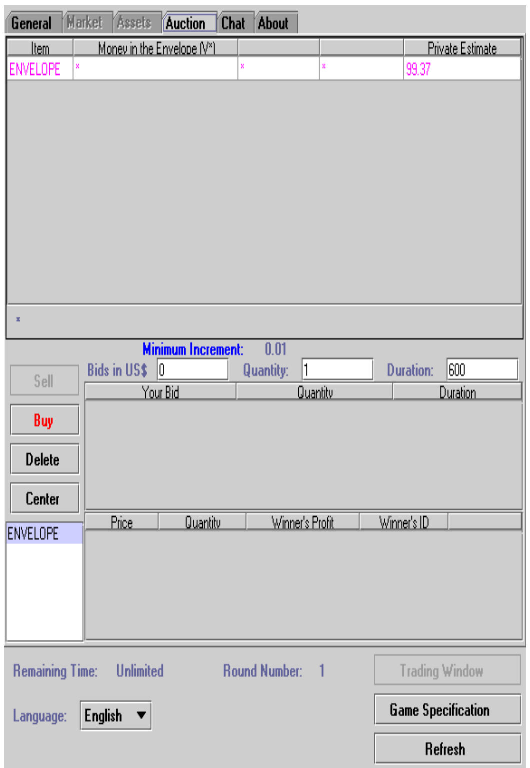
To familiarize yourself with the auction program, you will participate in ***two trial rounds*** that ***will not count towards your payment.***

(1) First start the auction on the computer, using the following steps:

- In your folders, there is a card with a four-digit ID number. Input the ID number as both your Username and Password, and click on "Launch the Game."

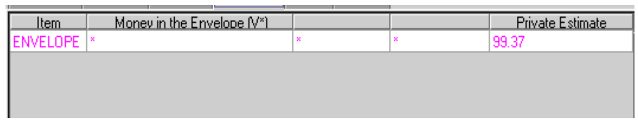
- Click on the "Auction" tab of the window that opens up

How is all the information presented on the computer screen and how will you make decisions?



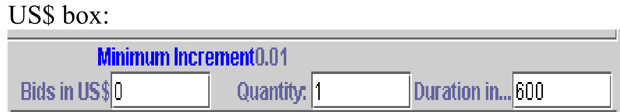
The market screen is divided into four parts:

1. ***Information about the envelope on your computer screen***
The top part shows Item: ENVELOPE that has been auctioned. You do not know the amount of money in the envelope (V*), and in the market window V* is covered with an asterisk (*) to remind you that it is unknown to you.



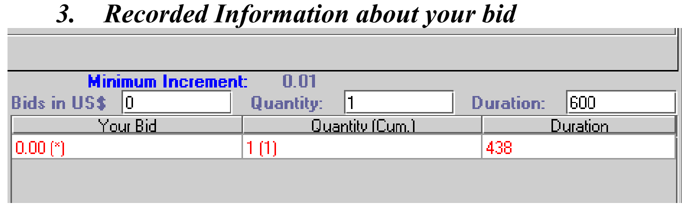
In the example above the Private estimate is $99.37, and ε is $1. Since V* lies between $100 and $300 respectively, the estimate can lie between ($100- ε) and ($300+ε).

2. ***Submitting your bid***
Decide an amount you are willing to bid for the money in the envelope. To submit a bid, write a dollar amount in the Bids in US$ box:



and click on the  button.
After you have pressed the buy button you can change your bid by selecting it, and clicking on  . When all subjects submit their bids for that round, we will make an announcement, and close the bidding.
The bid has to be greater than or equal to zero. The bids have to be rounded to the nearest cent (i.e. the minimum increment is 0.01). If you do not write your bid according to this rule, the computer will not accept the bid and you will receive the following message: "Invalid Bid Value". The Quantity auctioned is 1, and you cannot change it.

3. ***Recorded Information about your bid***

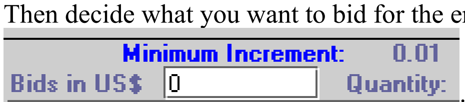


In the example above the typed bid is 0. You should not base your bids on the above example. After you press button  , the computer records on your screen that your bid was 0 for one unit. ***You do not see the bids of other players when you submit the bid.***

4. ***Recorded information about the winner's of the envelope***
The price, quantity, winner's profit, winner's ID is shown for each round.

- In your folder you will find a form like in the example below:

Round Number	If you win the auction, what will your profit be?
Trial round 1	
Trial round 2	
1	
2	
3	

- Based on the private estimate for that round, write down on the form what you think your profit will be if you win the auction.
- Then decide what you want to bid for the envelope. Type this number in the box 

. Bids must be rounded to the nearest cent so the Minimum Increment is 0.01.

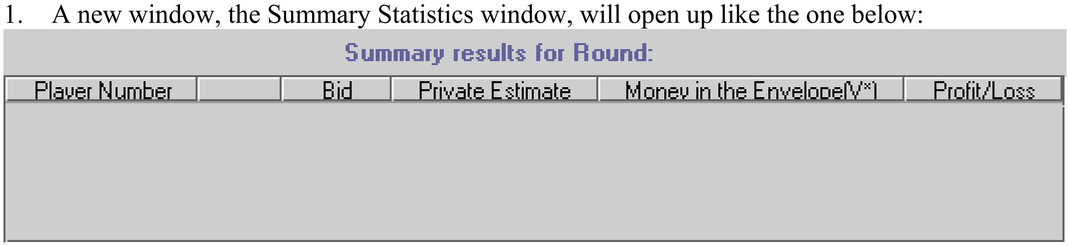
Note that you do not know the private estimates or bids of any other player at the time that you have to submit your own bids.

When all eight players have submitted their bids, the auction will be over. We will make an announcement before we close the bidding for that round.

What you see when the auction is over

There are two places where information will be provided to you once the auction is over:

1. A new window, the Summary Statistics window, will open up like the one below:



On this window, you will see the Player numbers, Bids and Private estimates for all eight players in your auction, including yourself. The bid that sets the price (the FIFTH highest bid) will be highlighted in blue. In addition, you will see the true value of V*, and the profit or loss earned by the winner.

2. On the Auction Information window, a transactions table will display for you key information about all auctions that took place in each round (remember that, if there are more than eight players on any given round, they will be randomly matched into different markets). Here, you will see the price paid by the winner, the ID number of the winner, and the profit or loss of the winner.

Now, let's look at the results of the trial round, and the information provided to you. As you see in your auction window, there were ___ markets in this round. In market 1, the price was $___, the winner ID number ___ was, and the winner earned a profit (loss) of $____.
In market 2 the price was $___, the winner ID number was ___, and the winner earned a profit (loss) of $____.

Next, look at the Summary Statistics window. In market 1, there were eight bidders. If you were in this market, you see their bids and private estimates, as well as V*. The bids, from highest to lowest, were $____, _____, _____, ____, ____, ____, ____, and _____. The private estimates were $____, ____, ____, ____, ____, ____, ____, and ____. Money in the envelope (V*) was $ ____. The winner was one of the four bidders, who had one of the four highest bids. The winner paid $____, the amount of the fifth highest bid (shown in blue).

In market 2, there were eight bidders. If you were in this market, you see their bids and private estimates, as well as V*. The bids, from highest to lowest, were $____, _____, _____, ____, ____, ____, ____, and _____. The private estimates were $____, ____, ____, ____, ____, ____, ____, and ____. Money in the envelope (V*) was $ ____. The winner was one of the four bidders, who had one of the four highest bids. The winner paid $____, the amount of the fifth highest bid (shown in blue).

Note that you only see information about your own market in the Summary Statistics window, but you see the results from all markets in the auction window.

We will now run a second trial round. The bids in this round also will not affect your payments. After this second trial round, your payment will be adjusted for profit or loss in the auction at each round.

Other elements of the experiment

You are not to reveal your bids or profits, nor are you allowed to speak to any other subject while the experiment is in progress.

Summary of main points of the experiment

a. In each round, EIGHT buyers always participate in the market.
b. The winner is selected among the FOUR highest bidders or 50% of the market. The winner earns the item and earns a profit= money in the envelope minus the ***fifth*** highest bid.
c. You do not see the bids of other players when you submit the bid.
d. Ten rounds will be chosen for payment at the end of the experiment. Your profit in these rounds will be added to the $30 you earn for participating in the experiment, and your losses will be subtracted from it. If the net amount is negative you will earn $0. Your balance at the end of experiment will be paid in cash.
e. Your private estimate is randomly drawn from the interval (V^* - ε) to (V^* + ε). Each estimate in this interval is equally likely. The amount of money in the envelope can never be more than (your private estimate + ε), or less than (your private estimate - ε). Values for ε are $0, $1, $20 and $30, and we will announce the value of ε before each round.
f. Each of you gets a different private estimate.
g. The amount of money in the envelope will always be between $100 and $300 inclusively, and each value in this range is equally likely.

Are there any questions?

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