Diffusion of Complementary Products with Network Effects:
A Model and Application

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ABSTRACT

In this study, we model the diffusion of two “interacting” products, taking into account how the complementary nature of these two products influences their diffusion. Extending the standard Generalized Bass-type diffusion model (GBM), we allow for the type of interaction to change over the course of the study period and incorporate how network externalities influence new product diffusion. To our knowledge this represents the first attempt in the marketing literature to incorporate both complementary and network effects into a single GBM framework. This is noteworthy since a wide range of high-technology markets are characterized by both cross-product interactions and network effects. Our methodology can be readily applied to any multi-product diffusion setting where cross-product and network effects are present. We demonstrate the applicability of our modeling framework in an empirical application that focuses on the diffusion of PCs and the Internet. In our application, we use a comprehensive data set that covers over two decades—the period from 1981 to 2005—and 19 countries cross Europe and North America. Our estimations confirm that our extended model generates a better fit than conventional models. We also generate substantive insights into the market for PCs and the Internet. Specifically, we find convincing evidence for (a) the existence of multi-product interactions, (b) the occurrence of a change in the nature of those interactions during our study period, and (c) the role of network externalities, each of which enhances our understanding of the evolution of these markets.

Keywords: multi-product diffusion, complementary products, network externalities, Bass diffusion model
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1. INTRODUCTION

Since the publication of the seminal Bass model in 1969, research on the diffusion of innovations has resulted in a very large body of literature (see, e.g., Mahajan, Muller, and Bass, 1990 or Mahajan, Muller and Wind 2000 for comprehensive treatments). In this paper, we address two aspects of diffusion that are often observed together in practice but rarely considered in one modeling framework: multi-product interactions that occur across product categories (e.g., hardware sales affecting software sales) and network externalities that occur within a category (e.g., the size of the adopter base for a particular software product impacting the utility and likelihood of future adoption).

Multi-product diffusion models, as compared to diffusion models more generally, have received relatively little attention. Despite the obvious managerial relevance of understanding the market dynamics of multiple, interacting product categories, diffusion researchers have typically considered only isolated cases of inter-product interactions, and even then within well-established paradigms. Bayus et al. (2000) provide an excellent review of the current status of growth models that incorporate multi-product interactions, provide a general taxonomy of such relationships, and outline potential directions for future research (also see Shocker et al. 2003). They divide existing research on multi-product interactions into two main streams: (1) research that extends single-product diffusion models to account for possible multi-product interactions (e.g., Peterson and Mahajan, 1978; Bucklin and Sengupta, 1993; Gupta, Jain, and Sawhney, 1999) and (2) models that concentrate on the diffusion of successive product generations (e.g., Norton and Bass, 1987, Danaher, et al. 2000).

Whereas multi-product interactions are often viewed in the context of competition between a new entrant and the mature product it substitutes, Bayus et al. (2000) contend that the interactions among multiple product categories are potentially much broader. To help address this void, they develop a conceptual framework that identifies a more complete range of the possible interactions among multiple products. This
framework builds on previous empirical research by Bayus (1987), who presents a practical method for estimating hardware and software sales, incorporating effects due to different market segment behaviors, pricing, awareness levels, and purchase intentions, applied to the compact disc prerecorded audio market.

The body of research on network externalities, by contrast, is considerably more developed. The concept 'network externalities', first defined and discussed by Rohlfs (1974), refers to the notion that the value of a product to one user depends in a systematic fashion on how many other users there are or, more specifically, the value of connecting to a network depends on the number of other people already connected to it (see Katz and Shapiro 1994, Economides 1996, and Shapiro and Varian, 1999 for comprehensive surveys). It is this 'bigger is better' aspect of networks that can give rise to 'increasing returns' or 'positive feedback effects' (e.g., Arthur 1989), which can lead to extreme outcomes. A range of studies show that network externalities can slow or speed adoption, and even lead to adoption of an inferior technology (e.g., Andolfatto and MacDonald 1998; Farrell and Saloner 1985; Geroski 2000; Irwin and Klenow 1994; Jovanovic and MacDonald 1994; Katz and Shapiro 1985; 1986). Examples of empirical applications include Brynjolfsson and Kemerer (1996), Gandal (1994) and Saloner and Shepard (1995). Models in this area often rely on herd-like adoption behavior once a particular (variant of a) new technology reaches a certain 'critical mass' or 'threshold level' (Bikhchandani, Hirshleifer and Welch 1992; 1998). For example, Goolsbee and Klenow (2002), employing a database on the computer ownership and purchase decisions of US households, find evidence of network externalities in the diffusion of PCs. Specifically, they find that people are more likely to buy their first home computer in areas where a high fraction of households already own computers or when a large share of their friends and family own computers.

We argue that in order to fully understand and predict the diffusion of products such as PCs and the Internet, which are likely to have both network effects and cross-product interactions, it is essential to consider both network externality and cross-product effects within a single modeling framework. It seems reasonable that the speed of adoption of PCs may have impacted the speed of Internet diffusion and vice versa. This phenomenon is common to many high-technology product categories (such as computers and software, or
HD-DVD and home theater systems.) and, accordingly, our framework can be adopted to fit a number of similar industries and applications.

In specifying our model, we build on ideas developed by Bayus (1987) and Bayus et al. (2000), and draw on literature in areas such as multi-product diffusion, diffusion with network externalities, and international diffusion. We estimate our model using data on PC and Internet penetration that cover over two decades (1981-2005) and 19 countries in North America and Europe. Figure 1 depicts the aggregate PC and Internet diffusion patterns for all countries in the dataset.

--- Figure 1 about here ---

In modeling the simultaneous diffusion of PCs and the Internet, we test whether PC adoption facilitates Internet adoption – the higher the number of PCs, the higher the Internet's growth prospects – and we test whether the reverse (also) holds – the more popular the Internet, the higher the growth prospects of the market for PCs. In doing so, we allow for the fact that there may be multiple 'nested' structures that need to be considered. By comparing alternative nested structures, we are able to test competing alternative theories regarding the nature of multi-product interaction.

We believe that our approach make several important contributions to the marketing literature. First, previous research on multi-product interactions in the case of complementary goods has been limited. More specifically, to the best of our knowledge, our study is the first to empirically analyze the comprehensive conceptual framework developed in Bayus et al (2000). We find evidence for the existence of multi-product interactions. Specifically, in the terminology of Bayus et al (2000), we find strong support for the idea that the interaction process starts with prior Internet adoption significantly facilitating the diffusion of PCs and this effect gradually changes, and at a critical point, the installed network of home PCs fosters new adoptions of the Internet (auxiliary effect). Second, by considering effects of network externalities, we build on recent work

--- Estimations in the analysis include the 19 countries with complete time series available for the periods studied: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom, and United States. ---
on cross-country diffusion (e.g., Putsis et al., 1997), addressing a call for such research made in a review by Dekimpe, Parker and Sarvary (2000). We show that incorporating network externalities leads to an improved model fit and an enhanced substantive understanding of the evolution of markets. Third, we consider effects on both the speed of diffusion and the total size of the market. Studies that allow for fluctuations in (potential) market size are rare – particularly those that adopt well-established diffusion models such as the Bass model. An exception is Kim, Chang and Shocker (2000), who develop a model that incorporate both inter-product category and technological substitution effects simultaneously. Their empirical application examines the pagers and cellular telephone markets, focusing on cross-category substitution and the role that pager market size/potential, for example, play in cellular telephone adoption. In our research, we recognize that cross-category complementary and network effects can be important in many product categories. In a number of markets, one market’s development can facilitate the development of another market – for example, the growth of the Internet may encourage individuals to adopt a personal computer and sign up for access to the Internet via an ISP. Our study shows that such an approach can reveal important insights into diffusion processes.

The paper proceeds as follows. We begin in the next section by presenting our model, focusing first on network externalities, and then on multi-product interactions. Next, we describe the data used in estimating the model as well as the estimation procedure, paying particular attention to the nested structure. We continue with a discussion of key findings. We end with conclusions, implications, and future research opportunities.

2. **THE MODEL**

In describing our modeling approach, we first extend the basic Generalized Bass Model (GBM) by incorporating within-product network externalities, second, incorporate cross-product effects (in which one product affects the diffusion of another product) and, third, allow for a change in those cross-product effects over time. We end this section by tailoring the general model to the context of PCs and the Internet.
2.1. The Basic Approach – Bass Diffusion with Network Externalities

We account for network externalities by considering endogenous growth in the potential market: the bigger the size of the network of users of a product, the higher the value of these products for potential consumers, and hence the more likely consumers are to adopt it.

Consistent with Rogers (1995) and Dekimpe et al. (1998), for country $i$, we consider a social system, $S_i(t)$, within which an innovation diffuses. Only a certain part of the social system considers the intrinsic utility derived from the innovation to be sufficiently large to adopt it. A bounded variable, $0 \leq C_i(t) \leq 1$, indicates the cumulative fraction of the social system willing to adopt the innovation at any time $t$. We can now define the potential market at any point of time, $M_i(t)$, as the proportion of the social system within which the innovation is eligible to diffuse:

$$M_i(t) = C_i(t)S_i(t)$$

In markets where network externalities are present, the utility that consumers derive from acquiring an innovation is likely to be, at least in part, a function of existing levels of adoption. Hence, in line with Katz and Shapiro (1994), who model effects of network externalities by considering a consumer's utility as a function of the network size (measured by the total number of consumers owning compatible units of the product), we extend the standard diffusion model for technological innovations by considering that for every period $t$, the proportion of the population with a positive 'network utility', $C_i(t)$, varies in a systematic fashion with the network size. The variable $N_i(t)$ denotes the cumulative number of adopters in country $i$ at time $t$. We assume that $C_i(t)$ depends in a systematic way on the network size, $N_i(t)$ and, in the interest of parsimony, that the relationship follows an exponential cumulative distribution. We express both the potential market and the

\[\text{\footnote{We can view } C_i(t) \text{ as a 'ceiling variable', capturing the cumulative proportion of the social system 'susceptible' to adopting the innovation at any point in time; see Putis and Srinivasan (1994) for a theoretical discussion of threshold utility in this context. } C_i(t), S_i(t) \text{ and hence } M_i(t) \text{ are all assumed to be monotonically increasing in } t.\]
effective network size relative to the social system to facilitate inter-model and inter-country comparisons.

The potential market for every country \( i \) then becomes:

\[
M_i(t) = C_i(t)S_i(t) = \left(1 - \theta \exp\left[-\phi \left(\frac{N_i(t)}{S_i(t)}\right)\right]\right)S_i(t) \tag{2}
\]

where 'network externality parameters' \( \theta \) and \( \phi \) capture the shape of the growth of the potential market as a function of the (existing) network size. Higher values of \( \theta \) (0 < \( \theta < 1 \)) go hand-in-hand with a smaller potential market size, and therefore a slower diffusion process. That is, the lower the number of 'initial adopters', the fraction of the social system unaffected by network externalities (1-\( \theta \)), the longer it takes for diffusion to take off and the lower the long-run adoption level. Also the parameter \( \phi \) provides an estimate of the strength and existence of network effects (\( \phi = 0 \) implies the absence of network effects).

In the interest of keeping our framework as general as possible, we begin by extending the Bass model by taking into account the endogeneity of the potential market.\(^3\) Assuming network externalities are present, the number of new adopters of an innovation can be expressed by the following equation:

\[
n_i(t) = \left[\alpha_i + \beta_i \frac{N_i(t-1)}{M_i(t-1)}\right][M_i(t-1) - N_i(t-1)] \tag{3}
\]

where, for every country \( i \), \( n_i(t) \) denotes the number of new adopters of an innovation at period \( t \), \( N_i(t-1) \) denotes the cumulative number of adopters of the innovation at the beginning of period \( t \), \( M_i(t-1) \) denotes the potential market (as expressed in equation 2), \( \alpha_i \) is the “usual” coefficient of external influence, and \( \beta_i \) is the “usual” coefficient of internal influence (Bass 1979). The standard Bass model assumes that the complete

\(^3\) Note that the basic framework that we set out for including network externalities can be extended to include most any cumulative distribution function (e.g., extending this framework to a Generalized Bass Model is straightforward, as we demonstrate in the next section).
social system $S_i(t)$ is willing to adopt the innovation from the very start of the diffusion process. In contrast, when network externalities are considered, the proportion of the social system willing to adopt an innovation is an increasing function of the network size. As a result, in our model, the role of external influence is likely to be smaller during the early stages of the diffusion process than it is in a standard Bass model. We note that the varying impact of network size could help explain the findings of Van den Bulte and Joshi (2007) regarding the time-varying nature of the diffusion process.

----- Figure 2 about here ----- 

The adoption process takes place within the segment of the non-adopting population $(M_i(t) - N_i(t))$, so the speed of the diffusion process is governed by the dynamics of this market. Figure 2 compares the potential market dynamics for two hypothetical examples: (1) without network externalities ($\phi = 0$) versus (2) with relevant network externalities. The vertical distance between the potential market function and the diagonal represent the fraction of the potential market within which the innovation diffuses, $M_i(t) - N_i(t)$, which determines the speed of the diffusion process. Note that in the absence of network effects, the fraction of potential adopters, $M_i(t) - N_i(t)$ is highest at the beginning of the process and decreasing linearly as the adoption grows.

----- Figure 3 about here ----- 

Figure 3 illustrates the potential market size and the diffusion process for different values of $\phi$. It demonstrates that, as described above, the network externality parameter $\phi$ relates directly to the slope of the potential market function. The marginal contribution of the network size, $N_i(t)$, to the growing potential market, $M_i(t)$, is positively related to $\phi$. For higher (lower) values of $\phi$, the network effects become relatively more (less) important and the potential market $M_i(t)$ grows more (less) quickly, approaching a higher (lower)
fraction of the social system $S(t)$. By comparison, for $\phi=0$, our model resembles the Dekimpe, Parker and Sarvary (1998) framework with an endogenous, but constant potential market—a potential market that is a constant fraction $(1-\theta)$ of the social system.

Different combinations of parameters $\theta$ and $\phi$ gives rise to different diffusion patterns. When values for both $\theta$ and $\phi$ are low, the potential market is fairly stable throughout different stages of the diffusion process—network externalities only play a marginal role. When values for both $\theta$ and $\phi$ are high, the potential market is small early in the diffusion process, but quickly grows to a high fraction of the social system—a context that is characteristic for markets with strong network externalities. In short, the diffusion patterns in a model with strong network effects differ significantly from those in a regular Bass model: adoption is slower in the early stages, but increases relatively rapidly once a certain threshold level of adopters has been reached. The larger the network effect, the later the diffusion process peaks.

2.2. Expanding the Model: Generalized Bass Diffusion with Product Interactions

We now turn to the role of cross-product interactions. We extend the diffusion-with-network-externalities model described above by considering the effect of the diffusion of good $x$ on that of good $y$, as well as the reverse effect. We aim to go beyond simply assuming that both complementary product effects and network effects are present – and that they work bi-directionally between PCs and the Internet – by testing for each type of effect in a nested fashion. We believe this is crucial to understanding the specifics of these markets. We describe our approach in modeling the effect of these cross-product interactions below, and provide more details about the nested structure in the “Data and Estimation” Section.

When we address cross-product effects in the diffusion process, the role of marketing mix variables becomes particularly important—a relative price decline in one product, for example, can play an important role in encouraging faster diffusion of the other. Hence, we need to not only account for cross-product interactions but also incorporate marketing mix variables into the framework. Consequently, we model the number of adopters in a certain period using a modified version of the Generalized Bass Model (GBM), as
proposed by Bass, Krishnan and Jain (1994). This model allows for the inclusion of a marketing-mix variable that affects the diffusion process. In addition, we assume that the adoption level of one good affects the speed with which another good diffuses. That is, we model the effect of the adoption of good $y$ on good $x$ by including an additional term in the model that expresses the speed of the diffusion of good $x$. Thus, diffusion levels for good $x$ in every period due to new adoptions are expressed as:

$$n_{xi}(t) = \alpha_x + \beta_x \frac{N_{xi}(t-1)}{M_{xi}(t-1)} + \gamma_x \frac{N_{yi}(t-1)}{S_{yi}(t-1)} (M_{xi}(t-1) - N_{xi}(t-1))Z_{xi}(t)$$

where for every country $i$, $n_{xi}(t)$ denotes the number of new adopters for good $x$ at period $t$, $N_{xi}(t-1)$ denotes the total number of existing adopters of good $x$ at the beginning of period $t$, $M_{xi}(t-1)$ denotes the potential market for good $x$ (as determined as in equation 2), $z_{xi}(t)$ denotes the set of marketing-mix variables that affect the diffusion process, the term $N_{yi}(t-1)/S_{yi}(t-1)$ denotes the adoption level for good $y$, $\alpha_x$ is the coefficient of external influence, and $\beta_x$ is the coefficient of internal influence. The terms $\gamma_x$ represent the coefficients of multi-product interaction. Specifically, $\gamma_x$ captures (and later allows us to test for) the role that adoption levels for good $y$ play in the adoption of good $x$.

2.3. Further Expanding the Model: Generalized Bass Diffusion with Changing Product Interactions

In many industries, it is quite possible that the pattern of interaction across two product categories evolves over time. Specifically, one could envision a scenario in which, early on, adoption levels for good $x$ stimulate adoption for good $y$ while, later in the products’ lifecycles, the reverse is true. For example, in the context of the application we discuss below, one could imagine that the lure of the Internet initially spurred PC adoption, but falling prices and a critical mass in PCs may have later stimulated Interned adoption. It is
important to account for the possibility that such a shift may have occurred. We assume a general framework with dynamic product interactions by allowing the parameter $\gamma$ to evolve in time:

$$n_{xi}(t) = \left[ \alpha_x + \beta_x \frac{N_{xi}(t-1)}{M_{xi}(t-1)} + \gamma_x(t) \frac{N_{yi}(t-1)}{S_{yi}(t-1)} \right] \left( M_{xi}(t-1) - N_{xi}(t-1) \right) Z_{xi}(t)$$

(5)

2.4. Applying the Model – PC versus Internet Diffusion

To demonstrate the substantive benefits of our modeling framework, we apply it to a high-technology setting where cross-product and network externalities are generally thought to have occurred: the market for PCs and the Internet. We modify our earlier notation, and now use the subscript $x$ to denote personal computers (PCs) and subscript $y$ to denote the Internet. Here, we capture the dynamic evolution of the interaction between both technologies by assuming a quadratic structure for the interaction term. Consequently, starting with the diffusion of PCs,

$$n_{xi}(t) = \left[ \alpha_x + \beta_x \frac{PC GDP_{s5}(t)}{M_{xi}(t-1)} + \left( \gamma_{x0} + \gamma_{x1} t + \gamma_{x2} t^2 \right) \frac{N_{yi}(t-1)}{S_{yi}(t-1)} \right] \left( M_{xi}(t-1) - N_{xi}(t-1) \right) Z_{xi}(t)$$

(6)

where for every country $i$, $n_{xi}(t)$ denotes the number of new households acquiring a PC at period $t$, $N_{xi}(t-1)$ denotes the total number of households already owning a PC at the beginning of period $t$, $M_{xi}(t-1)$ denotes the potential market for PCs, and $N_{yi}(t-1)/S_{yi}(t-1)$ denotes the Internet adoption level. As in equation (4) and (5), $\alpha_x$ is the coefficient of external influence, and $\beta_x$ is the coefficient of internal influence, while the polynomial expression, $\gamma_{x0} + \gamma_{x1} t + \gamma_{x2} t^2$, captures the evolving role that Internet adoption plays in PC
adoption. We focus on price as it is naturally the most salient of the marketing mix variables affecting the diffusion of the two focal products, and use the variable $z$ to control for prices in the diffusion of the PCs, as proposed by Bass et. al (1994):

$$Z_{yi}(t) = 1 + e \left( 1 + \frac{P_i(t) - P_i(t-1)}{P_i(t-1)} \right),$$

where the expression in parentheses captures the inflation rate at period $t+1$. Following Horsky’s (1990) intuition, we add an additional term, $PCGDP$, to denote per capita gross domestic product, which is expected to shift the diffusion curve up or down.4

In modeling Internet diffusion, we follow an analogous approach. If we incorporate the complementary nature of PCs and network externalities, the number of new Internet adopters in every period, $n_{yi}(t)$, can be expressed as:

$$n_{yi}(t) = \left[ \alpha_y + (\beta + \delta_{PCGDP})_{S_{yi},S_{yi}}(t) \right] \frac{N_{yi}(t-1)}{M_{yi}(t-1)} + \left( \gamma_{y0} + \gamma_{y1} t + \gamma_{y2} t^2 \right) \frac{N_{yi}(t-1)}{S_{yi}(t-1)} \cdot (M_{yi}(t-1) - N_{yi}(t-1))$$

(7)

3. **Data and Estimation**

We estimate six nested models to determine the effects of complementarities and network externalities on the diffusion processes of PCs and the Internet. Our objective in using and testing this nested structure is to develop a model that best describes the products and markets under investigation. We note that the methodology is applicable to any industry setting characterized by network externalities, multiple countries or segments, and the existence of complementary products—only the specific results pertaining to the nested tests are unique to our empirical setting.

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4 Note that, more generally, our approach is in line with Peterson and Mahajan (1978), has been used by others when modeling complementary or substitution effects in a multi-product framework (see Bayus et al., 2000 for a detailed discussion), and has been adopted in a number of related international diffusion models (Kalish, Mahajan, and Muller (1995) and Putsis et al. (1997) are prime examples).
In estimating the models, we use data that cover 19 countries in Europe and North America (see footnote 3). Annual data on PC penetration—the number of households owning a PC—are available for the period 1981-2005. Annual data for Internet penetration—the number of Internet users—are available for the period 1991-2005. We also obtained data on other relevant variables, such as the number of households, the total population size, the real per-capita disposable income (more specifically the real GDP measured at purchasing power parity, in 1995 US$), and an index of prices, all for all countries and for each year under investigation. We compiled the data using various Euromonitor reports, most notably Euromonitor’s annual 'European Marketing Data and Statistics Report' and 'World Marketing Data and Statistics Report'. In line with our measures of the diffusion of home PCs and the Internet, we use the total number of households and the total population size as measures of the respective social systems. We use real per-capita national product and prices as an explanatory covariate for the diffusion of home PCs.

We opt for a full information maximum likelihood (FIML) estimation procedure to account for the endogeneity of the regressors in equations (6) and (7) (Greene 1997). We assess the fit of equations (6) and (7) by calculating a system-wide measure of adjusted R², using the system-wide Sum of Squared Errors (SSE). Furthermore, we employ a standard log-likelihood ratio test to determine which of the six models in our nested model structure is preferred year-by-year and compared the log likelihood for each combination.

Our testing structure, based on the conceptual model by Bayus et al (2000) is depicted Table 2. Each model is a particular case of our general framework captured by equations (6) and (7). Based upon the model presented above, we have three different factors that need to be tested for: i) the presence or absence of network effects for each product (ψ), ii) the presence or absence of cross-product interactions (γ), and iii) whether or not any observed cross-product interactions change over the course of the study period. Thus, we estimated six models as follows:

\[
R^2 = 1 - \frac{\hat{e}^T (I - K)}{y' (I - \psi D_T - \gamma D_T') y / (T - 1)}, \quad \text{where} \quad D_T = I_T - jj' / T, \quad \text{with} \quad j = (1,1,...,1)'. \quad \text{The Matrix} \quad D_T \quad \text{transforms a given} \quad y_i \quad \text{from its original observations into deviations around its means.} \quad T \quad \text{and} \quad K \quad \text{are the sample size and the number of parameters estimated for each non-linear equation respectively (see Judge et al, 1985 for a detailed discussion on goodness of fit measures in equation systems).}
\]
Models 1A and 1B: No cross-product interactions ($\gamma_0 = \gamma_1 = \gamma_2 = 0$) without (1A) and with network effects (1B).

Models 2A and 2B: Constant cross-product interactions ($\gamma_0 = \gamma_2 = 0$) without (2A) and with (2B) network effects.

Models 3A and 3B: Dynamic cross-product interactions ($\gamma_0, \gamma_1, \gamma_2 \neq 0$) without (3A) and with (3B) network effects.

We refer to our benchmark model (1A and 1B) as the Independent Products model. Model 1A is a model without cross-product interaction and without network effects ($\phi_{xi} = \phi_{yi} = 0$). The latter model closely resembles the Dekimpe, Parker and Sarvary (1998) framework with an endogenous, but constant, potential market—a potential market not influenced by network effects. Model 1B is an Independent Products model without product interactions but with network externalities ($\phi_{xi} > 0$). We extend these base models by allowing for Constant Interactions over time, whereby PC diffusion positively influences Internet diffusion and Internet diffusion positively impacts PC diffusion ($\gamma_{x0} > 0 \cap \gamma_{0} > 0$). The version of this “Constant Interactions” model without network externalities is referred to as Model 2A; the one with network externalities as Model 2B. Finally, we allow for the possibility that the nature of the cross-product interaction changes in time. We refer to this as the Dynamic Interaction products model and consider this without (3A) and with (3B) network externalities.

4. FINDINGS

In our discussion of key findings below, we start with the preferred model selection procedure. We then briefly describe and interpret the coefficients of external and internal influence in order to establish some face validity and to put the results in a broader perspective. Next, seeking to confirm and elaborate on the preferred model selection procedure, we look at parameter estimates for the cross-product interaction...
coefficients in more detail. We end the Findings section with an examination of the estimates of—early and
long-term—potential market sizes.

4.1. Selection of the Preferred Model: Overall Fit and Log-Likelihood Ratio Tests

Recall that our model structure is based on the system of equations (6) and (7), and that we estimate six
models using FIML. Table 1 reports the complete set of results for all alternative specifications.

----- Table 1 about here -----

Several preliminary insights emerge from the system-wide adjusted R² values (reported at the bottom of the
table). Most importantly, the model fit improves significantly when we allow for a shift in the interaction
effects. Compared with models 2A and 2B (which assume a constant interaction effect), the adjusted R²
values for models 3A and 3B (which allow for a shift in the interaction effects) are significantly higher for
both the PC and Internet diffusion processes.

We employ log-likelihood ratio tests to obtain more formal insights into the network externalities and
product interaction effects at play. We select the preferred model by testing (1) the nature and significance of
the complementary interactions in PC and Internet diffusion, and (2) whether network externalities
significantly impact PC and Internet diffusion. Table 2 provides an overview of the results of both steps in
the model selection procedure.

----- Table 2 about here -----

As expected, the log-likelihood ratio tests confirm the preliminary results for the overall model fit already
implied in Table 1. First, as far as the product interactions are concerned, we find the following trends: Models
2A and 2B (with product interactions) outperform Models 1A and 1B (without product interactions), and
Models 3A and 3B (with dynamic interactions) outperform Model 2A and 2B (with constant product interactions). Thus, Models 3A and 3B, which account for dynamic product interactions, are preferred over Models 1 and 2. The log-likelihood ratio test rejects the hypothesis of constant and no interaction effects at a level of 0.1%. This result holds for both the specifications with and without network effects (Models B and A respectively). Consequently, we conclude that the cross-product interactions significantly vary along the diffusion processes for both technologies.

Second, tests for network externalities for the preferred product interaction specification (models 3A and 3B) lead us to reject the null hypothesis of no network externality effects, \( H_0: \phi_{xl} = \phi_{yl} = 0 \quad \forall i \), at a 0.1% significance level. That is, we find that the fit of our model with dynamic interaction effects (Model 3A) significantly improves if we also account for network externality effects (Model 3B). Consequently, we conclude that model 3B, which accounts for dynamic interaction effects and network externality effects, is the preferred model.

4.2. External Influence

The results for the coefficient of external influence are particularly interesting—they are consistent with traditional research in this area, but also provide an additional explanation for some of the recent research on the time-varying nature of the estimated coefficients (e.g., Van den Bulte and Joshi 2007). When no networks are considered (analogous to more traditional diffusion models), the coefficients of external influence in PCs (\( \alpha_i \)), ranging between 1% and 2% in the three relevant specifications (Model 1A, 2A and 3A), are entirely consistent with the large stream of previous research. However, for the models that incorporate network externalities, these values increase to between 16% and 20%. This significant increase is a consequence of the low potential market revealed in the early stages when network externalities are considered as well as the substantial growth rate once the network effects “kick in.”

Figure 4 depicts the evolution of the estimated potential markets. The depicted dynamics reveal that, in models with network externalities, the fraction of the social system willing to adopt the innovation is
significantly lower in the early stages than in the long run. As a result, the coefficients of external influence, which essentially represent the fraction of the potential market that adopts the innovation at an early stage, are significantly higher for the specifications with network externalities. This in turn implies that one would expect that the net impact of external influence varies in a systematic fashion over the course of the product's lifecycle, and is consistent with recent interesting findings by Van den Bulte and Joshi (2007) and, earlier, Van den Bulte and Lilien (1997). Our study thus adds to a growing body of research that indicates that diffusion researchers must necessarily be concerned with factors that influence market evolution (such as the network effect considered here) beyond the classic static framework.

Importantly, these effects may be product specific. Noteworthy, we find substantively different results for the Internet, where the coefficients of external influence ($\alpha$) are extremely small and not significant for any specification, except for the framework model 1A (only 0.5%). This result is consistent with the slow diffusion levels during the Internet's first years. The coefficient of internal influence, $\beta$, drives the speed of diffusion during the first years. When comparing both diffusion processes, we can see that the diffusion of home PCs, starting in 1981, reached 4% of households in 1984 while only 1% of the population had adopted the Internet after the third year of its introduction. These significant differences are shown in Figure 1. The strong network effects presented in the diffusion of the Internet may have also precluded a faster adoption process for the Internet during the first years (we will return to the network effects later). These results are in line with conventional wisdom about how both markets developed.

4.3. Internal Influence

The internal-influence coefficients for PCs ($\beta$) are small, ranging below 0.1 for all the six estimated models. These estimates strongly contrast with the internal-influence coefficients for the Internet ($\beta$), which are all
larger in magnitude and statistically significant, ranging between 0.15 and 0.8. Noteworthy, the specifications that account for network externalities (Models B), already capture the continuous growth of the product value as a result of the increasing size of the network. Thus, discounting for this network effect, internal influence coefficients isolates an important word-of-mouth effect – our findings suggest, not surprisingly, that word-of-mouth effects were considerably more influential for the Internet: knowing about the internet and its characteristics and possibilities was an important driving force behind its adoption. While not at all surprising, this provides additional face validity for our results.

4.4. Product Interactions

In order to provide more detailed insights into the complementary effects that are at play here, we analyze the product interaction functions $\gamma_x(t)$ and $\gamma_y(t)$ for Models 3A and 3B. This leads to interesting substantive insights into the markets at hand. Specifically, while the interaction processes start with Internet adoption significantly accelerating the diffusion of PCs, this effect gradually changes, and at some point, the large installed base of PCs fosters new adoptions of the Internet. Importantly, it appears the results are robust: both for the Models without and with network effects (Model 3A and 3B, respectively), and for both linear and quadratic specifications, as also illustrated in Figure 5.

----- Figure 5 about here -----
diffusion of PCs, but this effect gradually dissipated. In contrast, the growing auxiliary product interaction effects, \( \gamma_0 + \gamma_1 t + \gamma_2 t^2 \), reveal that the adoption of PCs significantly drives the adoption of the Internet during the last few years of the study period. Again, the results are intuitive.

4.5. Putting It All Together: The Three Effects Driving Adoption

Equations (4) and (5) include the three factors driving adoptions among the fraction of the potential market that has not yet adopted the innovations, \( M_i(t - 1) - N_i(t - 1) \). The first is captured by the coefficient of external influence, \( \alpha_x \), the second is captured by the coefficient of internal influence, \( \delta_x \), and the third effect by the product interaction dynamic, \( \gamma_0 + \gamma_1 t + \gamma_2 t^2 \).

In order to better understand the relative magnitude and the dynamics of these forces for both technologies, we calculate the one-period forecasted new adoptions using the estimated parameters for the preferred Model 3B. The aggregate number of new adoptions within the different countries and the forecasted effects are depicted in Figure 6.

----- Figure 6 about here -----
predict how that would have altered the diffusion of PCs from 1991 to 2005. Such an analysis reveals that without the Internet’s complementary effect the cumulative adoption level for PCs would be significantly lower.

4.6. Evolution of the Potential Market

The network externality effects appear robust. Recall that all specifications with network externalities (Models B) nest those without network externalities as a particular case (for which \( \phi = 0 \)). The estimates for the network effects, captured by \( \phi \), are all positive and significant at a 0.001% level. As shown in Figure 5, these results reflect a continuously growing potential market as both innovations diffuse.

**Early adoption levels:** The parameter \( (1-\theta) \) reflects the potential market for a 0% adoption level. Estimates for the early potential market range between 6% (for Models 1B and 2B) and 7% (for Model 3B) of the social system for PCs. For the Internet, the estimated potential market varies between 18% (for Model 1B) and 45% (for Model 3B). For our preferred specification (Model 3B), the early potential market for PCs and the Internet represents 7% and 45% of the social system, respectively.

**Long-term adoption levels:** For every specification of the product interaction—no interactions, constant interactions, and dynamic interaction—Models A and B predict the same long-run potential market. This follows from the fact that the time series are long, and adoption levels are already around 50% to 55% in 2005 for both technologies. As a result, the potential market functions (constant for Model A and exponential for Model B) converge to the same long-run values. For our preferred specification (Model 3B), the estimated long-term adoption levels for PCs and the Internet are 88% and 62%, respectively, which is slightly higher that those for Models 1 and 2.
5. CONCLUSIONS, IMPLICATIONS, AND FUTURE RESEARCH OPPORTUNITIES

5.1. Conclusions and Implications

In this study, we attempt to show how standard Generalized Bass-type diffusion models can be extended to incorporate both the complementary nature of two goods and related network effects within each of the two product categories. To our knowledge, this represents the first attempt in the marketing literature to incorporate both influences into a single framework of that kind. This is noteworthy since a number of high technology markets are characterized by both network effects and cross-product interactions.

Several substantive results of our empirical analysis, which focuses on PC versus Internet diffusion, stand out. First, estimations confirm the benefits of our modeling approach. Our model appears to fit the data better than benchmark models such as the Dekimpe, Parker and Sarvary (1998) model. This suggest that, in modeling PC and Internet diffusion, it is beneficial to incorporate the effects of product interactions and network externalities on both the speed of diffusion and the potential market size. Specifically, we find convincing evidence for (a) the existence of multi-product interactions, (b) the occurrence of a change in the nature of those interactions during our study period, and (c) the role of network externalities.

As such, our paper illustrates a useful way in which the comprehensive conceptual framework for multi-product interactions developed by Bayus et al (2000) can be empirically analyzed. Our methodology is applicable to any industry setting that involves interacting products or product categories, is characterized by network externalities, and consists of multiple countries or segments. Although conventional models can capture diffusion patterns presented by complementary innovations with network effects, a key feature of our nested modeling structure is that it explicitly selects the appropriate functional form. Crucially, this allows for a better understanding of how potential markets evolve throughout a product's diffusion, as well as map the way in which the diffusion of an innovation affects and is affected by the diffusion of a complementary innovation.
This, in turn, is critical to firms that seek to develop marketing strategies for products that are likely to be affected by complementarities. Information obtained using our modeling framework can assist them in a variety of ways. First, it can help them to better anticipate (changes in) growth for a particular product or product category, and thus aid them in efficiently and effectively allocating resources. For example, in the context of our paper, a consistent overestimation of long-run PC adoption levels could be costly for PC manufacturers, retailers, and other industry players. In addition, armed with a better understanding of market dynamics, firms can aim to actively influence growth. One possible scenario here is attempting to increase sales for one product by offering incentives to potential customers for a complementary, but less profitable, one. Again in the context of our empirical application, in the early stages of diffusion, PC manufacturers might have benefited from offering discounts on Internet service. Further building on this idea, our modeling efforts could assist firms in deciding whether or not to make a new technology compatible with an existing technology. For example, our finding that the rapid diffusion of the Internet revived the market for PCs may suggest to manufacturers of other hardware (e.g., mobile phones, PDAs) the need to ensure their products are able to connect to the Internet as well. Once early diffusion data are available, our model can be used to assess to what extent a decision regarding compatibility appears to have paid off. Finally, our approach could benefit the development of international product introduction strategies. Specifically, a grasp of how complementarities and network externalities affect the diffusion speeds and long-run market potentials can help firms in analyzing which markets to enter, in what order, and in what fashion.

5.2. Future Research Opportunities

In our view, the modeling approach described in this manuscript can be extended in four ways that are particularly fruitful. First, in modeling the diffusion of PCs, it appears useful to incorporate the distinction between first-purchase sales, replacements, and purchases of additional units. It is a well-known fact that, for most durable goods, the proportion of sales accounted for by replacement purchases increases significantly as the product matures (Mahajan et al 1990). Minor extensions of our model can help to incorporate this
distinction; to-date, data limitations have prevented us from estimating such a model. Second, while we currently assume that the 'ceiling variable' used to determine the total potential market size has an exponential distribution, further tests can be performed with other functional forms. Third, an effort can be made to incorporate the effect of additional marketing-mix variables. While several other researchers have given marketing-decision variables the attention they deserve, such research efforts are generally limited to the case of technological product substitution—not complementary goods. Fourth, we think it is worthwhile to test our model for other complementary product combinations, either related to the products studied here (e.g., PCs and spreadsheet software) or concerning other technologies (e.g., DVD players and discs, and video games and consoles). We believe that such analyses will further advance our understanding of multi-product interactions and network externalities.
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Figure 1. Adoption Levels for Home PCs and the Internet

Note: The figure depicts aggregate PC and Internet adoption levels for the 19 countries used in this study: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom, and the United States. PC adoption is expressed as the percentage of households that own a PC, while Internet adoption is expressed as the percentage of people that use the Internet.
Figure 2. Potential Adopters as a Function of the Level of Adoption

Without Network Externality Effects
(Models 1A, 2A, and 3A)

With Network Externality Effects
(Models 1B, 2B, and 3B)

Note: This figure compares dynamics in the potential market for two hypothetical examples: in the absence of network externalities ($\phi = 0$, in the graph on the left side) versus with relevant network externalities (in the graph on the right side). The vertical distance between the potential market function and the diagonal represent the fraction of the potential market within which the innovation diffuses. In the left-side graph, the fraction of potential adopters is highest at the beginning of the process and decreasing linearly as the adoption grows. In the right-side graph, the fraction of potential adopters is lower but fairly steady initially, and only decreases faster at higher levels of adoption.
Figure 3. Adoption Levels and Potential Market Sizes for Different Values of $\phi$

Note: This figure illustrates the potential market size and the diffusion process for different values of the externality parameter $\phi$. The graph on the left side depicts the evolution of the potential market as a function of the adoption level, while the graph on the right side captures the dynamics of the diffusion process. The graphs show that the network externality parameter relates directly to the slope of the potential market function.
Figure 4. Estimated Potential Markets

Note: This figure depicts the evolution of the estimated potential markets for PCs (on the left side) and the Internet (on the right side). The graphs capture the idea that in models with network externalities, the fraction of the social system willing to adopt the innovation is significantly lower in the early stages than in the long run.
Figure 5. Dynamic Cross-Product Interactions

Facilitating Effects
\[ \gamma_{x0} + \gamma_{x1} t + \gamma_{x2} t^2 \]

Effect of internet adoption level on the diffusion of PCs

Year


-0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0

Model 3B quadratic function
Model 3B linear function
Model 3A quadratic function
Model 3A linear function

Auxiliary Effects
\[ \gamma_{y0} + \gamma_{y1} t + \gamma_{y2} t^2 \]

Effect of PC adoption level on the diffusion of the Internet

Year


-0.1 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7

Model 3B quadratic function
Model 3B linear function
Model 3A quadratic function
Model 3A linear function

Note: This figure captures the product interaction effects, with linear and quadratic functions. The left-side graph captures facilitating effects (i.e., the effect of Internet diffusion on PC diffusion) over time, while the right side captures auxiliary effects (i.e., the reverse effect, of PC diffusion on Internet diffusion) over time. The graphs reveal that the facilitating product interaction effects start positive but gradually decrease over time, while the growing auxiliary product interaction effects start negative but gradually increase over time.
Figure 6. One-Period Forecast of New Adoptions

Note: This figure the one-period forecasted new adopters, based on the estimated parameters for the preferred Model 3B, for PCs (on the left side) and the Internet (on the right side). The estimated new adopters are plotted in gray, while the actual new adopters are plotted with a dashed line. The graphs suggest that PC diffusion is mainly driven by external influence, while Internet diffusion is primarily driven by internal influence. In each case, the effect of product interactions first increased, and then decreased.
Table 1. Parameter Estimates

<table>
<thead>
<tr>
<th></th>
<th>Without Network Externalities</th>
<th>With Network Externalities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No interactions (1A)</td>
<td>Constant interactions (2A)</td>
</tr>
<tr>
<td></td>
<td>(1B)</td>
<td>(2B)</td>
</tr>
<tr>
<td>External Influence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCs (α_x)</td>
<td>0.018 *** (0.001)</td>
<td>0.017 *** (0.002)</td>
</tr>
<tr>
<td>Internet (α_y)</td>
<td>0.005 *** (0.001)</td>
<td>-0.004 (0.004)</td>
</tr>
<tr>
<td>Internal Influence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCs (β_x)</td>
<td>0.065 ** (0.025)</td>
<td>0.075 ** (0.027)</td>
</tr>
<tr>
<td>Internet (β_y)</td>
<td>0.474 *** (0.047)</td>
<td>0.435 *** (0.089)</td>
</tr>
<tr>
<td>Per Capita GDP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCs (δ_x)</td>
<td>0.000025 *** (8.6547 e^-7)</td>
<td>0.000025 *** (9.558 e^-7)</td>
</tr>
<tr>
<td>Internet (δ_y)</td>
<td>0.000018 *** (1.486 e^-6)</td>
<td>0.000016 *** (1.482 e^-6)</td>
</tr>
<tr>
<td>Price (ε)</td>
<td>-0.783 *** (0.221)</td>
<td>-0.844*** (0.243)</td>
</tr>
<tr>
<td>Interactions:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Internet impacts PC</td>
<td>-</td>
<td>-0.022 (0.018)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>γ_y0 = -0.178 ***</td>
</tr>
<tr>
<td>Interactions:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PC impacts Internet</td>
<td>-</td>
<td>0.068 *** (0.015)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>γ_x2 = 0.001 **</td>
</tr>
<tr>
<td>Early Adopters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCs (1-θ_x)</td>
<td>0.795 *** (0.004)</td>
<td>0.798 *** (0.005)</td>
</tr>
<tr>
<td>Internet (1-θ_y)</td>
<td>0.6145 *** (0.00189)</td>
<td>0.618 *** (0.003)</td>
</tr>
<tr>
<td>Network Effect</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCs (φ_x)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Internet (φ_y)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Long-Run Market (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCs</td>
<td>79.54</td>
<td>79.78</td>
</tr>
<tr>
<td>Internet</td>
<td>61.45</td>
<td>61.80</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCs</td>
<td>0.887</td>
<td>0.887</td>
</tr>
<tr>
<td>Internet</td>
<td>0.893</td>
<td>0.898</td>
</tr>
</tbody>
</table>

Note: The table presents estimates for equations (6) and (7); * denotes p<0.05, ** p<0.01, and *** p<0.001.
**Table 2. Nested Structure and Log-Likelihood Ratio Tests**

<table>
<thead>
<tr>
<th>Interaction Effects</th>
<th>No Interactions ( \gamma_x = 0 ) ( \gamma_y = 0 ) (1)</th>
<th>Constant Interactions ( \gamma_x \neq 0 ) ( \gamma_y \neq 0 ) (2)</th>
<th>Dynamic Interactions ( \gamma_x = a_x + b_x t + c_x t^2 ) ( \gamma_y = a_y + b_y t + c_y t^2 ) (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network Externalities</td>
<td>Parameters</td>
<td>LL</td>
<td>Parameters</td>
</tr>
<tr>
<td>Without Network Externalities ( \phi_{x1} = 0 ), ( \phi_{y1} = 0 ) (A)</td>
<td>9</td>
<td>-11930.3</td>
<td>11</td>
</tr>
<tr>
<td>With Network Externalities ( \phi_{x1} \neq 0 ), ( \phi_{y1} \neq 0 ) (B)</td>
<td>11</td>
<td>-11916.6</td>
<td>13</td>
</tr>
</tbody>
</table>

**Goodness-of-Fit Tests**

<table>
<thead>
<tr>
<th>Interaction Effects</th>
<th>d.f.</th>
<th>( \chi^2 )</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network Externalities</td>
<td>(1B) vs (1A)</td>
<td>2</td>
<td>27.4</td>
</tr>
<tr>
<td></td>
<td>(2B) vs (2A)</td>
<td>2</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>(3B) vs (3A)</td>
<td>2</td>
<td>55.4</td>
</tr>
<tr>
<td>Interaction Effects</td>
<td>(2A) vs (1A)</td>
<td>2</td>
<td>24.4</td>
</tr>
<tr>
<td></td>
<td>(3A) vs (2A)</td>
<td>4</td>
<td>124.6</td>
</tr>
<tr>
<td></td>
<td>(3A) vs (1A)</td>
<td>6</td>
<td>149.0</td>
</tr>
<tr>
<td></td>
<td>(2B) vs (1B)</td>
<td>2</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>(3B) vs (2B)</td>
<td>4</td>
<td>162.8</td>
</tr>
<tr>
<td></td>
<td>(3B) vs (1B)</td>
<td>6</td>
<td>165.8</td>
</tr>
</tbody>
</table>

Note: The table presents the nested structure of the six models, and corresponding log-likelihood estimates. The goodness-of-fit tests reported in the lower table suggest that Model 3B (with dynamic interactions and network externalities) is the preferred model.