Market Culture:
How Norms Governing Exploding Offers Affect Market Performance

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Different markets have different rules, norms, and expectations about how and when offers will be made, accepted, and rejected. For example, in some labor markets, it is conventional for employers to make exploding offers, to which candidates must reply before receiving other offers, while in other markets it is customary for all offers to remain open long enough to allow candidates to compare multiple offers. Similarly, norms differ concerning the circumstances under which a candidate may honorably change his mind about an offer he has accepted. These differences--whether they are enshrined in legally enforceable rules, or simply in expected behavior, and whether they are dictated by the larger market environment, or constitute different equilibria within a given market—can influence who makes offers to whom, at what time, and what outcome is produced. And many markets have sought to change their market cultures, in these respects, in order to promote more orderly markets, and control the timing of the market until information is available to make more efficient matches between firms and applicants (see e.g. Roth and Xing 1994, 1997).

One market in which a good deal of effort has been spent shaping and discussing these aspects of culture is the market for graduate students. The Council of Graduate Schools has, since the mid 1960’s, attempted to establish norms concerning how graduate students are recruited. Over 350 American universities subscribe to its resolution, which is distributed to applicants to graduate programs, and states in part:

“Students are under no obligation to respond to offers of financial support prior to April 15; earlier deadlines for acceptance of such offers violate the intent of this Resolution. In those instances in which a student accepts an offer before April 15, and subsequently desires to withdraw that acceptance, the student may submit in writing a resignation of the appointment at any time through April 15. However, an acceptance given or left in force after April 15 commits the student not to accept another offer without first obtaining a written release from the institution to which a commitment has been made.”

The resolution is accompanied by some explanatory discussion of how the resolution should be honored in the breach, which reads in part as follows:

“Students may be waiting for offers from several institutions so that they can compare and make a decision. One of the complaints we hear is that some
departments make offers quite early and insist that students respond quickly or lose the offer. According to the Resolution, the option available to the student in this situation who wishes to review several offers is to accept each one and then, by April 15, resign from all but one. But this places the student in an awkward position and really violates the spirit of the Resolution, that is, that acceptances should not be made casually.

“A better approach is for institutions to give students until April 15 to make decisions regarding appointments. Students often consider multiple offers, and this option provides a reasonable opportunity for them to do so. This would not preclude institutions asking students to accept or reject offers in a timely manner.”

Note in particular that the resolution attempts to foster a market culture under which exploding offers are both discouraged directly, and are also indirectly discouraged by being made less enforceable. That is, the resolution suggests that a student who accepts an exploding offer with a deadline before April 15, but subsequently declines it before April 15, should not be thought of as behaving badly. This reduces the cost of reneging on (and hence also of accepting) an exploding offer, in a world with opportunities for repeated interactions, in which social norms may have some force.¹

Similar concerns, and attempts to alter market culture, have played large roles in attempts to organize entry level labor markets for doctors, for lawyers, and college admissions at the undergraduate level. For example, doctors engage in a medical match that attempts to inculcate certain norms of participation (Roth, 1984, 1991, Roth and Peranson 1999). One of these is that employers are not supposed to ask applicants to make commitments prior to the match, or to indicate how they will record their preferences in the match. Surveys of medical students reveal that when they are nevertheless asked for such indications and commitments, they feel free to answer encouragingly, without constraining their behavior in the match (see e.g. Anderson et al., 1999; Carek et al., 2000; Pearson and Innes, 1999; Teichman et al., 2000). In contrast, law students who apply for appellate court clerkships are frequently given exploding offers, and are almost

¹ And by marking acceptances on April 15 as more binding than those made before, it also makes it less attractive for departments to make new offers after the 15th. So, for example, the chair of the graduate student recruiting committee in the Economics department of a competitive New England university writes that “we do not make any offers after April 15, 5 pm. This year lots happened between 2 and 5 pm of April 15. Nobody on our waiting list had accepted other offers; they were told to wait until the last minute.”
never reported to renege on them (Roth and Xing 1994; Avery, Jolls, Posner, and Roth, 2001).²

Even in a given market, commitments made at different times may have different force. In the undergraduate college admissions process, students’ acceptance of an early offer of admission from an “early decision” college, made in the Fall of a student’s senior year in high school, is considered binding (Avery, Fairbanks, and Zeckhauser, 2002).³ In contrast, regular admits are required to reply to early April offers by May 1, but many students are left on “waiting lists” beyond that date, and are relatively free to cancel their acceptance when they are given an offer from a waiting list.⁴

In what follows, we report an experimental investigation of these issues. We study a baseline environment in which firms may choose to make either exploding offers or offers that remain open until the end of the market, and compare it with two alternative environments. In one of these, only open offers may be made, and in the other, either open or exploding offers may be made, but candidates may renege on their acceptance of an exploding offer at a small cost.

² Of course, some elements of a market’s culture regarding whether certain commitments are binding can be explained at least in part in terms of the amount of repeated interaction in these markets. Hospitals and new doctors are part of a large national market, and a doctor who goes to the West coast is unlikely to interact closely with a hospital on the East coast (whose offer he might have reneged on). Appellate judges are harder for lawyers to avoid. But repetition, or the lack of it, isn’t destiny. Market failures are sometimes fixed or avoided by changes in the culture. In many of the markets discussed in Roth and Xing (1994), for example, a good deal of effort is spent by professional organizations in trying to alter the market culture.

³ However, this has been subject to discussion, following a September, 2001 meeting of the National Association for College Admission Counseling (NACAC), at which it was resolved that students applying to a binding Early Decision program should still be allowed to apply to as many nonbinding Early Action programs as they wish (http://www.nacac.com/policies.html). In response, Harvard changed its early action requirements to indicate that students could apply early action to Harvard even if they were applying early elsewhere. It was initially unclear whether Harvard would allow people who were accepted to Harvard under its nonbinding early action program to attend if they were simultaneously accepted in a binding early decision program by another college. While Harvard has indicated that it would expect students to honor commitments made to other schools, it appears likely that some students in this situation might question whether their commitment to attend the other school is legally binding, and this might eventually make binding early decision programs less attractive to college admissions officers. In this connection, Princeton and Brown have indicated that they will not comply with the NACAC decision, and will continue to require students who apply to their early decision programs to indicate in the contract they sign, not only that they will attend if admitted, but also that they are not applying early elsewhere (see e.g. Hoover, 2002, Rosenheck, 2002).

⁴ For example, Harvard’s letter to students who have been placed on the waiting list states:

“We recognize that you must make plans at another college while you await our final decision. Please be assured that all colleges will understand your situation…”
We will see that, in the environments we explore, the market unravels to inefficiently early contracts when firms are free to make exploding offers, and acceptances are binding, but both the prevention of exploding offers, and the facilitation of reneges, change the market dynamics in a way that promotes later offers, greater efficiency, and the resulting matching has fewer blocking pairs of firms and applicants.

As we will discuss in the conclusion, each of these ways to fix a market that unravels entails different norms that govern exploding offers. On the one hand, organizations of market participants can try to change the market culture by outlawing exploding offers. However this may be difficult, and another alternative is to reduce the costs to applicants of reneging on exploding offers they may have accepted.

The Experimental matching markets

In each market there are five firms and six applicants. Each market lasts 9 periods. Firms and applicants are assigned “qualities,” but the qualities of applicants become known only gradually. In each market a firm can hire one applicant and an applicant can work for one firm. A matched firm and applicant each receive the product of their qualities as a payoff, unmatched market participants have a payoff of zero.

The qualities of firms are simply their assigned ID number (from 1 to 5), the qualities of applicants are revealed over time. In periods 1, 4 and 7 each applicant receives an integer signal from 1 to 10 (uniform iid). The final quality of each applicant is determined in period 7 through the relative ranking of the sum of their three signals. The applicant with the highest sum receives a quality of 6, the second highest a quality of 5, the lowest a quality of 1 (ties are broken randomly).

Firms will be referred to by their ID numbers 1-5, which are also their qualities. When we are referring to applicants by their ID numbers, we will denote them 1-6. When we refer to applicants by their quality, we will call them applicants A1-A6, or simply refer to them as “the applicant of quality k”. (That is, applicant A6, the applicant with quality 6,
may also be applicant 3, if the third applicant ends up being the applicant with the highest quality.)

Firms see all the applicants’ signals as they become available over time, but applicants only receive information about their own signals.⁵

Firms can make two types of offers:  
An *exploding* offer is an offer that the applicant can only accept right away, i.e. in the same period in which it was made; if it is not accepted immediately, it is rejected.

An *open* offer is an offer the applicant can also hold (until period 9). That is, an applicant who receives an open offer may accept or reject it immediately, or may hold it, to accept or reject at a later period. An applicant must reject a held offer if he wishes to hold or accept another offer.⁶

In a given period, first all the firms decide what offers they will make. Each firm that is unmatched, and has no open offer being held by an applicant, may decide to make at most one offer.⁷ Each applicant learns of all offers he receives in that period before having to decide how to respond to each of them. If an applicant accepts the offer of a firm, the applicant and the firm are matched, and all market participants are informed about this. Offers are made in private; i.e. until they have been accepted they are not announced to the other firms and workers.

We consider 3 treatments:

**Treatment 1: Exploding** and Open offers

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⁵ This feature of the experimental environment is motivated by the situation in many markets, in which firms see a whole pool of applicants, but applicants may have difficulty knowing how they compare with other applicants.

⁶ This is not an onerous constraint for the applicants, since they have strict, unchanging preferences over the firms. In our experimental environment, it reduces the cost to a firm of making an open offer, since it reduces the likelihood that an open offer will be held by an applicant who has no intention of taking it.

⁷ Note that following the release of each signal, firms have 3 periods to make offers, i.e. each firm can make 3 offers after each signal. This is to avoid exogenously imposed congestion in the market. Of course, congestion may develop endogenously, if applicants hold offers until late, or firms delay making offers.
Each firm can decide whether to make each offer open or exploding. Once an applicant accepts an offer, the acceptance is binding, and firms cannot make subsequent offers to an applicant who has already accepted an offer.

**Treatment 2: Open Offers Only**

Firms can only make open offers. Once an applicant accepts an offer, the acceptance is binding, and firms cannot make subsequent offers to an applicant who has already accepted an offer.

**Treatment 3: Renege**

In this treatment, firms can again decide whether to make open or exploding offers. However, an applicant who accepted an offer may still receive further offers. An applicant can renege on initial acceptances and accept a new offer at a cost of 1 point (that is subtracted from his final payment).

We conducted 7 sessions of the exploding offer treatment, and 6 sessions of the open offer and 6 of the renege treatment. Subjects only participated once. Participants kept their role, firm or applicant, for the whole experiment, and, for firms, also the firm ID and hence quality (from 1 to 5). The experiment was conducted at the Harvard Business School, with students, using the software z-Tree. Firms 1 and 2 received an additional amount of $5, and each participant received $0.10 for each point earned. All participants received a $10 show up fee.

We conjecture that both the ability to make exploding offers and the ability of applicants to make binding agreements are necessary for unraveling to occur.

**Some simple theory of these environments:**

All three treatments allow for a whole array of Bayesian Nash equilibria. For example, in each treatment, there is an equilibrium in which, after the first period, all the firms are

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8 The instructions stated that some participants, already determined in advance, would receive some additional payment (see instructions).

9 One can think of the applicants’ ability to make binding agreements as an agreement among firms to not make offers to applicants who accepted another firm’s offer. See, for example, footnote 3, and Harvard’s decision to honor other universities’ early decision acceptances.
matched, and in each treatment there are also equilibria in which all matches are made only after all information has become available.

For example, the following strategies constitute an equilibrium in which all firms are matched in Period 1.

*Strategies of firms:* Each firm $i$ makes an offer in period 1 to applicant $i$. A firm whose offer is rejected never makes an offer any more.

*Strategies of applicants:* Each applicant $i$ in period 1 accepts an offer from firm $i$ and rejects any other offers (i.e. offer from other firms, offers received in other periods).

These strategies constitute an equilibrium, as no firm has an incentive to deviate, given the strategies of applicants and vice versa. However, this equilibrium has the unattractive property of using weakly dominated strategies.

In the exploding offer treatment there is a sequential equilibrium in which all firms hire after period 7, and the outcome is efficient.

**Proposition 1:** In the exploding offer treatment, when the firms and applicants are all risk neutral, there is a sequential equilibrium in which all the firms $j$ for $j<5$ and workers match only in period 7, 8 or 9, that is after all the uncertainty is resolved. Firm 5 can make an offer as soon as there is an applicant such that it is certain the applicant will have quality 6.

The intuition behind this result is as follows. At equilibrium, all workers expect to be matched only after their qualities are known. So whenever an applicant receives an offer from firm $i$ in period 1-6, and the offer is not from the highest firm that is not yet matched, the applicant assumes that the firm is strictly better off from getting this offer accepted than waiting until all the uncertainty is resolved. For risk neutral firms and workers, this means that the worker has a higher expected value for waiting and receiving the equilibrium match, and so the worker rejects the offer.
In and after period 7, the applicant of quality $k$ accepts all offers he receives from firms of quality $k-1$ or higher, or from the highest quality firm that is still on the market.

A firm of quality $j$ that is the $k$th from the bottom unmatched firm in period 7 (in equilibrium, $k=j$) makes an offer in period 7 to the $k+1$ from the bottom unmatched applicant. Firm 5 can make an offer as soon as there is an applicant who is certain to be of quality 6. The formal proof is in the appendix.

Note that when the applicants are risk averse, than they are willing to accept early offers from firms of lower quality. However, when firms are risk averse, they will only make offer to applicants who are of even higher quality.

The following propositions indicate that in the open and renege treatments, simple equilibrium refinement suggests that idealized rational play leads to late matching, i.e. to matching only when all the information about match quality is available.

**Proposition 2:** In the open offer treatment, all Nash equilibria that survive iterated elimination of weakly dominated strategies are those in which, in period 7 (or 8 or 9), each firm $j$ for $j<5$ makes an offer to the applicant of quality $j+1$. Firm 5 can make an offer as soon as there is an applicant who is certain to have quality 6. Applicants hold the best offer they receive before period 7, applicants can accept the offer of a firm, if it is the firm of highest quality that is still unmatched.

**Proposition 3:** In the Renege treatment, all NE that survive iterated elimination of weakly dominated strategies are those in which, in period 7 (or 8 or 9), each firm $j$ for $j<5$ makes an offer to applicant of quality $j+1$. Firm 5 can make an offer as soon as there is an applicant who is certain to have quality 6.

Whenever an applicant receives an offer he prefers to an offer he has already accepted, he will renege on his former acceptance.
Results of the Experiment: Unraveling, Efficiency and blocking pairs

First we investigate whether different rules concerning exploding offers affect the timing of the market. How long do firms wait to extend offers, and hence how much information about an applicant’s quality do firms have when extending the offers that were eventually accepted? Since in all our treatments firms could make open offers, the time at which an offer was made doesn’t necessarily correspond to the time at which an offer was accepted. Therefore we first concentrate on the period in which final offers, i.e. offers that were eventually accepted, were extended. We will later examine the timing of acceptances.

A market experiences no unraveling if final offers are all be made after period 7, once all the uncertainty about applicants’ qualities is resolved. In our experiment subjects participated in 20 markets. Figure 1 shows the timing of final offers for all treatments over all 20 markets. The timing is presented in terms of how many signals had been revealed before the offers were made. So a value of 1 corresponds to offers made when only one signal was available (periods 1-3), 2 denotes offers made after 2 signals, i.e. offers made in periods 4-6, and 3 signals corresponds to the final quality of applicants being known, that is offers made in periods 7-9. For the renege treatment, we only consider an offer to be final if it was accepted and not reneged upon. The results are presented in blocks of five markets.

![Timing of final offers](image_url)

Figure 1. For each treatment, the time at which final offers were made (i.e. offers that eventually result in a match).
The first 5 markets in all treatments look similar, there is no significant difference in the average number of signals observed before firms make their final offers (that is offers that result in a match). A two sided Mann Whitney $U$ test on session averages on the average number of signals observed when making a final offer in the first five markets, gives $p$ values of 0.63 when comparing Open to Renege, 0.32 when comparing Renege to Exploding and 0.316 when comparing Open to Exploding. However, as participants gain experience, matches come to be made later in the open offer and the renege treatments, but not in the exploding offer treatment. In the last five markets (markets 16-20), final offers in the exploding offer treatment are made with significantly fewer signals than the renege treatment ($p = 0.003$) and the open offer treatment ($p = 0.003$),\(^\text{10}\) while the renege and the open treatment are not significantly different ($p = 0.63$).

We now investigate the timing of offers in more detail. Figure 2 shows, for each treatment, in the last five markets, the percentage of offers that were made when one, two or all three signals (3 signals = final quality) about the applicants’ quality were available.

Figure 2 shows that unraveling occurs when firms can make exploding offers and acceptances are binding. When firms are forced to make open offers, or when applicants can renege on their acceptance, the markets experience almost no unraveling. Open offers and the applicants’ ability to renege also help a market to defer the timing of

\(^\text{10}\) In fact, the highest average number of signals used in any of the last five markets in any session of the exploding offer treatment, is lower than the lowest average number of signals used in any of the last five markets in any session of either the open or the renege treatments.
contracts, even when the market begins with early appointments (as in the first five markets of Figure 1).

The fact that the open offer treatment and the exploding offer treatment are so different, suggests that firms make exploding offers, when they are given the opportunity to do so. Indeed, in the last five markets of the exploding offer treatment, only firm 1 makes an open offer in more than 10% of the markets (while firm 5 makes no open offers at all). Except for firms 3 and 4, every firm made an exploding offer in every one of the last five markets in each of the seven sessions of the exploding offer treatment (and firms 3 and 4 made exploding offers in 34 of these 35 markets). In the renege treatment, firms make somewhat more open offers, but the vast majority of firms (at least 67%) make an exploding offer in each of the last five markets.

Thus when firms could make open and exploding offers, the majority of offers were exploding. Firms made use of their ability to make exploding offers to put pressure on applicants. However, this effect was more pronounced when acceptances are binding. When applicants can renege on their acceptance the value of making an exploding offer seems smaller, and firms made less use of that option.

Figure 3, which shows the timing of final offers for firms 4 and 5 only (in the last five markets) for each treatment, shows an even clearer difference among conditions than Figure 2. This means that the unraveling in the exploding offer treatment is to a large extent driven by firms 4 and 5, the high quality firms, which, in the open and renege treatment predominately hire applicants after all the information is known.
Figure 3a. For each treatment, in the last five markets, the proportion of final offers that were made when one, two or all three signals (and hence the final quality) about applicants’ quality were available, by firms 4 and 5 only.

Figure 3b. For each treatment, in the last five markets, the proportion of first offers that were made when one, two or all three signals (and hence the final quality) about applicants’ quality were available, by firms 4 and 5 only.

For firm 4 and firm 5, the final offer corresponds mostly to the first offer made.

In the exploding offer treatment, lower quality firms also make early offers, but their offers are much more often rejected. In the open offer and renege treatment, firms of all qualities wait for more information on applicants, before making their first offer. This results in first offers also being extended mostly when there is complete information about applicants’ qualities.

**Transaction Times:**

So far we examined the timing of offers averaged across different markets, now we explore the timing within markets. We will look at transaction times. A transaction is made (and announced to the market) only when an offer is accepted. Markets with exploding offers not only experience early contracting on average, but a large majority of such markets have a first acceptance made when only one signal is available, that is in periods 1-3. The following table shows for each treatment the timing of acceptances in the last five markets (where we use only final acceptances that were not reneged upon for the renege treatment).
Table 1: For each treatment (in the last five markets) the proportion of markets whose first acceptance (which as not reneged upon in the renege treatment) was made when only one signal, 2 signals or the final quality of applicants was available.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>1 Signal (1-3)</th>
<th>2 Signals (4-6)</th>
<th>Final Quality (7-9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploding</td>
<td>.71</td>
<td>.39</td>
<td>0</td>
</tr>
<tr>
<td>Open</td>
<td>0</td>
<td>.23</td>
<td>.77</td>
</tr>
<tr>
<td>Renege</td>
<td>.30</td>
<td>.10</td>
<td>.60</td>
</tr>
</tbody>
</table>

Markets with exploding offers not only experience early contracting on average. 71% of the markets have their first acceptance with only signal 1 available. All 35 markets (the last five markets of all 7 sessions of the exploding offer condition) have their first acceptance before the final quality of applicants becomes available. In contrast, when firms can only make open offers, or when acceptances by applicants are not binding, 77% and 60% of the markets, respectively, experience their first acceptance only after all the uncertainty about applicants’ quality is resolved.

Even though markets with different rules concerning exploding offers experience a difference in timing of the first accepted offer, their last accepted offer is predominately in periods 7-9. In the open and renege treatment, not a single market (of the last five markets) ends before period 7, and in the exploding offer treatment, 89% (31 out of 35) of the last five markets finish after period 7. Note that, in each treatment, the last firm to be unmatched has strong incentives to wait and see which of the two remaining applicants is of higher quality. Thus it is unsurprising that at least one offer is made after period 7 (since it will often be the case that at least one firm remains unmatched after period 6, even when firms try to match early).

What are the costs of unraveling?

From this point on, we eliminate from our main analysis one outlier session of the renege treatment. In that particular session there was one applicant who never accepted an offer. No other applicant in any session of any treatment behaved in this way. In footnotes we will show the analysis that includes all renege sessions.
We have seen that the market unravels when firms can make exploding offers and acceptances are binding. Now we investigate the costs of unraveling. We evaluate the different treatments according to the quality of the resulting matches. How much use do firms make of the information about applicants that becomes available over time? We consider three benchmarks: assortative matching when only signal 1 is available, assortative matching with 2 signals, and assortative matching once all the uncertainty about applicants’ qualities is resolved (the efficient outcome). That is, we compare the efficiency of the observed market outcomes with the most efficient match that could have been achieved using only the information in signal 1, in the first two signals, and when all three signals are available.

We calculate the value of the assortative match after the 1st signal by producing an assortative match between firms and applicants according to the applicants’ first signal. We use the actual quality of applicants determined during the experiment to compute the value of this match. In case of ties in the first signal between two applicants, we take the average of the two possible outcomes. The value of the assortative match with 2 signals is computed analogously. Let “1 Signal” and “2 Signals” denote the value of the assortative match after the first and after the second signal respectively, and “Efficient” the value of the unique stable and efficient match once all signals are known. Figure 4 shows the averages across sessions and markets of (Actual Profits – 1 Signal)/(Efficient – 1 Signal), and (Actual Profits – 2 Signal)/(Efficient – 2 Signal).

Figure 4a and 4b. 4a shows for each treatment the value of (Actual Profits – 1 Signal)/(Efficient – 1 Signal) averaged across sessions. That is it shows the relative gains
of the actual match towards efficiency compared to assortative matching after 1 signal. Figure 4b shows the similar results for signal 2.

Figure 4a shows that all treatments achieve on average a social surplus higher than assortative matching with one signal. The gains towards efficiency are significantly lower in the exploding offer treatment than in open (p=0.063) and renege (p=0.004). The renege treatment achieves significantly higher gains than the open offer treatment (p=0.045). Assortative matching based on 2 signals (the second signal becomes available at period 4) would have resulted in a higher efficiency than the exploding offer treatment, but both, the Open and Renege treatment achieve higher efficiency levels than assortative matching after two signals. Furthermore, both the open offer and renege treatment achieve significantly higher gains towards efficiency than the exploding offer treatment (p=0.007 and p=0.06 respectively), while they are not significantly different from each other (p=0.465). When we look at absolute efficiency levels, the efficiency of the exploding offer treatment is significantly lower than of the open treatment p=0.03 and the renege treatment p=0.009, while the renege and the open offer treatment are not significantly different p=0.116 using a two sided Mann-Whitney U test with session averages.14 15

A different way to measure the functioning of a market is to ask how many participants, who are currently not matched to each other, would both prefer to be so, instead of remaining with their current match (or being unmatched). A firm and a worker in such a position are called a blocking pair to the current match. We consider potentially “disruptive” blocking pairs that involve at least one matched player. These are blocking

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11 When we include the outlier session in the renege treatment, the value of the proportion of gains from assortative matching after one signal towards efficiency is 0.69 (instead of 0.88). The exploding offer treatment still achieves significantly lower gains towards the efficient outcome starting from the assortative match after 1 signal than all the sessions in the renege treatment: p=0.032.

12 When we include all renege sessions, the p-value is p = 0.2.

13 When we include all renege sessions, the value of the proportion of gains from assortative matching after two signals towards efficiency is 0.14 (as opposed to 0.56). The p-value when we include all renege sessions is p = 0.25.

14 All treatments achieve high levels of efficiency (compared to the alternative of no firm being matched). The efficiency in the exploding offer treatment is 93% compared to 96% in the open offer treatment and 98% in the renege treatment. However, even a random allocation of the six applicants to the five firms achieves an efficiency of 75%. Average efficiency of assortative matching after one signal is 88% and after two signals it is 93%. We’ll see below that even small changes in efficiency can be associated with big changes in payoffs to differently ranked participants.
pairs that would disrupt the outcome of the market, had they the chance. (Blocking pairs that simply involve unmatched participants are much less disruptive, and in naturally occurring markets they often have a subsequent opportunity to match to one another.)

The next table shows, for the last five markets of each treatment, the average number of disruptive blocking pairs in each market. In the exploding offer treatment, there are significantly more blocking pairs than in both the open (p=0.003) and renege treatment (p=0.004).\textsuperscript{16} In the exploding offer treatment, in each market, there are, on average 3 (firm, applicant) pairs that would rather be matched to each other, than to their current partner. In the open and renege treatment, there is on average only one such pair per market. The maximum number of disruptive blocking pairs is 10.\textsuperscript{17}

\textbf{Number of disruptive blocking pairs}

<table>
<thead>
<tr>
<th>Exploding</th>
<th>Open</th>
<th>Renege</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.06</td>
<td>1.03</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Table 2: Average number of disruptive blocking pairs in each treatment in the last five markets involving matched applicants.\textsuperscript{18}

\textbf{Individual level consequences of unraveling}

We have seen the loss of social surplus when firms can make exploding offers that are binding, compared to when offers have to be open or applicants can renege on their acceptance. Now we investigate the value of the match for each applicant and each firm separately, for the last five markets of each treatment. The following figures show for each treatment, which applicant was (on average) matched to which firm. The efficient and only stable match is the one at which firm 5 hires applicant 6, firm 4 hires applicant 5, and so on, with firm 1 hiring applicant 2 and applicant 1 remaining unmatched.

\textsuperscript{15} When we include all renege sessions, the comparison with the exploding offer treatment has a p-value of 0.07, the comparison to the open offer treatment yields p = 0.37.
\textsuperscript{16} The open offer and renege treatment, do not differ significantly in the number of blocking pairs (p = 0.2245). When we use all the renege sessions, the p-values are 0.002 and 0.46, when comparing it to the exploding and open treatment respectively.
\textsuperscript{17} The highest number of blocking pairs using only matched applicants, namely 10, is achieved by anti-assortative matching. Then firm 5 generates 4 blocking pairs (using matched applicants), firm 4 generates 3, firm 3 generates 2 and firm 2 generates only 1.
\textsuperscript{18} When we include session 2-2, then the renege treatment has on average 0.8 blocking pairs involving matched participants.
Figure 5: For each treatment, in the last five markets, for each firm the proportion of applicants of each quality this firm hired. UM shows the proportion of applicants of each quality that remain unmatched.
The exploding offer treatment is far from reaching an assortative match. Higher quality firms do match to higher quality applicants, and vice versa, but only barely so.

**The Firms**

The following graph shows for each firm the average quality of the applicant they are matched to and the average quality of the applicant that remains unmatched.

![The quality of the firms' matches](image)

Figure 6: The average quality of the applicant each firm is matched to in the last five markets of each treatment. UM shows the quality of applicants who remain unmatched. Efficient shows for each firm the quality of the applicant in the unique stable and socially efficient match.

The exploding offer treatment significantly lowers the payoff of the highest quality firm, firm 5, by 16% compared to the open (p = 0.0056) and by 15% compared to the renege (p = 0.046) treatment. It is the low quality firms, firm 2 and firm 1 that achieve a significantly higher payoff in the exploding offer treatment, compared to the open offer treatment (p=0.062 and p=0.07 for firm 2 and firm 1 respectively) and the renege treatment (p=0.001 and p=0.099).\(^{19}\) In the exploding offer treatment, firm 2 gains 45% compared to the open offer and 34% compared to the renege treatment.

\(^{19}\) When we include all renege sessions, the p-values for firm 5, when comparing the exploding offer to the renege treatment is p=0.023, while it is p=0.025 for firm 2 and p=0.05 for firm 1.
The difference in the quality of applicants between firm 2 and firm 5 is 0.92 in the exploding offer treatment, which is significantly lower than in the open offer treatment, 3.07 (p=0.0025) and the renege treatment, 2.76 (p=0.026).

In all treatments higher quality firms hire higher quality applicants, on average, but not in each session. For each firm, Table 3 considers the average quality of applicants hired in the last five periods, and shows the proportion of times that each firm was the one with the highest average quality, in each treatment. This gives another graphic view of the inefficiency of the Exploding offer treatment: firm 5 has the best average quality hire in fewer than half the markets.

<table>
<thead>
<tr>
<th>Firms</th>
<th>Exploding</th>
<th>Open</th>
<th>Renege</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.43</td>
<td>1</td>
<td>0.8</td>
</tr>
<tr>
<td>4</td>
<td>0.29</td>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: For each firm, for each treatment, we show the proportion of times this firm matched to the highest average quality of applicant in the last five markets.

**The Applicants**

The following table shows for each applicant the average quality of the firm they are matched to in the last five markets for each treatment.

---

20 When we include all renege sessions the p-value is p = 0.014
21 Only in the open offer treatment is the average quality of firm 1’s match lower than the average quality of unmatched participants. This is mostly driven by the fact that firm 1 sometimes fails to match, so that there are two unmatched applicants. We investigate the reasons for this when we discuss congestion below.
Figure 7: The average quality of the firm the applicant is matched to in the last five markets of each treatment. If an applicant is unmatched, we treat him as being matched to firm 0. Efficient shows for each applicant the quality of the firm in the unique stable and socially efficient match, where UM is unmatched.

As for the firms, it is the high quality applicants, applicant A6 and A5 that receive a significantly lower match in the exploding offer treatment than in the open offer treatment (by 10% and 11% with p=0.045 and p=0.049 for firm 6 and 5 respectively) and the renege treatment (by 17% and 13% with p=0.019 and 0.023). And it is a medium quality applicant, applicant 3, who significantly gains from unraveling (by 49% and 57% with p=0.026 and 0.041 compared to the open and renege treatment respectively). In all treatments higher quality applicants are hired by higher quality firms, on average. The difference in the quality of firms between applicant 3 and applicant 6 is 1.2 in the exploding offer treatment, which is significantly lower than in the open offer treatment, 2.9 (p=0.0034) and the renege treatment, 3.04 (p=0.0044).

Table 4 considers the average quality of firms that hired each applicant in the last 5 markets, and lets us see what proportion of times the applicant with the highest average quality firm is applicant A6. It should be 1, if applicant A6 gets, at least on average, the

---

22 When we include all renege sessions, the p-values are for firms 6, 5 and 3: p=0.009, p=0.108 and p=0.037 respectively.

23 When we include all renege sessions, the p-value is p = 0.0026.
best firm. This is exactly what we see in the Open and Renege treatments, but not in the Exploding offer treatment.

<table>
<thead>
<tr>
<th>Applicants</th>
<th>A6</th>
<th>A5</th>
<th>A4</th>
<th>A3</th>
<th>A2</th>
<th>A1</th>
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</thead>
<tbody>
<tr>
<td>Exploding</td>
<td>0.72</td>
<td>0.14</td>
<td>0.14</td>
<td></td>
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</tr>
<tr>
<td>Open</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Renege</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: For each applicant, for each treatment, the proportion of times this applicant was matched to the highest average quality firm in the last five markets.

The inefficient matchings in these markets are costly for the high quality firms and applicants, while low quality firms and applicants tend to gain from early matches.

**Congestion**

So far we focused on the average quality of applicants matched to specific firms. Now we investigate the number of matched and unmatched applicants (and firms). This will also be the starting point to investigate reasons for unraveling in the exploding offer treatment, namely whether it is driven mostly by the need to avoid congestion in the final periods.

We say that a market is congested if some firms are unmatched because they run out of time to make offers that they would have liked to be able to make. (See Roth and Xing, 1997, for a study of endogenous congestion in an open-offer market for clinical psychologists.)

In the exploding offer treatment, in the last five markets, no firm is ever unmatched. In the renege treatment, in the five sessions and five markets, firm 1 is unmatched twice, that is 1.6 % of firms fail to match.\(^\text{24}\) In the open offer treatment, 9.3 % of firms are unmatched (firm 2 is unmatched twice and firm 1 twelve times – out of 30 markets with 5 firms each), which is significantly higher than in the exploding (p= 0.043) and renege treatment (p=0.047).\(^\text{25}\)

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\(^{24}\) We continue to exclude session 2-2 of the renege treatment, as we had one applicant who *never* accepted any offer, which resulted in a session with an unusual number of unmatched firms, namely 1.2 on average in the last five markets.

\(^{25}\) When we include all renege sessions the difference is not significant any more, the p-value is 0.217.
To start understanding why open offers lead to so many more unmatched firms than when firms can make exploding offers and also when applicants can renege on their acceptance, we investigate the timing of acceptances of offers (In the renege treatment, we only consider acceptances that were not reneged upon).

![Cumulative acceptances within a market](image)

Figure 8: For each treatment, for the last five markets, the average number of final acceptances up to the end of each period.

In the exploding offer treatment about 60% of firms are matched by the end of period 6, while 54% are matched by the end of period 7 in the renege treatment. In the open offer treatment, only by the end of period 8 are 56% of firms matched, that is, on average, more than 2 firms still have to match in the last period in the open offer treatment. One possible explanation for the difference in timing of acceptances is that applicants can hold offers in the open offer treatment, while they have to accept offers that are exploding in the renege treatment. However, the fact that on average 0.46 firms remain unmatched implies that not all firms made offers to applicants who eventually accept their offer.

Another difference between the open offer treatment, and the renege treatment, is that all market participants are informed about acceptances of offers, but not of offers. In the open offer treatment, an applicant who waits for a better offer has to hold his offer while waiting, while in the renege treatment, if an applicant receives an exploding offer, he has

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26 We have seen that most firms in the renege treatment use exploding offers.
to either accept or reject that offer and can then wait for better offers to come along. Thus the information available to firms is different in the open and renege treatments. In the open offer treatment the firm will not make offers to matched applicants, but may waste one of her few remaining offers on an applicant who is already holding a better offer. In the renege treatment, once an applicant accepted an offer from a given firm, all firms of lower quality can tell that they should not waste an offer to try to match to this applicant, by extending an offer in the last 3 periods of the market.

This difference in information adds congestion to the open offer treatment, in which firms cannot avoid wasting some offers. On average, in the renege treatment, all firms make together about 5.6 offers in the last 3 periods of the market. In the open offer treatment, firms make about 5.53 offers in the last 3 periods of each market, hence about the same number of offers. In the renege treatment, only about 0.07 offers are wasted, in the sense that the applicant already has an offer from a higher quality firm. In the open offer treatment, about .93 offers, that is almost one offer per market, is wasted in this way. This difference is significant (p=0.049).  

While the open offer treatment and the renege treatment experience differences in timing of acceptances and differences in the number of firms that match, the exploding offer treatment and the renege treatment only differ in the timing of acceptances.

Note that the renege treatment experiences virtually no endogenous congestion, and that on average only 5.3% of firms are matched by the end of period 6. That is, when firms can make exploding offers, 3 periods are enough for the market to clear. However, in the exploding offer treatment, by the end of period 6, already 60.5 % of firms are matched. It appears that the reason for unraveling in the exploding offer treatment is therefore not anticipated congestion.

A further piece of evidence for strategic concerns of unraveling comes from asking about the length of the market, i.e. the timing between the first and the last acceptance, in the exploding offer treatment. 32 out of 35 markets last for 5 or more periods (with 10 lasting

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27 When we include all renege sessions the p-value is 0.0032.
exactly 5 periods). The longest a market can last, if all the firms make offers at the same time, is 5 periods, when all firms make offers to the same applicants. The fact that the markets in the exploding offer treatment last so long, shows that unraveling, in this treatment, is not due to avoid congestion only.²⁸

Discussion

In many markets, there are organizations of market participants that spend a good deal of effort addressing how and when offers are made, accepted, and rejected.²⁹ The formal and informal rules, customs, and norms that result are reflected in widely different organization of the matching processes that participants in these markets experience. In some markets, exploding offers are the norm, and applicants for positions find themselves faced with offers that must be accepted or rejected before other offers may be received or considered. In others, exploding offers are discouraged, or made more difficult to use to advantage. Because these many markets (e.g. those mentioned in footnote 29) are quite different from one another in other respects also, it is natural to look to the laboratory for an investigation that seeks to isolate the effects of different rules and customs concerning exploding offers.³⁰

In our experiment, we see that firms choose to make early exploding offers when the rules allow them, and when applicants’ acceptances are binding. In consequence, in this condition of the experiment we see inefficiently early contracting, lower profits, and a higher number of blocking pairs. Both the possibility for firms to make exploding offers, and the possibility of binding acceptances of applicants are necessary for that outcome.

²⁸ In the open offers only and the renege treatment, the first final offers are made later, and the markets also last for a shorter time. In the renege treatment, 40% of the markets last for 5 or more periods, and for the open offer treatment, the number is 23%.

²⁹ For example, the Council of Graduate Schools (CGS) in connection with graduate admissions, the National Association for College Admission Counseling (NCAC) for undergraduate admissions, the National Resident Matching Program (NRMP) for entry level medical residencies, the Specialty Matching Services for advanced medical residencies and fellowships, the Association of Psychology Postdoctoral and Internship Centers (APPIC) for clinical psychology positions, The National Association for Law Placement (NALP) for positions in law firms, The Judicial Conference of the United States and various ad hoc committees of judges for federal judicial clerkships, the National Collegiate Athletic Association (NCAA) for recruitment of college athletes, etc. Less formally, national meetings of scholarly societies play a role in the organization and timing of entry level academic job markets in a number of disciplines.

³⁰ Also, different rules and customs need not exhaust the reasons why markets differ in the incidence of exploding offers, etc. i.e. this can also be affected by equilibrium considerations that differ due to factors other than rules and customs. But in the laboratory, we can isolate the effect of different rules.

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Eliminating either of these, results in a market in which matches are made later, with more information, which results in increased efficiency and fewer blocking pairs.

The laboratory environment makes it easy to manipulate these factors. In naturally occurring markets, making it possible for applicants to renege on acceptances of early exploding offers involves a whole complex of behaviors; for example firms need to be willing to make offers to applicants who have already accepted exploding offers (recall e.g. footnote 3, about the debate over rules for college admissions). Yet the results of the experiment have clear implications for many markets suffering from inefficiently early contracting.

For example, in March of 2002, a large majority of Federal appellate judges voted to approve a proposal stating that “…the hiring of law clerks in the Fall after the first year of law school is an unacceptable practice,” and that they therefore endorsed “a moratorium on law clerk hiring during the Fall of 2002…” However, the proposal calls for no changes in the market, and in fact a FAQ accompanying the letter by Judges Becker and Edwards includes the following question and answer:

Q Are judges forbidden from making "exploding offers," i.e., offers that require an applicant to respond promptly to an offer?
A The Plan does not purport to address how an offer is given by a judge. This is for each judge to determine. However, no applicant is obliged to act on an offer if the terms are unacceptable, nor is an applicant obliged to accept the first offer that he or she receives.

Given that law students are virtually never reported to renege on promises made to senior Federal judges (Avery et al 2000), the results of the present experiment give us a clear prediction: the problem of early contracting will not be solved by the current resolution. Rather, more fundamental changes in the market culture of judges and law clerks will be needed.  

More generally, many of the markets that have suffered market failures associated with early contracting and the unraveling of appointment times have attempted to fix these

32 On this point, see the discussion in Haruvy, Roth, and Unver 2002.
market failures with centralized clearinghouses. But there are many more markets that
have not suffered these market failures, and are not organized in any centralized way.
The present paper is meant to begin the investigation of the hypothesis that elements of
market culture may play a critical role in the susceptibility of markets to these kinds of
failures. Our results suggest that, when it comes to market design, manners may be as
important as machinery.
Appendix:

**Proposition 1**

Assume that the firms and the applicants are all risk neutral.

In the exploding offer treatment, there is a sequential equilibrium, where all the firms $j$ for $j<5$ and workers match only in period 7, 8 or 9, that is after all the uncertainty is resolved. Firm 5 can make an offer as soon as there is an applicant $i$ with all applicants having signals $x$ such that $p_{x,6}(i)=1$ (where $p_{x,y}(i)$ is the probability of applicant $i$ to be of quality $y$ when applicants received signals $x=(x_1,x_2,...,x_6)$ where applicant $i$ receives signal $x_i$.)

Whenever an applicant receives an offer from firm $i$ in period 1-6, and the offer is not from the highest firm that is not yet matched, the applicant assumes that the firm is strictly better off from getting this offer accepted than waiting until all the uncertainty is resolved, and rejects the offer.

In and after period 7, the applicant of quality $y$ accepts all offers he receives from firms of quality $y+1$ or higher, or from the highest quality firm that is still on the market.

A firm of quality $j$ that is the $k$-highest unmatched firm in period 7 (in equilibrium, $k=j$) makes an offer in period 7 to the $k+1$ highest unmatched applicant.

**Proof of Proposition 1:**

Suppose the market is in period 1-6, that is applicants do not yet know their quality, and not all the uncertainty about applicants quality is resolved. (Note that, as signals range from 1-10, that is, 2 signals range from 2 to 20, it can be that the market already knows, the earliest after 2 signals, who is the worst applicant, or who is the best applicant (though not both at the same time), but no other qualities about applicants can be known for certain.)

Suppose all firms $j$ make an offer in period 7 to applicant of quality $j+1$ respectively. Suppose in that in a period before 7, firm $j$ makes an offer to applicant $i$. Given that all applicants received signals $x$, applicant $i$ has some probabilities (unknown to applicant $i$ who doesn’t observe the other applicant’s signals) $p_{x,y}(i)$ to be of quality $y$. We will show that in order for both the firm to profit from making an early offer to applicant $i$ and
the applicant to accept the offer, that \( p_{x,j}(i) = 1 \). That is, the deviation is only then mutually profitable (weakly), if the applicant is the correct candidate for the firm.

For a risk neutral firm \( j \) it is profitable to make an early offer (that is accepted) to applicant \( i \) of signal \( x \) and probabilities \( p_{x,y} = p_{x,y}(i) \) to be of quality \( y \) whenever:

\[
p_{x,1} j + p_{x,2} j + p_{x,3} j + p_{x,4} j + p_{x,5} j + p_{x,6} j \geq j(j+1)
\]

(F)

The (risk neutral) applicant \( i \) of signal \( x_i \) with all signals being \( x \) and probabilities \( p_{x,y} = p_{x,y}(i) \) to be of quality \( y \), prefers to accept firm \( j \)'s offer whenever

\[
p_{x,1} 0 + p_{x,2} 1 + p_{x,3} 2 + p_{x,4} 3 + p_{x,5} 4 + p_{x,6} 5 \leq
\]

\[
p_{x,1} j + p_{x,2} j + p_{x,3} j + p_{x,4} j + p_{x,5} j + p_{x,6} j.
\]

(A)

We now show, for each firm \( j \) that (F) and (A) can only be fulfilled if \( p_{x,j}(i) = 1 \), as long as no applicant is yet matched (for convenience we denote \( p_{x,y} = p_{x,y}(i) \)).

It is clear that when \( j = 5 \), (F) cannot be fulfilled unless \( p_{x,6} = 1 \).

For \( j = 4 \):

(A) equals

\[
p_{x,2} 2 \cdot 1 + p_{x,3} 3 \cdot 2 + p_{x,4} 4 \cdot 3 + p_{x,5} 5 \cdot 4 + p_{x,6} 6 \cdot 5 \leq
\]

\[
p_{x,1} 4 + p_{x,2} 2 \cdot 4 + p_{x,3} 3 \cdot 4 + p_{x,4} 4 \cdot 4 + p_{x,5} 5 \cdot 4 + p_{x,6} 6 \cdot 4.
\]

\[\Leftrightarrow\] \[4p_{x,1} \geq -6p_{x,2} - 6p_{x,3} - 4p_{x,4} + 4p_{x,6} \cdot 4 \]

(F) equals

\[4p_{x,1} + 8p_{x,2} + 12p_{x,3} + 16p_{x,4} + 20p_{x,5} + 24p_{x,6} \geq 20 \]

We use that \( \sum p_{x,i} = 1 \) and obtain

\[4p_{x,1} \leq -3p_{x,2} - 2p_{x,3} - p_{x,4} + p_{x,6} \cdot 4 \]

Combining (A) and (F) implies:

\[-3p_{x,2} - 4p_{x,3} - 3p_{x,4} + 5p_{x,6} \leq 0 \]

Which implies that \(-15p_{x,2} - 10p_{x,3} - 5p_{x,4} + 5p_{x,6} < 0 \) if \( p_{x,i} > 0 \) for at least one \( i \) of \( \{2,3,4\} \). The last strict inequality, with (F) delivers that \( 20p_{x,1} < 0 \) : contradiction.

If \( p_{x,i} = 0 \) for \( i = 2,3,4 \), then the combination of (A) and (F) imply that \( p_{x,6} = 0 \), which using (F) implies that \( p_{x,1} = 0 \) that means \( p_{x,5} = 1 \).

For \( j = 3 \):
(A) equals
\[ p_{x,2}^2 + p_{x,3}^3 + p_{x,4}^4 + p_{x,5}^5 + p_{x,6}^6 \leq \]
\[ 3p_{x,1} + 6p_{x,2} + 9p_{x,3} + 12p_{x,4} + 15p_{x,5} + 18p_{x,6}. \]

\[ \Leftrightarrow \quad 3p_{x,1} \geq -4p_{x,2} - 3p_{x,3} + 5p_{x,5} + 12p_{x,6}. \]

(F) equals
\[ 3p_{x,1} + 6p_{x,2} + 9p_{x,3} + 12p_{x,4} + 15p_{x,5} + 18p_{x,6} \geq 12 \]

We use that \( \sum_i p_{x,i} = 1 \) and obtain \( 3p_{x,1} \leq -2p_{x,2} - p_{x,3} + p_{x,5} + 2p_{x,6}. \)

Combining (A) and (F) implies: \( -p_{x,2} - p_{x,3} + 2p_{x,5} + 5p_{x,6} \leq 0 \)

Which implies that \( -2p_{x,2} - p_{x,3} + p_{x,5} + 2p_{x,6} < 0 \) if \( p_{x,i} > 0 \) for at least one \( i \) of \( \{2,5,6\} \). The last strict inequality, with (F) delivers that \( 3p_{x,1} < 0 \): contradiction.

If \( p_{x,i} = 0 \) for \( i = 2,5,6 \), then (F) implies that \( p_{x,3} = 0 = p_{x,1} \), which means \( p_{x,4} = 1. \)

For \( j = 2 \):

(A) equals
\[ p_{x,2}^2 + p_{x,3}^3 + p_{x,4}^4 + p_{x,5}^5 + p_{x,6}^6 \leq \]
\[ 2p_{x,1} + 4p_{x,2} + 6p_{x,3} + 8p_{x,4} + 10p_{x,5} + 12p_{x,6}. \]

\[ \Leftrightarrow \quad 2p_{x,1} \geq -2p_{x,2} + 4p_{x,4} + 10p_{x,5} + 18p_{x,6}. \]

(F) equals
\[ 2p_{x,1} + 4p_{x,2} + 6p_{x,3} + 8p_{x,4} + 10p_{x,5} + 12p_{x,6} \geq 6 \]

We use that \( \sum_i p_{x,i} = 1 \) and obtain \( 2p_{x,1} \leq -p_{x,2} + p_{x,4} + 2p_{x,5} + 3p_{x,6}. \)

Combining (A) and (F) implies: \( -p_{x,2} + 3p_{x,4} + 8p_{x,5} + 15p_{x,6} \leq 0 \)

Which implies that \( -p_{x,2} + p_{x,4} + 2p_{x,5} + 3p_{x,6} < 0 \) if \( p_{x,i} > 0 \) for at least one \( i \) of \( \{4,5,6\} \). The last strict inequality, with (F) delivers that \( 2p_{x,1} < 0 \): contradiction.

If \( p_{x,i} = 0 \) for \( i = 4,5,6 \), then (F) implies that \( p_{x,2} = 0 = p_{x,1} \), which means \( p_{x,3} = 1. \)

For \( j = 1 \):

(A) equals
\[ p_{x,2}^2 + p_{x,3}^3 + p_{x,4}^4 + p_{x,5}^5 + p_{x,6}^6 \leq \]
\[ p_{x,1} + 2p_{x,2} + 3p_{x,3} + 4p_{x,4} + 5p_{x,5} + 6p_{x,6}. \]

\[ \Leftrightarrow \quad p_{x,1} \geq 3p_{x,3} + 8p_{x,4} + 15p_{x,5} + 24p_{x,6}. \]

(F) equals
\[ p_{x,1} + 2p_{x,2} + 3p_{x,3} + 4p_{x,4} + 5p_{x,5} + 6p_{x,6} \geq 2 \]

We use that \( \sum_i p_{x,i} = 1 \) and obtain \( p_{x,1} \leq p_{x,3} + 2p_{x,4} + 3p_{x,5} + 4p_{x,6}. \)

Combining (A) and (F) implies: \( 2p_{x,3} + 6p_{x,4} + 12p_{x,5} + 20p_{x,6} \leq 0 \)
Which implies that $p_{x,i} = 0$ for $i = 3, 4, 5, 6$, then (F) implies that $p_{x,1} = 0$, which means $p_{x,2} = 1$.

**Proposition 2:**
All NE that survive iterated elimination of weakly dominated strategies in the open offer treatment, are ones where in period 7 (or 8 or 9) each firm $j$ for $j < 5$ makes an offer to applicant of quality $j+1$. Firm 5 can make an offer as soon as there is an applicant $i$ with all applicants having signals $x$ such that $p_{x,6}(i) = 1$.

Applicants hold the best of all offers they receive before period 7, applicants can accept the offer of a firm, if it is the firm of highest quality that is still unmatched.

**Proof:**
First note that before period 7, each applicant will only accept an offer, if the offer is from the firm of highest quality that is still unmatched. Otherwise, the applicant will among all his offers hold the offer from the firm of quality highest. The reason is that receiving an offer from a firm is independent on the decision whether to accept, reject or hold the offer. But once the applicant accepted an offer, he cannot receive any more offers. Furthermore, before period 7, for all signal values, the applicant does not know his quality, that means for every offer he receives that is not from the highest firm that is still unmatched, there is a positive chance that he receives a higher offer (as he may be of a highest quality in period 7.)

Given these strategies of applicants, we show that no firm has an incentive to make an early offer (apart from making an offer to an applicant $i$ with $p_{x,6}(i) = 1$ before period 7.

We will show that for one firm after the other, starting with the highest quality firm.

Note that it is a weakly dominated strategy for firm 5 to make an offer to an applicant before it is clear that that applicant is of quality 6. (Firm 5 can only risk receiving a worse applicant. If firm 5 waits until the applicant of quality 6 is revealed and makes an offer at that time, (e.g. period 7, or possibly period 4), the applicant will still be
available, because we eliminated strategies of applicants that accept offers before either period 9, or before firm 5 is matched.)

This in turn implies that it is a weakly dominated strategy for firm 4 to make an offer only in period 7 or 8 or 9 to applicant of quality 5.

Suppose firm 4 makes an offer to applicant $i$ before period 7, who has probabilities $p_{x,y}(i)$ to be of quality $y$ in period 7, given that all applicants received signals $x$.

The alternative strategy is for firm 4 to wait until period 7 and make an offer to the applicant of quality 5. Applicant 5 will still be available, as he will not receive an offer from firm 5, and does not accept other offers before period 9 (see above.)

If, in period 7, applicant $i$ has quality $6 > 5$, then firm 5 will make applicant $i$ an offer. Applicant $i$ will reject firm 4’s offer, firm 4 did not benefit from making an early offer. If firm 5 makes the offer only in period 9, firm 4 will be displaced in period 9 without having a chance to make another offer: Firm 4 can be strictly worse off.

If, in period 7, applicant $i$ has quality 5, then firm 4 did not benefit from making an early offer.

If, in period 7, applicant $i$ has quality $y < 5$, then firm 5 (the only firm of quality higher than 4) will not make an offer to applicant $i$. Applicant $i$ will accept firm 4’s offer, firm 4 is strictly worse off by making an early offer.

Similarly we can show that this implies that it is a weakly dominated strategy for firm 3 to make an offer only in period 7 or 8 or 9 to applicant of quality 4.

This in turn implies the same thing for firm 2 and then for firm 1.

**Proposition 3:**

All NE that survive iterated elimination of weakly dominated strategies in the renege treatment, are ones where in period 7 (or 8 or 9) each firm $j$ for $j<5$ makes an offer to applicant of quality $j+1$. Firm 5 can make an offer as soon as there is an applicant $i$ and all applicants have signals $x$ such that $p_{x,6}(i) = 1$.

Whenever an applicant receives an offer when he has already accepted another offer, he will renege on his former acceptance.

**Proof of Proposition 3:**
First, let us be agnostic whether the applicant accepts or rejects exploding offers he receives before period 7. The applicant will accept any offer from a firm if that firm is the highest unmatched form still on the market. If the applicant receives only open offers, he will, among all his open offers hold the offer from the firm of highest quality. Whenever the applicant accepted an offer, and he receives an offer either from firm 5, or the highest quality firm that is still unmatched, he will renege on his former acceptance and accept that offer. If the applicant is in period 7 of quality $y$, then he will accept any offer from firm $y+1$, unless he already accepted or holds an offer from a higher quality firm.

Given these strategies of applicants, we show that no firm has an incentive to make an early offer (apart from making an offer to an applicant $i$ with $p_{x,6}(i) = 1$) before period 7. We will show that for one firm after the other, starting with the highest quality firm.

Note that it is a weakly dominated strategy for firm 5 to make an offer to an applicant before it is clear that that applicant is of quality 6. (Firm 5 can only risk receiving a worse applicant. If firm 5 waits until the applicant of quality 6 is revealed and makes an offer at that time, (e.g. period 7, or possibly period 4), the applicant will still accept that offer, because we eliminated strategies of applicants that do not accept offers they receive form the highest quality firm.

This in turn implies that it is a weakly dominated strategy for firm 4 to make an offer only in period 7 or 8 or 9 to applicant of quality 5.

Suppose firm 4 makes an offer to applicant $i$ before period 7, who has probabilities $p_{x,y}(i)$ to be of quality $y$ in period 7, given that all applicants received signals $x$.

The alternative strategy is for firm 4 to wait until period 7 and make an offer to the applicant of quality 5. Applicant 5 will accept this offer, as he will not have, nor receive an offer from firm 5. If the applicant holds the offer from another firm, he will renege on that firm, as it is of lower quality than 4 (see above.)

What is the expected payoff for firm 4 to make an offer before period 7.

If, in period 7, applicant $i$ has quality $6 > 5$, then firm 5 will make applicant $i$ an offer. Applicant $i$ will renege on firm 4’s offer, firm 4 did not benefit from making an early offer. If firm 5 makes the offer only in period 9, firm 4 will be displaced in period 9 without having a chance to make another offer: Firm 4 can be strictly worse off.
If, in period 7, applicant \( i \) has quality 5, then firm 4 did not benefit from making an early offer.

If, in period 7, applicant \( i \) has quality \( y < 5 \), then firm 5 (the only firm of quality higher than 4) will not make an offer to applicant \( i \). Applicant \( i \) may accept firm 4’s offer, firm 4 can be strictly worse off by making an early offer.

Similarly we can show that this implies that it is a weakly dominated strategy for firm 3 to make an offer only in period 7 or 8 or 9 to applicant of quality 4.

This in turn implies the same thing for firm 2 and then for firm 1.
**Bibliography:**


