# Two-Sided Platforms: Product Variety And Pricing Structures*

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**Abstract**

This paper provides a new modeling framework to analyze two-sided platforms connecting producers and consumers. In contrast to the existing literature, indirect network effects are determined endogenously, through consumers' taste for variety and producer competition. Three new aspects of platform pricing structures are derived.

First, the optimal platform pricing structure shifts towards extracting more rents from producers relative to consumers when consumers have stronger demand for variety, since producers become less substitutable. With platform competition, consumer preferences for variety, producer market power and producer economies of scale in multihoming also make platforms' price-cutting strategies on the consumer side *less* effective. This second effect on equilibrium pricing structures goes in the opposite direction relative to the first one.

Third, variable fees charged to producers can serve to trade off producer innovation incentives against the need to reduce a platform holdup problem.

**Keywords**: two-sided platforms, pricing structure, indirect network effects, product variety.

## 1 Introduction

An increasing number of industries in today's economy are organized around platforms, which enable consumers to access, purchase and/or use a great variety of products. The classic example is shopping malls: the mall developers have to attract retailers and shoppers. However, it is in industries at the core of the "new economy" that this form of market organization has become most important. Personal computers and an ever-expanding number of consumer electronics products such as personal digital assistants, smart mobile phones, television sets and even car navigation systems are built around operating system platforms (Windows, Symbian, Apple's iPhone OS,

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Google's Android and Linux), which allow consumers to acquire and use thousands of applications from third-party developers. Internet retailers like Amazon, eBay, iTunes and Rakuten[1] allow consumers to select from a variety of products and services offered by suppliers having obtained the right to be listed on the site. In the videogame market, users have to purchase consoles such as Sony's Playstation, Microsoft's XBox and Nintendo's Wii in order to have access to hundreds of games supplied by independent publishers. Likewise, digital media platforms - from wireless services such as NTT DoCoMo's i-mode in Japan, to Windows Media Player and RealPlayer, to interactive television platforms such as TiVo and Verizon FiOS - enable consumers to access a variety of content (games, news, music, movies, etc.) from thousands of independent providers.[2]

These platforms are "two-sided" in the sense that both sides - consumers and third-party producers - need to gain access to the same platform in order to be able to interact and the value of platform access to each side is higher, the more members are present on the other side[3]. In this context, a critical problem for platforms is to choose how much to charge each side for access (or membership in the language of Rochet and Tirole (2006)) in order to maximize profits.

This paper lays out a model which analyzes several economic and strategic factors driving optimal access pricing structures for two-sided platforms connecting consumers and producers, such that producers compete against each other. Its novelty consists in allowing consumers to have a variable preference for variety over sellers' products, which introduces substitutability (competition) among producers. The paper belongs to the very recent and quickly growing economics literature on two-sided markets (e.g. Armstrong (2006), Caillaud and Jullien (2003) and Rochet and Tirole (2003) and (2006)). Up to now, this literature has largely focused on the effects of the relative magnitudes of indirect network externalities, demand elasticities and coordination (chicken-and-egg) issues on platform pricing structures. My analysis confirms some insights from previous work but yields a few novel ones.

[1] The leading online shopping mall in Japan.

[2] Gawer and Cusumano (2002) and Evans Hagiu and Schmalensee (2006) provide detailed surveys of the business characteristics of some of these platforms.

[3] This definition is consistent with Rochet and Tirole (2006). They provide a formal definition of two-sidedness for the case of "pure usage externalities", i.e. when the numbers of customers (memberships) are fixed on both sides. A platform is two-sided in this context if and only if the total volume of transactions that take place on it depends on the platform's allocation of variable fees between the two sides (and not just on the sum of these fees). When memberships on the two sides are not fixed, they recognize that in virtually all cases of interest, the platform's choice of fixed (access) fees will affect memberships and therefore - provided each side values the other's presence positively - the corresponding platforms are two-sided.

The first main contribution is to show that consumer demand for product variety is a key factor determining the optimal platform pricing structures. For a monopoly platform, stronger preferences for product diversity shift the optimal pricing structure towards making a larger share of profits on third-party producers relative to consumers. The reason is that when consumers demand more product variety products become less substitutable, meaning that their suppliers are able to extract a larger share of the joint surplus created by the interaction of the two sides. For the same reason, the platform will seek to extract more profits from producers when the latter have more market power over consumers. These results suggest an explanation for some patterns of platform pricing structures observed in practice. For example, as will be argued below, the fact that videogame console makers make most of their profits on the game developer side (through royalties), while software platforms like Windows, Symbian and Palm OS make most of their profits on the consumer side, can be understood by noting that consumers demand for product variety is significantly higher for videogames than for software applications.

The second main contribution is to show that platform competition creates counterintuitive pricing effects which have not been identified up to now. The standard motivation in the existing literature for setting a lower price on the consumer side is accounting for indirect benefits created by additional consumers on the producer side. Here, I show that there exists an additional motivation for lowering prices to consumers: undercutting the rival platform and thereby stealing some of its consumers drives some producers away from it, resulting in even more consumers stolen, etc. In this context, consumer preferences for variety and producer market power acquire an additional effect which goes in the opposite direction relative to the one described above. The *stronger* consumers' preferences for variety and/or the *more* market power producers have over consumers, the *less* effective a given platform's price cutting on the consumer side is in driving producers away from the competing platform, and therefore the *smaller* the equilibrium consumer price cut. This effect shifts the optimal pricing structure for platforms towards making *less* money on producers relative to consumers. For the same reason, it is also possible that stronger economies of scale in multihoming on the producer side also result in platforms raising the share of profits made on consumers relative to producers.

Third, I also use my model to propose a novel interpretation of a second pricing decision faced by two-sided platforms: the choice between membership fees and usage fees (royalties) on the producer side of the market. I show that there are two conflicting objectives that determine the optimal level of usage fees: providing third-party producers with appropriate innovation incentives when the quality of their products is not contractible and mitigating a hold-up problem by the platform. This hold-up problem arises when producers make their platform adoption decisions before consumers and platforms cannot credibly commit *ex-ante* to consumer prices. The first objective tends to require a low usage fee whereas the second one requires a high one.

**Relationship to the literature**

In the two-sided market literature, the closest paper is Armstrong (2006). The setup of my duopoly model of two-sided platforms (section 4) is similar to his general framework of competitive bottlenecks (section 5 in his paper). However, his general analysis is conducted for the case without *intra*-platform competition. Subsequently, he only uses a stylized example to discuss the scenario with competition among producers on the same platform, without explicitly modelling platform adoption decisions by producers and without deriving the resulting pricing structures. Specifically, he shows that if platforms can charge consumers for membership then they will allow some competition among producers, whereas if for exogenous reasons they cannot then they will restrict competition among producers in order to drive up the revenues they can extract from them. He does not fully derive the optimal pricing structures, nor does he obtain the novel effects of producer market power and consumer preferences for variety that I derive in this paper. Armstrong and Wright (2007) also model two-sided competitive bottlenecks but there is no intra-platform competition on either side in their model. A model of two-sided platforms which can charge usage fees to the producer side of the market can also be found in Hagiu (2006). However, in that paper usage fees solely serve to check producer market power.

My model can also be viewed as an extension of the earlier economics literature on product variety, free entry and social efficiency (Mankiw and Whinston (1986), Kiyono and Suzumura (1987), Spence (1976), Dixit and Stiglitz (1977) and Salop (1979)) in two dimensions. First, those models are "one-sided" in the sense that the number of consumers participating in the market is fixed and only the number of producers is variable. The authors therefore focus exclusively on *direct* competitive (business-stealing) effects on the producer side and abstract from the positive *indirect* network effects between the consumer side and the producer side, which are central to my paper. Second, the two-sided platforms controlling market access through membership fees charged to both consumers and producers constitute a novel form of market organization, which has not been analyzed by the literature on product variety.

Finally, this paper is related to the literature on indirect network effects, especially Church and Gandal (1992) and Church, Gandal and Krause (2002). Both papers study two-sided technology (or platform) adoption, however in both models, platforms are assumed to be entirely "passive", i.e. unable to charge fees. Nocke, Peitz and Stahl (2007) use a similar model of a monopoly platform connecting sellers and buyers, however they restrict attention to the case in which the platform can only charge the seller side of the market (their focus is on various forms of platform ownership).

The remainder of the paper is organized as follows: the next section lays out the model and sets up the optimization problem for a monopoly two-sided platform. Section 3 derives the optimal pricing structure for a monopoly platform and discusses the implications for understanding pricing structures of software and videogames platforms. Section 4 derives platform pricing structures in a symmetric duopoly setting. Section 5 uses an extended version of the monopoly platform model to show how royalties charged to the producer side of the market allow the platform to trade off the need to preserve investment incentives and the need to avoid a form of hold up. Section 6 concludes.

# 2 Modelling Framework

The two-sided platforms considered in this paper are bottlenecks between consumers and producers[4] in the sense that a consumer can purchase and use a seller's product if and only if both join the same platform. Moreover, consumers are interested in buying a variety of products so that their value from accessing a platform is increasing in the number of products it supports and conversely, producers' profits from joining a platform are increasing in the number of consumers that have adopted it. The platform controls access on both sides of the market through membership fees. I take the bottleneck characteristic of the platform as exogenously given. In reality, it can be the

[4] Throughout the paper I will use the terms "producers" and "sellers" interchangeably. These could be independent software or content developers for digital media platforms or product suppliers for brick-and-mortar platforms. Each producer offers only one product.

result of a number of microfoundations, such as matchmaking and marketmaking functions, as modeled in Spulber (2006)[5].

Net surplus for a consumer from joining a platform which charges her $P^U$ and is supported by $n$ producers is $u(n) - P^U - \theta$, where $u(n)$ is the surplus obtained from the $n$ products, *net of the prices charged by product sellers* and $\theta$ is a horizontal differentiation parameter distributed over a support $[0, \theta_H]$ with twice continuously differentiable cumulative distribution $F(\theta)$ and density $f(\theta)$. I denote by $\varepsilon_F$ the elasticity of $F$, which is to be interpreted as the "elasticity" of consumer demand for the platform[6]:

$$\varepsilon_F(\theta) = \frac{\theta f(\theta)}{F(\theta)} > 0$$

Similarly, net profits for a producer from joining a platform which charges $P^D$ and is adopted by all consumers $\theta \leq \theta^m$ are $\pi(n) F(\theta^m) - P^D - \phi$, where $\pi(n)$ is the profit *per platform consumer net of variable costs* and the parameter $\phi$ is the fixed cost of making one's product available for the platform. $\phi$ is distributed on $[0, \phi_H]$ with twice continuously differentiable cumulative distribution $H(\phi)$ and density $h$. The elasticity of producer demand for the platform is:

$$\varepsilon_H(\phi) = \frac{\phi h(\phi)}{H(\phi)} > 0$$

As suggested by this formulation integer constraints will be ignored and $n$ will be treated as a continuous variable throughout the paper. The reason is that in the markets that my analysis applies to there are hundreds or even thousands of producers. Continuity also renders the analysis very convenient by making it possible to use demand elasticities as parameters of the model.

There are three important assumptions embedded in the expressions of consumer surplus and producer profits above. First, all consumers have the same marginal valuation for products, i.e. there is no vertical differentiation among them. Second, all producers are identical and fully interchangeable from the point of view of every consumer and producers are also differentiated only horizontally by their fixed cost. These assumptions greatly simplify the analysis, however the main

[5] In particular, Spulber (2006) relies on network theory to model the choice of buyers and sellers to either search in a decentralized market or to go through a (centralized) intermediary. The intermediary offers some advantages over the decentralized search and interaction alternative, in the form of more efficient matchmaking and/or marketmaking.

[6] Note that elasticity here is defined with respect to net utility rather than to price as is usually the case. Armstrong (2006) uses a similar notion of elasticity, whereas Rochet and Tirole (2003) and (2006) use price elasticities.

insights hold for more general formulations. Third, platforms only charge membership fees. In the basic version of the model, nothing would change if the platform were allowed to also charge usage fees: the two pricing instruments are perfect substitutes. In section 5 however, it is shown that usage fees play an important role when one worries about producers' investment incentives.

Let:

$$V(n) = u(n) + n\pi(n)$$

denote the gross surplus created by $n$ products for each platform consumer.

I make the following assumption:

**Assumption 1** *$u(n)$ is strictly increasing, $\pi(n)$ is strictly decreasing and $V(n)$ is strictly increasing and concave.*

This assumption is quite reasonable: it simply says that net consumer surplus $u(n)$ is increasing in the number of products she has access to, that each producer's profits per consumer are decreasing in $n$ (intra-platform competition) and that the gross consumer surplus created by $n$ products is increasing at a decreasing rate (the 100th product is less valuable than the 10th).

Denote by $\varepsilon_V$ the elasticity of $V$:

$$\varepsilon_V(n) = \frac{nV'(n)}{V(n)} \in ]0,1[$$

The elasticity $\varepsilon_V$ plays a central role in my model: it measures the intensity of consumers' preference for variety. The higher $\varepsilon_V$, the less concave $V(.)$ and therefore the higher the marginal contribution of an additional product to gross consumer surplus. $\varepsilon_V$ can also be interpreted as the degree of substitution between producers: a higher $\varepsilon_V$ means that products are less substitutable, therefore competition among producers is less intense (and vice versa).

Also, let:

$$\lambda(n) = \frac{\pi(n)}{V'(n)}$$

denote the ratio between producer profits and the marginal contribution of an additional producer to gross consumer surplus. $\lambda$ can be interpreted as a measure of the market power producers have over consumers: the higher $\lambda$, the greater the ability of producers to extract the surplus their products create.

Note that although $\lambda$ and $\varepsilon_V$ may be related, one does not necessarily fully determine the other (cf. example 1 below). In particular, imagine that $\varepsilon_V$ increases but that there is competition *within* each product variety, such that the equilibrium price for products is $p = c + m$, where $m$ is a fixed margin. In other words, the price charged by producers is determined by intra-variety competition rather than by inter-variety competition. In this case, holding $n$ fixed, when $\varepsilon_V$ increases, $\lambda$ decreases (since $\pi(n)$ stays constant and $V'(n)$ increases).

Finally, it remains to clearly specify the timing of the pricing game:

- Stage 1) the platform(s) set(s) membership fees $P^U$ and $P^D$ for consumers and producers
- Stage 2) consumers and producers make their adoption decision simultaneously
- Stage 3) producers set prices for consumers and those consumers who have joined the platform in the second stage decide which products to buy.

The slightly odd-sounding assumption that consumers decide whether or not to buy the platform *before* producers set their prices is made in order to simplify the analysis of the two-sided pricing game. Given that producers are atomistic in my model, it is entirely harmless: producers ignore the effect of their pricing decision on total consumer demand for the platform anyway.

Two specific examples are useful in order to understand how the reduced forms $u(n)$, $\pi(n)$ and $V(n)$ are obtained:

***Example* 1** Suppose each consumer's gross utility has the Spence-Dixit-Stiglitz form $G\left(\int_{i=0}^{n} v(q_i)\,di\right)$, where $q_i$ is the quantity of product $i$ consumed, $v(0) = 0$, $v'(.) > 0$ and $v''(.) < 0$ and $G'(.) > 0$, $G''(.) < 0$. Producers set prices $p_i$ simultaneously. Each producer takes the market price $G'\left(\int_{i=0}^{n} v(q_i)\,di\right)$ as given when setting his price. Consumer maximization implies that the quantity $q_k$ demanded by each platform consumer from producer $k$ charging $p_k$ satisfies $p_k = v'(q_k)\,G'\left(\int_{i=0}^{n} v(q_i)\,di\right)$. Consequently, the stage 3 pricing equilibrium among producers is symmetric and defined by:

$$v'(q_n)\,G'(nv(q_n)) = p_n = \arg\max_p \left\{(p - c)\,v'^{-1}\left(\frac{p}{G'(nv(q_n))}\right)\right\}$$

Then: $\pi(n) = (p_n - c) q_n$, $u(n) = G(nv(q_n)) - np_n q_n$ and $V(n) = G(nv(q_n)) - ncq_n$. Letting $v(q) = q^\sigma$ and $G(z) = z^{\frac{\alpha}{\sigma}}$, with $0 < \alpha < \sigma < 1$, I obtain:

$$\pi(n) = (1-\sigma)\alpha\left(\frac{\alpha\sigma}{c}\right)^{\frac{\alpha}{1-\alpha}} n^{-\frac{\sigma-\alpha}{\sigma(1-\alpha)}} \quad ; \quad u(n) = (1-\alpha)\left(\frac{\alpha\sigma}{c}\right)^{\frac{\alpha}{1-\alpha}} n^{\frac{\alpha(1-\sigma)}{\sigma(1-\alpha)}}$$

$$V(n) = (1-\sigma\alpha)\left(\frac{\alpha\sigma}{c}\right)^{\frac{\alpha}{1-\alpha}} n^{\frac{\alpha(1-\sigma)}{\sigma(1-\alpha)}}$$

$$\varepsilon_V = \frac{\alpha(1-\sigma)}{\sigma(1-\alpha)} \in ]0,1[ \quad ; \quad \lambda = \frac{\sigma(1-\alpha)}{1-\sigma\alpha} \in ]0,1[$$

***Example 2*** Suppose consumers have unitary demand for each products (i.e. buy either 0 or one unit) and gross benefits from consuming $n$ products for an individual consumer are $V(n)$ with $V'(.) > 0$, $V''(.) < 0$. Again, producers set their prices simultaneously. In this case the stage 3 price equilibrium is: $p_n = V'(n)$ leading to: $\pi(n) = V'(n)$, $u(n) = V(n) - nV'(n) > 0$ and $\lambda = 1$. Letting $V(n) = An^\beta$, with $0 < \beta < 1$, I obtain[7]:

$$\pi(n) = \beta A n^{\beta-1} \quad ; \quad u(n) = (1-\beta) A n^\beta > 0$$

$$\varepsilon_V = \beta \quad ; \quad \lambda = 1$$

Both of these examples satisfy the assumption above regarding $u(.)$, $\pi(.)$ and $V(.)$.

# 3 Optimal Pricing Structure For A Monopoly Platform

The key question I wish to study is that of the optimal pricing structure for the type of two-sided platforms modeled above: what are the economic factors determining whether platforms should seek to extract more profits from consumers relative to producers or vice versa?

I start by setting up the platform optimization problem. Given the platform's membership fees $P^U$ and $P^D$, it is indeed an (interior) equilibrium for $n$ producers and $F(\theta^m)$ consumers to adopt

[7] This example is used by Church Gandal and Krause (2002).

the platform in stage 2 only if the following two conditions hold:

$$\pi(n) F(\theta^m) - P^D - H^{-1}(n) = 0 \tag{1}$$

$$u(n) - P^U = \theta^m \tag{2}$$

The first condition says that in equilibrium all profit opportunities are exhausted for producers (assuming the supply of producers is large enough) and the second condition says that the marginal consumer $\theta^m$ must be indifferent between adopting and not adopting the platform.

Equation (1) determines producer demand $n$ as a function $N(\theta^m, P^D)$ of consumer demand and the price charged to producers, whereas equation (2) determines the marginal consumer $\theta^m$ (and therefore consumer demand $F(\theta^m)$) as a function $\Theta(n, P^U)$ of producer demand and the price charged to consumers. Note that these two-way demand interdependencies or indirect network externalities are positive: $N(., P^D)$ and $\Theta(., P^U)$ are both increasing.

Plugging (2) into (1), I obtain $n$ as an implicit function of the platform's fees $P^D$ and $P^U$:

$$\pi(n) F(u(n) - P^U) = H^{-1}(n) + P^D \tag{3}$$

This expression makes clear that on the producer side of the market there are both positive *indirect* network effects, contained in the term $F(u(n) - P^U)$, and *negative direct* competition effects, contained in the term $\pi(n)$.

For simplicity and without any loss of substance the platform's marginal costs on both sides are normalized to 0. The expression of platform profits is then:

$$\Pi^P = P^U F(\theta^m) + nP^D = (V(n) - \theta^m) F(\theta^m) - nH^{-1}(n) \tag{4}$$

which depends only on $(\theta^m, n)$. Therefore, rather than maximizing platform profits over $(P^U, P^D)$ I will do so directly over $(\theta^m, n)$. The first-order conditions determining the optimal $(\theta_p^m, n_p)$ are:

$$\frac{V(n) - \theta^m}{\theta^m} = \frac{1}{\varepsilon_F(\theta^m)} \tag{5}$$

$$V'(n)\,F(\theta^m) = nH^{-1\prime}(n) + H^{-1}(n) \qquad (6)$$

Given the profit-maximizing $(\theta_p^m, n_p)$, the corresponding profit maximizing prices $(P_{2sp}^U, P_{2sp}^D)$ are then *uniquely* determined by (1) and (2).

For completeness, the following two assumptions are also needed:

**Assumption 2** *The expression of platform profits (4) is concave in $(\theta^m, n))$ and, given prices $(P_{2sp}^U, P_{2sp}^D)$, the market configuration $(\theta_p^m, n_p)$ is a stable equilibrium*[8]*.*

**Assumption 3** *If (1) and (2) have multiple stable solutions $(\theta^m, n)$ given $(P^U, P^D) = (P_{2sp}^U, P_{2sp}^D)$, then the platform is able to coordinate consumers and producers on its most preferred configuration, i.e. $(\theta_p^m, n_p)$.*

Assumption 1 ensures that the first order conditions (5) and (6) define indeed a global maximum for $\Pi^P$. In the appendix I explicitly derive sufficient conditions for satisifying this assumption for the case when the elasticities $\varepsilon_F$, $\varepsilon_H$, $\varepsilon_V$ and the parameter $\lambda$ are all constant. Assumption 2 is less restrictive than it might appear at first glance. Even when there are multiple stable equilibria[9], it is reasonable to expect consumers and producers will coordinate on the stable equilibrium with the highest levels of entry on both sides of the market[10], otherwise, in the absence of any entry restrictions, there would be strictly positive rents available to coalitions of consumers and producers which are left out of the market. Therefore the only potentially problematic case is when the platform's preferred stable equilibrium is not the one with the highest levels of entry. But then the platform can simply adopt a policy of entry restriction on either side, inducing both sides to coordinate on its most preferred equilibrium.

Throughout the paper I will calculate the pricing structure as the ratio between the share of total profits $\Pi^P$ made on producers, $\Pi^{PD} \equiv nP^D$, and the share made on consumers, $\Pi^{PU} \equiv P^U F(\theta^m)$.

[8] Graphically, stability means that at the point $(n_p, \theta_p^m)$, the curve $n = N(\theta^m, P_{2sp}^D)$ crosses the curve $\theta^m = \Theta(n, P_{2sp}^U)$ from below in a $(n, \theta^m)$ plane.

[9] If $\varepsilon_F$, $\varepsilon_H$, $\varepsilon_V$ and $\lambda$ are constant and satisfy the conditions derived in the appendix, then there is only one stable market configuration.

[10] Note that because $N(., P^D)$ and $\Theta(., P^U)$ are increasing, if $(\theta_m, n)$ and $(\theta'_m, n')$ are two equilibria given the same prices $(P^U, P^D)$ then $\theta_m < \theta'_m$ if and only if $n < n'$, so that it makes sense to talk about the highest level of entry on *both* sides.

Then, using (1), (2) and (6):

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{n\pi\left(n\right)F\left(\theta^{m}\right) - nH^{-1}\left(n\right)}{\left(u\left(n\right) - \theta^{m}\right)F\left(\theta^{m}\right)} = \frac{nV'\left(n\right)\left(\frac{\pi(n)}{V'(n)} - 1 + \frac{nH^{-1\prime}(n)}{V'(n)F(\theta^{m})}\right)}{V\left(n\right)\left(1 - \frac{\pi(n)}{V'(n)}\frac{nV'(n)}{V(n)} - \frac{\theta^{m}}{V(n)}\right)}$$

Finally, using the first order conditions (5) and (6), and recalling that $\frac{nH^{-1\prime}(n)}{H^{-1}(n)} = \frac{1}{\varepsilon_H(H^{-1}(n))}$, $\varepsilon_V\left(n\right) = \frac{nV'(n)}{V(n)}$ and $\lambda\left(n\right) = \frac{\pi(n)}{V'(n)}$, I obtain the following proposition.

**Proposition 1** *Assuming the optimization program is well-behaved, the optimal platform pricing structure is given by*[11]*:*

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{\varepsilon_V\left(1+\varepsilon_F\right)\left(1-\left(1-\lambda\right)\left(1+\varepsilon_H\right)\right)}{\left(1+\varepsilon_H\right)\left(1-\lambda\varepsilon_V\left(1+\varepsilon_F\right)\right)} \tag{7}$$

*If* $\lambda \leq \frac{\varepsilon_H}{1+\varepsilon_H}$ *then the platform subsidizes producers (*$P^D < 0$*)*

*If* $\lambda \geq \frac{1}{\varepsilon_V(1+\varepsilon_F)}$ *then the platform subsidizes consumers (*$P^U < 0$*)*

*If* $\frac{\varepsilon_H}{1+\varepsilon_H} < \lambda < \frac{1}{\varepsilon_V(1+\varepsilon_F)}$ *then the platform makes positive profits on both sides of the market. In this case the share of profits made on producers relative to the share of profits made on consumers is decreasing in* $\varepsilon_H$ *and increasing in* $\varepsilon_F$*,* $\varepsilon_V$ *and* $\lambda$*.*

■

The result that $\frac{\Pi^{PD}}{\Pi^{PU}}$ is increasing in $\varepsilon_F$ (the elasticity of consumer demand for the platform) and decreasing in $\varepsilon_H$ (a measure of the elasticity of producer demand) is not surprising and has been obtained (under various forms) in other theoretical models of two-sided markets, Armstrong (2006) and Rochet and Tirole (2003) in particular. It simply says that the share or profits made on one side of the market relative to the other side is higher the easier it is to attract the former and the more difficult it is to attract the latter.

My model yields however two new results. First, the platform makes more profits on producers relative to consumers when producers extract a larger share $\lambda$ of their marginal contribution to gross consumer surplus (per platform consumer). Second, producers account for a larger share of platform profits when $\varepsilon_V$, the "intensity" of consumers' preferences for variety, is higher.

[11] We omit function arguments in order to avoid clutter. Note that the results in the proposition hold for any functional forms, not just for the model with constant elasticities.

To provide some intuition for these results, I will isolate each of these two effects in turn. First, assume $\pi(.)$ is constant equal to $\pi > 0$ and $u(.)$ is linear, i.e. $u(n) = un$, where $u > 0$. In this case $V(n) = (\pi + u)n$, so that $\varepsilon_V = 1$ and $\lambda = \frac{\pi}{u+\pi}$. Thus, given that a producer creates total surplus equal to $u + \pi$ per consumer, $\lambda$ is simply the share of this surplus captured by the producer and $1 - \lambda = \frac{u}{u+\pi}$ is what remains to the consumer.[12] The expression of the optimal structure ((7) above) becomes:

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{\frac{1}{1+\varepsilon_H} - \frac{u}{\pi+u}}{\frac{1}{1+\varepsilon_F} - \frac{\pi}{\pi+u}} = \frac{\frac{1}{1+\varepsilon_H} - (1-\lambda)}{\frac{1}{1+\varepsilon_F} - \lambda}$$

This means that the platform is adjusting its pricing structure to account for the relative division of economic surplus between consumers and producers: it extracts relatively more profits from the side that has more "power" over the other. For example, if competition among producers becomes more intense, then consumers are left with more surplus, hence the platform should charge them more than producers[13]. Armstrong (2006) contains a similar version of this insight, however he derives the result using a stylized example, which does not explicitly model producers' platform adoption decisions. More importantly, as we will see in the next section, with platform competition both $\lambda$ and $\varepsilon_V$ acquire novel and somewhat counterintuitive effects, which are not captured in Armstrong (2006).

Second, it is useful to take a closer look at the effect of consumers' preferences for variety by using example 2: $V(n) = An^{\beta}$, $\pi(n) = V'(n)$, so that $\varepsilon_V = \beta$ and $\lambda = 1$. Then the optimal pricing structure is:

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{\beta(1+\varepsilon_F)}{(1+\varepsilon_H)(1-\beta(1+\varepsilon_F))} \tag{8}$$

which is clearly increasing in $\beta$, the measure of consumer preferences for variety. This is because, when consumers demand more variety, producers' products become less substitutable, which means that, all other things equal - in particular, keeping the number of producers constant - they are able to charge higher prices. Thus, although *total* surplus from consumers-producers transactions ($An^{\beta}N$ here) increases (memberships on both sides - $n$ and $N$ - increase), the share of this surplus

[12] Note that in this case $\lambda$ is fixed; by contrast, when $\pi(.)$ and $u(.)$ are not constant, the relative magnitude of indirect network effects is endogenous - they depend on the numbers of producers and consumers who enter.

[13] One can easily put further structure onto our model by assuming that $n$ stands for different types of products (rather than different individual producers) and that for any given product category, there can be one or more competing producers (the same number across product types in order to maintain the symmetry). It is straightforward to show in this context that the share of profits made on producers is decreasing in the number of producers of the same type, which is a proxy for the intensity of competition.

extracted by consumers ($(1-\beta)$) decreases, while the share extracted by producers ($\beta$) increases, which means the platform should adjust its pricing structure to extract a higher share of rents from producers relative to consumers.

Clearly this result does not depend on producers being able to extract exactly their marginal contribution to consumer surplus ($\pi(n) = V'(n)$): all that matters is that stronger consumer preferences for product variety increase producers' rent-extraction power: $\pi(n) = \lambda V'(n)$ or $\frac{n\pi(n)}{V(n)} = \lambda\varepsilon_V$. Again, the underlying principle is that platforms should extract relatively more rents from the side that has relatively more "power" over the other.

**Empirical relevance**

The insights derived above can be used to make sense of some patterns of platform pricing structures observed in practice.

For example, Evans, Hagiu and Schmalensee (2006) document that, despite numerous economic similarities, two radically different pricing structures prevail in computer-based industries. On the one hand, vendors of operating systems for PCs, workstations, PDAs and smartphones have chosen to subsidize or earn a very small share of profits on the producer side of the market (applications developers). Despite investing large amounts of money every year in "developer support", Microsoft, Apple, Symbian, Palm, Sun and others derive virtually all of their profits by licensing their platforms to users (directly or through OEMs). At the other end of the spectrum, in the videogame market, all console manufacturers make the bulk of their profits through per-game royalties charged to publishers-developers[14] and sell their consoles at or below cost to users.

The preceding analysis suggests the following explanation. While there is no empirical evidence that the elasticities of developer and consumer demand for platforms are markedly different across computer-based industries, there are good reasons to believe that consumer demand for product variety is significantly higher for videogames than for productivity-oriented software. The most important such reason is durability: games get "played out", whereas software with practical applications is theoretically infinitely durable (technological obsolescence notwithstanding of course). As pointed out by Campbell-Kelly (2003):

[14] For example, Sony's Playstation 3, Nintendo's Wii and Microsoft's XBox 360 charge roughly \$7 royalties per game to independent game publishers.

*"[...] Thus, while the personal computer market could bear no more than a few word processors or spreadsheet programs, the teenage videogame market could support an indefinite number of programs in any genre. In this respect, videogames were, again, more like recorded music or books than like corporate software..."*[15]

Given this difference, my model predicts that the pricing structures should be such that videogame platforms make a larger relative share of profits on developers than the other software platforms, which is precisely what is observed.

Another example is the evolution of Microsoft's Windows pricing structure over time[16]. While there has been no dramatic reversal (application developers contribute less than 10% to the profits Microsoft derives from Windows whereas the rest comes from users), the price of Windows to users has increased during the last 15 years by most measures (controlling for quality increases)[17]. At the same time, Microsoft has become ever more generous in its support of software developers. Again, this is entirely consistent with the predictions of the model, given the "commoditization" of the PC software application industry (with a few exceptions, including word processing and spreadsheet software) that Microsoft has induced over the same period.

In the following two sections I provide two extensions of the basic model: platform competition and allowing the monopoly platform studied up to here to charge royalties to developers when the latter make investments in product quality.

# 4 Pricing Structure With Competing Platforms

Platform competition is modeled in the following way. The consumer horizontal differentiation parameter $\theta$ is assumed to be uniformly distributed on a Hotelling segment $[0, 1]$; unit transportation costs are $t$ and there is one platform situated at each of the two extremities. Thus, the utility of a consumer located at $\theta$ from adopting platform 1 exclusively is $u_0 + u(n_1) - t\theta - P_1^U$, whereas that from adopting platform 2 exclusively is $u_0 + u(n_2) - t(1-\theta) - P_2^U$, where $u_0$ is the standalone value of each platform for consumers and $n_i$ is the number of producers supporting platform $i = 1, 2$. Although in

[15] Campbell-Kelly (2003), p. 281.
[16] I am grateful to an anonymous referee for suggesting this example.
[17] Yoffie et al (2006).

principle both consumers and producers could multihome, I restrict them to always coordinate on the equilibria in which no consumer multihomes: this corresponds to a pattern frequently observed in practice.[18] To show that this is consistent with equilibrium behavior, I assume it for now and I show below that given the resulting behavior by producers, no consumer will indeed have any incentive to multihome.

Denote by $D_i^U$ total consumer demand for platform $i$. A producer with fixed development cost $\phi$ makes profits $\pi(n_i) D_i^U - P_i^D - \phi$ by joining platform $i$ exclusively and, given that no consumer multihomes, $\pi(n_1) D_1^U + \pi(n_2) D_2^U - P_1^D - P_2^D - 2\phi$ by multihoming. Thus, for each producer, the decision to adopt platform 1 is independent of his decision to adopt platform 2, *given* $D_1^U$ *and* $D_2^U$[19]. There are two assumptions implicit in this set-up: i) there is no horizontal differentiation between platforms from the producers' perspective[20]; ii) there are no "economies of scale" in developing for multiple platforms. At the end of this section I discuss the effects of relaxing these assumptions.

Producer demand $n_i$ for platform $i \in \{1, 2\}$ is then implicitly defined by:

$$\pi(n_i) D_i^U - P_i^D - H^{-1}(n_i) = 0 \tag{9}$$

This implies that if $n_1 \geq n_2$ as defined by (9) then *all* producers who join platform 2 also join platform 1 (conversely if $n_2 \geq n_1$). Consequently, a consumer $\theta$ who multihomes obtains utility $u_0 + u(\max(n_1, n_2)) - P_1^U - P_2^U - t$. If $P_1^U \geq 0$ and $P_2^U \geq 0$ then this is always less than the utility from being exclusive with one of the two platforms, so that no consumer will ever want to multihome. Finally, if platforms incur a sufficiently large marginal cost $c > 0$ per consumer then they would never set $P_i^U < 0$ (note that this does not preclude them from incurring negative *margins* on consumers, i.e. $P_i^U < c$).

I will now derive the symmetric candidate equilibrium of the simultaneous pricing game between

[18] Most PC users use only one operating system; most videogame users buy only one console; most consumers generally visit one shopping mall. There are however markets in which consumers multihoming is widespread: for instance, most consumers carry two or more credit cards; many Internet users go to two or more online commerce sites and use two or more digital media players.

[19] This is because developers are atomistic, so that each individual developer does not take into account the effect of his adoption decision on $D_1^U$ and $D_2^U$ through the indirect network effect mechanism.

[20] The assumption that platforms are differentiated from the point of view of consumers but are perfect substitutes for producers simplifies the analysis and is also plausible in most real world settings. Indeed, at equal platform quality, producers usually care only about the respective installed bases of consumers and, compared to the latter, they are relatively less likely to have intrinsic preferences for one platform over the other (e.g. applications developers are less likely to be die-hard Windows or MacIntosh fans than PC users).

platforms: each platform attracts half the consumers exclusively, whereas all producers who enter multihome. I show afterwards that if the model is sufficiently close to one with $V(n)$ linear and if the consumer differentiation parameter $t$ is sufficiently large, this symmetric equilibrium exists and is unique.

In all that follows $u_0$ is assumed to be large enough so that the consumer market is always entirely covered. Given that consumers single-home, we have $D_1^U + D_2^U = 1$ and:

$$D_1^U = \frac{1}{2} + \frac{u_1 - u_2}{2t} \tag{10}$$

where $u_i \equiv u(n_i) - P_i^U$.

Platform 1's profits can then be written as:

$$\Pi_1^P = \left(P_1^U - c\right) D_1^U + P_1^D n_1 = (V(n_1) - u_1 - c)\left(\frac{1}{2} + \frac{u_1 - u_2}{2t}\right) - n_1 H^{-1}(n_1)$$

In order to find the symmetric candidate pricing equilibrium without explicitly deriving the two-dimensional best-response functions, I use a "trick" developed by Armstrong (2006) and Choi (2004). In the symmetric equilibrium, $u_1 = u_2 = u$ and $D_1^U = D_2^U = \frac{1}{2}$. Consider then varying $P_1^D$ while maintaining $u(n_1) - P_1^U$ fixed equal to $u$:

$$\Pi_1^P = (V(n_1) - u - c)\frac{1}{2} - n_1 H^{-1}(n_1)$$

Meanwhile, (9) defines a 1-to-1 relationship between $n_1$ and $P_1^D$:

$$\pi(n_1)\frac{1}{2} - P_1^D = H^{-1}(n_1)$$

so that I can optimize directly over $n_1$. I obtain that the number $n_c$ of producers who enter in the symmetric candidate equilibrium is defined by[21]:

$$V'(n_c)\frac{1}{2} = n_c H^{-1\prime}(n_c) + H^{-1}(n_c) \tag{11}$$

[21] If the right-hand side of (11) is increasing then $n_c$ is unique. This is the case when $H$ has constant elasticity.

and the corresponding price charged by platforms to producers is:

$$P_c^D = \pi(n_c)\frac{1}{2} - H^{-1}(n_c) \tag{12}$$

In order to determine the equilibrium price for consumers, consider varying $P_1^U$ keeping all other prices constant. Recalling that $D_2^U = 1 - D_1^U$, (9) applied for $i = 1, 2$ defines $n_1$ and $n_2$ as strictly increasing, respectively decreasing functions of $D_1^U$. Indeed, using the implicit function theorem:

$$\frac{dn_1}{dD_1^U} = \frac{\pi(n_1)}{H^{-1\prime}(n_1) - \pi'(n_1) D_1^U}$$

$$\frac{dn_2}{dD_1^U} = \frac{-\pi(n_2)}{H^{-1\prime}(n_2) - \pi'(n_2) D_2^U}$$

Then, plugging these functions into (10), one obtains a 1-to-1, decreasing relationship between $D_1^U$ and $P_1^U$ *holding all other prices constant* (for $t$ large enough, which we assume). It then turns out to be more convenient to optimize $\Pi_1^P$ over $D_1^U$ rather than over $P_1^U$:

$$\max_{D_1^U} \left\{ (V(n_1) - u_1 - c) D_1^U - nH^{-1}(n_1) \right\}$$

yielding the first-order condition:

$$V(n_1) - u_1 - c + D_1^U \left( V'(n_1)\frac{dn_1}{dD_1^U} - \frac{du_1}{dD_1^U} \right) = \left( n_1 H^{-1\prime}(n_1) + H^{-1}(n_1) \right) \frac{dn_1}{dD_1^U}$$

From (10), I have:

$$\frac{du_1}{dD_1^U} = 2t - u'(n_2)\frac{\pi(n_2)}{H^{-1\prime}(n_2) - \pi'(n_2) D_2^U}$$

Plugging this expression into the first order condition above and using the fact that in equilibrium $D_1^U = \frac{1}{2}$ and $n_1 = n_2 = n_c$, where $n_c$ satisfies (11), I obtain the symmetric equilibrium consumer price:

$$P_c^U = c + t - n_c \pi(n_c) - \frac{u'(n_c)\pi(n_c)}{2H^{-1\prime}(n_c) - \pi'(n_c)} \tag{13}$$

Thus, the candidate equilibrium price on the consumer side of the market with competing platforms is equal to the standard Hotelling price $c + t$ discounted by two positive terms. The first term, $n_c\pi(n_c)$, is the indirect network benefit created for the other side of the market (producers) by the gain of an additional consumer. The second term is specifically due to platform competition: the additional consumer attracted by platform 1 is in fact "stolen" from platform 2, which lowers the number of platform 2 producers by $\frac{\pi(n_c)}{H^{-1\prime}(n_c)-\pi'(n_c)\frac{1}{2}}$[22], which in turn lowers the total utility of all consumers of platform 2 by $\frac{1}{2}u'(n_c)\times\frac{\pi(n_c)}{H^{-1\prime}(n_c)-\pi'(n_c)\frac{1}{2}}$.

The following proposition summarizes the preceding analysis:

**Proposition 2.1** *With symmetric competing platforms, the symmetric pricing equilibrium* $(P_c^U, P_c^D, n_c)$ *with consumers singlehoming necessarily satisfies (11), (12) and (13).*

■

The following proposition proves that, for a specific class of surplus functions and a range of model parameters, the necessary conditions (11), (12) and (13) are also sufficient, i.e. they define the unique symmetric pricing equilibrium.

**Proposition 2.2** *Assume that producers for the same platform compete as in example 1, with* $V(n)=An^\beta$, $\pi(n)=A\lambda\beta n^{\beta-1}$, $u(n)=(1-\lambda\beta)An^\beta$, *where* $\beta,\lambda\in(0,1)$[23], *and* $H$ *has constant elasticity* $\varepsilon_H$, *i.e.* $H^{-1}(n)=Bn^{\frac{1}{\varepsilon_H}}$. *Then, for all* $\beta$ *and* $\varepsilon_H$ *sufficiently close to 1 and* $t$ *large enough,* $(n_c, P_c^U, P_c^D)$ *determined by (11), (12) and (13) represents the unique symmetric pricing equilibrium in the platform competition game. In this equilibrium, consumers split equally among the two platforms and there are* $n_c$ *producers who enter and multihome, where* $n_c$ *is given by (11)*

**Proof** See appendix.

■

Using the model described in Proposition 2.2 (with $\beta$ and $\varepsilon_H$ close to 1), (11), (12) and (13) yield closed form expressions for the pricing equilibrium and the resulting platform pricing structure:

$$n_c=\left[\frac{\beta A\varepsilon_H}{2B(1+\varepsilon_H)}\right]^{\frac{1}{1+\frac{1}{\varepsilon_H}-\beta}}\quad\text{(increasing in }\beta\text{)}$$

$$P_c^D=\left(\lambda-\frac{\varepsilon_H}{1+\varepsilon_H}\right)\frac{\beta An_c^{\beta-1}}{2}\text{ and }P_c^U=c+t-\lambda\beta An_c^\beta\left(1+\frac{1-\lambda\beta}{\frac{1}{1+\varepsilon_H}+\lambda(1-\beta)}\right)$$

[22] Recall indeed that this factor is equal to $\frac{dn_1}{dD_1^U}$.

[23] To simplify notation, we have denoted $\beta=\frac{\alpha(1-\sigma)}{\sigma(1-\alpha)}$ and $\lambda=\frac{\sigma(1-\alpha)}{1-\sigma\alpha}$.

These expressions yield the following platform pricing structure:

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{\left(\lambda - \frac{\varepsilon_H}{1+\varepsilon_H}\right)\beta A n_c^\beta}{t - \lambda\beta A n_c^\beta\left(1 + \frac{1-\lambda\beta}{\frac{1}{1+\varepsilon_H}+\lambda(1-\beta)}\right)} \tag{14}$$

There are two noteworthy observations to be made when comparing (14) with (7) (setting $\varepsilon_F = 1$ and $\varepsilon_V \equiv \beta$).

First, the condition under which the producer side gets subsidized (i.e. $\Pi^{PD} < 0$) is the same as in the previous section with a monopoly platform: $\lambda < \frac{\varepsilon_H}{1+\varepsilon_H}$. Its interpretation is the same as above. For the rest of the discussion, I assume $\lambda > \frac{\varepsilon_H}{1+\varepsilon_H}$ and $t$ large enough so that $\Pi^{PD} > 0$ and $\Pi^{PU} > 0$.

Second, $\lambda$ and $\beta$ now have two effects on the equilibrium pricing structure. The first one is contained in the terms $\left(\lambda - \frac{\varepsilon_H}{1+\varepsilon_H}\right)\beta A n_c^\beta$ in the numerator and $\lambda\beta A n_c^\beta$ in the denominator. It is very similar to and goes in the same direction as the one in the monopoly platform case: higher $\lambda$ and $\beta$ make producers richer and therefore induce the platform to extract more profits from them. The second effect is new and goes in the opposite direction, through the factor $\frac{1-\lambda\beta}{\frac{1}{1+\varepsilon_H}+\lambda(1-\beta)}$. Recall that, say for platform 1, this factor is due to the *indirect competitive effect* of a decrease in $P_1^U$ contained in the term $\frac{u'(n_2)\pi(n_2)}{2t\left(H^{-1\prime}(n_2)-\pi'(n_2)D_2^U\right)}$. In addition to the usual *direct competitive effect* $\frac{1}{2t}$, a price cut by platform 1 on the consumer side, by stealing consumers from platform 2, drives away some producers from platform 2, which in turn induces even more consumers to shift from 2 to 1 and so on and so forth. Through this effect, both $\lambda$ and $\beta$ tend to *decrease* the share of profits made on producers relative to consumers (indeed, $\frac{1-\lambda\beta}{\frac{1}{1+\varepsilon_H}+\lambda(1-\beta)}$ is decreasing in $\lambda$ everywhere and decreasing in $\beta$ whenever $\lambda > \frac{\varepsilon_H}{1+\varepsilon_H}$). The reason is the following: the effectiveness of the consumer price cut in driving producers away from platform 2 by "stealing" some of its consumers is *lower* when producers extract a *larger* share of the total surplus they create (through higher $\lambda$ and/or $\beta$) because a larger proportion of producers finds it profitable to sell to the remaining platform 2 consumers. Consequently, the optimal price cut on the consumer side for platform 1 will be lower, i.e. the share of profits made on consumers relative to producers will increase.

The second effect may well dominate when platforms are not very differentiated from consumers' perspective. By contrast, it would be reduced if I allowed for consumer market expansion, by introducing hinterlands at each end of the Hotelling segment for example.

**Economies of scale in multihoming**

It is now interesting to ask what would happen to the strategic effects described above if I introduced economies of scale in supporting multiple platforms for producers. To fix ideas, suppose that the cost of supporting a single platform for a producer $\phi$ is still $\phi$, but his fixed costs of multihoming are now $(1+\gamma)\phi$, where $0 \leq \gamma < 1$ is the same for all producers. In other words, the lower $\gamma$, the larger the economies of scale in multihoming ($\gamma = 1$ corresponds to the case analyzed above). Heuristically, when $\gamma$ is *lower*, price cuts by a platform on the consumer side become *less effective* in driving producers away from its rival[24]. In other words, the magnitude of the second effect of $\lambda$ and $\beta$ on the pricing structure identified above is reduced. Consequently, when economies of scale in multihoming are sufficiently strong, I expect the share of profits made on producers relative to consumers to *increase* in both $\lambda$ and $\beta$, just like in the monopoly case.

While a full treatment of the pricing equilibrium for all values of $\gamma$ is beyond the scope of this paper, this intuition can be confirmed by looking at the polar case $\gamma = 0$ (strongest possible economies of scale in multihoming). In this case, at the symmetric equilibrium all producers multihome and we have $\pi(n_0) - 2P^D = H^{-1}(n_0)$. However, unlike the case with $\gamma = 1$, when, say, platform 1 deviates slightly from the equilibrium prices $\left(P^U, P^D\right)$, the number of producers varies but all producers who enter continue to multihome. This means that both platforms continue to be supported by equal numbers of producers, so that $D_1^U = \frac{1}{2} + \frac{P^U - P_1^U}{2t}$ and $\pi(n) - P^D - P_1^D = H^{-1}(n)$. Therefore, for $\left(P_1^U, P_1^D\right)$ close to $\left(P^U, P^D\right)$:

$$\Pi_1^P = \left(P_1^U - c\right) D_1^U + nP_1^D = \left(P_1^U - c\right)\left(\frac{1}{2} + \frac{P^U - P_1^U}{2t}\right) + n\pi(n) - nP^D - nH^{-1}(n)$$

Thus, at the symmetric equlibrium we necessarily have $P^U = c + t$ and $P^D = \frac{\pi(n_0) - H^{-1}(n_0)}{2}$,

[24] To see this, take any quadruple of prices $\left(P_i^U, P_i^D\right)$, $i = 1, 2$ and assume without loss of generality that in the resulting equilibrium market configuration we have $\pi_1(n_1) D_1^U - P_1^D \leq \pi_2(n_2) D_2^U - P_2^D$. Then all producers who enter will join platform 2 and among them, those with sufficiently low costs $\phi$ will multihome. More precisely, a producer $\phi$ who enters multihomes if and only if $\pi_1(n_1) D_1^U - P_1^D - \gamma\phi \geq 0$. Thus, the lower $\gamma$, the higher the proportion of multihoming producers.

where $n_0$ is given by:

$$\frac{\pi(n_0)}{2} = n_0 H^{-1\prime}(n_0) + H^{-1}(n_0) - \left( n_0 \pi'(n_0) + \frac{H^{-1}(n_0)}{2} \right) \tag{15}$$

Using the same functional forms as in Proposition 2.2, we obtain $n_0 = \left( \frac{\lambda A \beta (2\beta - 1)}{B\left(1 + \frac{2}{\varepsilon_H}\right)} \right)^{\frac{1}{1+\frac{1}{\varepsilon_H} - \beta}}$ which is increasing in $\lambda$ and $\beta$ (provided $n_0 > 1$, which we assume). The resulting pricing structure is:

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{\lambda A \beta n_0^{\beta} \left(1 - \frac{2\beta - 1}{1 + \frac{2}{\varepsilon_H}}\right)}{t} > 0 \tag{16}$$

This ratio is increasing in both $\lambda$ and $\beta$ (again, provided $n_0 > 1$). The reason is that, as anticipated, the indirect competitive effects of $\lambda$ and $\beta$ have disappeared so that the comparative statics are again the same as in the monopoly platform case[25].

Note however that the comparison between (16) and (14) is not clear cut: it is not obvious whether - keeping all other parameters fixed - a lower $\gamma$ shifts the pricing structure towards making more or less profits on producers relative to consumers. Indeed, while $\Pi^{PU}$ has unambiguously increased (because consumer price cuts are less effective in driving producers away from the rival platform), $\Pi^{PD}$ may increase or decrease. For example, if $\lambda$ and $\beta$ are close to 1, then it is easily verified that $n_0 > n_c$ and $\Pi^{PD}(\gamma = 0) > \Pi^{PD}(\gamma = 1)$. The increase in $\Pi^{PD}$ corresponds then to the standard intuition according to which when one side is more likely to multihome, platforms will compete less vigorously for it and therefore will extract more profits from that side relative to the other (cf. Armstrong and Wright (2007) and Rochet and Tirole (2003)). In our model however, the novel indirect competitive effect operating through $P^U$ makes it possible that larger economies of scale in multihoming for producers may result in platforms making a larger share of profits on consumers relative to producers.[26]

[25] The stark result that $P^U = c + t$ (the standard Hotelling price in a one-sided market) is due to our simplifying assumption that there is no market expansion on the consumer side. Together with costless multihoming, this implies that around the equilibrium the price $P_1^U$ charged to consumers does not have any effect on the number of producers $n$. If market expansion were introduced, we would obtain the exact same comparative statics, but $P^U$ would be lower (accounting for the familiar indirect network benefit bestowed on the producer side), possibly below cost.

[26] The same line of reasoning can be applied to the case when horizontal differentiation is introduced on the producer side. There again, stronger horizontal differentiation has two effects that go in opposite directions. On the one hand platforms will charge producers higher prices (they have more market power over them) but on the other hand, they will also increase their prices to consumers since it is now more difficult to use consumer price cuts to make producers defect from the rival platform.

The preceding discussion can be used to uphold the logic of the pricing structure observed in the videogame market that I laid out at the end of section 3. While it is true that the videogame market has always been characterized by intense inter-platform competition[27], it is also important to note that the bulk of fixed costs incurred by videogame developers is platform-independent and most developers do indeed multihome[28]. Therefore, in light of the preceding analysis, the effects of consumer preferences for variety and producer market power on the optimal platform pricing structure should be the ones described in section 3 and are therefore consistent with what we observe.

# 5 The Role Of Royalties: Commitment vs. Innovation Incentives

In the basic model presented in sections 2 and 3 above platforms were restricted to charging membership fees on both sides. In reality however, many platforms charge usage fees on at least one side of the market: videogame console manufacturers charge third-party developers royalties per game copy sold, shopping mall developers levy a percentage of sales revenues of the retail shops located on their property and wireless carriers charge usage fees to both consumers and content providers. The most obvious reason for charging usage fees is price-discrimination through the familiar razor-and-blades logic. In this section however, I wish to focus on an alternative, novel explanation for levying usage fees, which arises in two-sided contexts when platforms need to contract with producers *before* consumers enter the market and producer investment incentives are important. This is particularly relevant for the case of software platforms (Windows, Symbian, Palm OS) and videogame consoles. If, for technological or other reasons, platforms have to contract with a critical mass of third-party producers *before* selling to consumers, a hold-up-like problem arises. Indeed, the platform alone determines the extent of consumer adoption, which in turn determines the payoffs producers derive from being affiliated with the platform. The platform does not internalize these payoffs unless it is able to extract compensation from the producers' sales through usage fees. On the other hand, the

[27] Sony PlayStation 3 vs. Nintendo Wii vs. Microsoft Xbox 360 in the current generation.
[28] Chapter 7 of Evans Hagiu and Schmalensee (2006) documents this fact.

platforms' economic success depends heavily on the quality of the products they support. Thus, in order to preserve producers' incentives to make quality-enhancing investments, platforms cannot charge excessive usage fees.

Accordingly, this section modifies the monopoly platform model by endogenizing the choice of product quality by producers and allowing the platform to charge producers both membership and usage fees (or royalties) and shows how the optimal royalty is determined by the two conflicting purposes I have just described.

The quality $q(i)$ of application $i$ is modeled as the proportion of consumers who are interested in it. Suppose then that the platform is supported by $n$ applications and $F(\theta^m)$ consumers. Then each platform consumer is interested in $\int_0^n q(i)\,di$ applications and values them at $V\left(\int_0^n q(i)\,di\right)$. Conversely, each application producer $i$ knows that his application is desired by $q(i)\,F(\theta^m)$ consumers. I assume producers compete as in example 2, so that in equilibrium each producer will charge[29] $V'\left(\int_0^n q(i)\,di\right)$ and sell to $q(i)\,F(\theta^m)$ consumers.

I assume producers' fixed cost of producing an application of quality $q$ is $c(q)+\phi$, where, as before, the number of producers with quality-independent fixed cost lower than $\phi$ is $H(\phi)$. However, we assume for simplicity $c(q)$ is the same for all producers, i.e. all producers have the same marginal cost of quality provision. I assume $c(.)$ is strictly increasing and convex and $c(0)=0$.

The platform can charge producers both membership fees $P^D$ and usage fees $\rho$, which for clarity of exposition I assume are proportional to the price charged by producers to consumers. Thus, a producer with fixed cost $\phi$ choosing quality $q$ makes profits:

$$(1-\rho)\,V'\left(\int_0^n q(i)\,di\right) qF(\theta^m) - P^D - \phi - c(q)$$

## 5.1 Simultaneous entry and first-best choices

I start by assuming the same timing of the pricing game as before, i.e. that producers and consumers make their platform adoption decisions simultaneously and then those producers who enter choose the qualities and prices of their respective products simultaneously and non-cooperatively. Then,

[29] Given that all other developers charge this price, an individual developer cannot charge a price higher than the marginal increase in utility his product offers to users which are interested in it. A lower price would not make sense, since it already sells to all potentially interested users.

given $P^D$, $\rho$, $n$, $\theta^m$ and the other producers' quality choices, the optimal quality choice $q$ for a producer is defined by $(1-\rho)\, V'\left(\int_0^n q\,(i)\, di\right) F\left(\theta^m\right) = c'\left(q\right)$. In the symmetric equilibrium, all producers who enter choose the same quality $q$, defined implicitly by:

$$(1-\rho)\, V'\left(nq\right) F\left(\theta^m\right) = c'\left(q\right) \tag{17}$$

Thus, in the adoption stage, consumers and producers enter until the following two equations hold, where $q$ is the function of $(n, \theta^m, \rho)$ defined by (17):

$$(1-\rho)\, V'\left(nq\right) qF\left(\theta^m\right) - P^D - c\left(q\right) - H^{-1}\left(n\right) = 0 \tag{18}$$

$$V\left(nq\right) - nqV'\left(nq\right) - P^U - \theta^m = 0 \tag{19}$$

Consequently, given $\left(P^U, \rho, P^D\right)$, $(n, \theta^m, q)$ are simultaneously given by (17), (18) and (19). Platform profits are then:

$$\Pi^P = P^U F\left(\theta^m\right) + \rho nqV'\left(nq\right) F\left(\theta^m\right) + P^D n = \left(V\left(nq\right) - \theta^m\right) F\left(\theta^m\right) - nc\left(q\right) - nH^{-1}\left(n\right) \tag{20}$$

which the platform maximizes over $\left(P^U, \rho, P^D\right)$ subject to (17), (18) and (19) or, equivalently, over $(n, \theta^m, \rho)$, subject to (17). Note that in the absence of quality investment concerns, i.e. if $q$ were exogenously given (say, equal to 1), then constraint (17) would disappear and, since expression (20) does not depend on $\rho$, (18) implies that $\rho$ and $P^D$ are perfect pricing substitutes for the platform. This case is identical to the one studied in section 3. Here however, although $\rho$ does not appear in expression (20), it determines $q$ as a function of $\theta^m$ and $n$ through (17), whereas $P^D$ (together with $\rho$) determines $n$.

Let us first determine the platform's first best levels of consumer adoption, product variety and quality, i.e. the triplet $\left(\theta^m_{fb}, n_{fb}, q_{fb}\right)$ which maximizes its profits *if* it were able to choose product quality *directly*, or equivalently, if the latter were contractible. The first–order conditions determining $\left(\theta^m_{fb}, n_{fb}, q_{fb}\right)$ are:

$$\theta^m = \frac{\varepsilon_F V\left(nq\right)}{1+\varepsilon_F} \tag{21}$$

$$qV'\left(nq\right) F\left(\theta^m\right) = c\left(q\right) + nH^{-1\prime}\left(n\right) + H^{-1}\left(n\right) \tag{22}$$

$$V'(nq)\,F(\theta^m) = c'(q) \tag{23}$$

It is then easily seen that in this simple model the platform can attain the first-best level of profits even when product quality is not contractible, by setting $\rho = 0$. Indeed, this implies that (17) is identical to the first-order condition (23), so that given $(\theta^m, n)$, the choice of product quality by producers is optimal from the platform's point of view.

## 5.2 Producers enter before consumers

I now further modify the basic model by assuming producers arrive before consumers. Specifically:

- Stage 1) the platform sets $P^D$ and $\rho$; producers decide whether or not to enter
- Stage 2) participating producers decide their investments in product quality
- Stage 3) the platform sets $P^U$; consumers decide whether or not to adopt
- Stage 4) producers set prices for consumers; consumers decide which products to buy.

I solve the pricing game backwards, starting with the fourth stage. In stage 4, a producer having joined the platform and chosen product quality $q$ makes revenues $(1-\rho)\,V'\left(\int_0^n q(i)\,di\right) qF(\theta^m)$. In stage 3 the platform takes as given the number $n$ of producers having entered, their product quality choices $q(i)$ for $i \in [0,n]$ and the royalty rate $\rho$ it had set in stage 1. It maximizes its profits from stage 3 onwards:

$$\max_{P^U} \left\{ \left(P^U + \rho Q V'(Q)\right) F\left(V(Q) - QV'(Q) - P^U\right) \right\}$$

where $Q = \int_0^n q(i)\,di$. This is equivalent to optimizing directly over $\theta^m = V(Q) - QV'(Q) - P^U$, which yields:

$$\theta^m = \frac{\varepsilon_F\left(V(Q) - (1-\rho)\,QV'(Q)\right)}{1+\varepsilon_F} \equiv \theta^m(Q,\rho) \tag{24}$$

In stage 2, each individual producer takes $Q$ as given and chooses $q$ to maximize his future profits. Therefore, in equilibrium all producers choose the same quality $q$, defined implicitly by:

$$(1-\rho)\,V'(nq)\,F(\theta^m(nq,\rho)) = c'(q) \tag{25}$$

Together, (24) (with $Q = nq$) and (25) determine the equilibrium product quality and consumer adoption as functions of producer demand $n$ and the royalty rate $\rho$: $q(n, \rho)$ and $\theta^m(n, \rho)$.

Finally, in stage 1, given $P^D$ and $\rho$, producer demand $n$ is implicitly determined by:

$$(1-\rho) V'(nq(n,\rho)) q(n,\rho) F(\theta^m(n,\rho)) - c(q(n,\rho)) - H^{-1}(n) - P^D = 0 \tag{26}$$

Platform profits from the perspective of stage 1 are the sum of the membership fees collected at this stage and the usage fees collected in stage 4:

$$\Pi^P = nP^D + (V(nq) - (1-\rho) nqV'(nq) - \theta^m) F(\theta^m)$$

where $q = q(n, \rho)$, $\theta^m = \theta^m(n, \rho)$ and $n$ is given by (26).

Plugging in (26), I obtain that the platform maximizes:

$$(V(nq(n,\rho)) - \theta^m(n,\rho)) F(\theta^m(n,\rho)) - nc(q(n,\rho)) - nH^{-1}(n) \tag{27}$$

over $\left(P^D, \rho\right)$ subject to (26), or equivalenty, over $(n, \rho)$ directly.

This time it is clear that the platform is unable to achieve the first-best level of profits. The reason is that now $\rho$ plays two conflicting roles: on the one hand it helps mitigate the platform's hold-up problem in stage 3 (this calls for a high $\rho$), while on the other hand it has to preserve producer innovation incentives (this calls for a low $\rho$).

First, assume that product quality is not an issue, i.e. it is contractible *ex-ante* for example. Then (25) disappears and it is easily seen from (24) that the platform can still achieve the first best by charging $\rho = 1$. In other words, by committing to take over all producer revenues in stage 4, it effectively commits itself to the first-best consumer price $P^U$ -or equivalently the first-best level of consumer adoption $\theta^m_{fb}$- from the perspective of stage 1, which in turn allows it to induce the first-best level of producer adoption $n_{fb}$.

Second, assume the platform were able to credibly commit in stage 1 to the consumer price it will charge in stage 3 to consumers or that it could get together will all producers in stage 3 and negotiate the consumer price in order to maximize joint profits. In this case the first-best level of profits is once again attainable by setting $\rho = 0$, as in the case when consumers and producers arrived simultaneously. This is because (24) disappears and the platform can guarantee in stage 1 the first-best level of consumer adoption $\theta^m = \frac{\varepsilon_F V(nq(n,\rho))}{1+\varepsilon_F}$ given $n$ and $\rho$.

In the case when quality is not contractible and the platform is neither able to credibly commit to its consumer price *ex-ante*, nor negotiate with all producers in stage 3, the platform will have to settle for a second-best level of profits. The corresponding optimal royalty rate will then be strictly between 0 and 1, reflecting the tradeoff I have just explained.

The following proposition summarizes this discussion:

**Proposition 3** *Assume producers have to make their platform adoption decision before consumers. Then the platform can achieve the first-best level of profits if and only if the quality of producers' products is contractible (the optimum is attained by setting $\rho = 1$) or if the platform is able to credibly commit ex-ante to its consumer price (the optimum is attained by setting $\rho = 0$). Otherwise, the first-best level of profits is not attainable and the optimal royalty rate $\rho$ is strictly between 0 and 1.*

■

Proposition 3 reveals that with sequential entry of the two sides, an indirect "hold-up" problem arises, since the consumer market for producers is "made" by the platform and at this stage the platform no longer has an incentive to internalize producers' payoffs. Specifying usage fees in the initial contractual agreement between the platform and its producers helps mitigate this hold-up issue to a certain extent. Ideally, from a joint profit-maximizing perspective, the platform should internalize *all* revenues producers derive from their interaction with consumers. This is however not feasible when producers need to make further platform-specific investments to improve the quality of their products. Because usage fees play this dual role, the first-best level of profits is generally not attainable by the platform and the optimal royalty rate lies strictly between 0 and 1: the more acute the hold-up problem and the less important investment incentives are, the higher this rate, and vice versa.

As an example, consider the case with linear consumer demand for the platform, i.e. $F(\theta) = B\theta$; quadratic cost of quality provision, i.e. $c(q) = \frac{cq^2}{2}$; and $V(Q) = AQ^\beta$, $\beta \in ]0,1[$. In the appendix we show that in this case the optimal royalty rate is $\rho = \frac{\beta}{1+\beta}$.

# 6 Conclusion

This paper has derived three key insights concerning the pricing behavior of two-sided platforms connecting consumers and producers in markets in which consumers are interested in purchasing a variety of products and producers compete against each other.

First, I have identified the intensity of consumer preferences for variety as a key factor driving platform pricing structures. I have shown that, when consumers care more about product variety, the optimal pricing structure for a monopoly platform shifts towards making a larger share of profits on producers. This is because stronger consumer preferences for variety are equivalent to less competition between producers and therefore higher rent extraction power by producers, an effect which a two-sided platform naturally internalizes in its pricing structure. This result can be used to make sense of differences between pricing structures observed in practice, such as that between software platforms and videogame consoles.

Second, I have shown that platform competition may create dynamics which are counterintuitive and run against the conventional wisdom derived from previous models. Specifically, an additional effect of consumer preferences for variety and producer market power appears, which goes in the opposite direction relative to the one derived under monopoly. The stronger consumer preferences for variety and/or producer market power, the less effective a given platform's price cutting strategies on the consumer side are in driving producers away from the competing platform, and therefore the higher the equilibrium price charged to consumers. Introducing economies of scale in multihoming on the producer side makes such price cutting strategies even less effective, therefore it may also be that platforms find it optimal to make higher profits on producers relative to consumers when it is less costly for producers to multihome.

Third, I have rationalized the use of usage fees by platforms as stemming from a conflict between two objectives: providing appropriate investment incentives to producers and reducing a hold-up problem by the platform, which arises when producers make their platform adoption decisions before consumers.

There are several extensions of my analysis that seem promising for future research. One would be a complete study of the effects of scale economies in multihoming and market expansion on equilibrium platform pricing structures building on the model provided at the end of section 4.

Another would be an analysis of equilibrium choices (e.g. the tradeoffs between variety and quality on the producer side of the market; investment decisions by platforms, etc.) other than prices in a context with competition between the type of two-sided platforms modeled in this paper.

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# 7 Appendix

**Derivation of the necessary and sufficient conditions for the market configuration $\left(\theta_p^m, n_p\right)$ to be a global maximum for $\Pi^P$ and stable given prices $\left(P_{2sp}^U, P_{2sp}^D\right)$**

Assume $F$, $H$ and $V$ are defined on $[0,+\infty]$ and have constant elasticities, i.e. $F(\theta) = \theta^{\varepsilon_F}$, $H^{-1}(\phi) = B\phi^{\frac{1}{\varepsilon_H}}$, $V(n) = An^{\varepsilon_V}$ and that $\pi(n) = \lambda V'(n)$. With these functional forms, (5) and (6) are equivalent to:

$$\theta_p^m = \frac{\varepsilon_F}{1+\varepsilon_F} A n_p^{\varepsilon_V}$$

$$\varepsilon_V A^{1+\varepsilon_F} \left(\frac{\varepsilon_F}{1+\varepsilon_F}\right)^{\varepsilon_F} n_p^{\varepsilon_V(1+\varepsilon_F)} = B\left(1+\frac{1}{\varepsilon_H}\right) n_p^{\frac{1}{\varepsilon_H}}$$

and they admit a unique solution interior to $[0,+\infty]^2$ if $\varepsilon_V(1+\varepsilon_F) - 1 < \frac{1}{\varepsilon_H}$. This solution is a global maximum for $\Pi^P(\theta^m, n)$ if and only if the second order condition holds, i.e. if and only if the Hessian matrix of $\Pi^P(\theta^m, n)$ evaluated at $\left(\theta_p^m, n_p\right)$ is semi-definite negative. We have:

$$\frac{\partial^2 \Pi^P}{(\partial \theta^m)^2}\left(\theta_p^m, n_p\right) = \left(V'(n_p) - \theta_p^m\right) f'\left(\theta_p^m\right) - 2f\left(\theta_p^m\right) = -f\left(\theta_p^m\right)\left(1+\frac{1}{\varepsilon_F}\right) < 0$$

$$\frac{\partial^2 \Pi^P}{\partial n^2}\left(\theta_p^m, n_p\right) = V''\left(n_p\right) F\left(\theta_p^m\right) - \left(nH^{-1}\left(n\right)\right)''\left(n_p\right) < 0$$

because $V$ is concave and $\left(nH^{-1}\left(n\right)\right)'' = B\left(1+\frac{1}{\varepsilon_H}\right)\frac{1}{\varepsilon_H} n^{\frac{1}{\varepsilon_H}-1} > 0$. Also:

$$\frac{\partial^2 \Pi^P}{\partial \theta^m \partial n}\left(\theta_p^m, n_p\right) = V'\left(n_p\right) f\left(\theta_p^m\right) > 0$$

It therefore remains to check that $\frac{\partial^2 \Pi^P}{\partial \theta^m \partial n}\left(\theta_p^m, n_p\right) < \frac{\partial^2 \Pi^P}{\left(\partial \theta^m\right)^2}\left(\theta_p^m, n_p\right) \times \frac{\partial^2 \Pi^P}{\partial n^2}\left(\theta_p^m, n_p\right)$, which, using the expressions above and omitting arguments of some functions in order to avoid clutter, is equivalent to:

$$V'^2 f < \left(1+\frac{1}{\varepsilon_F}\right)\left(-V''F + B\left(1+\frac{1}{\varepsilon_H}\right)\frac{1}{\varepsilon_H} n_p^{\frac{1}{\varepsilon_H}-1}\right)$$

But (6) implies $B\left(1+\frac{1}{\varepsilon_H}\right)\frac{1}{\varepsilon_H} n_p^{\frac{1}{\varepsilon_H}-1} = \frac{1}{\varepsilon_H}\frac{V'(n_p)}{n_p} F\left(\theta_p^m\right)$ so that the inequality above is equivalent to:

$$V'^2 < \left(1+\frac{1}{\varepsilon_F}\right)\left(-\frac{V''\theta_p^m}{\varepsilon_F} + \frac{1}{\varepsilon_H}\frac{V'\theta_p^m}{n_p}\frac{1}{\varepsilon_F}\right)$$

or, using (5) and $\frac{V''V}{V'^2} = \frac{\varepsilon_V - 1}{\varepsilon_V}$:

$$\varepsilon_V\left(1+\varepsilon_F\right) < 1 + \frac{1}{\varepsilon_H} \tag{28}$$

The condition for stability is justified by postulating a dynamic adjustment process for fixed $\left(P_{2sp}^U, P_{2sp}^D\right)$ of the following type. Starting from any $\left(n_0, \theta_0^m\right)$, assume the market configuration $\left(n_t, \theta_t^m\right)$ evolves according to[30]:

$$\begin{pmatrix} \dot{n}_t \\ \dot{\theta}_t^m \end{pmatrix} = \begin{pmatrix} B\left(\pi\left(n_t\right) F\left(\theta_t^m\right) - P_p^D - H^{-1}\left(n_t\right)\right) \\ A\left(u\left(n_t\right) - P_p^U - \theta_t^m\right) \end{pmatrix}$$

where $A$ and $B$ are two positive constants. The necessary and sufficient condition for stability is that this process converges for any initial configuration $\left(n_0, \theta_0^m\right)$. This is equivalent to:

$$\frac{\partial \Theta}{\partial n}\left(n_p, \pi\left(n_p\right) F\left(\theta_p^m\right) - H^{-1}\left(n_p\right)\right) < \frac{1}{\frac{\partial N}{\partial \theta^m}\left(\theta_p^m, u\left(n_p\right) - \theta_p^m\right)}$$

[30] This formulation is a two-dimensional version of the one-dimensional process postulated by Kiyono and Suzumura (1987) in analyzing the social efficiency of free-entry.

Using the implicit function theorem, this condition can also be written:

$$u'(n_p) < \frac{H^{-1\prime}(n_p) - \pi'(n_p) F\left(\theta_p^m\right)}{\pi(n_p) f\left(\theta_p^m\right)} \tag{29}$$

Using the first order conditions (5) and (6) and recalling that $\pi(n) = \lambda V'(n)$ and $u(n) = V(n) - \lambda n V'(n) = (1 - \lambda \varepsilon_V) V(n)$, (29) is equivalent to:

$$(1 + \varepsilon_F) \frac{n_p (1 - \lambda \varepsilon_V) \lambda V'}{V} + \frac{\lambda V'' n_p}{V'} < \frac{1}{1 + \varepsilon_H}$$

or:

$$(1 + \varepsilon_F) \lambda \varepsilon_V (1 - \lambda \varepsilon_V) - \lambda (1 - \varepsilon_V) < \frac{1}{1 + \varepsilon_H} \tag{30}$$

Thus, concavity of (4) in $(\theta^m, n))$ and stability of the equilibrium $\left(\theta_p^m, n_p\right)$ are equivalent to (28) and (30).

■

**Proof of Proposition 2.2** We will first prove Proposition 2.2 for $\beta = 1$, $\varepsilon_H = 1$ and $\lambda \in (0, 1)$. With these parameters, we now have a two-sided model with linear demands on both sides. (9) and (10) become (recall that we always focus on the equilibrium in which no consumer multihomes):

$$\lambda A D_i^U - P_i^D = B n_i$$

$$D_1^U = \frac{1}{2} + \frac{(1 - \lambda)(n_1 - n_2) + P_1^U - P_2^U}{2t}$$

$$D_2^U = 1 - D_1^U$$

Given $\left(P_i^U, P_i^D\right)$, these equations determine a unique consistent demand configuration $\left(D_i^U, n_i\right)$, which we can solve for directly[31]:

$$D_1^U = 1 - D_2^U = \frac{1}{2} + \frac{P_2^U - P_1^U}{2tk} + \frac{A(1 - \lambda)\left(P_2^D - P_1^D\right)}{2tkB}$$

[31] The key reason we are able to obtain a unique consistent demand configuration, unlike in Armstrong and Wright (2007), is that in our model the decisions of any producer to join a platform is independent of his decision to join the other *and* there is horizontal or vertical differentiation on both sides of the market.

$$n_i = \frac{\lambda A}{B} D_i^U - \frac{P_i^D}{B}$$

where $k \equiv 1 - \frac{A^2 \lambda (1-\lambda)}{tB}$ is assumed to be strictly positive. Note that this simply means that the transportation costs are large enough relative to the product of indirect network effects on the two sides - a familiar condition in the two-sided market literature.

Platform $i$ maximizes its profits $\left(P_i^U - c\right) D_i^U + P_i^D n_i$ taking its rival's profits as given. Platform $i$'s profit function is strictly concave in $\left(P_i^U, P_i^D\right)$ if $\frac{A^2[1+\lambda(1-\lambda)]}{2tB} < 1$ (which implies $k > 0$). This ensures that the first order conditions define the unique simultaneous pricing equilibrium. Given the symmetry of demand functions, this equilibrium is symmetric. Straightforward calculations yield:

$$P^U = c + t - \frac{3A^2 \lambda (1-\lambda)}{4B} - \frac{A^2 \lambda^2}{4B} \quad \text{and} \quad P^D = \left(\lambda - \frac{1}{2}\right) \frac{A}{2}$$

These prices are exactly equal to the ones determined by (11), (13) and (12) when $V(n) = An$, $\pi(n) = \lambda A$, $u(n) = (1-\lambda) An$ and $H^{-1}(n) = Bn$. Therefore, the candidate equilibrium defined by (11), (13) and (12) is indeed the unique (symmetric) equilibrium for $\beta = 1$ and $\varepsilon_H = 1$. By continuity, for $\beta$ and $\varepsilon_H$ close to 1, (11), (13) and (12) still define the unique equilibrium (in the main text, we have shown that they are *necessary* conditions to be satisfied in equilibrium). ■

**Example illustrating Proposition 3** The equations defining $\theta^m(n,\rho)$ and $q(n,\rho)$ (24) and (25) become:

$$\theta^m = \frac{A(1-\beta(1-\rho))(nq)^\beta}{2}$$

$$A\beta(1-\rho)(nq)^{\beta-1} B\theta^m = cq$$

Letting $\lambda = \beta(1-\rho)$ and solving these two equations simultaneously, we get:

$$q(n,\lambda) = \left(\frac{AB}{2c}\right)^{\frac{1}{2-2\beta}} n^{\frac{2\beta-1}{2-2\beta}} \left[\lambda(1-\lambda)\right]^{\frac{1}{2-2\beta}}$$

(27) then becomes:

$$\begin{aligned}
& A\left(nq\left(n,\lambda\right)\right)^{\beta}\left(1-\frac{1-\lambda}{2}\right)\frac{cq\left(n,\lambda\right)}{A\lambda\left(nq\right)^{\beta-1}}-n\frac{cq\left(n,\lambda\right)^{2}}{2}-nH^{-1}\left(n\right) \\
= & \frac{ncq\left(n,\lambda\right)^{2}}{2\lambda}-nH^{-1}\left(n\right) \\
= & \frac{c}{2}\left(\frac{AB}{2c}\right)^{\frac{2}{2-2\beta}}n^{\frac{2\beta}{2-2\beta}}\lambda^{\frac{2\beta}{2-2\beta}}\left(1-\lambda\right)^{\frac{2}{2-2\beta}}-nH^{-1}\left(n\right)
\end{aligned}$$

which has to be maximized over $\lambda$ and $n$. Maximizing over $\lambda$ is equivalent to maximizing $\lambda^{\beta}\left(1-\lambda\right)$, which yields the optimal solution $\lambda=\frac{\beta}{1+\beta}$, or $\rho=\frac{\beta}{1+\beta}$. ■