Relationships, Competition, and the Structure of Investment Banking Markets*

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This version: January 2005
Original version: January 2003

Abstract

It is well known that competition can destroy incentives to invest in firm-specific relationships. This paper examines how the tension between relationships and competition is resolved in the investment banking market, which for decades has been characterized by both relationships and competition. The model studies the impact on relationships of four different dimensions of competition: non-exclusive relationships, competition from arm’s-length intermediaries, non-price competition, and endogenous entry. The analysis shows how market equilibrium adjusts so that relationships are sustained in the face of such competition. Banks are shown to establish relationships without either local or aggregate monopoly power. The model rationalizes two distinct empirical regularities of market structure: the invariance of market concentration to market size; and a pyramidal market structure with an oligopoly comprising similar-sized players at the top and a large number of small banks at the bottom. The analysis may also shed light on the industrial organization of other professional service industries.

JEL classification: G20, L22
Key words: investment banking, loose linkage, relationships, sunk costs.

* We thank two anonymous referees, an editor, colleagues and seminar participants at various universities, the Econometric Society Meetings (Buenos Aires) and the National Bureau of Economic Research (Summer Institute meetings) for helpful comments. Anand gratefully acknowledges the financial support of the Division of Research at Harvard Business School. Galetovic gratefully acknowledges the financial support of Fondecyt (project C1970340), Fundación Andes, the Hewlett Foundation and the Mellon Foundation.
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1. Introduction

The industrial organization of professional service markets has been largely ignored. A central feature of these markets is the existence of client-firm relationships that are costly to establish and difficult to appropriate. But, professional service firms also compete vigorously in many dimensions. This raises an intriguing question: how are the incentives to invest in relationships preserved in the face of competition? Indeed, a key insight from other relationship-based markets like commercial banking is that competition should weaken incentives to invest in relationships, and that some market power is needed to preserve these investments (see, for example, Petersen and Rajan 1994). This paper examines how the tension between relationships and competition is resolved in a particular professional service industry: investment banking. The analysis may shed light on several outstanding puzzles about this and related industries.

Our focus on the US investment banking market is motivated by the fact that there is a vast literature establishing that, for many decades, it has been characterized by both relationships and competition. Competition occurs along four dimensions. First, there are a large number of banks (more than 1,000). Second, each firm is also involved with many different banks which offer similar services, so there is no exclusive dealing. Third, regulation does not “contaminate” the observed market structure and there is free entry and exit. Fourth, industry accounts typically argue that there is brutal nonprice competition.

Some observers have also pointed out, however, that competition in prices is soft. Firms rarely choose underwriters through competitive bidding, and investment banks cooperate on each deal via syndicates. Moreover, prices are unusually stable, non-negotiable, and, importantly, do not depend on firm- or deal-specific characteristics: Matthews (1994, p.161) notes that spreads on high-quality, long-term corporate bonds have been 7/8% of capital raised for several decades. Similar practices are observed in Britain, where, for decades, underwriting fees have been equal to 1.25% of capital raised. And, recently, Chen and Ritter (2000) documented that 90% of initial-public-offerings deals between 1995 and 1998 that raised between $20 and $80 million had gross spreads of exactly 7%. All this suggests that .

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1 One reason is that relationships are often embodied in human capital that can move between firms, taking these relationships with them.
2 Relationships between investment banks and corporations have always been important in the US investment banking market, and the sunk costs incurred by investment banks in establishing and maintaining each relationship are large; see Nanda and Warther (1998), Wilhelm (1999), and Wilhelm and Downing (2001).
4 Rolfe and Troob (2000, p.103) argue in a colorful recent account of investment banking, spreads have stayed high “[... ] because there has always been an unspoken agreement among the bankers that when it comes to underwritings they won’t compete on price. The spreads are sacrosanct. He who cuts spreads will himself become an outcast [...]. The community of investment banks has always been small enough so that if one bank were to break ranks on the pricing issue, the others would quickly join forces and squash the offender [...]. Every banker knows that the pricing issue is a slippery slope best avoided because once the price cutting begins, there’s no telling where it will end.”
These facts raise interesting questions. First, how does competition impact relationships? Second, how do the various forms of competition interact? For example, how is soft price competition sustained with a large number of banks, and entry; and, does non-price competition dissipate rents? Third, how can these facts on firm conduct be reconciled with those on market structure? Specifically, the structure of investment banking is both remarkably stable over time, and displays the following salient features:

- **Concentration is unrelated to market size:** Market size has grown significantly over time but concentration has not changed. Consider, for example, Figure 1, which plots the C8 ratio (left vertical axis) and volume underwritten (right vertical axis, in logs) between 1950 and 1986. Market size grew about twenty fold, but concentration barely changed (and even slightly increased). As another example, Figure 2, which plots the same concentration ratio for mergers and acquisitions between 1987 and 1998, shows the same regularity.

- **A two-layered pyramidal structure:** with an oligopoly of similar-sized players at the top, few mid-sized banks, and a large number of small banks. Since the late nineteenth century a group of between six and ten banks with similar market shares (the so-called ‘bulge-bracket’ banks) has consistently underwritten more than 70% of securities issued. According to Hayes et al. (1988, p.17): “As at virtually every stage of its evolution, investment banking exhibited an oligopolistic industry structure that was roughly pyramidal in shape, with a handful of powerful firms at the apex.”

To explain these regularities we start from the observation that relationships are characterized by what Eccles and Crane (1988) call the “loose linkage” between relationship costs and deal revenues: each investment bank incurs costs to set up and nurture a relationship, but does not directly charge for it nor receives any contractual assurance that it will be selected by the firm on its deals. Instead, an investment bank collects fees only when it does a deal. But then how can investment banks be assured that the fees charged ex-post will cover the ex-ante cost of relationships?

We study this question with a model where infinitely-lived investment banks sell services, which we call “deals”, to firms. A firm is characterized by its deal size \( v \). Banks can employ one of two technologies to “do deals”: an arm’s-length technology or a relationship technology. All banks can do arm’s-length deals but to use the relationship technology, a bank has to sink a one-time

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See also “Overcharging Underwriters” *The Economist* (June 27, 1998), where it is noted that “… studies in both countries suggest issuing companies are overcharged, and that they are stung for more in America.” Similar attributions to bankers can be found elsewhere, as noted by Chen and Ritter (2000, p. 1,106). For an empirical analysis of the IPO market see Hansen (2001).

\(^{5}\) By “deal” we mean, for example, a security flotation, a merger or an acquisition.
entry cost, \( E \). We call banks who invested in the relationship technology as relationship banks; banks who only can do arm’s length deals are termed fringe banks.

The cost of implementing an arm’s-length deal increases linearly in deal size (i.e. a deal of size \( v \) costs \( \beta v \)), as seen in Figure 3. By contrast, each bank-firm relationship involves a cost \( R \) which is independent of \( v \) and is sunk by the time the deal is done and fees are collected. A second feature of relationships is that they are non-verifiable, so that banks cannot directly charge for the sunk costs \( R \). Instead, they can only charge fees when they do a deal—i.e., there is loose linkage.\(^6\)

Third, relationships are not excludable: banks can free-ride on the information gathered through others’ relationships, so that the deals for a firm that has relationships can be implemented at a very low cost by other relationship banks who did not sink \( R \).\(^7\) Last, each firm establishes \( k > 1 \) relationships.

The first set of results is about competition between relationship and fringe banks. Does it erode the incentives to establish relationships? The key result is that scale economies inherent to relationships provide relationship banks with a cost advantage when providing services to large firms, because \( R < \beta v \) for firms whose \( v \) is large enough. Competition from fringe banks will not threaten relationships because it cannot: any fee that makes profits for a fringe bank also makes ex-post profits for a relationship bank. Hence relationship banks can and do charge lower fees in equilibrium. It follows that any firm, regardless of deal volume, would prefer to do its deals with relationship banks. However relationship banks would make losses for firms with small deal volumes. Thus, all firms want relationships, whereas relationship banks ration smaller firms out. In other words, the market will be vertically segmented: relationship banks will serve large firms and fringe banks will serve small firms. This separation is similar to the common distinction in practice between “bulge bracket” banks, which serve predominantly large corporations, and the rest, which serve smaller firms.

The second set of results characterizes competition among relationship banks. To recoup the investments in relationships made each period, price competition must be soft in equilibrium, which is sustainable with an implicit contract between relationship banks not to undercut. This implicit contract yields an oligopoly such that relationship banks have similar market shares and concentration cannot fall below a given bound. In addition, we show that this lower bound on concentration does not fall as market size increases. Last, because the implicit contract serves an

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\(^6\) The assumption that relationships are nonverifiable distinguishes this setting from the large body of work that studies investment incentives, incomplete contracting and the hold-up problem (see Hart 1995). In those settings, ex-post hold up can be moderated by specific ex-ante allocation of decision rights over the assets: examples include the allocation of ownership rights over the asset as in Grossman and Hart (1986) and Hart and Moore (1990), or the contractual right of one party to block the use of the asset in a transaction with a third party, as in Segal and Whinston (2000).

\(^7\) Relationship banks can free ride on each other’s efforts by copying their financial products, poaching competitors’ employees, or by getting information on rivals’ ideas from client firms themselves. Section 2.1 discusses these sources of free-riding in more detail.
efficiency role, it is robust to entry despite yielding rents to banks in equilibrium.\textsuperscript{8,9}

Imperfect competition can be traced to the non-verifiability and non-excludability of relationships. We show that if banks could charge $R$ directly, the relationship segment could be perfectly competitive and there would be no tension between relationships and competition despite scale economies from relationships. The reason is that scale economies exist only at the level of each bank-firm relationship, i.e., at the “local” level. On the other hand, aggregate relationship costs increase (linearly) with the number of relationships that the intermediary establishes. We also show that nonexcludability of relationships is necessary to explain why the investment banking market is an oligopoly. If, as a counterfactual, relationships were nonverifiable but excludable, implicit contracts would still be needed to soften price competition, but would not impose any restrictions on aggregate market structure.

The comparative static exercises in section 3 distinguish changes in the number of banks from a change in the intensity of price competition.\textsuperscript{10} The general message from these exercises is that when competition gets more intense, the endogenous adjustment in fees or in the number of banks undoes their deleterious effects on the incentives to establish relationships. In addition, vertical segmentation suggests caution when assessing how concentrated or competitive the investment banking industry is. Specifically, market segmentation implies that looking at the industry as a single market will unduly deflate traditional measures of concentration (e.g., the inverse of a Herfindahl index). Section 4 discusses additional welfare implications.

Our paper is related to a large literature on relationships in financial markets. Petersen and Rajan (1995) were the first to point out that imperfect competition is necessary to maintain relationships.\textsuperscript{11} That conclusion still holds here, but the analysis shows why neither local monopoly power nor aggregate monopoly is necessary to establish relationships. Aggregate monopoly power is not necessary because relationships imply local scale economies, not aggregate ones.

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\textsuperscript{8}The study of self-enforcing norms in the relationship segment is methodologically related to Dutta and Madhavan (1997) who study implicit collusion in broker-dealer markets. The collusive equilibrium in that model rationalizes a striking series of practises which have been empirically documented. Unlike that model, the implicit contract here is not a purely facilitating device, but supports rents that are necessary for a relationship segment to exist. Consequently, a self-enforcing norm is sustainable in equilibrium even with free entry.

\textsuperscript{9}Pichler and Wilhelm (2001) obtain a similar efficiency result in their analysis of investment banking syndicates. They study how the relationships and the potential for free-riding on information-gathering, shapes syndicates. Membership stability across deals create entry barriers that provide banks with quasi-rents, which stimulate investments in information production.

\textsuperscript{10}This distinction follows Sutton (1991) and others who note that in models with endogenous entry the number of economic actors is an uninformative measure of competition.

\textsuperscript{11}See also Mayer (1988) and Hellwig (1991) for conceptual treatments of this issue. There is a large theoretical literature that explores the benefits and costs of relationships; for surveys of this literature see Berger (1999), Boot (2000) and Ongena and Smith (2000). For example, Berglöf and von Thadden (1994), Boot and Thakor (1994), Chemmanur and Fulghieri (1994), Diamond (1991), Rajan (1992) and von Thadden (1995) all model the benefits of long-term bilateral relationships. Several papers in the literature have also studied the cost of exclusive relationships that come from the exploitation of market power when banks can hold up firms (e.g., Greenbaum et al. 1989, Rajan 1992, Sharpe 1990 and von Thadden 1998).
power is not necessary because endogenous entry and exit together with soft price competition can undo the deleterious effect of multiple relationships. This “possibility result” also contrasts with previous studies which have generally posited the need for either exclusive relationships (see, for example, the discussion in Hellwig 1991) or aggregate market power (following Petersen and Rajan 1995).

Free-riding problems in information production by investment banks and the facilitating role of market structure are also studied in Benveniste et al. (2002). Market power allows banks to bundle IPOs in the same industry, that are sold to a similar investor pool. This in turn allows the “smoothing” of underpricing across a wave of IPOs. The burden of compensating investors for costly information production is then shared more equitably between pioneers and followers. Empirical support for the theory is provided in Benveniste et al. (2003). The analysis in that pair of papers takes the existence of market power in investment banking as given, whereas this study examines where market power comes from.

This paper is also related to Anand and Galetovic (2000), which presents a model to explain why intermediaries may finance the production of assets over which they cannot establish property rights (such as information and certain types of human capital). That paper shows that investment in non-excludable assets requires soft price competition, which may be the equilibrium of a repeated game between intermediaries. By contrast, the focus in this paper is the tension between relationships and competition. While soft price competition is a feature of investment banking, most observers describe this industry as brutally competitive in almost all other dimensions. In this model we study these other dimensions—firms have multiple relationships, there is a large fringe of banks that do not establish relationships in equilibrium, and relationship banks can exert sales efforts—and endogenously derive market structure. We show why relationships are not deleteriously impacted by price competition from fringe banks (because of vertical segmentation), nor by entry and non-price competition (because market structure endogenously adjusts to preserve investment incentives).

Finally, our paper is related to Boot and Thakor (2000), who study how commercial banks that make relationships are affected by competition from banks that do not (“arm’s-length banks”).12 Both the characteristics and the consequences of bank-client relationships are different in their model from the one studied here. The reason for these differences is, quite simply, that the source of relationships in the two markets are different. We discuss in more detail these differences between commercial and investment banking, and its consequences for how the tension between relationships and competition is solved, in section 4.2.

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12 Yafeh and Yosha (2001) have also recently studied how competition from arm’s length loans affect relationship lending. Their focus, however, is not on the canonical intertemporal problem caused by sunk relationship investments, but on intratemporal competition between the arm’s-length and relationship segments and the strategic use of relationships as an entry-deterrence device.
The rest of the paper is organized as follows. Section 2 documents the importance of relationships in investment banking, motivates the formal model and shows the well known tension between competition and relationships. Section 3 describes how the tension between relationships and competition is solved, and presents the results of the model. Section 4 discusses several extensions and robustness checks. Section 5 discusses other industries where relationships are important, and their similarities and differences from investment banking. Section 6 concludes.

2. The model

2.1. Motivation

2.1.1. Relationships in investment banking

Relationships between investment banks and firms provide banks with access to firm-specific information that can be used to structure deals or price securities. Crane and Eccles (1993, p. 136) note that “access and information exchange are the key elements in the definition of relationships” between investment banks and client firms. Consequently, suppliers with relationships often have “preferred vendor status” because without such information they would, for example, be “making virtually random blue-book pitches with little chance of hitting the target.” Similarly, in a recent survey, Boot (2000, p.7) points out that even the task of underwriting public issues involves absorbing credit and placement risk which may be “facilitated by the proprietary information and multiple interactions that are the hallmark of relationship banking.” And, Wilhelm and Downing (2001) note that while changes in information technology might commoditize those investment banking services that have to do mostly with the storage and dissemination of information to investors, the information needed for corporate advisory services still rests largely on bank-firm relationships.

Relationships are well-documented for the US market. Until about 25 years ago, the rule in the industry was that a corporation would establish a relationship with only one investment bank. While relationships have varied in strength over time, they still remain important today. In a recent Institutional Investor survey of 1,600 chief financial officers of firms made in August 2001, 44% of those who prefer “specialized institutions” for their different needs, and 64% of those who prefer one-stop banks, stated that their primary reason for choosing a bank was “prior relationships”

13 Crane and Eccles (1993, pp.131-136).
14 See Wilhelm and Downing (2001) for an overview.
15 Eccles and Crane (1988, pp. 53 and 54) attribute the end of exclusive relationships to the increasing sophistication of capital markets. Specifically, the capital markets offer more product variety and, as firms have themselves become financially more sophisticated, they now encourage solicitations from more than one bank.
16 See Nanda and Warther (1998) for an analysis of the trends in the strength of underwriting relationships. Crane and Eccles (1993, p.132) note that relationships were even more important in the early 1990s than they were in the previous decade.
with it. Moreover, evidence on firms’ choices of investment banks points indirectly to the strength of relationships as well. For example, Baker (1990) examined ties between investment banks and corporations with market value of more than $50 million between 1981 and 1985. He reports that the 1,091 corporations that made two or more deals during this period used three lead banks on average (these firms made eight deals on average). All but nine granted more than 50% of their business to their top three banks and, on average, 59% of the business was allocated to the top bank. Similarly, Eccles and Crane (1988, ch.4) report that among the 500 most active corporations in the market between 1984 and 1986, 55.6% used predominantly one bank to float their securities, and the rest maintained relationships with only a few banks. They did not find any corporation selecting underwriters on a deal-by-deal basis. James (1992) finds that in the first common stock security offering after an initial public offering (IPO), 72% of firms choose the same lead bank as before; for debt offerings, 65% of issuers do not switch banks. And, Krigman et al. (2001) show that 69% of firms that made an IPO between 1993 and 1995 and a secondary equity offering within three years of the IPO, chose the same lead underwriter in both transactions.

It has been argued that relationships may subject the firm to a hold up from the intermediary with whom it has a relationship. As we will see now, however, the opposite seems to be more relevant in the case of investment banks: firms may find it too easy to switch investment banks once they have established the relationship.17

2.1.2. The technology of relationships

Three characteristics of relationships have been extensively documented. First, firms and investment banks interact constantly in the course of a relationship, but the bank is paid only when a deal is made. Eccles and Crane (1988) call this the ‘loose linkage between costs and fees’. It implies that investment banks recoup the costs incurred to set up and nurture a relationship only if selected to do a deal. As Eccles and Crane (1988, pp. 39-40) point out, one reason for loose linkage is that it is difficult for business firms to evaluate the quality of the advice provided, unless deals are done. In other words, it is not merely an industry practise that can be changed, but a technological feature of relationships which stems from non-verifiability.18

Second, banks incur substantial costs to establish and nurture a relationship and these are sunk by the time they compete for doing the deal of a given firm. As Crane and Eccles (1993, p. 142) point out, “[...] the strategy of investment banks [is] to incur substantial costs in delivering value—

17 Ongena and Smith (2001) study the duration of firm-bank relationships in Norway and find that firms are more likely to leave a given bank as the relationship matures, thus suggesting that firms do not get locked into relationships in commercial banking either.

18 Indeed, because banks “are willing to incur current costs in the hope of getting future fees, [t]his gives the customer an opportunity to receive services that he or she may never have to pay for” (Crane and Eccles, 1993, p. 143). The extreme case of loose linkage is the “analysis” function of investment banks, where banks earn most of their commissions from investors who trade the firm’s security.
in the form of advice, special studies, and market information—as a way of creating obligations that are hopefully converted into transaction fees in the future.” Moreover, a significant fraction of this sunk cost is incurred by the investment bank because most of the exchange of information takes place through direct interaction with the bank’s staff person (often referred to as a “relationship manager”).

Third, investment banks often cannot establish property rights because information is not excludable. To begin, because most of the exchange of information takes place through direct interaction between the firm and the investment bank’s staff person, the relationship-specific knowledge often walks with employees when they are hired away. As an example, Deutsche Bank built a global investment bank in a year (Deutsche Morgan Grenfell) by hiring away staff en masse from other major banks. At the same time, Eccles and Crane (1988, pages 61-62) note that banks often fear that firms will take their ideas to be implemented by rival banks for less money. Copying is quite easy: Tufano (1989) notes that most product innovations by banks are copied by rivals within a day of introduction. And, as is the case in commercial banking, most firms have more than one relationship.

2.2. Model description

There are many identical and risk-neutral investment banks and a continuum of firms of measure $f$ which wants to do deals. Each firm is described by its deal volume $v$ which is uniformly distributed in the interval $[0, \overline{v}]$, with density function $g(v) = \frac{1}{\overline{v}}$ and corresponding cdf $G(v) = \int_0^v \frac{1}{\overline{v}} du = \frac{v}{\overline{v}}$. Thus $\frac{v}{\overline{v}}$ is the measure of firms whose deal volume is at most $v$.

Banks can implement deals using either a relationship or an arm’s-length technology. All banks can do arm’s-length deals, but to use the relationship technology a bank must sink an entry cost $E$ once. Banks who invest in this technology will be called “relationship banks.” Banks who don’t will be called “fringe banks.”

Firms that do deals with a relationship bank establish $k > 1$ relationships (i.e. relationships are not exclusive). Following Eccles and Crane (1988) we call this the firm’s group of “$k$ core banks”.

\footnote{A good or service is excludable if the owner can prevent others from using it at a very low cost.}
\footnote{See Anand and Galetovic (2000).}
\footnote{There are other examples as well: Wilhelm and Downing (2001) note that when “Bruce Wasserstein and Joseph Perella walked away from First Boston’s top-ranked M&A business in 1988...virtually overnight First Boston fell from the ranks of serious contenders for new M&A business” (pages 68-69).}
\footnote{Tufano (1989) estimates the costs of designing a security, including product development, marketing and legal expenses to be between $0.5$ million and $5$ million. These products cannot be patented and all details become publicly available once the offering is filed with the SEC. For a model of product innovation in investment banking with weak property rights, see Bhattacharya and Nanda (2000) and Persons and Warther (1997).}
\footnote{See Eccles and Crane (1988). Moreover, in their survey on relationships in commercial banking Ongena and Smith (2000) conclude that multiple relationships are a common feature of nearly all countries for which evidence has been collected.}
\footnote{Explicitly modeling the entry cost $E$ also allows us to endogenize the number of relationship banks $m$ later on.}
For simplicity, $k$ is assumed to be exogenous and the same for all firms (in section 3.5 we discuss why this assumption is not restrictive). Note that we assume away any differentiation among core relationship banks—any of the $k$ banks is as good as any other to do the deals of the firm. Figure 3 shows the investment banking market from the perspective of a firm with relationships.

Each time a bank establishes a relationship with a given firm, it incurs a sunk cost $R$ which is independent of volume $v$. Hence, there are scale economies at the level of each relationship (or local level). Banks incur no sunk cost when they do an arm’s-length deal, but such a transaction imposes a transaction cost on firms. The magnitude of the transaction cost depends on (i) whether the firm has a group of $k$ core banks; and (ii) the type of bank it transacts with. Specifically:

**Assumption 2.1.** When the firm does not have any relationships, then implementing the deal with any bank imposes a transaction cost $\beta v$ on the firm.

**Assumption 2.2.** When the firm has a group of $k$ core banks, then implementing a deal with a non-core relationship bank imposes a transaction cost $\alpha v$, with $\alpha \in [0, \frac{R}{v}]$ (that is, $\alpha$ is “small”). On the other hand, a deal implemented by a fringe bank imposes a transaction cost $\beta v$, with $\alpha < \frac{R}{v} < \beta < 1$.

Assumption 2.1 says that firms pay a transaction cost which grows with $v$ when they do an arm’s length deal. One reason is that without the knowledge that is gathered in a relationship it is less likely that the right deal structure will be chosen. Thus mistakes are more likely (see Eccles and Crane [1988] for an elaborate account) and their costs should be roughly proportional to the size of the deal. At the same time, this assumption implies that the relationship technology is efficient for firms with large enough deal volume. It would be hard to justify the contrary: if arm’s length technologies were always more efficient, then relationships would not be observed in the first place.25 Last, because we assume that the arm’s length technology does not exhibit economies of scale at the firm level (larger deals are more costly), there is no loss of generality in assuming that banks incur no cost when using it—competition would ensure that firms pay any cost incurred by banks in equilibrium.

Assumption 2.2 says that relationships create an externality that reduces the cost of doing a deal with a non-core relationship bank. Moreover, the benefit is substantial because $\alpha$ is “small.” This captures the idea, as we saw above, that information is not excludable because senior bankers can switch or the firm can approach another bank to implement the ideas suggested by a relationship bank.

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25 Ljungqvist et al. (2001) examine 2,143 IPOs by issuers in 65 countries outside the United States between 1992 and 1999. They find that firms selling their securities to US. investors through US. banks typically charge higher direct fees, but including underpricing they are considerably cheaper. They argue (p.25) that “[...] the more sophisticated capital markets outside the US. have only recently begun to develop[...] the relationships that link key intermediaries in the venture capital and the private equity markets to the primary and secondary markets.”
The assumption that $\alpha < \frac{R}{v} < \beta$ says that while non-core banks can free-ride on relationship investments, fringe banks cannot. This asymmetry captures the idea that banks that employ the relationship technology are differentiated from banks that do not. Recall that to establish and exploit the advantages of relationships requires two types of costs: the entry cost $E$ of setting up a “relationship infrastructure,” and the cost $R$ of gathering firm-specific information through a relationship with it. While this latter information can be used to better structure deals and offer value-added services that cater to the firm’s needs, implementing these deals and services requires the additional infrastructure costs $E$. In practice, this “infrastructure” may involve investments in a broad product line, experience with particular product specialties, and a large retail distribution network—all of which allow banks to more efficiently implement the complex deals that relationships invoke.\textsuperscript{26} Differences between non-core banks (that entered the relationship segment but do not have a relationship with a particular firm) and fringe banks are thus embodied in $E$, making it cheaper for non-core banks to free-ride on relationships by core banks than it would be for fringe banks to do so.\textsuperscript{27}

Last, it is also assumed that each time a relationship or a fringe bank $i$ does a deal (but only then), it charges a fee that is a proportion $\lambda^i \in [0,1)$ of the dollar value of the deal. Hence, total fees charged by bank $i$ to a firm that generates a deal of size $v$ are $\lambda^i v$. For simplicity, we assume that each relationship bank charges the same proportional fee to all the firms in its core group with whom it does a deal, regardless of $v$.\textsuperscript{28} Below we extend the analysis to consider non-linear fee schedules and show that this does not change our conclusions.

Now in each period the timing of the game is as follows:

1. Each firm establishes a relationship with $k$ investment banks, at which point banks incur the sunk relationship cost $R$.

2. Investment banks offer a fee which is a fraction $\lambda$ of the dollar value of the deal.

3. The firm chooses an investment bank

\textsuperscript{26}Eccles and Crane (1988, pages 100-108) describe in detail the different investments that banks with relationships make. For example, they note the banks’ “concern about the consequences to customer relationships of not having a full product line”; the “experience expertise in the many product specialties required to serve the needs of these customers”; the fact that the “availability of a large retail network is undoubtedly a strength when soliciting certain kinds of customers, such as companies with household names that issue securities”; and that “those with a full product line, such as all six of the special bracket firms, believe that there are strong interdependencies across the various financial markets and that they can serve some customers better by being able to meet all their needs.”

\textsuperscript{27}In practice, bulge-bracket banks are similar multiproduct firms which specialize in doing the deals of large corporations (see, for example, Hayes et al., 1983). Thus, it is not difficult for Goldman Sachs to execute a deal that has been designed by Merrill Lynch. By contrast, fringe banks tend to be small boutiques which do not have the infrastructure which is typical of bulge-bracket banks.

\textsuperscript{28}In practice, there is evidence that smaller deals tend to pay higher fees as a proportion of deal size (see Ritter [1987] and Lee et al. [1996]).
4. Deals are implemented, fees paid and the game ends.

2.3. The well-known tension in a one-period model

We can now examine the well-known tension between relationships and competition in the one-period game. Assume that there are \( m > k \) relationship banks (below we endogenize \( m \)). The equilibrium of this game is straightforward.\(^{29}\) Non-exclusive relationships imply that any relationship bank that is a member of the firm’s core group can do the deal at zero cost after the relationship cost \( R \) has been sunk. Hence, in a one-period game every bank undercuts, and Bertrand competition drives the equilibrium fee to zero.

It is important to note that even if relationships were exclusive (\( k = 1 \)), competition from non-core banks would drive fees to \( \alpha \), which is not enough to recover \( R \) because \( \alpha < \frac{R}{\delta} \)—this is nonexcludability. Loose linkage, in turn, implies that investment banks cannot charge for establishing relationships. Anticipating all this, no investment bank will establish a relationship in the first place.

In this setting, relationships can emerge only if competition is imperfect. This is the well-known tension between relationships and competition. In what follows we characterize this imperfectly competitive equilibrium market structure when relationship banks compete with each other and with fringe banks.

3. Relationships and competition

3.1. The repeated game

Banks will invest in relationships only if they anticipate that they will not be undercut. Among the mechanisms that can restrain price competition are regulations, frictions like informational monopolies, contracts, and self-enforcing norms.\(^{30}\) Neither of the first three seem very relevant in restraining price competition in the investment banking market. On the other hand, as mentioned earlier, many accounts of the industry suggest that price competition is restrained by informal unwritten rules. This suggests that a repeated-game is appropriate to study relationships and competition. Thus assume:

**Assumption 3.1.** (i) Banks are infinitely lived and by sinking \( E \) they can enter into the relationship at the beginning of any period thus remaining a relationship bank forever.

(ii) Relationship banks play the one-period game an infinite number of times.

(iii) Banks discount the future with discount factor \( \delta \in \left( \frac{k-1}{k}, 1 \right) \).

\(^{29}\)The formal proof is in Appendix A.

\(^{30}\)See Aoki and Dinc (1997).
(iv) $E < \frac{1}{1-\delta} f^r \left( \frac{\beta V}{x} - R \right)$, with $f^r$ as defined below.

Assumptions (iii) and (iv) ensure that an implicit contract that sustains relationships exists: assumption (iii) ensures that the implicit contract can be sustained with $k$ core banks, and assumption (iv) says the sunk entry cost can be recovered when the fee is at its maximum, $\beta$. In addition, we make the following simplifying assumption.

**Assumption 3.2.** Each generation of firms lives for only one period.

Assumption 3.2 is made for simplicity. It may seem surprising at first because in practice relationships tend to last for years and information can be reused to some extent (see, for example, James 1992). But, even there, the key point is that banks constantly interact with the firm—so that established relationships must be nurtured over time—whereas deals occur only at discrete moments. In a more elaborate model, time would be continuous, relationship expenditures by banks would be made every instant and depreciate over time and deals would randomly arrive at discrete moments. Thus, at each moment, relationship banks would have a stock of sunk relationship investments that they would lose if they engage in a price war. Our one period assumption captures this with a much simpler structure.31

Now as is well known from the theory of repeated games (see Fudenberg and Maskin 1986), multiple equilibria are endemic. As Sutton (1991, 1998) has shown in a different context, however, it can still be very useful to characterize the bounds on the variables under study that obtain in equilibrium. Following the approach of Sutton, we will characterize the bounds on fees, market shares, and concentration that define the range over which cooperative equilibria exist. To do so, we consider equilibria with the strongest feasible punishments. Specifically, any deviation by an intermediary is assumed to destroy the implicit contract forever.32 Moreover, we study an equilibrium where entry into the relationship segment is accommodated as long as it is profitable.33 Because it is tedious to specify strategy combinations that yield a subgame perfect equilibrium, this is relegated to the appendix. Here we only characterize the equilibrium path.

3.2. Four key conditions

This section describes four conditions that must hold in an equilibrium with relationships. Assume that a symmetric equilibrium with relationships exists where all relationship banks charge fee $\lambda^c$ (where the superscript ‘c’ stands for ‘core bank’).34

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31 A different but related interpretation is that $R$ is sunk in the sense that it does not affect short-run price competition, as in Sutton (1991).
32 In section 3.5 we explain why this assumption is exactly what is needed to obtain lower bounds on concentration and fees.
33 That is, we rule out equilibrium threats such as “if you enter we will no longer cooperate.”
34 Strategy combinations that ensure this is an equilibrium are derived in Appendix B. In section 3.5 it is shown that results do not change if banks charge nonlinear fees.
Competition by fringe banks  The first condition says that the fee charged by a relationship bank cannot exceed $\beta$, that is

$$\lambda^c \leq \beta. \quad (3.1)$$

Because the arm’s length technology exhibits constant returns to scale, and all transactions costs are borne by firms, competition ensures that the fee charged by fringe banks, call it $\lambda^f$, will equal zero in equilibrium. But a firm characterized by volume $v$ which does its deals with a fringe bank would incure a transaction cost $\beta v$, from which condition (3.1) follows. (Notice that even if banks incurred the transaction cost $\beta v$, firms would be charged this full cost in equilibrium, hence the assumption on whether banks or firms bear the arm’s-length cost is not central.)

Relationships and deal size  The second condition says that banks will not establish relationships with firms that generate low volumes of deals. To see this note that since each firm’s core group contains $k$ relationship banks, each bank in that group wins a given deal with probability $\frac{1}{k}$. Therefore, banks will establish relationships only with those firms with volume $v$ such that

$$\frac{1}{k} \lambda^c v - R \geq 0,$$

from which a lower bound $v = \frac{kR}{\lambda^c}$ follows. Sunk set up costs introduce scale economies at the level of each deal. Since these set up costs are incurred by relationship banks that cannot charge for them directly, banks will choose not to establish relationships with firms that generate a low volume of deals. Note that since $\lambda^c \leq \beta$ (from 3.1), it follows that $v \geq v_\beta \equiv \frac{kR}{\beta}$.

Conditions (3.1) and (3.2) are depicted in Figure 3, which plots deal volume on the horizontal axis and the total cost of doing a deal of a given volume on the vertical axis. Given $\lambda^c$, firms whose total deal volume does not exceed $v \geq v_\beta$ are rationed out by relationship banks. On the other hand, note that in equilibrium any firm such that $v \geq v_\beta$ will get $k$ relationships, because banks make profits in expected value.

For future reference it is useful to note that the measure of firms establishing relationships equals $f \cdot \left( \frac{v - v'}{\beta} \right) \equiv f^* ;$ and the average size of a deal done by relationship banks is $V = \frac{v + v'}{2}$.  

Relationships and imperfect competition  Consider a relationship bank who has already sunk this period’s relationship costs and now has to decide whether to undercut rivals. The bank will compare the one-time gains of undercutting to increase its market share today from $\eta_i$ to 1; against the cost of losing the long run gains from cooperation from the next period on.

We start by computing the value of the implicit contract. Bank $i$ will compete for deals with $k - 1$ other banks in each core group of which $i$ is a member. Thus bank $i$ will make deals of value $\frac{V}{k}$ on average. Each firm will pay $\lambda^c V$ in fees on average and total costs will be $R$ per firm, regardless of the number of deals done. Hence, profits per firm are $\frac{\lambda^c V}{k} - R$ on average. If bank $i$
has relationships with a fraction \( \eta_i \) of all \( f^r \) firms that establish relationships (with \( \eta_i \in [0, 1] \) and \( \sum_{j=1}^{m} \eta_j = k \)),\(^{35}\) its long-run profits of cooperation are

\[
\frac{1}{1-\delta} f^r \eta_i \left( \frac{\lambda^c V}{k} - R \right).
\]

The payoff from undercutting is obtained as follows. If the bank-undercuts in period \( t \), it will get the one-time gains of undercutting on top of \( f^r \eta_i \left( \frac{\lambda^c V}{k} - R \right) \). The gains from undercutting are obtained as follows. By setting \( \lambda^c \) slightly below \( \lambda^c \), bank \( i \) can get an additional \( \lambda^c V - \frac{\lambda^c V}{k} = (1 - \frac{1}{k}) \lambda^c V \) on average from firms with whom it has a relationship. Moreover, by setting \( \lambda_{nc}^c \) (where the superscript ‘nc’ stands for ‘non-core bank’) slightly below \( \lambda^c - \alpha \), bank \( i \) can win deals from the remaining \((1 - \eta_i) f^r \) firms with whom it does not have a relationship, thus obtaining slightly less than \((\lambda^c - \alpha) V\) per firm. If undercutting destroys the implicit contract forever, the unilateral deviator gets

\[
f^r \eta_i \left( \frac{\lambda^c V}{k} - R \right) + f^r \left[ +\eta_i \left( 1 - \frac{1}{k} \right) \lambda^c + (1 - \eta_i)(\lambda^c - \alpha) \right] V.
\]

Therefore, after some trivial algebra the implicit contract condition reads:

\[
\frac{\delta}{1-\delta} f^r \eta_i \left( \frac{\lambda^c V}{k} - R \right) \geq f^r \left[ \eta_i \left( 1 - \frac{1}{k} \right) \lambda^c + (1 - \eta_i)(\lambda^c - \alpha) \right] V. \tag{3.3}
\]

For future reference it is useful to note that when all banks establish relationships with the same number of firms, \( \eta_i = \frac{k}{m} \). Then, condition (3.3) can be rewritten as

\[
\frac{\delta}{1-\delta} \frac{f^r}{m} k \left( \frac{\lambda^c V}{k} - R \right) \geq \frac{f^r}{m} [(m - 1) \lambda^c - (m - k) \alpha] V. \tag{3.4}
\]

The implicit contract condition (3.3) is, of course, equivalent to the standard collusion condition in oligopoly. Nevertheless, here “collusion” or “cooperation” is necessary for the existence of a market with relationships, unlike in standard oligopolies where it is merely a collusive device. In other words, here collusion serves an efficiency role. For this reason, henceforth we will use the more neutral term “implicit contract” when we refer to condition (3.3) and use “cooperation” most of the time.

\(^{35}\) Note that \( \eta_i \) is not a market share. Bank \( i \) may have a relationship with all firms and yet not be a monopoly, since each firm has relationships with \( k \) banks. There is a direct relation between \( \eta_i \) and \( i \)’s market share, however. If on average banks get a fraction \( \frac{1}{k} \) of deals made by firms with whom they have a relationship, bank \( i \) will make a fraction \( \mu_i \equiv \frac{k}{m} \) of all deals, with \( \sum_{j=1}^{m} \mu_j = 1 \). Thus, \( \mu_i \) is bank’s \( i \) market share.
Entry into the relationship segment and sustainability  The fourth condition says that relationship banks must make enough profits in present value to pay the entry cost $\mathcal{E}$. Thus

$$\frac{1}{1-\delta} \left[ \frac{\lambda^c V}{k} - R \right] \geq \mathcal{E}. \quad (3.5)$$

Note that this is a standard sustainability condition: the permanent flow of profits if relationships are sustained must be enough to pay the sunk costs of entering the relationship segment.

3.3. Investment banking structure

We are now ready to show that the four conditions rationalize observed market structure. In particular, the market separates into two segments which do not compete with each other: in one, relationship banks serve large firms; in another, fringe banks serve small firms. Conditions (3.1) and (3.2) jointly explain vertical segmentation. Conditions (3.4) and (3.5), in turn, explain market structure, competition and entry into the relationship segment. Last, we discuss how observed market structure rules out alternative explanations.

Vertical segmentation  The claim is that vertical segmentation implies that fringe banks do not effectively compete with relationship banks. The argument proceeds in two parts. The first part of the argument says that low-volume firms are rationed out of relationships by relationship banks. That is, these firms would like to establish a relationship but relationship banks will not do so with them. To see this, note that condition (3.1) implies that the maximum that can be charged by relationship or arm’s length banks for doing a firm’s deals is $\beta v$. Since each firm establishes $k$ relationships, relationship banks will not establish a relationship with a firm whose volume is less than $v_\beta \equiv kR$, because in that case $\frac{\beta v}{k} < R$. But, on the other hand, since $\lambda^c \leq \beta$, these firms would like to establish a relationship. It follows that low-volume firms must be rationed out from relationships by banks.

The second part of the argument says that relationship banks are not threatened by competition from fringe banks. This is because of scale economies inherent in the relationship technology. Since $\beta v > kR$ for $v > v_\beta$, it follows that relationship banks will have a cost advantage which grows with volume; this is seen clearly in Figure 3. Thus, arm’s-length banks cannot compete for the business of large-volume firms because they are inherently more costly (and they must be, otherwise there would be no value to relationships).

In conclusion, low-volume firms would like to be served by relationship banks, but will be rejected by them. On the other hand, the relevant banking market for high-volume firms is the relationship segment. The cost advantage of relationship banks in serving these firms is large enough that fringe banks are not meaningful competitors, despite the fact that relationship banks make
rents. This analysis yields a central result on why relationships are sheltered from competition from arm’s-length banks:

**Result 3.3 (Vertical segmentation).** There are two different markets: fringe banks serve low-volume firms and a few large relationship banks serve large-volume firms. Fringe banks do not compete with relationship banks.

Result 3.3 stems from the assumption that firms differ in their deal volume. Therefore, relative to arm’s length lending, relationship lending (which uses a high sunk cost–low marginal cost technology) is more advantageous the larger the volume of deals. This difference protects relationship lending from arm’s length competition at the margin.\(^{36}\)

**The relationship segment** We now study the relationship segment. The implicit contract conditions (3.3) and (3.4) impose restrictions on competition and market structure. The first result indicates that the relationship segment is an oligopoly:

**Proposition 3.4.** Relationships will be established only if there are few relationship banks with similar market shares.

**Proof.** Fix the equilibrium number of banks, \(m\). Then:

\[
\eta \leq \eta_i \leq k - (m - 1)\eta,
\]

where

\[
\eta = \frac{(\lambda^c - \alpha)V}{(\lambda^c - \alpha)V + \frac{\alpha}{1-\delta} \left( \frac{\lambda^c V}{k} - R \right) - (1 - \frac{1}{\gamma}) \lambda^c V} < 1.
\]

Since \(\mu_i \equiv \frac{\eta_i}{k}\), condition (3.6) also imposes a lower and upper bound on market shares:

\[
\underline{\mu} \leq \mu_i \leq 1 - (m - 1)\underline{\mu} \equiv \overline{\mu}.
\]

It is straightforward to see that the upper bound on \(\mu_i\) must be less than 1.

At the same time, the lower bound on the market share of any given bank implies that the maximum number of banks, \(\overline{m}\), that is consistent with relationships is given by \(\frac{1}{\underline{\mu}} = \frac{k}{\overline{\mu}}\). Thus, in equilibrium relationship banks must be “few”. ■

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\(^{36}\)This result contrasts with Boot and Thakor (2000), who find that more intense competition from arm’s length lenders (similar to a lower \(\beta\) here) reduces relationship lending by commercial banks at the margin. The reason is that: (a) firms differ in quality, but the size of each loan is the same for all; and, (b) although relationship lending adds value, the increment in value is smaller for higher quality firms. Consequently, when the cost of arm’s length lending falls, firms switch at the margin from bank to capital market lending.
Proposition 3.4 implies that relationship banks must have a similar number of relationships. On the one hand, if one becomes too small and establishes relationships with few firms, then cheating becomes profitable. On the other hand, if one relationship bank becomes too large, then there will be too few relationships left for the other relationship banks, which would like to deviate from the implicit contract. Notice that the lower bound on market shares also imposes an upper bound on the number of relationship banks, thus a lower bound on concentration. Figure 4 plots \( L \), the set of pairs of \( \lambda^c \) and \( m \) such that condition (3.4) holds. It is seen that the upper bound \( \overline{m} \) is increasing with the equilibrium fee \( \lambda^c \).

Note that we have deduced an upper bound on the number of firms that comes from an implicit contract condition, not a standard entry condition like (3.5). Of course, in equilibrium the number of banks in the relationship segment must also be small enough to pay the entry cost \( \mathcal{E} \). Thus the second result:

**Result 3.5.** In a subgame perfect equilibrium with symmetric market shares where entry is accommodated as long as it is profitable the number of relationship banks is at most equal to \( \min\{\overline{m}, m^{zp}\} \), with \( m^{zp} \) defined by

\[
\frac{1 - \delta}{1 - \delta m^{zp} k} \left( \frac{\lambda^c V}{k} - R \right) \equiv \mathcal{E}. \tag{3.7}
\]

(To simplify the notation, we henceforth assume that \( \overline{m} \), and \( m^{zp} \) are integers.) Equation (3.7) is a standard zero-profit condition, but it does not necessarily determine the number of relationship banks. The result says that if \( \overline{m} < m^{zp} \) then there exist equilibria where relationship banks make profits net of entry costs, and yet there can be no further entry because cooperation (or collusion) would no longer be sustainable.

Note that \( \mathcal{E} > 0 \), however small but strictly positive, is needed to ensure that entry stops when \( m = m^{zp} \). If \( \mathcal{E} = 0 \), banks would be indifferent between remaining outside or entering an industry whose only equilibrium is such that no relationships are ever established. Thus, \( m = m^{zp} \) as \( \mathcal{E} \to 0 \), but it would be indeterminate if \( \mathcal{E} = 0 \).

**Two important implications** Proposition 3.4 and Result 3.5 yield two important implications which we can use to check the model against observed market structure. The first is that one can rule out the fact that the observed oligopolistic investment banking structure is a consequence of standard scale economies from exogenous sunk costs. To see why, note that \( m^{zp} \) falls with market size \( f \). This is a standard result in models with exogenous sunk costs: as the market grows in size concentration falls in equilibrium (see Mas Collel et al. [1995, ch. 12] for a rigorous proof of this assertion). By contrast, it can be seen from condition (3.3) that \( \overline{m} \) is independent of market size \( f^r \).

Why is that so? Note that both sides of condition (3.3) are multiplied by \( f^r \), the number of firms that establish relationships. Thus, when the size of the market increases both the gains from
cooperation and the gains from cheating increase in the same proportion. Hence, any combination \((\lambda^c, m)\) satisfying condition (3.3) in the smaller market will also satisfy it in the larger market. It follows that concentration should not vary with market size—the regularity depicted in Figures 1 and 2.

The second implication concerns the role of nonexcludability of relationships (i.e. \(\alpha\) must be much smaller than \(\lambda^c\)) in explaining observed market structure. To see this, assume, to the contrary, no externalities between core and non-core banks, so that \(\lambda^c = \alpha\). Then (assuming equal market shares for simplicity) the implicit contract condition (3.3) reduces to

\[
\frac{fr}{m \delta} (\lambda^c V - kR) \geq \frac{fr}{m} \left( 1 - \frac{1}{k} \right) \lambda^c V. 
\]

(3.8)

and the sustainability condition is

\[
\frac{fr}{m \delta} (\lambda^c V - kR) \geq E. 
\]

As before, the implicit contract condition is not affected by market size \(fr\) as it appears on both sides. But now an increase in the number of relationships no longer makes cooperation harder, because both the gains from cooperating and cheating fall as the number of firms increases. For this reason, a relationship bank with a small market share is no longer a threat to cooperation: if you are small and establish few relationships, then you can gain little market share by undercutting. By contrast, when relationships are not excludable, the gains from cheating increase as market share falls because the unilateral cheater grabs the whole market. Because the implicit contract condition no longer restricts \(m\) when relationships are excludable, only the sustainability condition matters; but then concentration falls as market size increases.

In other words, the regularities depicted in Figures 1 and 2 restrict possible explanations of market structure quite tightly. Worth noting is that these two regularities are similar to those obtained in markets with endogenous aggregate sunk costs (Sutton, 1991). Here, however, the exogenous sunk cost \(R\) is incurred only at the local level. As a result, the aggregate technology exhibits constant returns to scale. Hence, sunk costs are naturally “escalated” when the size of the market and the number of firms increases.

3.4. Relationships and the intensity of competition

So far we have studied how market equilibrium embodies three faces of competition: multiple relationships, entry, and arm’s-length deals. We now examine the effect of changes in the intensity of competition: a fall in \(\alpha\), a firm’s cost of switching from a core to a non-core bank, and an increase in the number of relationships \(k\). Since we have shown that fringe and relationship banks serve different segments, we can ignore fringe banks in this analysis and examine how the upper bound
on \(\bar{m}\) and the lower bound on \(\lambda^c\)—i.e. the set \(L\) in Figure 4—varies with changes in exogenous parameters. This “bounds” approach follows Sutton (1991).

Consider a fall in \(\alpha\). Because relationships are not excludable, both core and non-core banks have a temptation to undercut. If \(\alpha\) falls the gains from undercutting increase because non-core banks need to discount their fees by a smaller amount to compete with core banks. Then, the implicit contract becomes less attractive unless fees increase. To confirm this intuition, totally differentiate (3.4) and rearrange to obtain:

\[
\frac{d\lambda^c}{d\alpha} = -\frac{(m-k)\alpha}{\delta (1-\delta) - (m-1) \left(1 - \varepsilon_{V,\lambda^c} \right) - \frac{\alpha \varepsilon (m-k)}{\lambda^c}}.
\]

where \(\varepsilon_{V,\lambda^c}\) is the elasticity of average volume to a change in the fee \(\lambda^c\) (see Appendix C for the details of the derivation). This derivative is negative as long as the no-undercutting locus \(CC\) derived from (3.3) is upward sloping. Hence, when \(\alpha\) falls, locus \(CC\) shifts leftward (see Figure 5). Similarly,

\[
\frac{d\bar{m}}{d\alpha} = \frac{\bar{m} - k}{\lambda^c - \alpha} > 0.
\]

**Result 3.6.** When switching costs from core to non-core banks fall, fees tend to be higher for a given number of banks \(m\). Conversely, concentration increases for a given fee \(\lambda^c\).

Moreover, if \(\alpha\) falls from \(\alpha'\) to \(\alpha'' < \alpha'\) then \(L(\alpha'') \subset L(\alpha')\). Therefore, the implicit contract is harder to sustain when competition from non-core banks is more intense.

Result (3.6) appears counterintuitive because lower switching costs for firms (or easier free-riding) are thought to decrease market power of banks. That effect, while true, ignores the equilibrium consequences. The reduced profit from each relationship is counteracted by exit of relationship banks, and therefore higher aggregate market shares for those that stay. In other words, increased competition for deals is offset by a decrease in competition for relationships.

Consider now what happens when \(k\) increases. For a variety of reasons, firms have tended over the last two decades to increase the number of investment banks with which they have relationships. Eccles and Crane (1988, ch. 4) term this a shift from a “dominant bank model” to a “core group model.” Firms may increase the number of relationships because of “increased information flow and ideas from multiple relationships”, an increase in underwriting and corporate restructuring business that can be allocated amongst more banks, or a desire to increase competition among relationship banks. Some observers question whether relationships can survive this trend.

Do multiple relationships weaken the incentives to establish them? Assume that \(k\) increases, so that firms establish relationships with more banks. To study the equilibrium effects on fees,
substitute $\lambda^c$ into condition (3.4), let it hold as an identity, and totally differentiate with respect to $\lambda^c$ and $k$ (see Appendix C for the details of the derivation). Rearranging yields

$$\frac{d\lambda^c}{dk} = \frac{[(m - 1)\lambda^c - (m - k)\alpha] \frac{\partial V}{\partial V} + \alpha V + \frac{\delta}{1 - \delta} \lambda^c \frac{\partial V}{\partial V}}{\left\{ \left( \frac{\alpha}{\lambda^c} - (m - 1) \right) (1 - \varepsilon_{V,\lambda}) + \frac{\alpha}{\lambda^c} (m - k) \varepsilon_{V,\lambda} \right\} V},$$

(3.9)

where we used that $\frac{\partial V}{\partial k} = \frac{V}{k}$. This derivative is positive as long as $\alpha$ is sufficiently small (again, see Appendix C), which implies:

**Result 3.7.** For a given number of banks, $m$, fees tend to be higher when firms establish more relationships.

Multiple relationships are often thought to toughen price competition. Result 3.7 runs counter to this intuition. While the analysis confirms that the effect of an increase in $k$ is to reduce the probability of winning a deal, and therefore the net margin per firm to each bank, $\frac{\lambda V}{k} - R$, it shows that the gains of unilaterally undercutting also increase. Thus, given $m$, each bank wants to establish relationships only if fees increase. Similarly,

$$\frac{d\pi^*}{dk} = -\frac{[(m - 1)\lambda^c - (m - k)] \frac{\partial V}{\partial V} + \alpha V - \frac{\delta}{1 - \delta} \left( \lambda^c \frac{\partial V}{\partial V} - R \right)}{(\lambda^c - \alpha) V},$$

which is negative. An increase in $k$ therefore reduces $\mathcal{L}$, the set of pairs $(\lambda, m)$ that can be sustained in equilibrium. Multiple relationships reduce firm-specific rents to banks since revenues per firm fall while relationship costs do not. The increased profits from which to recoup relationship costs can then be created by increasing prices or inducing exit.

An increase in $k$ will, for a given $\lambda^c$, not just affect the concentration of banks in the relationship segment of the market, but the size of the relationship segment itself. To see this, note that the lower bound $v$ on firm volume, $kR/\lambda^c$, increases in $k$. This will both reduce the number of firms that establish relationships with banks, $f^r$, and the aggregate volume of deals intermediated by relationship banks, $f^r \cdot \frac{1 + \pi}{2}$. Thus, the effect of firms establishing more relationships is to increase concentration of relationship banks on the one hand, while increasing the size of the market served by the competitive fringe on the other. This apparent increase in both competition and concentration might explain why the effects of such changes often appears puzzling to observers.

To summarize, both these results stress that one cannot analyze the impact of a change in market conditions at the local firm-bank level. Market equilibrium involves adjustments at the

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38 Boot and Thakor (2000) find a somewhat similar result in the context of commercial bank relationships. They argue that the effect of increased interbank competition on relationship lending by commercial banks includes both a negative *absolute* effect on volume of loans lent through relationships but a positive *relative* (substitution) effect on the capacity devoted by banks to relationship lending.
aggregate level—in fees or market concentration—to preserve the incentives to incur the sunk costs of relationships.

**Result 3.8.** An increase in the intensity of competition need not destroy relationships as long as the implicit contract remains sustainable.

Result 3.8 illustrates a general lesson: the deleterious effects on relationships of changes in the intensity of one type of competition may be partially undone by changes of market structure and the intensity of other types of competition.

3.5. Robustness and extensions

In this section we examine the robustness of the results to several modeling assumptions and explore the consequences of other extensions.

3.5.1. Which assumptions drive the results?

**Non-verifiability of relationships** To examine the role of loose linkage for market structure, assume the reverse: that is, banks can directly charge a firm for a relationship. Then banks would compete and charge \( R \) for establishing a relationship and a firm with volume \( v \) would establish a relationship only if \( kR \leq \beta v \). Moreover, a firm with \( v = v_\beta \) would be indifferent between either type of bank, not rationed by relationship banks. Hence:

**Result 3.9 (Relationships and perfect competition).** If relationships were verifiable and investment banks could charge directly for them, then relationships could be sustained in a perfectly competitive market.

Result 3.9 implies that imperfect competition is a consequence of relationships being nonverifiable, not a consequence of the economies of scale exhibited by the relationship technology.

**Game-theoretic issues** We have used a grim trigger strategy to sustain the implicit contract—i.e. one deviation destroys the equilibrium forever—which turns out to be the strongest feasible punishment in this case.\(^39\) One may wonder whether the bounds on \( m \) and \( \lambda^c \) we derived remain valid for weaker punishments.

Consider an alternative strategy that yields to intermediaries \( f^r \eta_i v_p > 0 \) in present value in the punishment phase (i.e., the payoff during a punishment is larger if the relationship segment \( f^r \)

\(^39\)It is the strongest feasible punishment because relationship banks can not be forced to get a payoff of less than zero.
is larger, or if the bank has a larger market share). In that case, the implicit-contract condition would read

$$\frac{\delta}{1 - \delta} f^r \eta_i \left( \frac{\lambda c V}{k} - R \right) \geq f^r \left[ \eta_i \left( 1 - \frac{1}{k} \right) \lambda c + (1 - \eta_i)(\lambda c - \alpha) \right] V + \delta f^r \eta_i v_p, \quad (3.10)$$

which implies, after some algebra, that the bounds are now determined by the equation

$$\frac{\lambda c V - k R - (1 - \delta)v_p}{\lambda c V} = \frac{1 - \delta}{\delta} \left[ (1 - \frac{1}{k}) + (m^w - 1) \frac{\lambda c - \alpha}{\lambda c} \right].$$

The weaker the punishment, the harder it is to fulfill the implicit contract condition (3.10). To restore incentives to cooperate, the gains from cheating must fall relative to the gains from cooperating. Hence, either the maximum number of intermediaries, call it $m^w$, must be smaller than $\overline{m}$ or $\lambda c$ must increase—that is, minimum concentration and the maximum fee are higher with a weaker punishment. Therefore, the set of pairs $(m, \lambda)$ in $\mathcal{L}$ with the strongest feasible punishment contains the set $\mathcal{L}^w$ with punishment $f^r \eta_i v_p$.

One may also be concerned that banks may renegotiate out of the strongest feasible punishment. Nevertheless, the bounds we derived are still the same, because we have shown that any set of pairs $(m, \lambda)$ sustainable with weaker punishments will be a subset of the set of pairs $(m, \lambda)$ in $\mathcal{L}$—if anything, fees and concentration will be higher with a renegotiation-proof implicit contract.

**Nonlinear fees** Linear fees are not necessary to any of our results. First, rationing follows from non-verifiable relationships, because relationship banks cannot charge more than $\beta v$ to any firm, regardless of how this sum is collected—whether through a linear fee, a two-part tariff or a more complex nonlinear schedule. Similarly, the result that high-volume firms are served only by relationship banks follows purely from $\beta v > k R$. Indeed, this condition allows, for example, not just a linear fee such that $\beta v \geq \lambda c v \geq k R$, but nonlinear schedules as well, call them $\mathcal{F}(v)$, such that $\frac{\mathcal{F}(v)}{k} \geq R$. Again, market separation follows from non-verifiable relationships, a problem that cannot be solved by charging non-linear fees.

Of course, whether relationship banks charge linear or nonlinear fees may affect $v$, the cutoff volume below which relationship banks ration firms. As seen, if a linear fee $\lambda c$ is charged, then $v = \frac{k R}{\lambda c}$. Thus, the exact value of $v$ only affects the relative sizes of the relationship and arm’s-length segments, not the result that banks in these segments effectively do not compete.

Second, condition (3.3) could be substituted by any fee schedule $\mathcal{F}(v)$, and then it would read

$$\frac{\delta}{1 - \delta} f^r \eta_i \left( \frac{E_{v \geq v}[\mathcal{F}(v)]}{k} - R \right) \geq f^r \left[ \eta_i \left( 1 - \frac{1}{k} \right) E_{v \geq v}[\mathcal{F}(v)] + (1 - \eta_i)E_{v \geq v}[\mathcal{F}(v, \alpha)] \right], \quad (3.11)$$
where $E_{v \geq \underline{\lambda}}[\mathcal{F}(v)]$ is the expected fee income generated from a firm that established a relationship and $E_{v \geq \underline{\lambda}}[\mathcal{F}(v, \alpha)] \leq E_{v \geq \underline{\lambda}}[\mathcal{F}(v)]$ is the expected income generated from a firm that is poached after undercutting. In other words, rents sustain the implicit contract in equilibrium and these need not come from linear fees.

**Are fees observable in practice?** The implicit contract condition (3.3) assumes that relationship banks observe the fees that have been offered by rivals and act upon them in the next period when they make their decision whether to continue cooperating or punish. But because deals between firms and investment banks contain complex transactions, it might seem unrealistic to assume that fees are observable.

Nevertheless, pricing in this industry is quite simple since fees are a proportion of the dollar amount of the deal and companies like Securities Data Company record the fees of every deal made and sell the information. Moreover, in practice underwriting is done through syndicates where top banks regularly meet. A deviator would need to eschew syndicates, which is observable. More important from the point of view of the modeling, fees need not be observable to define a strategy that punishes deviators. For example, banks could condition their actions on their market shares, and enter into a punishment phase when they fall below a certain threshold.

**3.5.2. Other extensions**

**Endogenous number of relationships** We have assumed that $k$ is exogenous. But, one might expect from equation (3.9) that by lowering $k$, a firm could lower the fee that it pays. As we explain below, there are several reasons why this “endogeneity” can be ignored in the analysis.

First, notice that equation (3.9) is a market-level relationship, not a firm level one. Hence, firms are price takers—they take the equilibrium market fee $\lambda^c$ as given when optimizing $k$. Second, there still could be an endogeneity concern when firms are price takers, if the level of fee $\lambda^c$ would affect the optimal number of relationships chosen by a firm (in that case, $k$ and $\lambda^c$ would be simultaneously determined in equilibrium). There are obviously many factors that should affect a firm’s optimal choice of $k$ (for example, access to a wider array of ideas and approaches, and the cost of time and attention of management). But, there is no reason that optimal $k$ must vary with fees. To see why, suppose that a change in some other parameter alters the equilibrium fee $\lambda^c$. Would this cause firms to change the number of relationships that they establish? No. The reason is that the cost and benefit of an extra *relationship* by a firm is not affected by fees, since these are paid only when a *deal* is done.

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40 For example, Eccles and Crane (1988, p. 94) report that of the 6,327 domestic security issues led by one of the six top banks between 1984 and 1986, 60.4% were comanaged by another top six bank.

41 For example, in Green and Porter’s (1984) classic model of collusion firms choose unobservable quantities but condition their strategies on observable prices.
Last, consider a more general case where, in equilibrium, banks offer a fee schedule such that lower fees are charged to a firm that established fewer relationships (the fee schedule could be linear or non-linear in $k$). In this case, a firm will optimize the number of relationships since fewer relationships can reduce the costs of doing any deal. Our market level analysis would not be affected, however. To see why, note that then a firm with volume $v$ that chooses $k$ relationships would generate, say, $\mathcal{F}(v, k)$ dollars in fees, and the bank would make her deal with probability $1/k$. Then only firms such that $\frac{\mathcal{F}(v,k)}{k} \geq R$ would get deals in equilibrium, and the implicit contract condition would be

$$\frac{\delta}{1-\delta} f^r \eta_i \left( E_{k,v \geq \underline{v}(k)} \left\{ \frac{[\mathcal{F}(v,k)]}{k} \right\} - R \right)$$

$$\geq f^r \left[ \eta_i E_{k,v \geq \underline{v}(k)} \left\{ \left( 1 - \frac{1}{k} \right) [\mathcal{F}(v,k)] \right\} + (1 - \eta_i) E_{k,v \geq \underline{v}(k)} [\mathcal{F}(v,k,\alpha)] \right].$$

As with fees that are nonlinear in $v$, $v$, the cutoff volume below which relationship banks ration firms, may be affected by variation in $k$. But the exact value of $v$ only affects the relative sizes of the relationship and arm's-length segments. The essential point is that endogenizing $k$ does not change either the result on vertical segmentation nor the predictions on market structure.

**Nonprice competition** Investment banks compete in various nonprice dimensions. They incur sales expenditures, advertise, provide “free” advice on other financial and investment matters. Indeed, it is often claimed that banks price some services below their cost in order to get access to clients. One might ask how nonprice competition would alter the previous results. In particular, is it the case that nonprice competition might dissipate the rents that banks get in equilibrium, thus undermining the incentive to establish relationships? In the appendix we extend the model to include nonprice competition among banks. Call $E$ the amount spent in sales efforts per firm by each bank in a symmetric equilibrium. In that case (as is shown in the appendix), the implicit contract condition would read

$$\frac{\delta}{1-\delta} f^r \left[ \frac{k}{m} \left( \frac{\lambda c V}{k} - R \right) - E \right] \geq f^r \left[ (m - 1)\lambda c - (m - k)\alpha \right] V,$$

which is almost the same as before except that costs $E$ are now taken into account as well.

The central point is that nonprice competition does not change the requirement that banks make excess profits in equilibrium in order to establish relationships. In many models ex-ante nonprice competition is a mechanism to dissipate ex-post rents. This does not occur here because relationships play an efficiency role (and require rents) and the gains from cheating are independent of sales efforts. Thus, sales efforts do not do away with the need for soft price competition, which is the source of rents in this model. Moreover, it can be easily seen that the bounds on concentration
and fees are, if anything, tighter—concentration and fees tend to be higher with sales efforts.

Might sales effort competition be dissipative, and thus inefficient? It certainly may be, for standard reasons. On the other hand, sales efforts may also allow firms access to better ideas to do deals. Last, even though nonprice competition is not a means to compete away rents, it does restrain bank market power by imposing an upper bound on the fee $\lambda_c$ that banks can charge in equilibrium (see the appendix for details).

**The multiproduct nature of investment banks** Bulge-bracket banks are multiproduct firms, so that concentration in any one market may mask the fact that leading banks differ across products. Nevertheless, the top, bulge bracket banks tend to be the same in most product lines (see, for example, Eccles and Crane 1998, Santomero and Babel, 2001, p. 500). Moreover, as we show in the appendix, as long as the economics of the technology exhibits these characteristics in some segments of the investment banking industry, then the implicit contract condition must hold across products. In particular, this implies that multiproduct competition cannot dissipate rents—otherwise cooperation would no longer be self-enforceable.

**Welfare** A central implication of the analysis is that collusion among relationship banks serves an efficiency role. Nevertheless, there might be typical concerns that accompany market power, such as inefficient exclusion or exploitation by intermediaries. Can one say anything about the overall welfare consequences of relationships?

First, inefficient exclusion is unlikely. The reason is that in equilibrium small volume firms who want relationships with banks are rationed out of the relationship segment—when $\lambda^c v - kR < 0$. If the fee $\lambda_c$ increases, fewer firms are rationed out of relationships and the size of the relationship segment increases with $\lambda^c$. Hence, exclusion falls as the fee increases, for exactly the same reason that in a market with pre-existing rationing (e.g., a market with a price ceiling fixed below the equilibrium level), sales increase when the price increases.

Second, the cost of market power (e.g., exploitation of firms by banks) is unlikely to overturn the beneficial aspects of relationships. To see why, notice that the appropriate welfare comparison is between a market with no relationships and one with imperfect competition. Now, because firms always have the choice of doing arm’s-length deals, it seems reasonable to think, by revealed preference, that a market with relationships and imperfect competition Pareto-dominates a market with no relationships.
4. Discussion

4.1. Off-equilibrium episodes

A useful question to ask is whether investment banks ever “undercut” and how such episodes resolve. As discussed, free-riding can take many forms: price undercutting, human capital poaching, or copying of ideas. While unilateral changes in price schedules have not been observed, there are some accounts of unusual poaching behavior. Even though the movement of bankers from one bank to another is fairly common, it is unlikely that these are construed as “off-equilibrium” actions. However, a few episodes do stand out. Perhaps the most famous is Deutsche Bank’s attempt to gain status as a “bulge bracket” bank in the early 1990s by en masse hiring of staff from other banks—this series of hirings, including that of “the flamboyant technology star Frank Quattrone,” was described as having “brought the baseball term ‘free agent’ to Wall Street.”42 Several years later, “when UBS AG’s Warburg Dillon Read lured veteran healthcare star Benjamin Lorello away from Citigroup’s Salomon Smith Barney to orchestrate its global healthcare investment banking practice, the headline made front-page news.”43

The reason that each of these cases was construed as a “poaching raid” was probably both because of the number of bankers that accompanied these departures (in the Deutsche Bank case, it involved more than 200 senior bankers) and because of the large increase in salaries. More interesting is that, even in an industry where banker mobility is fairly common, the Lorello hiring was described by one headhunter, as “an auction, completely without regard for the rules.”44 Each of these episodes was followed by competitor retaliation: following Deutsche Bank’s hiring spree, CS First Boston offered Quattrone and his 100-member high-tech group up to $1 billion to leave; one account noted that “rarely has a poaching raid aroused as much schadenfreude among top investment bankers”45 as their subsequent defection. Similarly, when Deutsche Bank poached numerous traders in over-the-counter derivatives and fixed-income trading from Commerzbank in 1998, Commerzbank retaliated two years later by hiring away many of Deutsche’s own traders, “including some who were originally poached from Commerzbank by Deutsche and are now being rehired on what one London banker claimed would be ‘pretty massive bonus promises’.”46 In response, Deutsche Bank cut off its lines of business with Commerzbank in the derivatives and fixed income markets in a move that “both hoped was temporary.”47 One industry account in 2000 noted that the net result of these raids and retaliations was that, in its attempt to achieve

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43 Ibid.
44 Ibid.
47 Ibid.
bulge-bracket status, “Deutsche Bank in New York (was still) not even near the starting gate.”

4.2. Applicability to Other Industries

Can one apply this model to explain how relationships are sustained in other industries? Here, we briefly discuss both the generalizability and specificity of the investment banking problem.

4.2.1. Professional service industries

The analysis here may be usefully extended to explore other professional service industries (e.g., consulting, law, advertising) that share characteristics with investment banking. Consider, for example, advertising agencies. Agencies make significant investments in client-firm relationships. And, these relationships are often difficult to appropriate because they are embodied in human capital that can move to other firms. Further, like investment banking, entry is unregulated.

Both firm conduct and market structure bear close resemblance to that of investment banking as well. On the one hand, agencies compete fiercely for market share and in sales efforts to get relationships. On the other hand, price competition was restrained: for a long time, advertising agencies would charge clients flat 15% commissions that were independent of deal volume, and did not vary with project characteristics. Moreover, the market is is dominated by a few firms with mostly similar market shares; concentration is largely invariant to market size; and, market structure is “pyramidal” and vertically segmented, with the few large firms co-existing with hundreds of boutiques that serve smaller clients. A more thorough analysis of this and other professional service markets that builds on the approach here may be fruitful.

4.2.2. Commercial banking

It is well known that relationships are important in commercial banking and it has long been debated whether competition hurts them. Are relationships in commercial banking “protected” by a similar industrial organization?

There are two significant differences which suggest not. For one, commercial banks maintain a relationship only while they lend money. Because loans are kept in the bank’s balance sheet, they generate a steady flow of interest income while the relationship is ongoing. Thus, it is unlikely

49 In 2004, the worldwide share of advertising revenues accruing to the top five firms was 74.1%, with their individual market shares being 18.3% (Interpublic), 18.2% (Omnicon), 16.5% (Publicis), 15.5% (WPP), and 5.6% (Havas). Data from Advertising Age, http://www.adage.com/images/random/agrpt04_piechart.pdf
50 For example, the share of worldwide advertising revenue accruing to the top five advertising groups was 61% in 1990 and 74% in 2004, a period during which market size grew by over 20% (Advertising Age, various years).
51 Thus, for example, Collis (1992) describes the industry as “hourglass shaped, with the mega-agencies at the top, a thin middle, and a profusion of specialist, boutique, and new agencies at the bottom.” See also King et al (2003).
that bank-firm relationships are characterized by loose linkage. For another, the cost of switching
to the equivalent of a non-core banks is comparably more important in commercial banking. As
Boot and Thakor (2000, p. 683) point out, commercial banks enjoy some degree of local market
power because of the illiquidity of each loan due to its information sensitivity.\textsuperscript{52} And condition
(3.8) shows that relationships impose no restriction on aggregate market structure if banks without
relationships (non-core banks) incur similar lending costs. Not surprisingly, the pyramidal structure
that characterizes investment banking is not a feature of commercial banking structure.

5. Conclusion

The tension between relationships and competition is a long-standing one. This paper examines how
this tension is resolved in the investment banking market, that has for decades been characterized
by both relationships and competition.

Perhaps the central message of the paper is that competition need not kill relationships. This
insight emerges clearly when one embeds relationships in an equilibrium analysis. The investment
banking market offers an example of how market equilibrium adjusts so that valuable relationships
can still be provided. Thus, for example, when competition gets more intense in some dimensions
(for example, firms increase the number of relationships that they establish, or sales efforts pitched
at establishing relationships increase), the endogenous adjustment in fees or in the number of banks
undo their deleterious effects on the incentives to establish relationships.

The second insight about competition and relationships comes from the technology of rela-
tionships. Specifically, price competition from intermediaries doing arm’s length deals does not
hurt relationships with large firms, because arm’s length deals are inherently more costly for these
firms. Relationships are therefore sheltered from such competition by the scale economies intrinsic
to relationships.

Last, our results caution against examining the effects of different types of competition in
isolation. Investment banking shows that different ways of competition interact. The technology
of relationships imposes restrictions on this interaction, and on the industrial organization of the
industry. These insights are probably useful to understand market structure in other professional
service industries where relationships are important.

\textsuperscript{52}See also James (1987), Kang et al. (2000), Lummer and McConnell (1989), and Shockley and Thakor (1997).
Appendix

A. Equilibrium in the one-period game

We now rigorously describe the timing of actions, and then characterize the equilibrium of the one-period game. The
time line is as follows:

1. Each firm randomly contacts $k$ relationship banks.
2. Each bank chooses those firms with which it wants to establish relationships, and incur the corresponding sunk
cost $R$. If $k$ banks establish relationships with a given firm, then that is the core group of banks of the firm.
3. Firms announce deals.
4. (Fee offers) Each relationship bank $i$ simultaneously makes a price offer $\lambda^e_i \in [0, 1] \cup \{\infty\}$ to all firms with whom it has a relationship, an offer $\lambda^{nc}_i \in [0, 1] \cup \{\infty\}$ to all firms in a core group that does not include $i$, and an offer $\lambda^{nr}_i \in [0, 1] \cup \{\infty\}$ to firms that have no relationships (superscript ‘c’ stands for ‘core’, superscript ‘nc’ for ‘non
core’ and superscript ‘nr’ for ‘no relationship’). These offers are expressed as a fraction of deal volume (thus,
they represent commissions or percentage fees). $\lambda = \infty$ means that no offer was made. Obviously $\lambda^e_i = \infty$ if
bank $i$ is in no core group and $\lambda^{nc}_i = \infty$ if it has a relationship with every firm.

Simultaneously, each fringe bank $j$ makes a price offer $\lambda^f_j \in [0, 1] \cup \{\infty\}$ to all firms.
5. Each firm chooses the bank offering the lowest fee net of transaction costs. If $x > 1$ banks tie, then each bank
wins the deal with probability $\frac{1}{x}$.
6. Deals are implemented, fees paid and the game ends.

To define bank strategies let $H$ be the set of possible histories right before banks make fee offers. A strategy by
a relationship bank $i$ is a tuple $(R_i, \Lambda_i)$. $R_i : [0, \infty] \rightarrow \{0, R\}$ is a function that indicates whether bank $i$ will establish
a relationship with those firms that selected $i$ to form part of the core group. Since firms are completely described
by $v$, bank $i$’s decision can be conditioned on firm type. $\Lambda_i = [\lambda^e_i, \lambda^{nc}_i, \lambda^{nr}_i]$ is a three-dimensional vector function
$\Lambda_i : H \rightarrow ([0, \infty] \cup \{\infty\})^3$. In turn, a strategy by fringe bank $j$ is a function $\Lambda^f_j : H \rightarrow [0, 1] \cup \{\infty\}$. Proposition A.1
characterizes the set of subgame perfect equilibria of this game. (See Appendix B for a strategy combination that is
a subgame perfect equilibrium of the one-period game.)

**Proposition A.1.** In any subgame perfect equilibrium, no relationships are established and $\lambda^f = 0$.

**Proof.** Suppose, by way of contradiction, that bank $i$ establishes a relationship with a firm. Suppose first that $k > 1$.
In any subgame with relationships, Bertrand competition for deals between core banks drives $\lambda^e$ to 0 in equilibrium.
On the other hand, if $k = 1$, Bertrand competition with relationship banks drives $\lambda^{nc}$ to $\alpha$, and hence $\lambda^e_i$ to $\alpha < \frac{R}{v}$
in equilibrium in any subgame where bank $i$ is the firm’s only relationship bank. Hence, in both cases bank $i$ loses
money if it becomes a relationship bank, therefore $\lambda^e_i = \infty$. Finally, note that since there are no variable cost of
doing deals, Bertrand competition among fringe banks drives $\lambda^f$ to 0 in equilibrium.

Hence no relationships are established in equilibrium, only banks that do not establish relationships are active,
and each deal costs $\beta$s to firms. Moreover, since there are no variable costs of doing deals, fees are driven to zero in
equilibrium. For firms with $v$ such that $\beta < \frac{R}{v}$ (i.e. firms that do small and infrequent deals) this equilibrium is
efficient. By contrast, firms with $v$ such that $\beta > \frac{R}{v}$ would want to establish $k$ relationships and compensate banks
for the incurred sunk costs. This can be summarized in the following result:

**Result A.2.** The equilibrium of the one-shot game is efficient form firms that do infrequent or small deals. It is
inefficient for firms that do large or frequent deals.

A straightforward implication of Result A.2 is that low-volume firms will never establish relationships. Moreover,
Result A.2 shows that establishing relationships is not efficient for every type of client. Firm volume will
determine which technology is efficient.

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B. A subgame perfect equilibrium with no undercutting

In this appendix, we study a symmetric subgame perfect equilibrium where all relationship banks charge the same fee $\lambda^r > 0$ period after period, all fringe banks compete and charge $\lambda^f = 0$, relationships are profitable and there is free entry into the relationship segment. The outcome of this equilibrium are the conditions examined in the text.

We start by defining a strategy combination that is a subgame perfect equilibrium of the one-period game.

**Definition B.1.** Call strategy combination $P$ (for ‘punishment’) the following combination of strategies

- For all relationship banks $i$
  1. $R_i(v) = 0$ for all $v \in [0, \bar{v}]$;
  2. $(\lambda^r_i, \lambda^{nc}_i, \lambda^{nr}_i) = \begin{cases} (0, 0, 0) & \text{if } i \text{ is member of at least one core group and not member of at least another core group;} \\ (\infty, 0, 0) & \text{if } i \text{ is member of no core group but at least one core group exists;} \\ (\infty, \infty, 0) & \text{if no core groups are established.} \end{cases}$

- For all fringe banks $j$, $\lambda^f_j = 0$.

Part (i) of the strategy of relationship banks implies that no bank establishes a relationship. Part (ii) implies that $i$ undercuts other relationship banks on all histories such that $i$ is in a core group. Finally, part (iii) implies that bank $i$ always undercuts when not in a core group. We are now ready to prove the following lemma:

**Lemma B.2.** Strategy combination $P$ is a subgame perfect equilibrium of the one period game.

**Proof.** Consider first histories where at least one firm establishes relationships with $k$ banks and forms its core group. According to $P$, for these histories we have to distinguish three cases:

$$(\lambda^r_i, \lambda^{nc}_i, \lambda^{nr}_i) = \begin{cases} (0, \infty, 0) & \text{if bank } i \text{ is member of all core groups;} \\ (0, 0, 0) & \text{if there is at least one core group where bank } i \text{ is not a member but bank } i \text{ is member of at least one core group;} \\ (\infty, 0, 0) & \text{if bank } i \text{ is member of no core group.} \end{cases}$$

In any of these three cases, any unilateral deviation by bank $i$ setting $\lambda^r_i > 0$ or $\lambda^{nc}_i > 0$ or $\lambda^{nr}_i > 0$ as the case may be will not increase its payoff, since it would get no deals.

Consider next histories where no firm forms a core group. Then, $(\lambda^r_i, \lambda^{nc}_i, \lambda^{nr}_i) = (\infty, \infty, 0)$ according to $P$. Setting $\lambda^{nr}_i > 0$ will not increase $i$’s payoff since it would get no deals.

Last, setting $R_i(v) = R$ for one or more $v$’s will not increase $i$’s payoff because according to strategies no other firm establishes relationships.

The following corollary follows from Proposition A.1 and Lemma B.2.

**Corollary B.3.** All banks receive a payoff equal to $0$ in the one period game.

Thus, since playing strategy combination $P$ forever is clearly a subgame perfect equilibrium in the infinitely repeated game, it follows that it can be used to construct a subgame perfect punishment. We now define ‘undercutting.’

**Definition B.4.** Let $\lambda^r$ be the fee charged in an equilibrium with relationships. Then there is undercutting in period $t$ if $\min(\lambda^r_i(t), \lambda^{nr}_i(t) + \alpha) < \lambda^r$ for at least some $i$.

Note that $\lambda^{nr}$ and $\lambda^f$ are not part of the definition. We are assuming that neither “undercutting” in the fringe segment, nor fringe banks setting fees such that $\lambda^f + \beta < \lambda^r$, destroys cooperation.

We now specify a strategy combination such that cooperation is a subgame perfect equilibrium. To do so, it is useful to assign each possible history of the game into one of two disjoint sets.
Definition B.5. We say that the history of the game at period \( t \) is ‘cooperative’ if

(a) \( m \leq \overline{m}(\lambda^c) \);
(b) no undercutting has occurred so far. That is, for all \( \tau < t \), \( \min\{\lambda^c_i(\tau), \lambda^{nc}_i(\tau) + \alpha\} \geq \lambda^c \).

Any other history is non–cooperative.

Notation B.6. We denote the state of the game at period \( t \) by \( \phi_t \). The state of the game after a history with no undercutting is cooperative and is denoted by \( \phi^c \). Any other state of the game is ‘non-cooperative’ and is denoted by \( \phi^{nc} \).

Note that this definition implies that the initial state of the game is cooperative. Next we define some notation we need to define strategies:

Notation B.7. As in the text, \( \eta_i \) denotes the share of firms with \( v \geq \overline{v} \) such that bank \( i \) is in their core group. Furthermore, we denote by \( \eta_{-i} \) the fraction of firms with \( v \geq \overline{v} \) that have a core group where relationship bank \( i \) is not a member.

Note that \( 1 - \eta_i - \eta_{-i} \) is the fraction of firms with \( v \geq \overline{v} \) who did not form a core group. Hence, if all firms formed a core group then \( \eta_{-i} = 1 - \eta_i \). Furthermore, \( 1 - \eta_i - \eta_{-i} = 1 - \eta_j - \eta_{-j} \) for all \( i, j \). We can now define the symmetric strategy combination \( C \) (for ‘cooperative’).

Definition B.8. Call strategy combination \( C \) the following combination of strategies:

1. **Entry into the relationship segment (only fringe banks move):**
   
   1. If \( \phi_t = \phi^c \),
      
      - (i) enter if \( i = m + 1, \ldots, \min[m^{zp}, \overline{m}] \);
      
      - (ii) remain a fringe bank if \( i > \min[m^{zp}, \overline{m}] \).
   
   2. Otherwise do not enter the relationship segment.

2. **For all relationship banks \( i \)**
   
   1. (Establishing relationships)
      
      - If \( \phi_i = \phi^c \) then play
      
      \[
      \mathcal{R}_i(v) = \begin{cases} 
      R & \text{for } v \geq \overline{v}, \\
      0 & \text{for } v < \overline{v}.
      \end{cases}
      \]
      
      - Otherwise, play according to \( P \).
   
   2. (Fee offers)
      
      - If \( \phi_i = \phi^c \) and
      
      \[
      \frac{\delta}{1 - \delta} f^c \frac{k}{m} \left( \frac{\lambda^e V}{k} - R \right) \geq \max_j \left\{ f^c \left[ \eta_j \left( 1 - \frac{1}{k} \right) \lambda^e + \eta_{-j}(\lambda^e - \alpha) \right] V \right\}
      \] (B.1)
      
      holds, then play
      
      \[
      (\lambda^e_i, \lambda^{nc}_i, \lambda^{nr}_i) = \begin{cases} 
      (\lambda^e, \infty, 0) & \text{if } \eta_i > 0; \\
      (\infty, \infty, 0) & \text{if } \eta_i = 0.
      \end{cases}
      \]
      
      - Otherwise play according to \( P \).

3. **For all fringe banks \( j \) play \( \lambda^f_j = 0 \).

Condition (B.1) says that bank \( i \) will not undercut in period \( t \) provided that continued cooperation is more profitable than undercutting, given period’s \( t \) ex ante fee offers. Lemma B.9 characterizes the outcome path induced by \( C \):
Lemma B.9. Along the path induced by C
(i) all entry occurs in the first period;
(ii) all relationship banks i play \((\lambda_i, \lambda_i^{nc}, \lambda_i^{nr}) = (\lambda^c, \infty, 0)\) for all \(t\);
(iii) all relationship banks have the same market share;
(iv) all fringe banks play \(\lambda_f^i = 0\) for all \(t\).

Proof. The proof is straightforward and we leave it to the reader.■

We now state and prove the main result of this appendix:

Proposition B.10. Let \((\lambda^c, m) \in L\). Then, strategy combination C is a subgame perfect equilibrium in the infinitely repeated game.

Proof. To prove this Proposition, we show that players’ strategies are optimal after any history. Since this is a repeated game with bounded payoffs, it suffices to show that one-step unilateral deviations from strategies are not profitable after any history (Hendon et al. 1996).

Now according to C histories can be classified into two groups, cooperative and non cooperative.

- **Non-cooperative histories** \((\phi_i = \phi^{nc})\):
  (i) At the beginning of the period it is optimal for any fringe bank not enter the relationship segment, which would leave long-run losses equal to \(E\).
  (ii) We know that when all other relationship banks are playing according to \(P\) in the one-period game, it is optimal for relationship bank \(i\) to do the same. Since all relationship banks will play according to \(P\) forever after, it is also optimal for relationship bank \(i\) to play according to \(P\) in any period of the repeated game.
  (iii) Last, note that playing \(\lambda_f^i = 0\) is optimal for fringe banks in the one-period game, hence it is also optimal to play so in the repeated game.

- **Cooperative histories** \((\phi_i = \phi^c)\):
  (i) It is optimal for fringe banks \(m + 1, \ldots, \min[m^{zp}, \bar{m}]\) to enter the relationship segment, since according to strategies, there will be cooperation in the future. For fringe banks \(\min[m^{zp}, \bar{m}], \ldots\) it is optimal not to enter the relationship segment, since further entry would switch the state to non-cooperative, in which case long-run profits gross of entry cost \(E\) are zero; or else raise \(m\) above \(m^{zp}\).
  (ii) Next consider histories after which the state of the game is cooperative \((\phi = \phi^c)\) and relationship banks must decide whether to establish relationships. Clearly a relationship bank cannot gain by deviating and setting \(R(v) = 0\) for firms such that \(v \geq v\) (it would lose \(\left(\frac{\lambda^c}{k} - R\right)\) per firm in the current period according to strategies) or by sinking the relationship cost with a firm such that \(v < \frac{\eta}{c}\) (since such a firm will not be successful in establishing a core group according to strategies because \(\left(\frac{\lambda^c + \alpha}{k} - R\right) < 0\)). Hence setting \(R(v) = R\) for \(v \geq v\) and \(R(v) = 0\) for \(v < \frac{\eta}{c}\) is optimal.
  (iii) Now consider decision by relationship bank \(i\) if condition (B.1) does not hold. Then all relationship banks play according to \(P\), from which we know it is optimal not to deviate.
  (iv) Now consider histories after which the state of the game is cooperative \((\phi_i = \phi^c)\), relationship banks must make fee offers, \(\eta_i > 0\) and condition (B.1) holds. Then bank \(i\) can not gain by undercutting (as condition [B.1] implies). On the other hand, if bank \(i\) would set \(\lambda_i^c > \lambda^c\) or \(\lambda_i^{nc} \in (\lambda_i^c + \alpha, \infty)\) it would not get any further deals; and setting \(\lambda_i^{nr} > 0\) would not get any further deals either. Thus, playing \((\lambda_i^c, \lambda_i^{nc}, \lambda_i^{nr}) = (\lambda^c, \infty, 0)\) is optimal. Moreover, if \(\eta_k = 0\) but \(\eta_{i-1} > 0\) it would not gain deviating from setting \((\lambda_i^c, \lambda_i^{nc}, \lambda_i^{nr}) = (\infty, \infty, 0)\). Last, if \(\eta_i = \eta_{i-1} = 0\) relationship bank \(i\) cannot gain by undercutting.
  (v) Last, note that playing \(\lambda_f^i = 0\) is optimal for fringe banks in the one-period game, hence it is also optimal to play so in the repeated game. This completes the proof. ■

C. Comparative equilibria

In this appendix we obtain the comparative equilibria derivatives that are presented in the text. All are obtained by totally differentiating the identity

\[
\frac{\delta}{1 - \delta} k \left(\lambda^c V - R\right) - [(m - 1)\lambda^c - (m - k)\alpha] V \equiv 0
\]
which is derived from the no undercutting condition (3.4). Totally differentiating this identity with respect to $\lambda^c$, $m$, $k$ and $\alpha$, recalling that $\varepsilon_{V,\lambda} \equiv -\frac{\partial V}{\partial \lambda}$, and simplifying yields

$$
\left\{ \left[ \frac{\delta}{1-\delta} - (m-1) \right] (1 - \varepsilon_{V,\lambda^c}) - \frac{\alpha}{\lambda^c} (m-k) \varepsilon_{V,\lambda^c} \right\} V d\lambda^c - (\lambda^c - \alpha) V dm
- \left\{ (m-1)\lambda^c - (m-k) \right\} \frac{\partial V}{\partial k} + \alpha V - \frac{\delta}{1-\delta} \left( \lambda^c \frac{\partial V}{\partial k} - R \right) \right\} dk + (m-k)V d\alpha \equiv 0,
$$

which can be rewritten as

$$
Ad\lambda^c - Bdm - Cdk + Dd\alpha \equiv 0. \quad \text{(C.1)}
$$

It will be useful to sign the coefficients in identity (C.1). Clearly $B > 0$ (since $\lambda^c > \alpha$) and $D > 0$ (since $m > k$). To sign $C$ note first that $(m-1)\lambda^c - (m-k) = k - \lambda^c > 0$. Moreover, since $\frac{\partial V}{\partial k} = \frac{\mu}{2k} = \frac{v}{2k} > 0$ it follows that

$$
\lambda^c \frac{\partial V}{\partial k} - R = \lambda^c \frac{v}{2k} - R < 0
$$

since $\lambda^c \frac{v}{2k} - R = 0$ by the definition of $\mu$. It follows that $C > 0$. Finally, noting that $\varepsilon_{V,\lambda^c} = \frac{\mu}{v_{\lambda^c}}$, $A$ an be rewritten as

$$
\frac{1}{\sigma + \frac{v}{\mu}} \left\{ \left[ \frac{\delta}{1-\delta} - (m-1) \right] \sigma - \frac{\alpha}{\lambda^c} (m-k) v \right\},
$$

whose sign is ambiguous but positive if $\alpha$ is sufficiently small. Now if $A > 0$ then the following result follows.

**Proposition C.1.** If $A > 0$ then:

$$
\frac{dm}{d\lambda^c} = A \frac{B}{B} > 0;
\frac{d\lambda^c}{dk} = C \frac{A}{A} > 0;
\frac{d\lambda^c}{d\alpha} = -D \frac{A}{A} < 0;
\frac{d\sigma}{dk} = -C \frac{B}{B} < 0;
\frac{d\sigma}{d\alpha} = D \frac{B}{B} > 0.
$$

**Proof.** By direct substitution. $\blacksquare$

## D. Nonprice competition

### D.1. Nonprice competition I: analysis

To study nonprice competition, it is assumed that at the beginning of each period, each relationship bank spends a total amount $E_i \equiv f^r E_i$ in sales expenditure (or, ‘sales effort’) to contact firms. Sales efforts result in contacts with firms according to the following assumptions:

**Assumption D.1.** (i) A relationship bank spends at least $E = f^r E$ in sales expenditures to contact firms.

(ii) Firms contact relationship banks who have spent the $k$th highest amounts in sales expenditures. If $y > 1$ banks spend the $k$th highest amount, then each bank contacts a firm with probability $\frac{1}{y}$.

(iii) If fewer than $k$ banks spend $E$ or more, then firms contact no bank.

It will be useful to number relationship banks according to their sales expenditures and adopt the following notational convention: $E_1 \geq E_2 \geq \ldots \geq E_k \geq \ldots \geq E_m$. That is, relationship bank $1$ spends the (weakly) largest amount in sales expenditures. Note that according to this convention $E_k$ is by definition the (possibly not unique) $k$th highest sales expenditure.

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The thrust of assumption D.1 is to make the marginal gains in relationships very sensitive to sales expenditures at the margin. As will be seen below, this assumption is not extreme. Specifically, one consequence of nonprice competition is to yield an upper bound on \( \lambda^c \). It turns out that any assumption that makes the marginal benefits to relationships less sensitive to sales effort at the margin yields an upper bound on \( \lambda^c \) that is smaller.

We now study a symmetric equilibrium where all relationship banks spend \( E \geq \lambda \) (strategy combinations that support this equilibrium are rigorously constructed below). Ceteris paribus, sales efforts reduce the profits from cooperating on prices by \( E \) every period. Hence, the long-run payoff from sticking to the implicit contract is

\[
\frac{\delta}{1-\delta} f^{\prime}\left[ \frac{k}{m} \left( \frac{\lambda^c V}{k} - R \right) - E \right] \geq \frac{f^{\prime}}{m} \left[ (m - 1) \lambda^c - (m - k) \alpha \right] V.
\]

This condition is very similar to (3.4) except for term \( E \) on the left-hand-side. Call \( \mathcal{L}^E \) the set of points in the space \((\lambda^c, m)\) such that condition D.1 holds. Then \( \frac{\partial \lambda^c}{\partial E} > 0 \) and \( \frac{\partial m}{\partial E} < 0 \), so that \( \mathcal{L}^E \subset \mathcal{L} \). Thus:

**Proposition D.2.** When nonprice competition by banks increases, fees tend to be higher for a given number of banks \( m \). Conversely, the market tends to be more concentrated for a given \( \lambda^c \).

**Proof.** To prove this result, let condition (D.1) hold as an identity and then totally differentiate with respect to \( \lambda^c \), \( m \) and \( E \). This yields

\[
A d\lambda^c - (\lambda^c - \alpha + \frac{\delta}{1-\delta} E) dm - \frac{\delta}{1-\delta} m dE = 0,
\]

where \( A \) is defined as in Appendix B. Setting \( dm = 0 \), straightforward manipulations yield

\[
\frac{d\lambda^c}{dE} = \frac{\alpha}{1-\delta} > 0.
\]

Similarly, setting \( d\lambda^c = 0 \) and rearranging yields

\[
\frac{dm}{dE} = -\frac{\delta}{1-\delta} \frac{m}{\lambda^c - \alpha + \frac{\delta}{1-\delta} E} < 0.
\]

This completes the proof. \( \blacksquare \)

The intuition behind Proposition D.2 should be clear by now: sales efforts, and, more generally, any sunk expenditures, reduce the gains from the implicit contract but not those of cheating. Adhering to the price norm must then be made more attractive, which is achieved either by exit (and increased concentration) or higher fees. The invariance of the gains of cheating to sunk expenditures has another implication:

**Result D.3.** Relationship banks do not compete away rents with sales efforts.

In many models ex-ante nonprice competition is a mechanism to dissipate ex-post rents. This does not occur here because the incentive to make relationship-investments relies on rents. Thus, sales efforts do not do away with soft price competition, which is the source of rents in this model.

Result D.3 has another interesting implication on cross-subsidies among different lines of business for multi-product banks. Suppose that one way to attract clients to do “high margin” deals is to sell them other commodity services at fees below cost. This cross-subsidization should not dissipate rents in the high margin activities performed by the bank. If that were the case, then relationship banks would want to unilaterally deviate from price norms, thus undermining relationships.

There is one sense, however, in which nonprice competition restrains the market power of banks. As is shown now, nonprice competition imposes an upper bound on the fee that investment banks can charge in equilibrium. To see this, note that an additional way for a relationship bank (say, bank 1) to deviate from equilibrium is by escalating sales expenditures. In that case bank 1 establishes a relationship with every firm and increases the proportion of firms in its portfolio of relationships, \( \eta_1 \), from \( \frac{A}{m} \) to 1. If this occurs, so that bank 1 establishes a relationship with
every firm and $E_2 = E_m = E$, there are $k - 1$ relationships left for $m - 1$ banks for any given firm. $\eta_i$ then falls from $\frac{m}{k}$ to $\frac{k-1}{k}$ for $i = 2, 3, ..., m$. Thus, if every bank sticks to $\lambda_i^* = \lambda^{c}$, and then every bank, including bank 1, pays $E_i = E$ from $t + 1$ on, then bank 1 makes a one-time gain of slightly less than

$$f'' \left( \frac{\lambda^{c} V}{k} - R \right) - E_1$$

$$f' \left[ \frac{k}{m} \left( \frac{\lambda^{c} V}{k} - R \right) - E \right] = f'' \left( \frac{\lambda^{c} V}{k} - R \right) \left( 1 - \frac{k}{m} \right)$$

$$\equiv f'' \left( \frac{\lambda^{c} V}{k} - R \right) \Delta \eta_1 > 0,$$

since relationship bank 1 sets $E_1$ a shade above $E$. But, of course, if this is so, then playing $E_1 = E$ forever cannot be the outcome of an equilibrium, since every bank would have an incentive to unilaterally increase sales efforts every period. It follows that relationship banks will not escalate sales expenditures only if that makes undercutting be the outcome of an equilibrium, since every bank would have an incentive to unilaterally increase sales efforts whenever everybody else’s market share. But this, of course, cannot occur in equilibrium because then every relationship bank would like to unilaterally increase sales efforts, and that no relationships will be established. Thus, as before, the equilibrium in the one period game can be used.

It is straightforward to show that

$$\text{Result D.4. Competition for establishing relationships sets an upper bound on fees charged by relationship banks.}$$

The intuition behind Result D.4 is as follows. When $\lambda_i^*$ is too high, adhering to the implicit contract is very attractive and it is not profitable to undercut even if a unilateral deviation in bank’s 1 sales efforts reduces everybody’s else’s market share. But this, of course, cannot occur in equilibrium because then every relationship bank would like to unilaterally increase sales effort. This determines the upper bound on $\lambda_i^*$. In other words, fees that are too high make it profitable to escalate sales efforts.

Note that the upper bound on $\lambda_i^*$ would be smaller had a unilateral increase in sales effort yielded a smaller increase in the market share of bank 1. This follows because the gains from undercutting (the left hand side in condition (D.2) would then be correspondingly smaller. Therefore, there is no loss of generality in assuming that a marginal increase in sales effort will enable the escalating relationship bank to grab all relationships.

This role of nonprice competition in reducing rents is similar to that obtained in standard models, where ex-ante nonprice competition can eliminate ex-post rents. The difference is that here nonprice competition only restricts the size of such rents, and does not eliminate them. The reason, again, is that rents are necessary to support efficient relationship investments, a feature that is absent when rents are merely a consequence of market power.

D.2. Nonprice competition II: a formal game

This subsection presents a formal game that underpins the analysis in the previous subsection. To analyze nonprice competition we replace the first stage of the one period game. Instead of firms randomly choosing $k$ relationship banks, we have relationship banks choosing sales effort $E_i$.

It is easy to show that in the one-period game relationship banks will not spend anything in sales efforts and that no relationships will be established. Thus, as before, the equilibrium in the one period game can be used as a subgame perfect punishment. Call again this subgame perfect punishment $P$. Next define ‘undercutting’ and ‘cooperative’ and ‘non-cooperative’ states exactly as in the previous section.\footnote{Note that this implies that the state of the game is determined only by the pricing behavior of relationship banks, and not by their sales efforts.} Last, we need one piece of additional notation to keep track of the fraction of firms that contact relationship bank $i$ in response of $i$’s sales effort:
Notation D.5. We denote by $\gamma_i$ the fraction of firms with $\nu \geq \nu$ that contact relationship bank $i$ after $i$ has chosen $E_i$.

Recall that, by definition, $E_1 \geq E_2 \geq ... \geq E_m$. Hence, our assumptions imply that $\gamma_i$ is a function $\gamma_i : R^m \rightarrow [0,1]$ such that

$$\gamma_i(E_1, ..., E_m) = \begin{cases} 0 & \text{if } E_i < E \text{ or } E_i < E_k; \\ \frac{1}{m} & \text{if } E_i = E_k \geq E \text{ and } y \text{ banks make the kth largest sales effort;} \\ \frac{1}{m} & \text{if } E_i = E_m \geq E; \\ 1 & \text{if } E_i > E_k \text{ and } E_i \geq E. \\ \end{cases}$$

(D.3)

Function $\gamma_i$ summarizes how banks sales efforts bring about contacts with firms. Note that $\sum_{i=1}^m \gamma_i = k$ if $E_k \geq E$. We can now define a strategy combination that is a subgame perfect equilibrium in the game with sales effort.

Definition D.6. Call strategy combination $C$ the following combination of strategies:

- For all relationship banks $i$
  1. (Sales effort)
     - If $\phi_i = \phi^c$ then play $E_i = E \geq E$
     - Otherwise, play $E_i = 0$.
  2. (Establishing relationships)
     - If $\phi_i = \phi^c$ and
       $$\frac{\delta}{1-\delta} f^r \left[ \frac{k}{m} \left( \frac{\lambda V}{k} - R \right) - E \right] \geq \max_j \left\{ f^r \left[ \gamma_j \left( 1 - \frac{1}{k} \right) \lambda^c + (1 - \gamma_j)(\lambda^c - \alpha) \right] V \right\}$$
       then play
       $$R_i(v) = \begin{cases} R & \text{for } v \geq \nu, \\ 0 & \text{for } v < \nu. \\ \end{cases}$$
     - Otherwise, play $R_i(v) = 0$ for all $v \in [\nu, \bar{\nu}]$.
  3. (Fee offers)
     - If $\phi_i = \phi^c$ and
       $$\frac{\delta}{1-\delta} f^r \left[ \frac{k}{m} \left( \frac{\lambda V}{k} - R \right) - E \right] \geq \max_j \left\{ f^r \left[ \eta_j \left( 1 - \frac{1}{k} \right) \lambda^c + \eta_j(\lambda^c - \alpha) \right] V \right\}$$
       then play
       $$\left( \lambda^c, \lambda^{nc}, \lambda^{nr} \right) = \begin{cases} (\lambda^c, \infty, 0) & \text{if } \eta_i > 0; \\ (\infty, \infty, 0) & \text{if } \eta_i = 0. \\ \end{cases}$$
     - Otherwise play according to $P$.
   - For all fringe banks $j$ play $\lambda_j^f = 0$.  

Like in the previous section, condition (D.5) says that bank $i$ will not undercut in period $t$ provided that continued cooperation is more profitable than undercutting. Note that this no-_undercutting condition is exactly the same as condition (B.1) in the previous section, except for the fact that sales effort expenditures $E$ are included in the left-hand side of condition (D.5). Lemma B.9 characterizes the outcome path induced by $C$.

Lemma D.7. Along the path induced by $C$
(i) all relationship banks $i$ select $E_i = E$;
(ii) all relationship banks $i$ play $(\lambda_i^c, \lambda_i^{nc}, \lambda_i^{nr}) = (\lambda^c, \infty, 0)$ for all $t$;
(iii) all relationship banks have the same market share;
(iv) all fringe banks play $\lambda_j^f = 0$ for all $t$.  

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Proof. The proof is straightforward and we leave it to the reader. □

We now state and prove the main result of this section:

Proposition D.8. Let $(\lambda^c, m) \in \mathcal{L}^E$. Then strategy combination $C$ is a subgame perfect equilibrium in the infinitely repeated game with sales effort. Moreover,

$$\lambda^c \leq \lambda^c < \overline{\lambda} = \frac{\delta}{1-f} \left[ \frac{k}{m} \left( \frac{f}{m} R + E \right) - \frac{1}{m} (m - k) \alpha V \right],$$

with $\rho \equiv \frac{m k - 1}{m - 1}$.

Proof. To prove this proposition, we show that the players’ strategies for the repeated game are optimal after any history. Again, since this is a repeated game with bounded payoffs, it suffices to show that one-step unilateral deviations from strategies are not profitable after any history.

Now as before, according to $C$ histories can be classified in two groups, cooperative and non-cooperative. Consider, then, histories after which the state of the game is non-cooperative ($\phi_t = \phi_{nc}$). We know that when all other banks are playing according to $P$ in the one-period game, it is optimal for bank $i$ to do the same. Since all banks will play according to $P$ forever after, it is also optimal for bank $i$ to play according to $P$ in any period of the repeated game.

Now consider histories after which the state of the game is cooperative ($\phi_t = \phi_c$) and relationship banks must make fee offers. Then, with the exception of the sales effort $E$ on the left-hand side of (D.5), the continuation game’s strategies look exactly as in the game without sales effort. Hence, one-shot deviations from strategies are unprofitable.

Next consider histories after which the state of the game is cooperative and relationship banks must decide whether to establish relationships with firms. If condition (D.5) holds, and all relationship banks conform to strategies, then $\gamma_j = \eta_j$ (that is, all relationship banks establish relationships with all firms they contacted) and $1 - \gamma_j = \eta_{-j}$ for all $j$. Hence condition (D.5) also holds and cooperation continues. A unilateral deviation by relationship bank $i$ not establishing relationships is therefore unprofitable (see the proof of Proposition B.10). On the other hand, if condition (D.4) does not hold, then no relationship bank establishes relationships, and so it is optimal for $i$ not to establish them either.

Next consider sales effort decisions when the state of the game is cooperative. According to strategies all relationship banks play $E_j = E$. Hence, a unilateral deviation is to play $E_1 \neq E$. If $E_1 > E$ then $E_1 > E_2 = \ldots = E_m$. It follows from (D.3) that $\gamma_1 = 1$ and $\gamma_2 = \ldots = \gamma_m = \frac{k-1}{m-1}$. Such unilateral deviation is unprofitable if

$$\frac{\delta}{1-f} \left[ \frac{k}{m} \left( \frac{f}{m} V - R \right) - E \right] < \frac{f}{m} [(m - \rho) \lambda^c - (m - k) \alpha] V,$$

otherwise it would pay to deviate to increase market share for one time. Now some straightforward algebra shows that condition (C.4) holds if and only if $\lambda^c < \overline{\lambda}$.

Last, consider playing $E_m < E$. Then $\gamma_m = 0$ and clearly condition (D.5) does not hold, since it holds with equality with $\lambda^c = \overline{\lambda}$ and $\min_j \gamma_j = \frac{k-1}{m-1}$. Hence, if relationship bank $m$ deviates selecting $E_m < E$, then no relationships are established in that period and profits are foregone. This completes the proof. □
References


Figure 1. Concentration and Volume in Underwriting
“C8-Ratio” is the share of total volume of securities underwritten in any given year by the top eight investment banks. Full credit is given to lead manager. “Volume” is the logarithm of total volume of securities underwritten in any given year (volume data is in real terms). Volume increased seventeen-fold between 1950 and 1986.
Figure 2. Concentration and Volume in Mergers and Acquisitions

Source: Author’s processing of data from Securities Data Company.

“C8-Ratio” is the share of total deal value of mergers and acquisitions brokered by the top eight investment banks in any given year. Full credit is given to the acquiror’s lead bank. The sample of M&A deals is restricted to those made by firms that do at least three such deals in the 12-year period 1987-1998. Maximum and minimum volume over this time period differ by a factor of eight.
Figure 3. The cost of doing a deal with alternative intermediation technologies.
Figure 4. $L$ is the set of pairs $(\lambda^c, m)$ such that the no undercutting condition (3.3) is satisfied. $\overline{m}$ is the number of relationship banks, and $\lambda^c$ is the fee charged by these banks in an equilibrium with relationships. $\lambda^c$ cannot exceed $\beta$ because firms would switch to fringe banks charging $\lambda^f=0$. Locus CC traces the maximum number of relationship banks, $\overline{m}$, for any given admissible fee $\lambda^c$; or, conversely, the lower bound on the fee, $\underline{\lambda}^c$, for any admissible number of relationship banks.
Figure 5. The effect on the no-undercutting locus CC of a smaller switching cost from a core bank to a non-core relationship bank. Locus CC' traces the effect of a smaller switching cost on the maximum number of relationship banks, \( m \), for any given admissible fee \( \lambda^c \). The shaded region indicates the set of pairs \((\lambda^c, m)\) such that the no-undercutting condition (4.1) is satisfied after a firm’s cost to switch from a core to a non-core bank falls.
Figure 6. Non-price competition and the upper bound on fees. Locus LHS plots a relationship bank’s long-run profits from continued cooperation as a function of the fee $\lambda^c$. Locus RHS plots a relationship bank’s one-time profits when unilaterally undercutting after another relationship bank has unilaterally escalated sales efforts. The intersection of LHS and RHS is the upper bound on the fee that can be charged in any cooperative equilibrium.