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HBS

May 26th 2009

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- How many users join the platform at price $P$?

$$n = U_0 + u(n) - P$$

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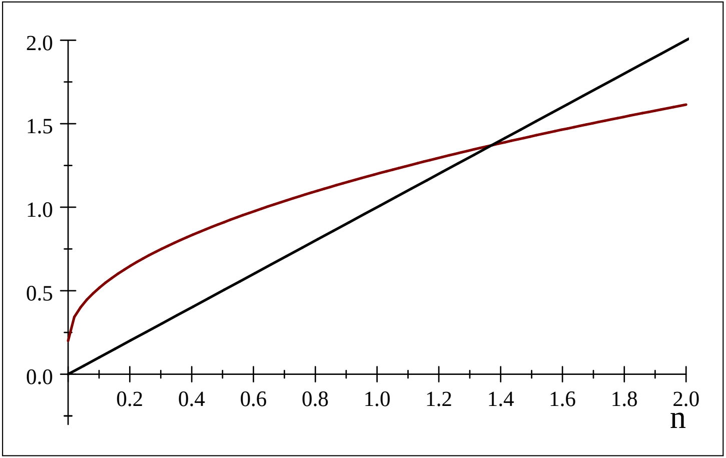
- Suppose $U_0 > P$:



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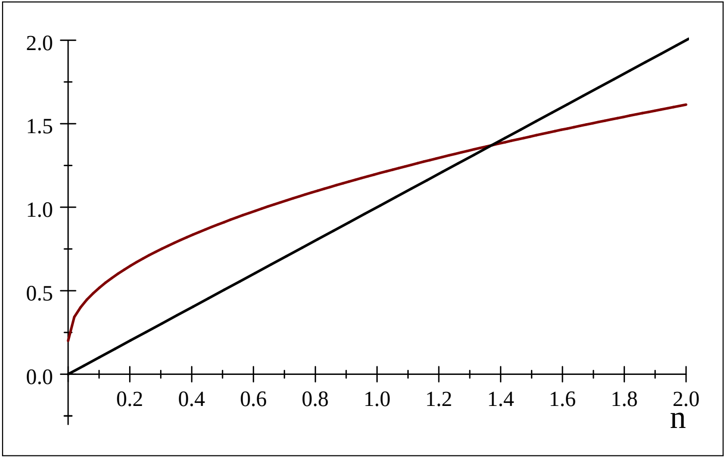


- One equilibrium with $n > 0$ (stable)

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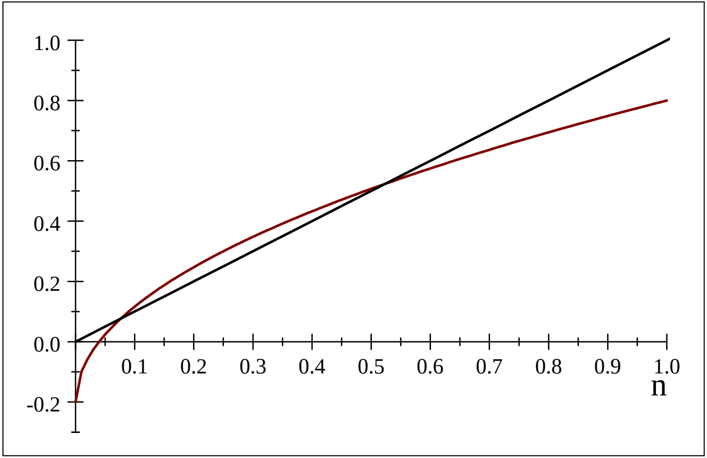

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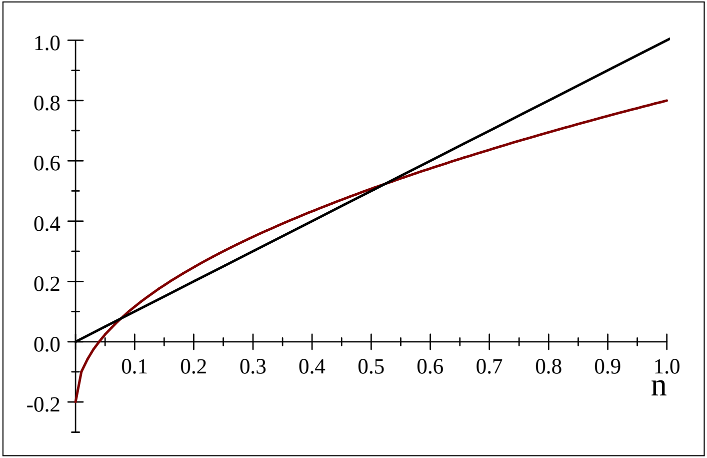


- 2 equilibria with $n > 0$: low $n$ ("unstable") and high $n$ ("stable")

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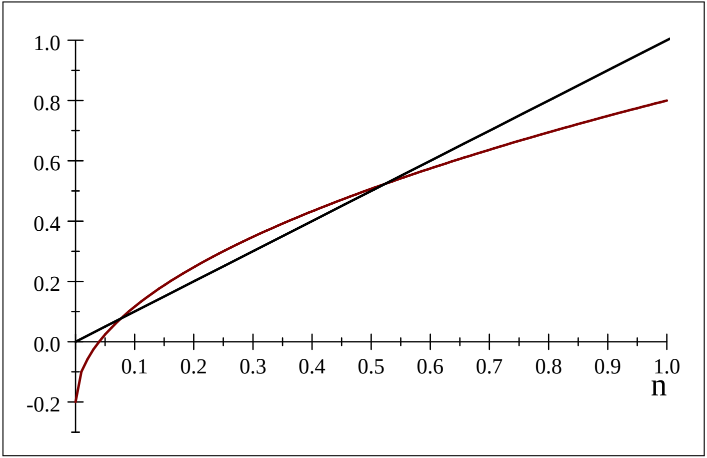


- 2 equilibria with $n > 0$: low $n$ ("unstable") and high $n$ ("stable")
- BUT... $n = 0$ is also an equilibrium!! And it is stable.

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- Platform or network has two groups of customers, 1 and 2:

$$\begin{aligned} U_1 &= U_{01} + u_1(n_2) - P_1 - \theta_1 \\ U_2 &= U_{02} + u_2(n_1) - P_2 - \theta_2 \end{aligned}$$

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- $u_1(n_2)$ and $u_2(n_1)$ *increasing* and concave
- How many users join the platform on each side at prices $(P_1, P_2)$?

$$n_1 = U_{01} + u_1(n_2) - P_1$$
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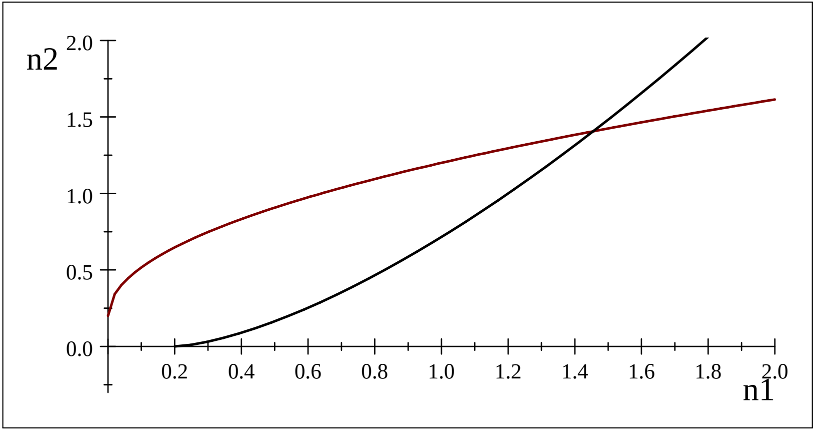
- Suppose $U_{01} > P_1$ and $U_{02} > P_2$:



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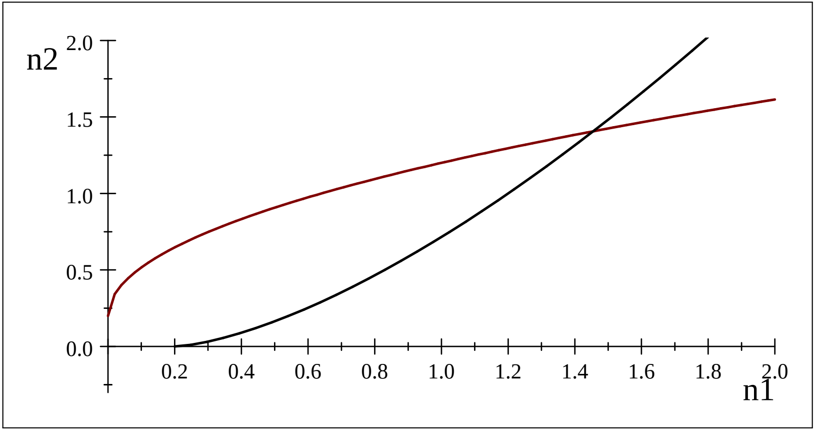


- One equilibrium (stable)

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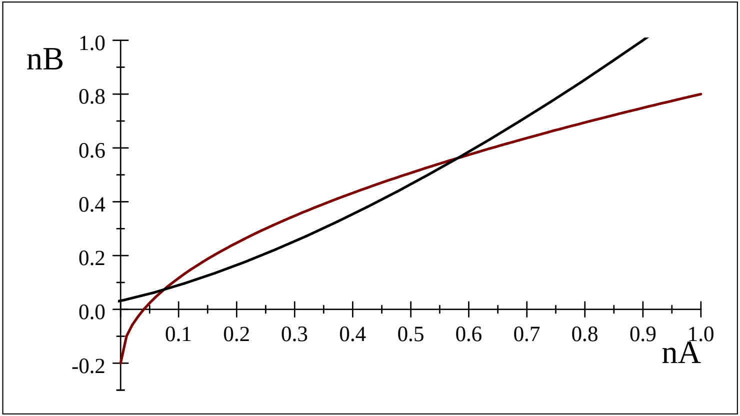

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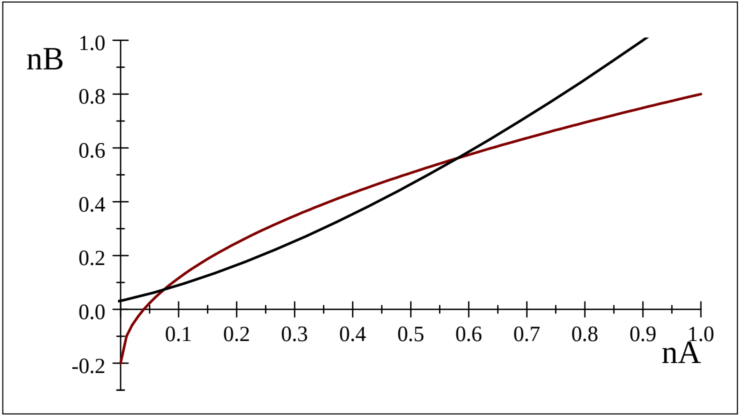


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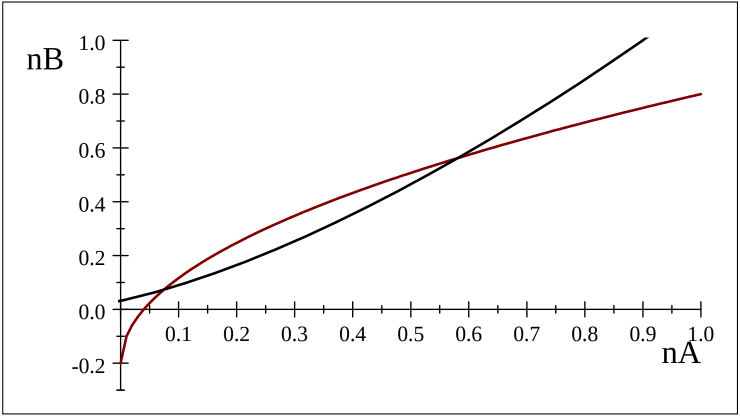
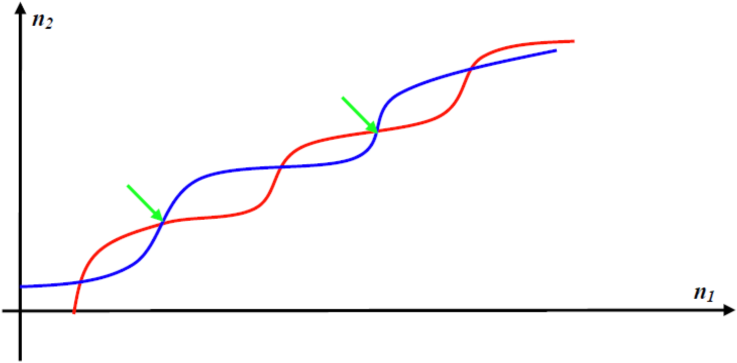



- Two equilibria with $n_1 > 0$ and $n_2 > 0$ : one with "high" demands and stable; one with "low" demands and unstable
- BUT ... $n_1 = 0$ and $n_2 = 0$ is also a stable equilibrium!!

# Indirect (i.e. two-sided) network effects

- It can be worse/uglier:



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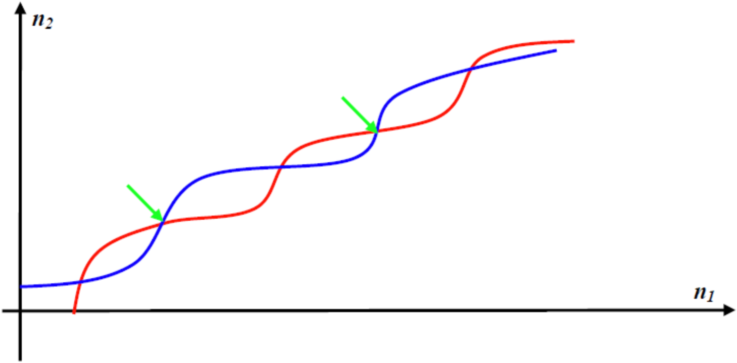


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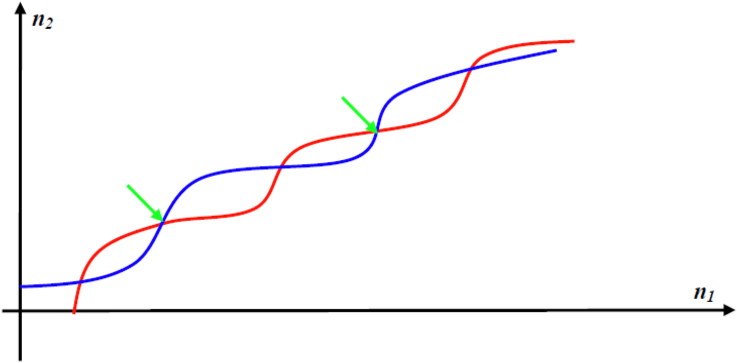


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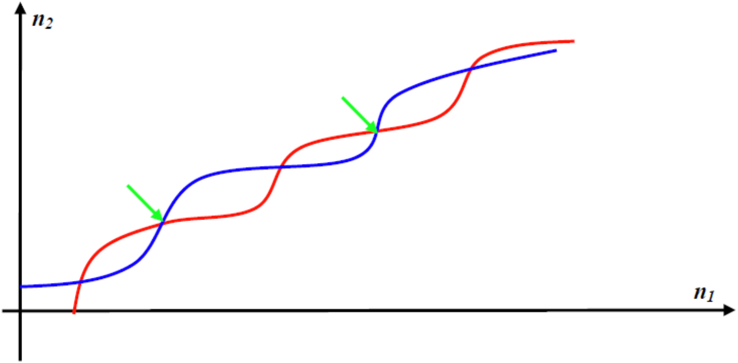


- Multiple stable equilibria: how does one choose?
  - Assume platform(s) can coordinate the two sides on its(their) most preferred equilibria
  - Assume away such pathological functional forms (e.g. assume $u_1(n_2)$ and $u_2(n_1)$ are concave)

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  - Platforms might use other coordination instruments (e.g. game consoles sometimes supply their own games or restrict entry of third-party game developers even though they would be willing to pay)
- Consequence: prevailing equilibrium not necessarily economically "efficient" (i.e. the best network does not necessarily win)

# User expectations - two-sided network effects

- Monopoly two-sided platform, $N_1$ *identical* users on side 1 and $N_2$ *identical* users on side 2. Utilities when $(n_1, n_2)$ users join:

$$U_1 = U_{01} + \alpha_1 n_2 - P_1 \quad \text{with } \alpha_1 > 0$$
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- attracts all users on both sides and yields profits:

$$\Pi^F = N_1 U_{01} + N_2 U_{02} + N_1 N_2 (\alpha_1 + \alpha_2)$$

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- Interpretation? Applications?

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  - Can do better with price discrimination! Offer $P_1 = U_0$ to $n_1$ users and $P_2 = U_0 + \alpha n_1$ to $n_2 = N + 1 - n_1$ users:

$$\Pi^{UF} = n_1 U_0 + n_2 (U_0 + \alpha n_1) = (N+1) U_0 + n_1 (N+1-n_1) \alpha$$

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$$\Pi^{UF} = n_1 U_0 + n_2\,(U_0 + \alpha n_1) = (N+1)\,U_0 + n_1\,(N+1-n_1)\,\alpha$$

  - maximized for $n_1 = \frac{N+1}{2}$, yielding $\Pi^{UF} = (N+1)\,U_0 + \alpha\frac{(N+1)^2}{4} > (N+1)\,U_0$

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    - $P_2 = U_0 + \alpha ...$

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- Unfavorable expectations and suppose can offer different prices to all users:
  - $P_1 = U_0$
  - $P_2 = U_0 + \alpha$...
  - $P_j = U_0 + (j - 1)\,\alpha$

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- Yields: $\Pi = (N+1)U_0 + \frac{N(N+1)}{2}\alpha$ (better than with just two prices)

Thank you for your attention.